Gravitational Wave Detection

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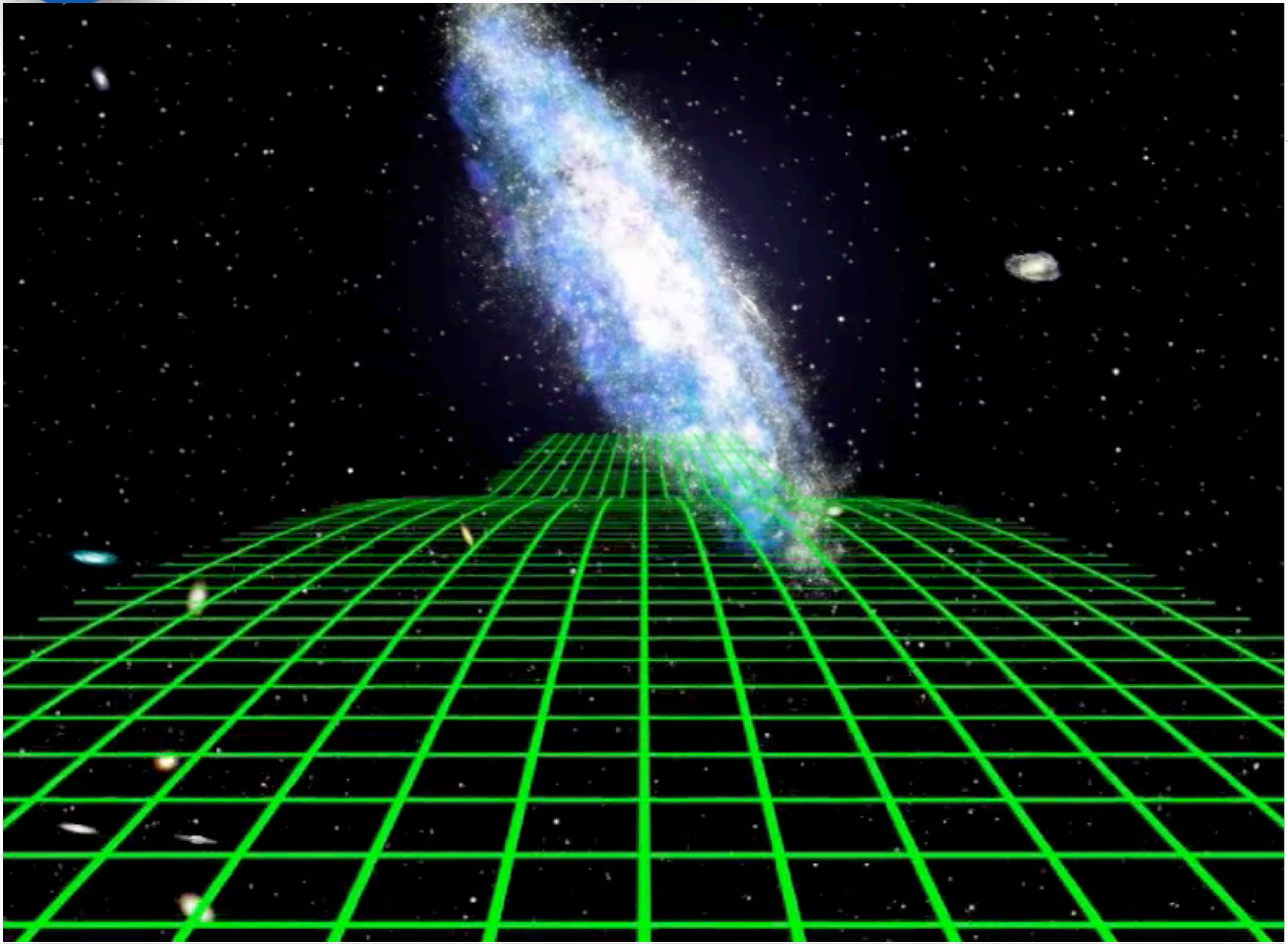
Overview

- Bayesian inference
- Search strategies for stochastic and deterministic signals
- Assessing detection significance



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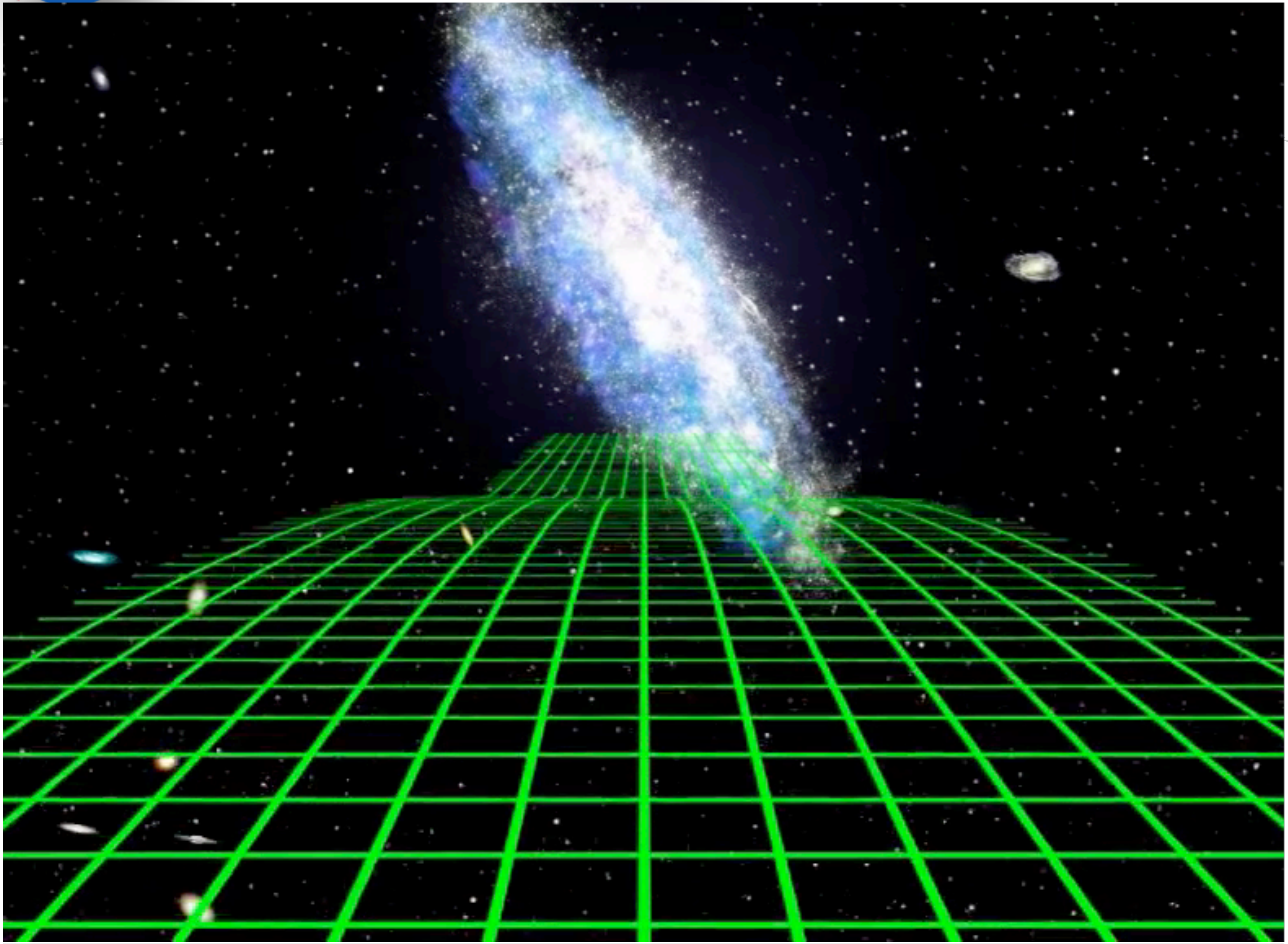

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Bayesian Inference

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Bayesian Inference

Bayes' Theorem

$$p(A, B) = p(A)p(B|A) = p(B)p(A|B)$$

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)}$$

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likelihood

prior knowledge

$$p(\vec{\theta}|\vec{d}) = \frac{p(\vec{d}|\vec{\theta})p(\vec{\theta})}{p(\vec{d})}$$

posterior probability

"Evidence" (more later)

Bayesian Inference

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Bayesian Inference

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- Bayesian inference recovers <u>probability</u> distributions — *measures the spread in our belief.*

- Frequentist inference recovers <u>frequency</u> distributions — *measures the long-timescale spread of experiments.*

Bayesian Inference

Example:

- A disease test is accurate 99% of the time.
- But the disease is quite rare: only affects 1 in 10,000 people.

- If a test comes back positive, what is the probability that you actually have the disease?

- <u>Intuition might lead us to think that, since the test is 99% accurate, then there is a good chance we are infected!</u>

Bayesian Inference

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Bayesian Inference

$$p(\text{infected}|\text{positive test}) = \frac{p(\text{positive test}|\text{infected})p(\text{infected})}{p(\text{positive test})}$$

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$$\sim 0.01$$

ONLY 1% !!!

The Pulsar-timing Likelihood

Two basic types of signals

- (1) **stochastic** — characterize through statistical properties, e.g. standard deviation
- (2) **deterministic** — characterize through amplitude, phase, etc.

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The only difference between treating stochastic signals and deterministic signals is through our prior on $\mathbf{s}$

Searching for single GW sources

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Searching for single GW sources

Deterministic signal $$\delta\mathbf{t} \rightarrow \delta\mathbf{t} - \mathbf{s}(t)$$

Searching for single GW sources

Deterministic signal

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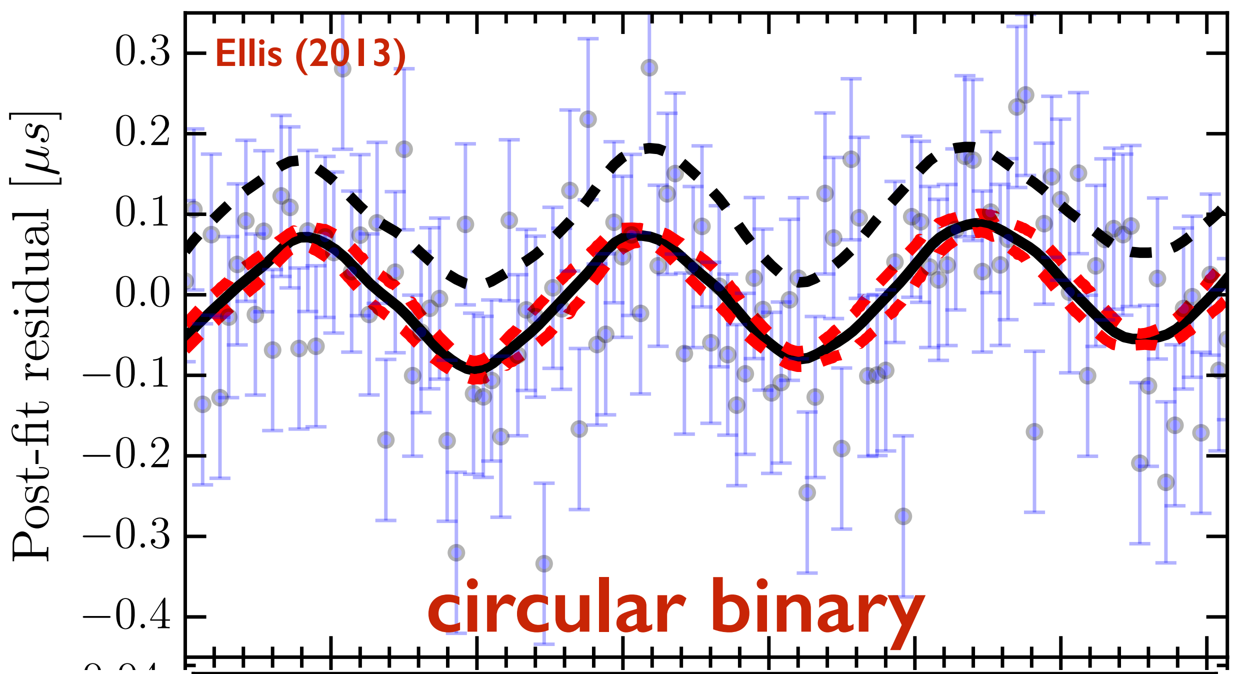

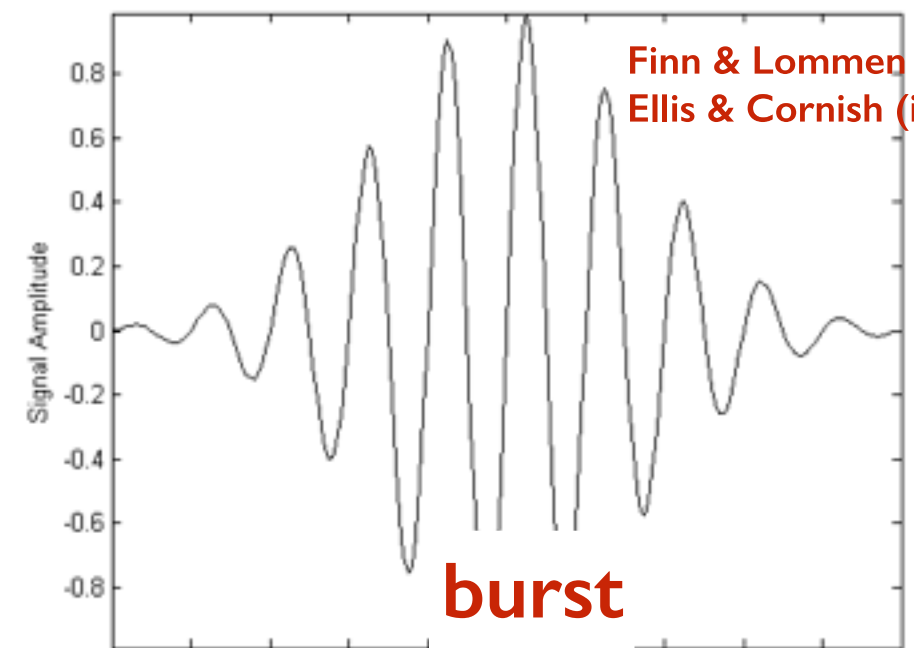

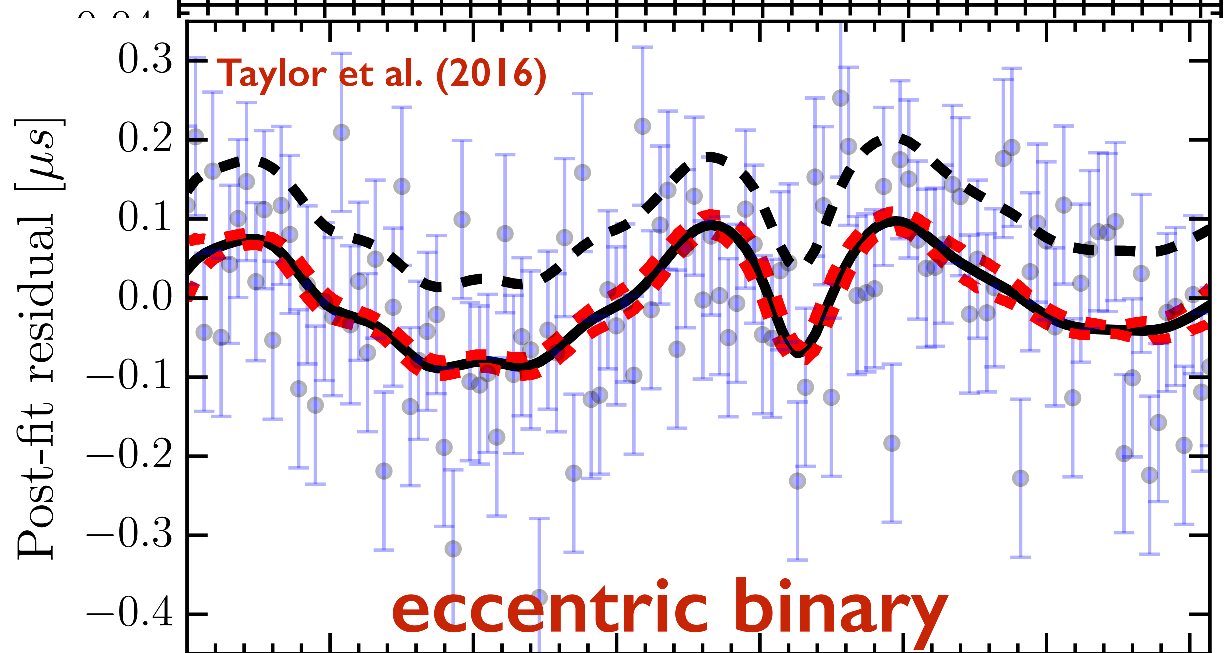

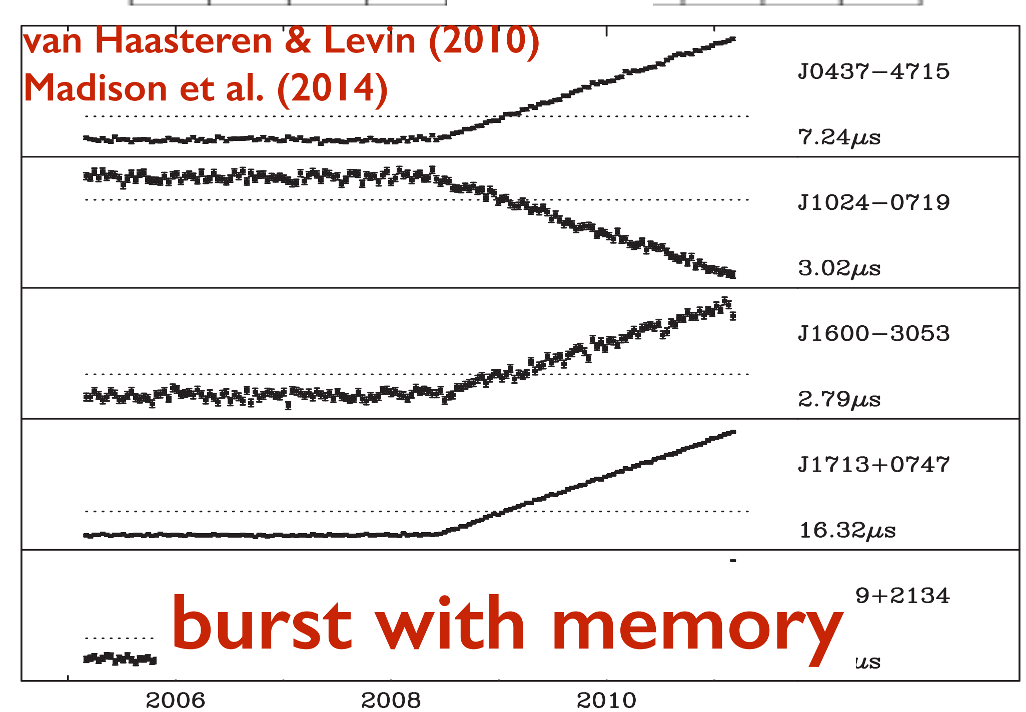

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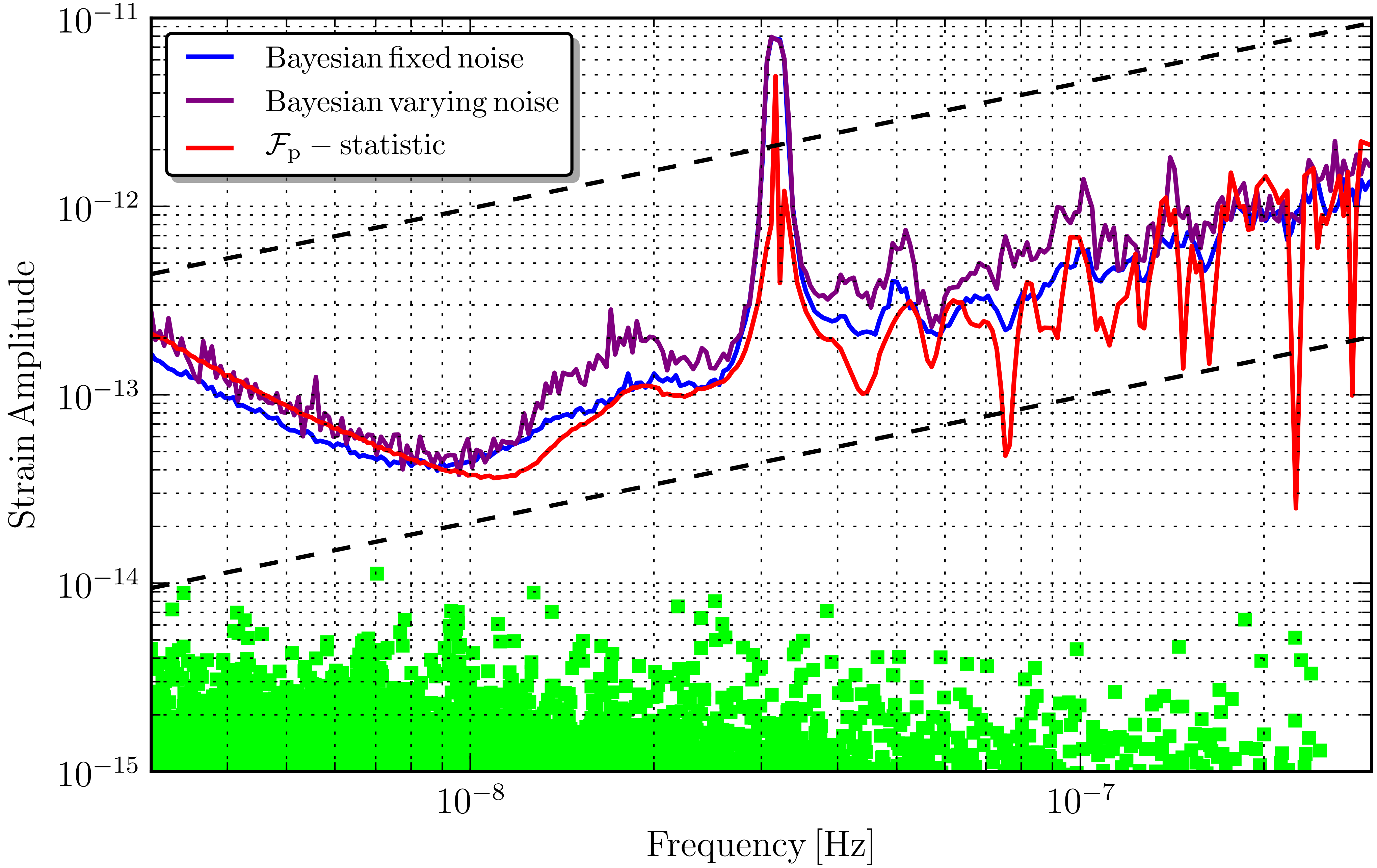
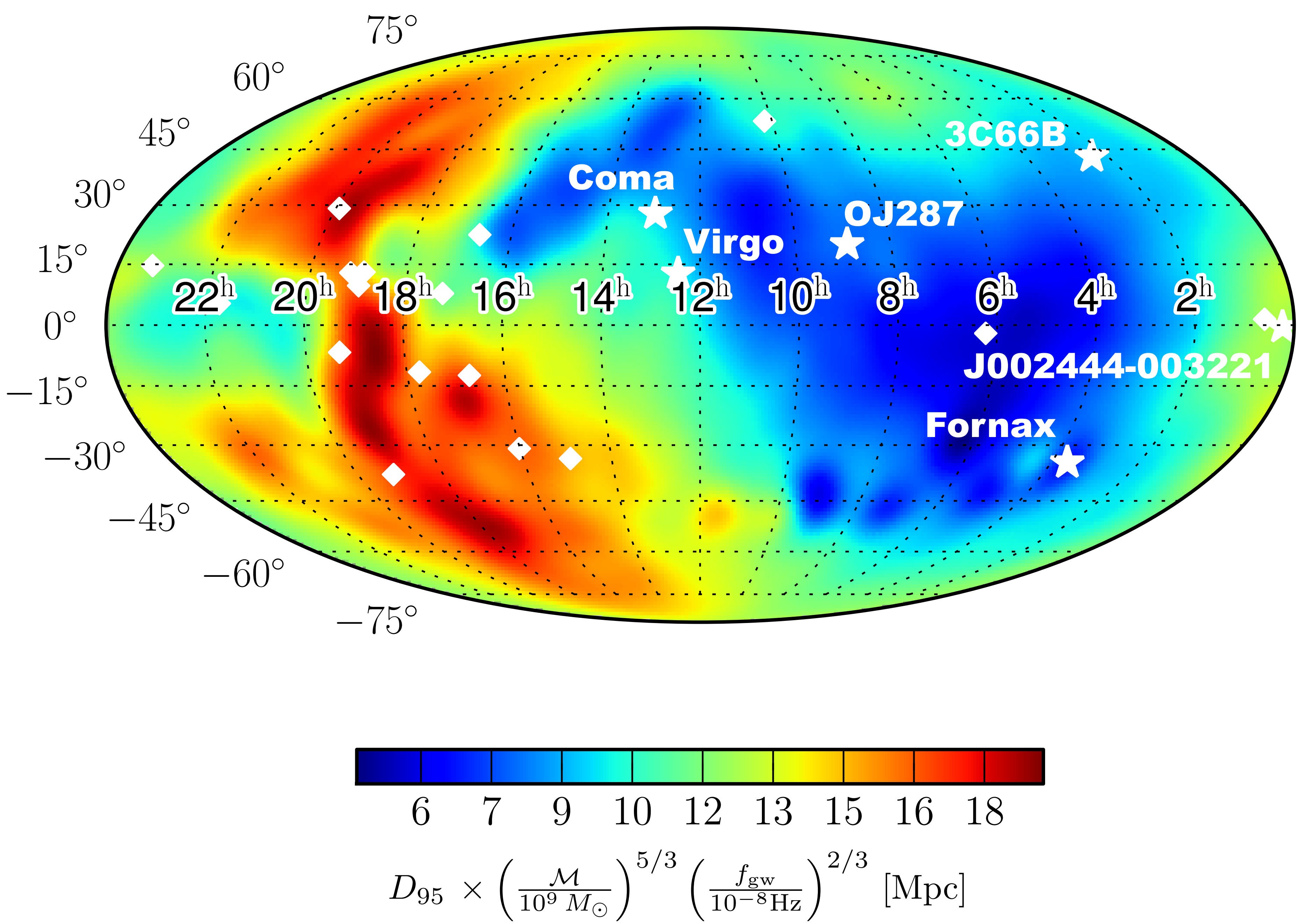


Arzoumanian et al. (2014)

Searching for single GW sources



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- ***We have lots of binaries, so we can't track all of them.***

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$$\mathbf{s} = T\mathbf{b}$$

$$p(\delta\mathbf{t}|\mathbf{b}) = \frac{\exp\left(-\frac{1}{2}(\delta\mathbf{t} - T\mathbf{b})^{\mathrm{T}} N^{-1}(\delta\mathbf{t} - T\mathbf{b})\right)}{\sqrt{\det(2\pi N)}}$$

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$$p(\mathbf{b}|\boldsymbol{\eta}) = \frac{\exp\left(-\frac{1}{2}\mathbf{b}^T B^{-1}\mathbf{b}\right)}{\sqrt{\det(2\pi B)}}$$

Searching for a GW background

$$p(\boldsymbol{\eta}, \mathbf{b}|\delta\mathbf{t}) \propto p(\delta\mathbf{t}|\mathbf{b})p(\mathbf{b}|\boldsymbol{\eta})p(\boldsymbol{\eta})$$

hierarchical modelling

$$p(\boldsymbol{\eta}|\delta\mathbf{t}) = \int p(\boldsymbol{\eta}, \mathbf{b}|\delta\mathbf{t})\mathrm{d}\mathbf{b}$$

(analytically!) marginalize over coefficients

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marginalized likelihood

$$p(\boldsymbol{\eta}|\delta\mathbf{t}) \propto \frac{\exp\left(-\frac{1}{2}\delta\mathbf{t}^T C^{-1}\delta\mathbf{t}\right)}{\sqrt{\det(2\pi C)}}p(\boldsymbol{\eta})$$

$$C = N + TBT^T$$

Searching for a GW background

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what are we actually doing here?

$$[F\phi F^T]_{ij} \simeq \int \mathrm{d}f\, S(f) \cos(2\pi f |t_i - t_j|)$$

this is just the Wiener-Khinchin theorem!

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Woodbury lemma

$$C^{-1} = (N + TBT^T)^{-1}$$
$$= N^{-1} - N^{-1}T(B^{-1} + T^T N^{-1} T)^{-1} T^T N^{-1}$$

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The Pulsar-timing Likelihood

- Without cross-pulsar correlations [~ms]
- With cross-pulsar correlations [~0.1s]

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$$[\phi]_{(ak),(bl)} = \Gamma_{ab}\rho_k\delta_{kl} + \kappa_{ak}\delta_{ab}\delta_{kl}$$

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ORF

GWB PSD

Intrinsic red-noise PSD

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Intrinsic red-noise PSD

$$\rho_k = S(f_k)\Delta f$$
$$\rightarrow \frac{A_{\mathrm{gwb}}^2}{12\pi^2 T_{\mathrm{obs}}} \left(\frac{f_k}{\mathrm{yr}^{-1}}\right)^{-\gamma} \mathrm{yr}^2$$

Lentati, Taylor et al. (2015)

- Simultaneously fit for stochastic GW background, correlated clock-noise, dipole process due to imprecise solar-system ephemerides, and intrinsic low-frequency pulsar noise.
- Investigated consistency of constraints with astrophysical predictions from Sesana (2013). Upper limit cuts out 5% of plausible amplitude distribution.
- Detailed analysis of constraints on possible cosmic-string network and primordial GWs

Shannon et al. (2015)

- Best published constraints to date.
- Used 4 pulsars, but limit dominated by J1909-3744 which is exceptionally well-timed with no measured low-frequency noise.
- Limit is in tension with our basic astrophysical predictions. Excludes 91 - 99.7% of range of basic predictions.

Arzoumanian et al. (2016)

- Detailed investigations of consistency of upper limits with Sesana (2013) and McWilliams, Ostriker, Pretorious (2014) predictions.
- Searched with a **generalized turnover model** to investigate "final-parsec" processes.
- Detailed analysis of constraints on possible cosmic-string network and primordial GWs

Parametrizing the GWB angular power

$$\Gamma_{ab} \propto (1+\delta_{ab}) \int \mathrm{d}^2\hat{\Omega}\, P(\hat{\Omega}) \left[F(\hat{\Omega})_a^+ F(\hat{\Omega})_b^+ + F(\hat{\Omega})_a^\times F(\hat{\Omega})_b^\times \right]$$

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$$\boldsymbol{\Gamma} = \boldsymbol{R} \cdot \boldsymbol{P} \cdot \boldsymbol{R}^T$$

$$\boldsymbol{\Gamma} \rightarrow [N_{\rm psr} \times N_{\rm psr}]$$

$$\boldsymbol{P} \rightarrow \mathrm{diag}(2N_{\rm pix})$$

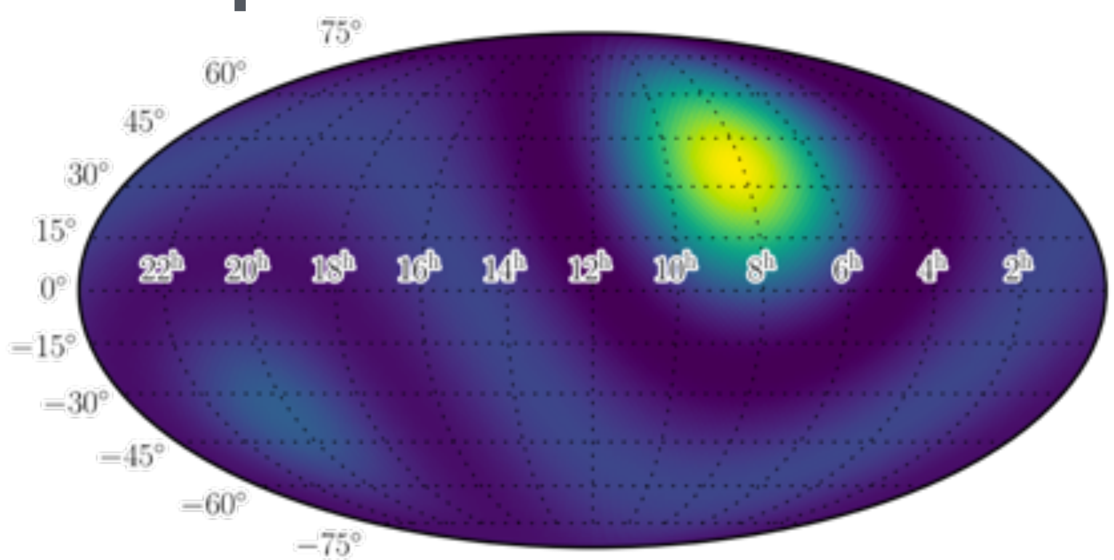
$$\boldsymbol{R} \rightarrow [N_{\rm psr} \times 2N_{\rm pix}]$$

- **R** = pulsar response matrix [fixed]
- **P** = power in each pixel [parametrize]

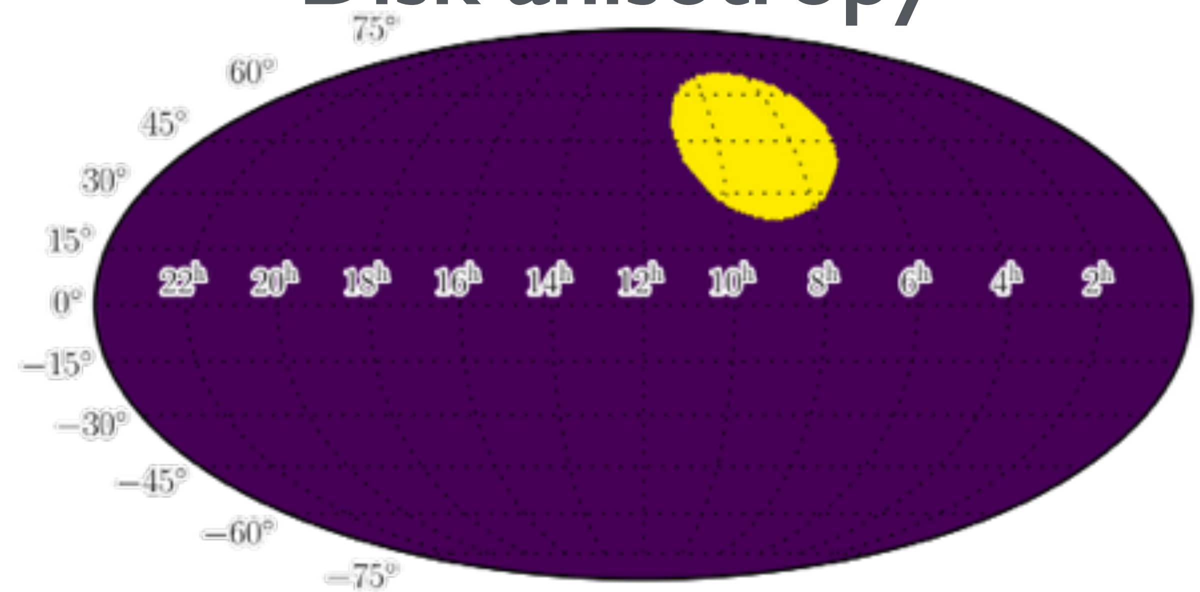
Mingarelli et al. (2013)
Taylor & Gair (2013)
Taylor & van Haasteren (in prep.)

Parametrizing the GWB angular power

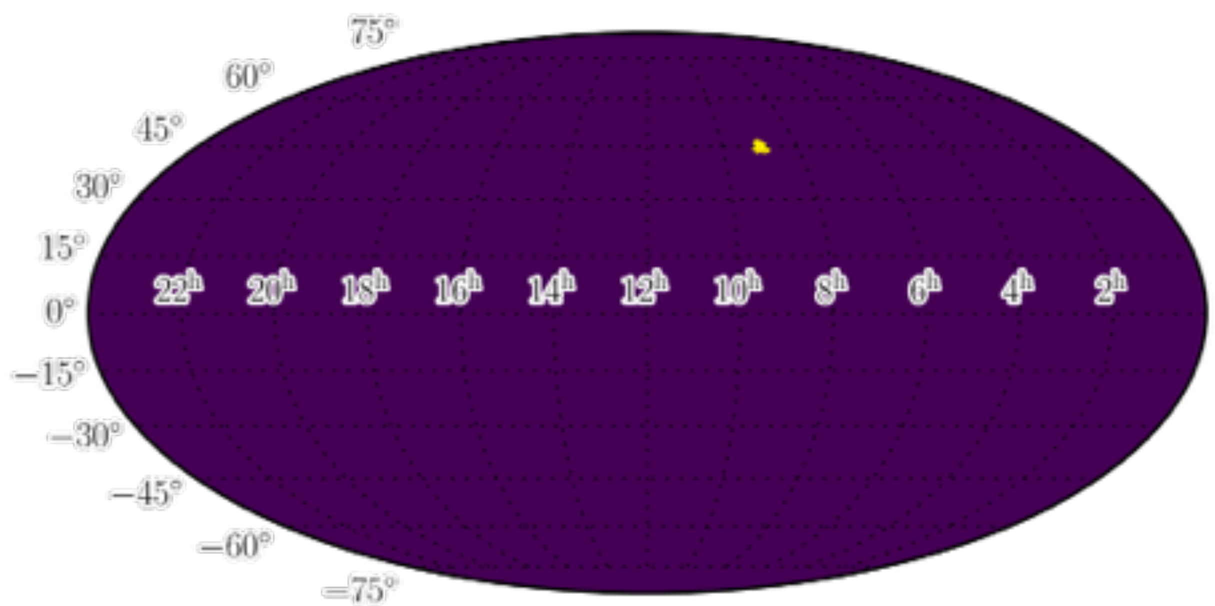
Spherical harmonics



Disk anisotropy



Point source



Detection

- Detection is a model-selection problem.
- We need to prove the presence of spatial correlations between pulsars.
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$$\mathcal{P}_{12} = \frac{p(\mathcal{H}_1|\mathbf{d})}{p(\mathcal{H}_2|\mathbf{d})} = \frac{p(\mathbf{d}|\mathcal{H}_1)}{p(\mathbf{d}|\mathcal{H}_2)}\frac{p(\mathcal{H}_1)}{p(\mathcal{H}_2)}$$

Posterior odds ratio | Bayes factor | Prior odds ratio

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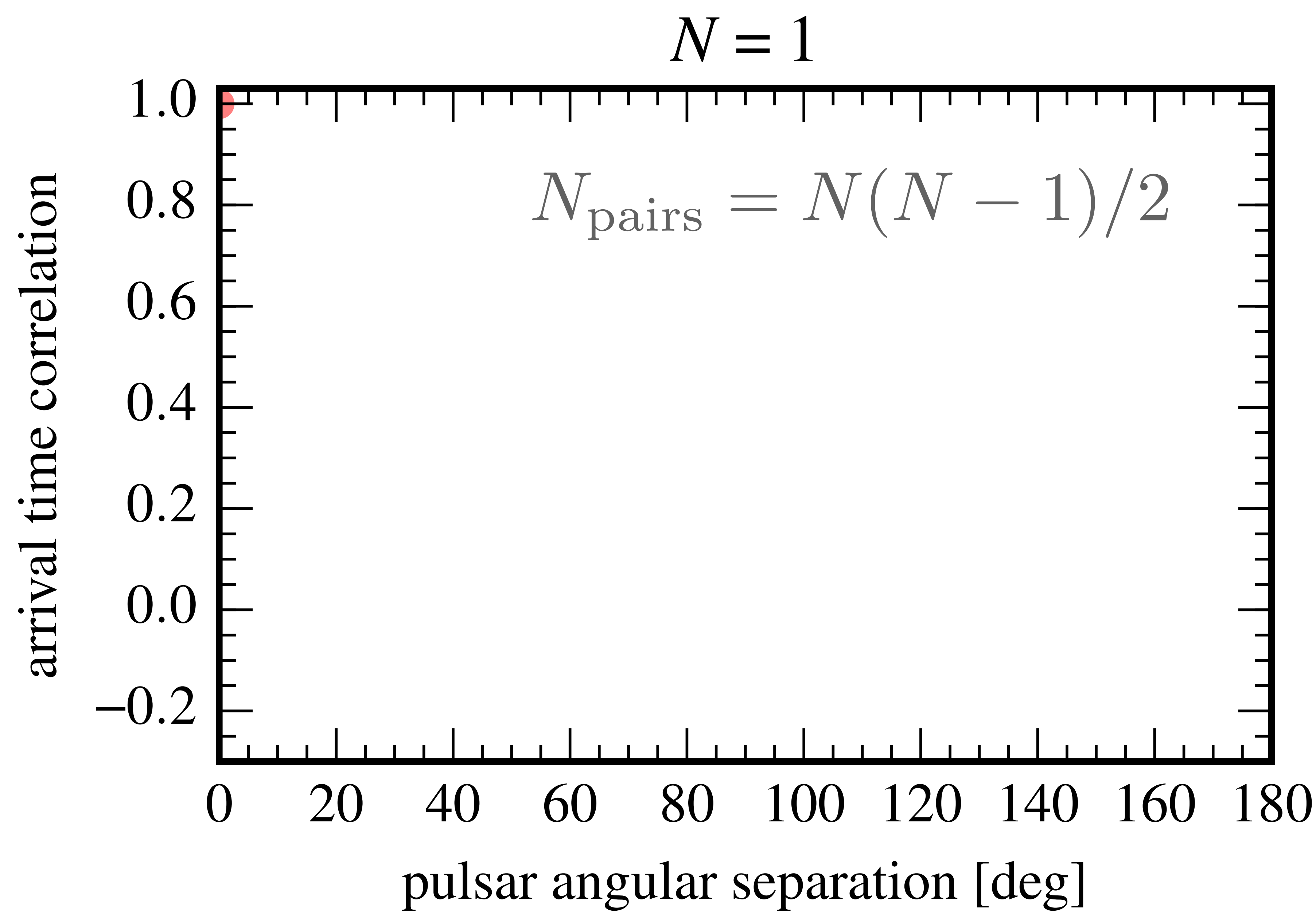
$$\mathcal{P}_{12} = \underbrace{\frac{p(\mathcal{H}_1|\mathbf{d})}{p(\mathcal{H}_2|\mathbf{d})}}_{\text{Posterior odds ratio}} = \underbrace{\frac{p(\mathbf{d}|\mathcal{H}_1)}{p(\mathbf{d}|\mathcal{H}_2)}}_{\text{Bayes factor}} \underbrace{\frac{p(\mathcal{H}_1)}{p(\mathcal{H}_2)}}_{\text{Prior odds ratio}}$$

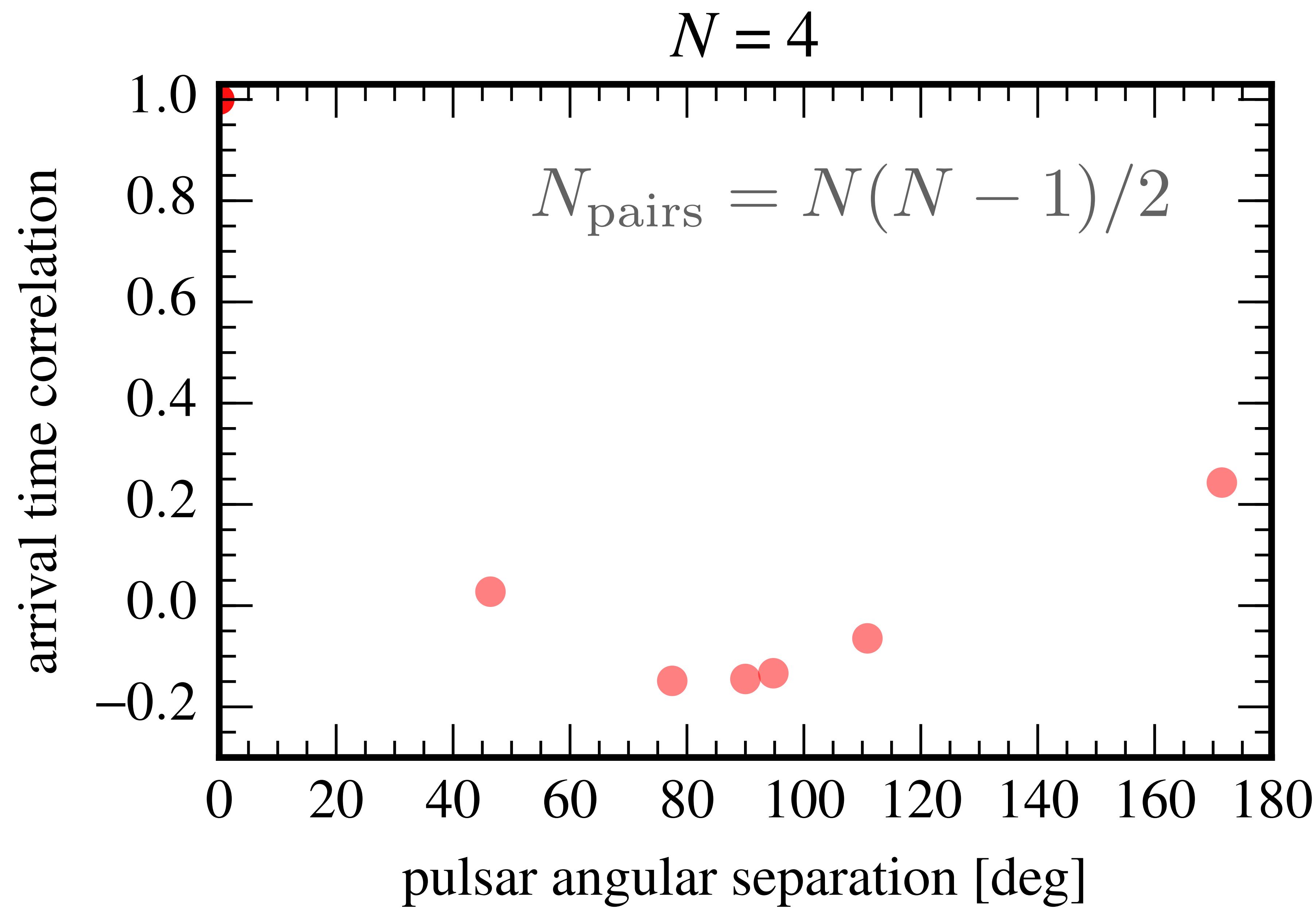
- MultiNest
- Thermodynamic integration
- RJMCMC
- Savage-Dickey ratio
- **Product space**

Detection



$N = 1$
$N_{\text{pairs}} = N(N-1)/2$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140
160
180
arrival time correlation
pulsar angular separation [deg]

Detection



$N = 4$
$N_{\rm pairs} = N(N-1)/2$
arrival time correlation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140
160
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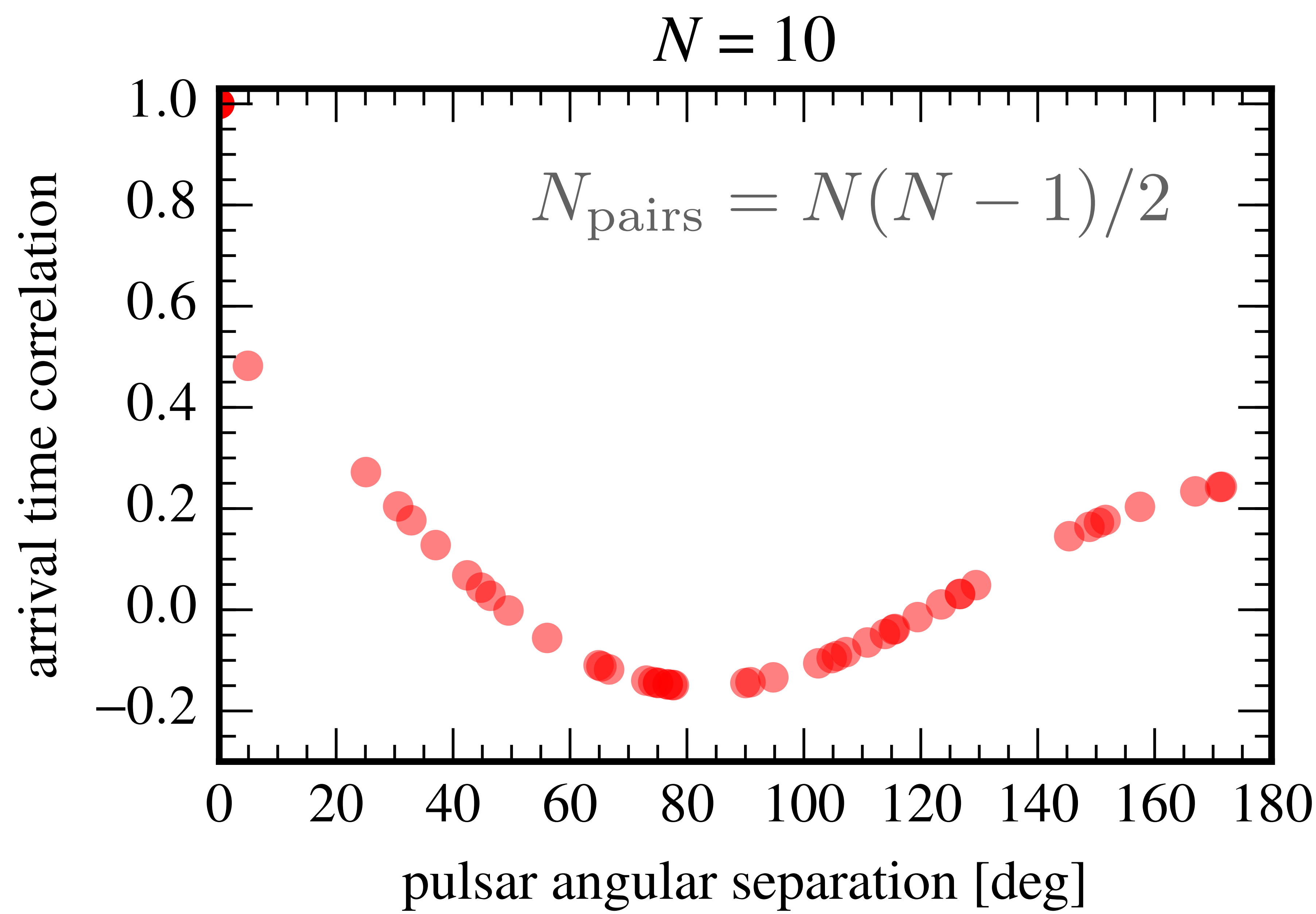
Detection



$N = 10$
$N_{\mathrm{pairs}} = N(N-1)/2$
arrival time correlation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
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60
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160
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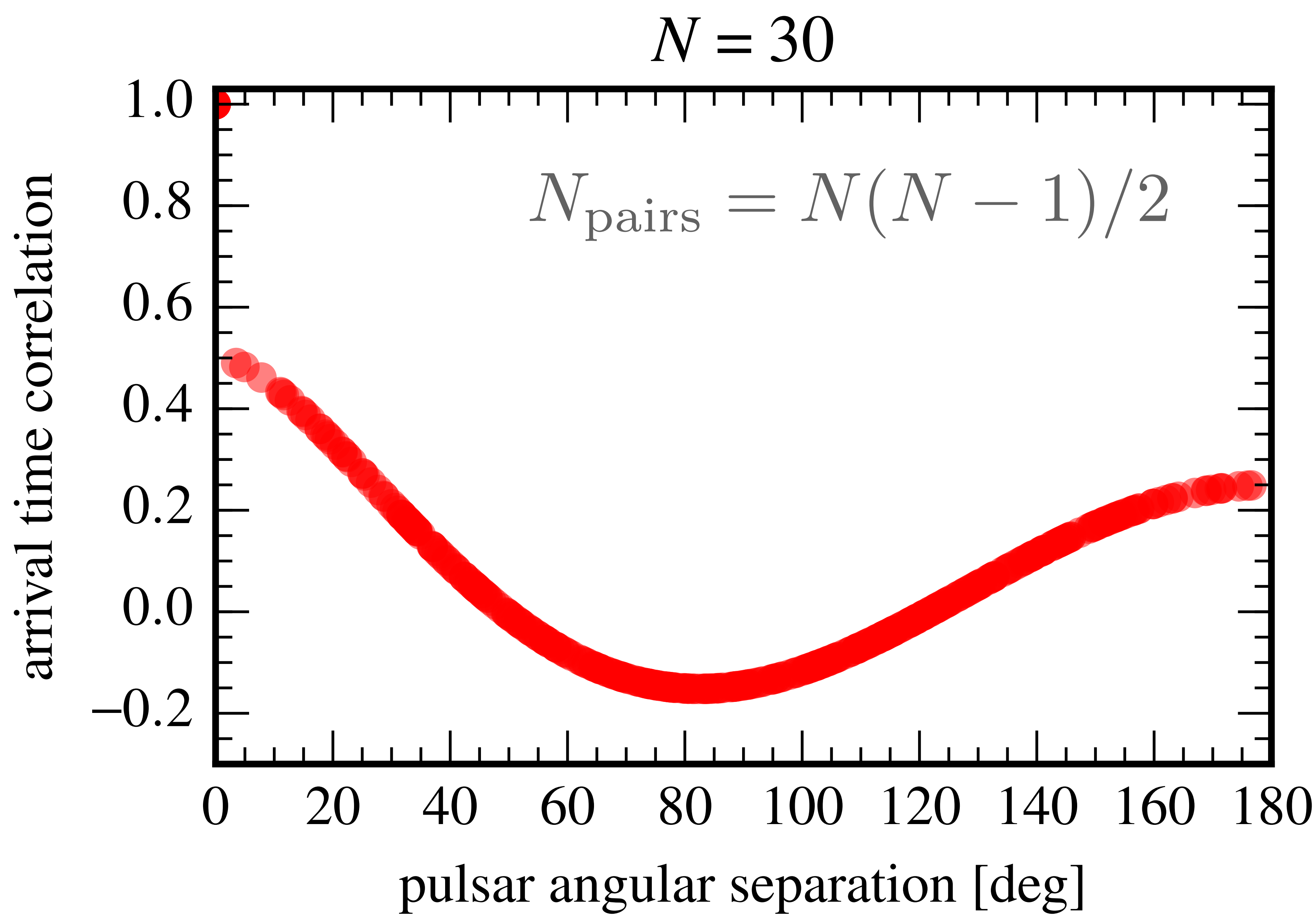

$N = 30$
$N_{\text{pairs}} = N(N-1)/2$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
arrival time correlation
0
20
40
60
80
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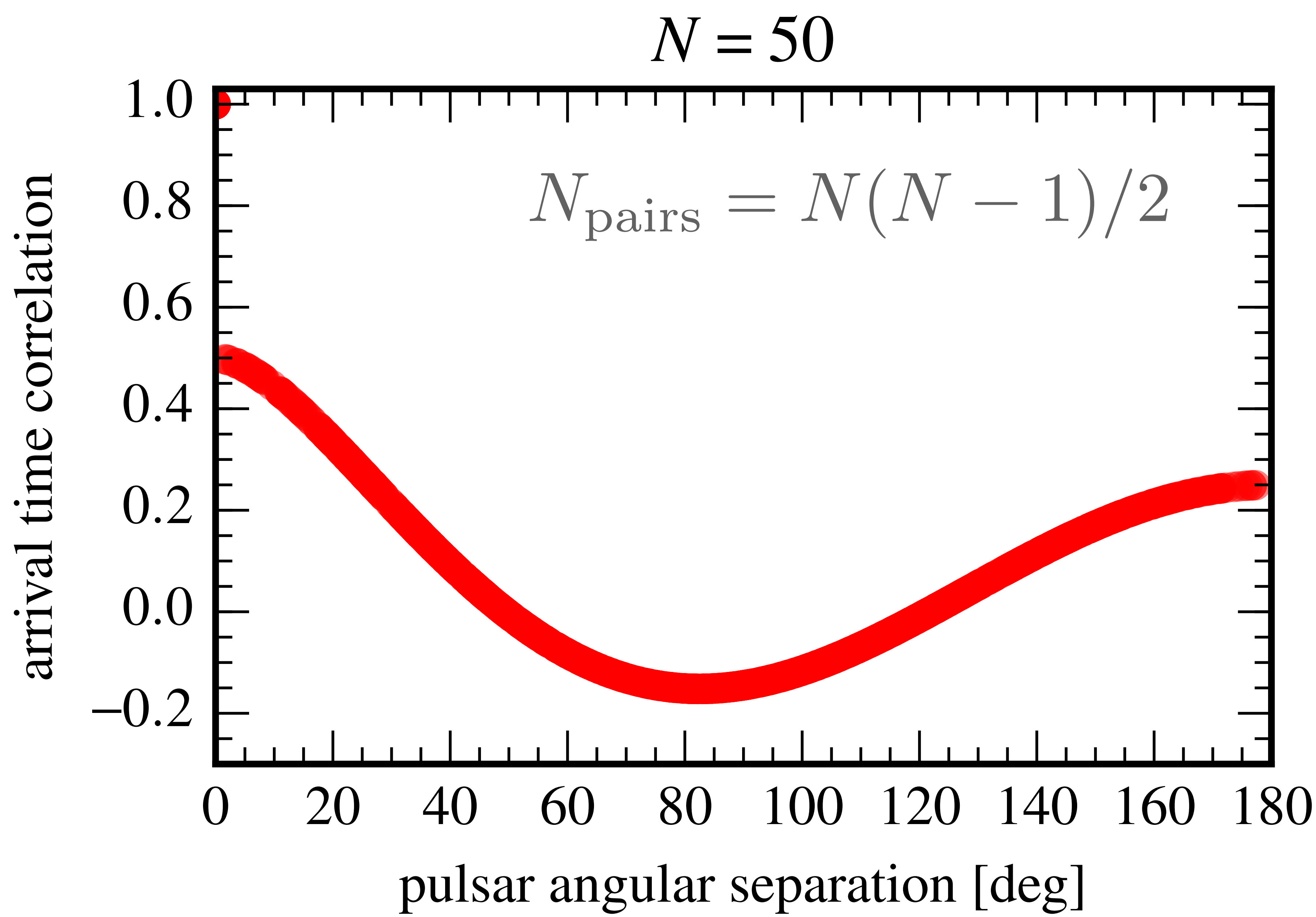
120
140
160
180
pulsar angular separation [deg]

Detection



$N = 50$
$N_{\rm pairs} = N(N-1)/2$
arrival time correlation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140
160
180
pulsar angular separation [deg]

Summary

- We can do single-source searches and stochastic background searches all together in our pipelines.
- Current limits are cutting into astrophysically interesting territory.
- We can probe the dynamics and environments of supermassive black-holes binaries.
- Within the next 10 years, we should have convincing signs of nanohertz GWs with pulsar-timing arrays.