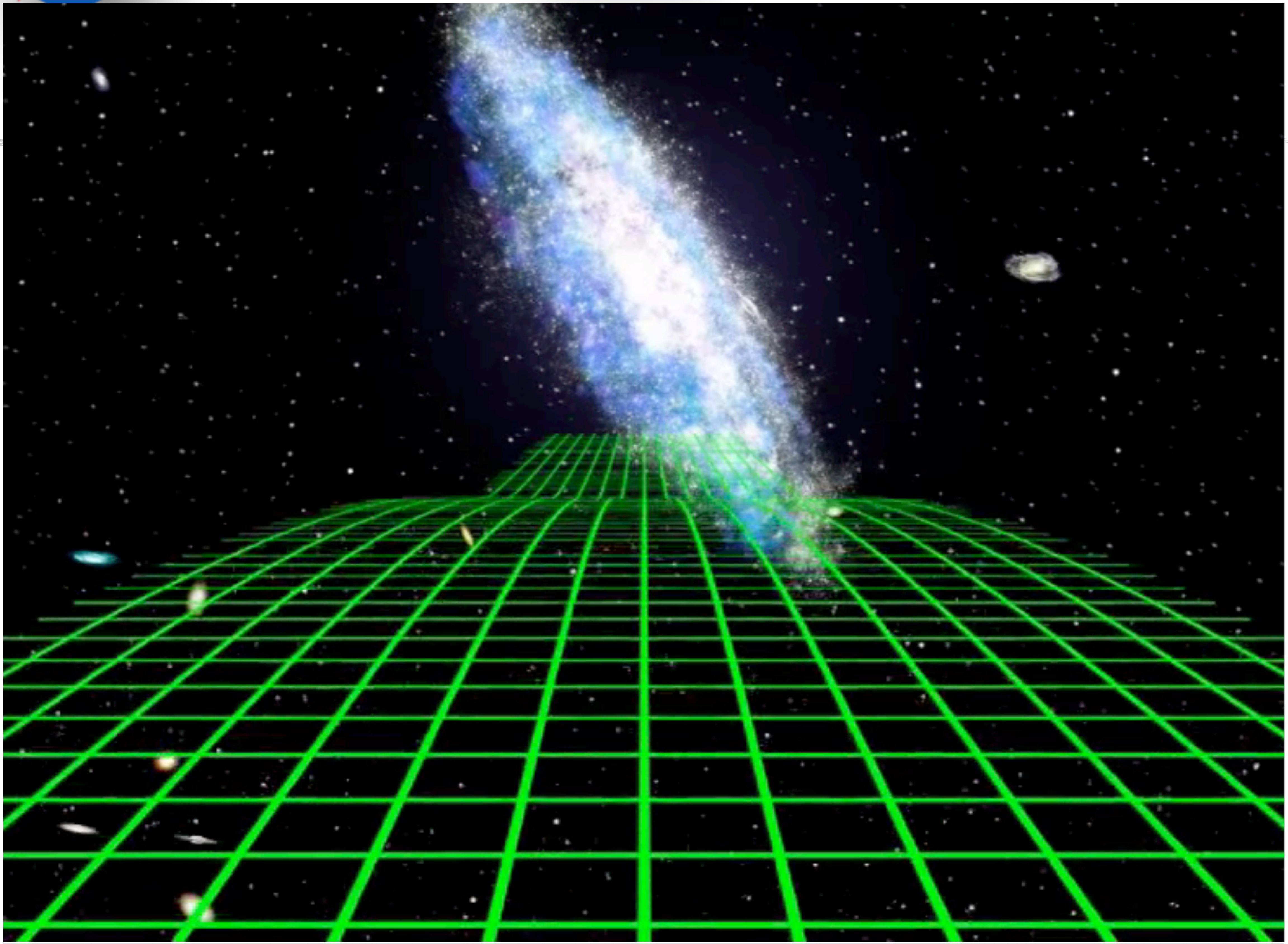
Gravitational Wave Detection

Stephen R. Taylor

JET PROPULSION LABORATORY,
CALIFORNIA INSTITUTE OF TECHNOLOGY

Overview

- Bayesian inference
- Search strategies for stochastic and deterministic signals
- Assessing detection significance



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Bayesian Inference

Bayes' Theorem

$$p(A,B) = p(A)p(B|A) = p(B)p(A|B)$$

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)}$$

$$A = \text{data}$$

$$B = \text{parameters we care about}$$

likelihood

prior knowledge

$$p(\vec{\theta}|\vec{d}) = \frac{p(\vec{d}|\vec{\theta})p(\vec{\theta})}{p(\vec{d})}$$

posterior probability

"Evidence" (more later)

Bayesian Inference

- **Bayesian inference recovers <u>probability</u> distributions — *measures the spread in our belief.***
- **Frequentist inference recovers <u>frequency</u> distributions — *measures the long-timescale spread of experiments.***

Bayesian Inference

Example:

- A disease test is accurate 99% of the time.
- But the disease is quite rare: only affects 1 in 10,000 people.

- If a test comes back positive, what is the probability that you actually have the disease?

- <u>Intuition might lead us to think that, since the test is 99% accurate, then there is a good chance we are infected!</u>

Bayesian Inference

$$p(\text{infected}|\text{positive test}) = \frac{p(\text{positive test}|\text{infected})p(\text{infected})}{p(\text{positive test})}$$

$$p(\text{infected}|\text{positive}) = \frac{0.99 \times 0.0001}{0.99 \times 0.0001 + 0.01 \times 0.9999}$$

$$p(\text{infected}|\text{positive}) = \frac{0.99 \times 0.0001}{0.99 \times 0.0001 + 0.01 \times 0.9999}$$

$$\sim 0.01$$

ONLY 1% !!!

The Pulsar-timing Likelihood

Two basic types of signals

- (1) **stochastic** — characterize through statistical properties, e.g. standard deviation
- (2) **deterministic** — characterize through amplitude, phase, etc.

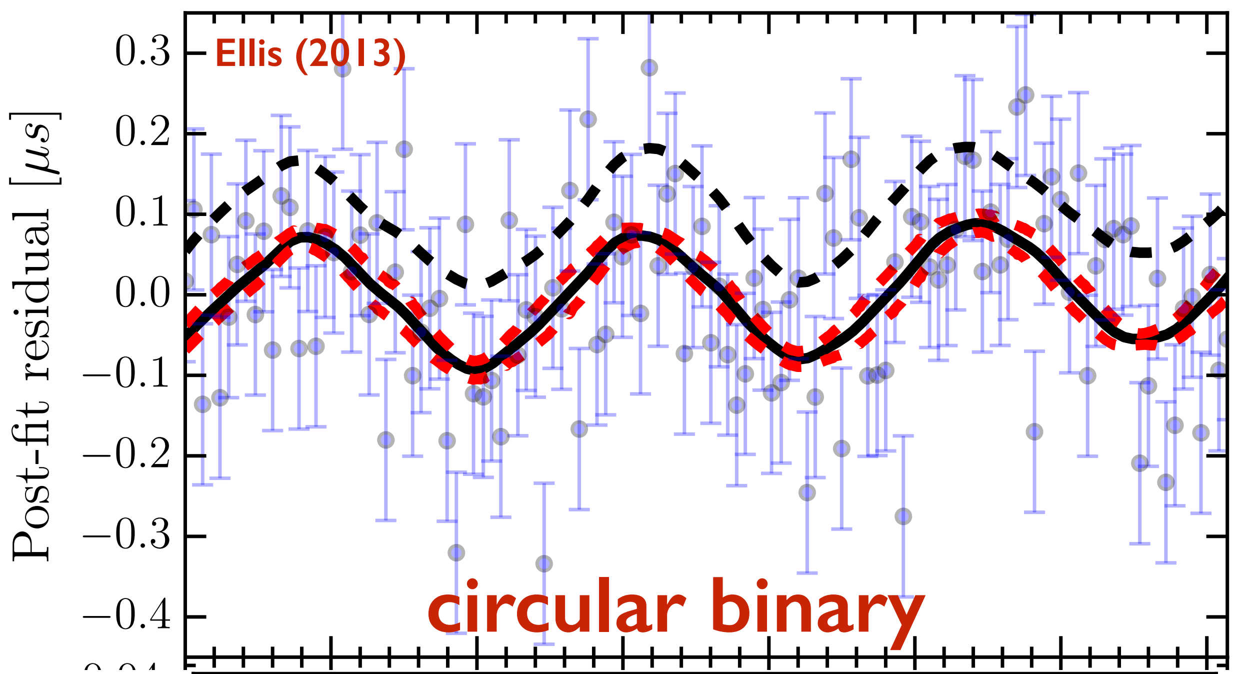
$$p(\delta\mathbf{t}|\mathbf{n}) = \frac{\exp\left(-\frac{1}{2}(\delta\mathbf{t}-\mathbf{s})^T N^{-1}(\delta\mathbf{t}-\mathbf{s})\right)}{\sqrt{\det(2\pi N)}}$$

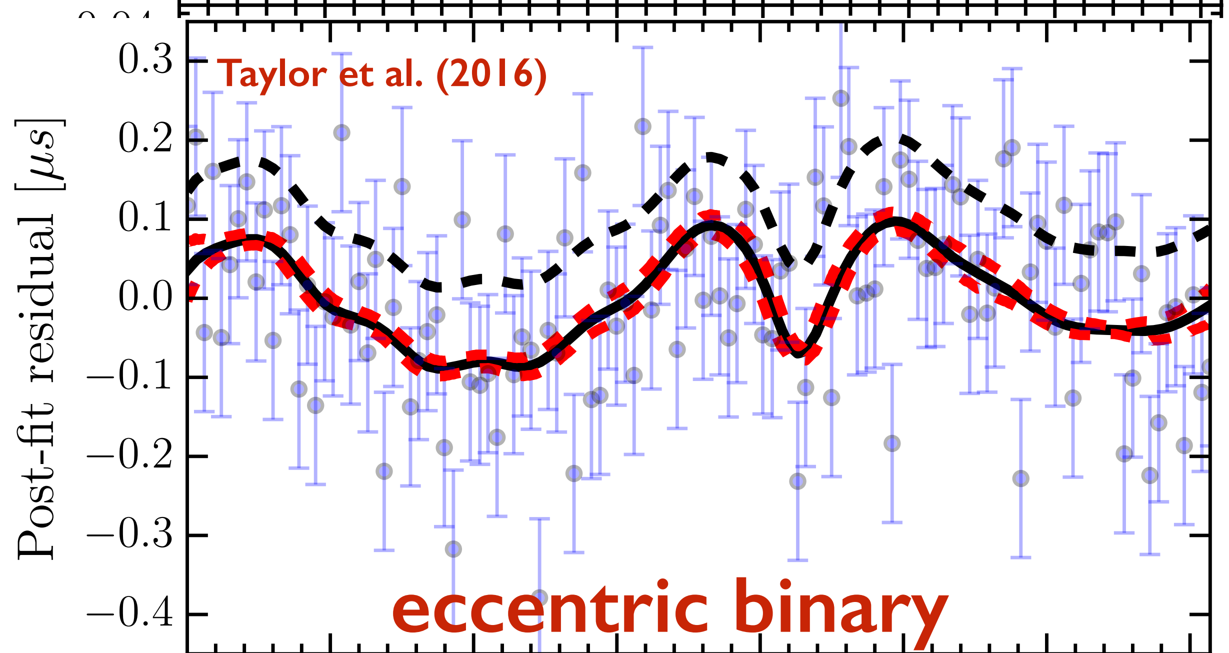
The only difference between treating stochastic signals and deterministic signals is through our prior on $\mathbf{s}$

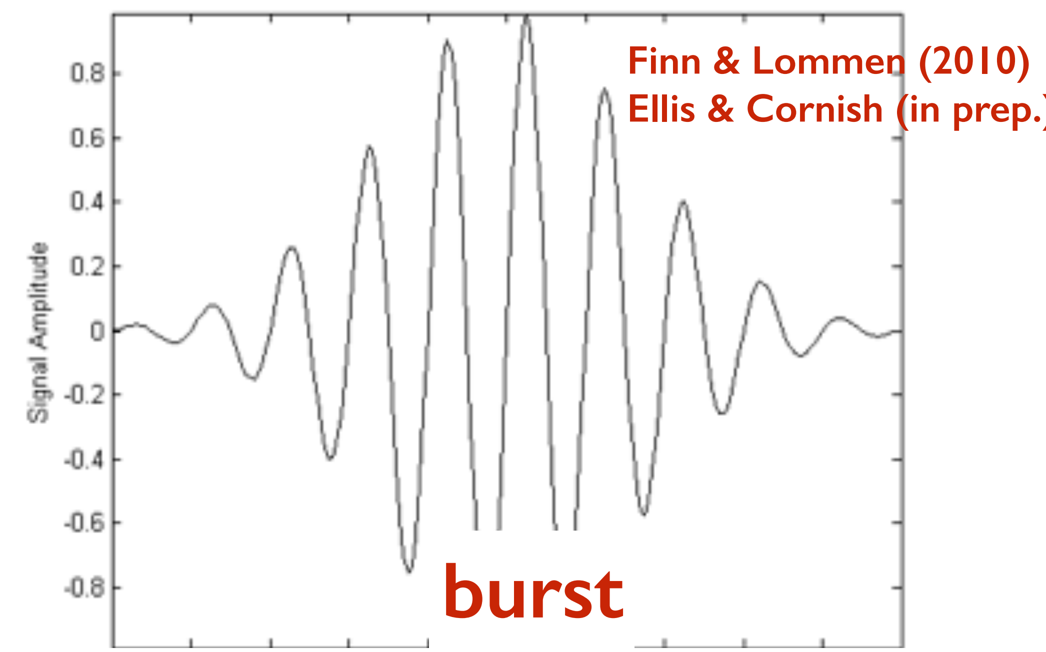
Searching for single GW sources

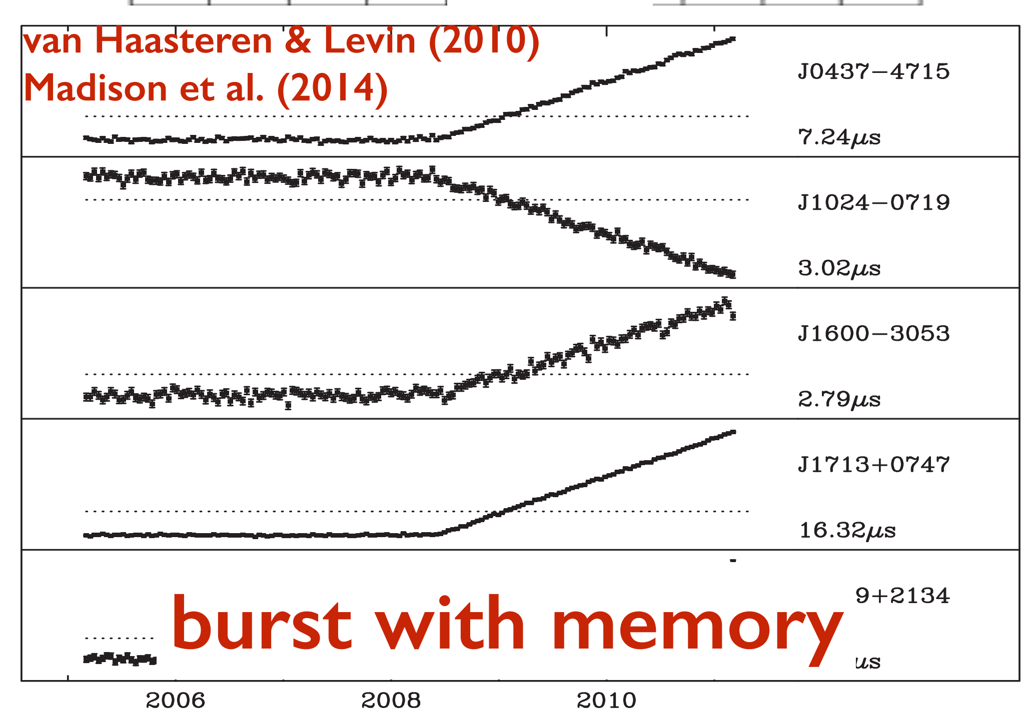
Deterministic signal

$$\delta \mathbf{t} \rightarrow \delta \mathbf{t} - \mathbf{s}(t)$$









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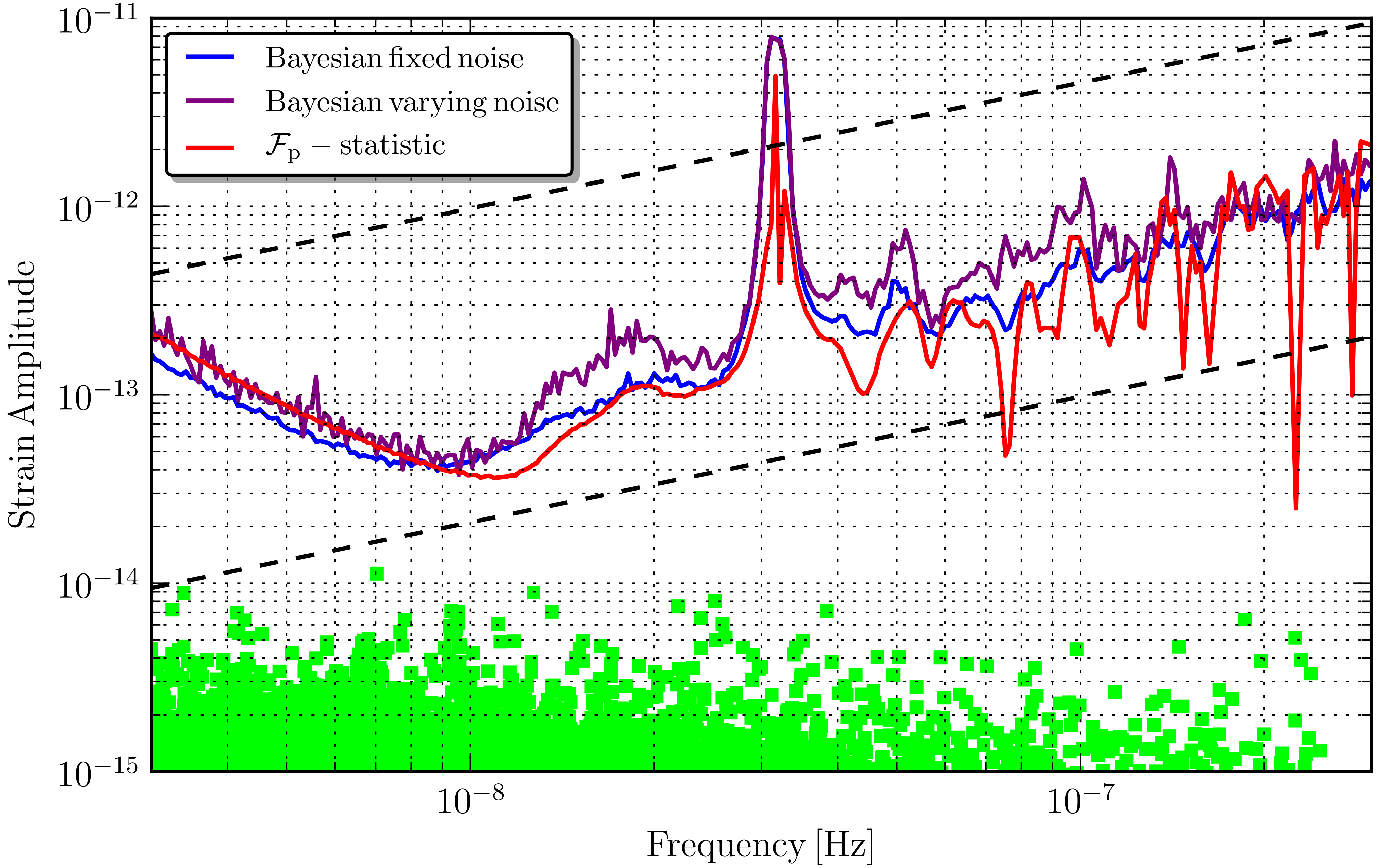
Searching for single GW sources



Arzoumanian et al. (2014)

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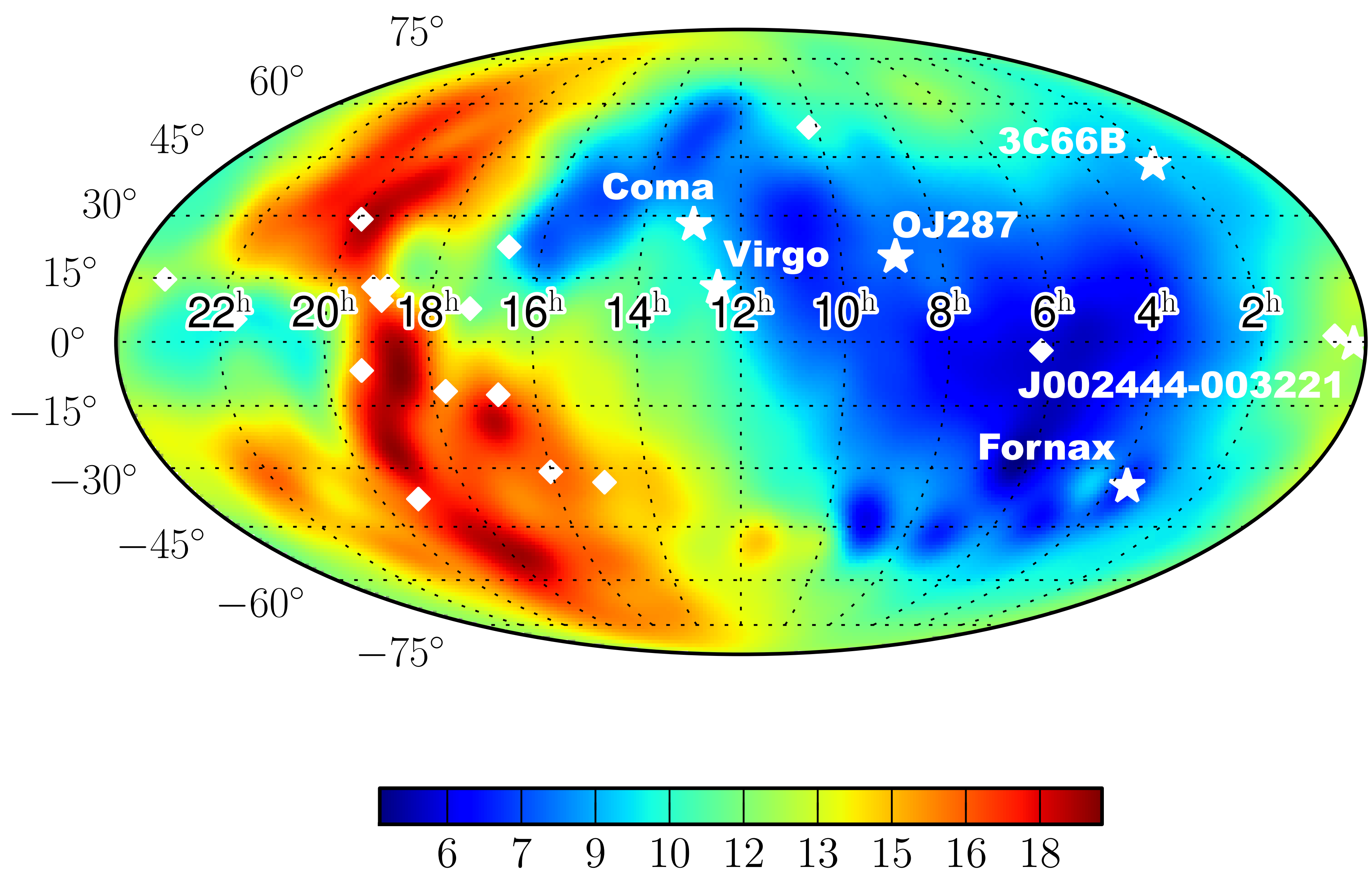
Searching for single GW sources



$$D_{95} \times \left(\frac{\mathcal{M}}{10^9\,M_\odot}\right)^{5/3}\left(\frac{f_{\rm gw}}{10^{-8}\,{\rm Hz}}\right)^{2/3}\ [{\rm Mpc}]$$

Arzoumanian et al. (2014)

Searching for a GW background

- *We have lots of binaries, so we can't track all of them.*
- *We just look at the statistical properties of the signal, e.g. the variance.*
- *All of the information on the background is in the signal variance and cross correlations between pulsars.*

- *Let's do a simple Fourier analysis of the background.*

$$\mathbf{s} = T\mathbf{b}$$

$$p(\delta\mathbf{t}|\mathbf{b}) = \frac{\exp\left(-\frac{1}{2}(\delta\mathbf{t} - T\mathbf{b})^{\mathrm{T}} N^{-1}(\delta\mathbf{t} - T\mathbf{b})\right)}{\sqrt{\det(2\pi N)}}$$

Searching for a GW background

- *Put a Gaussian prior on the signal amplitude coefficients.*
- *Variance is proportional to the power spectrum of the background.*
- *We can parameterize that power spectrum whichever way we want…*

$$p(\mathbf{b}|\boldsymbol{\eta}) = \frac{\exp\left(-\frac{1}{2}\mathbf{b}^T B^{-1}\mathbf{b}\right)}{\sqrt{\det(2\pi B)}}$$

Searching for a GW background

$$p(\boldsymbol{\eta}, \mathbf{b}|\delta\mathbf{t}) \propto p(\delta\mathbf{t}|\mathbf{b})p(\mathbf{b}|\boldsymbol{\eta})p(\boldsymbol{\eta})$$

hierarchical modelling

$$p(\boldsymbol{\eta}|\delta\mathbf{t}) = \int p(\boldsymbol{\eta}, \mathbf{b}|\delta\mathbf{t})\mathrm{d}\mathbf{b}$$

(analytically!) marginalize over coefficients

marginalized likelihood

$$p(\boldsymbol{\eta}|\delta\mathbf{t}) \propto \frac{\exp\left(-\frac{1}{2}\delta\mathbf{t}^T C^{-1}\delta\mathbf{t}\right)}{\sqrt{\det(2\pi C)}}p(\boldsymbol{\eta})$$

$$C = N + TBT^T$$

Searching for a GW background

$$C = N + TBT^T$$

what are we actually doing here?

$$[F\phi F^T]_{ij} \simeq \int \mathrm{d}f\, S(f)\cos(2\pi f|t_i - t_j|)$$

this is just the Wiener-Khinchin theorem!

Woodbury lemma

$$\begin{aligned} C^{-1} &= (N + TBT^T)^{-1} \\ &= N^{-1} - N^{-1}T\underbrace{(B^{-1} + T^T N^{-1} T)^{-1}}_{\text{easy!}}T^T N^{-1} \end{aligned}$$

The Pulsar-timing Likelihood

- Without cross-pulsar correlations [~ms]
- With cross-pulsar correlations [~0.1s]

The Pulsar-timing Likelihood

$$[F\phi F^T]_{ij} \simeq \int \mathrm{d}f\, S(f) \cos(2\pi f |t_i - t_j|)$$

$$[\phi]_{(ak),(bl)} = \Gamma_{ab}\rho_k\delta_{kl} + \kappa_{ak}\delta_{ab}\delta_{kl}$$

ORF

GWB PSD

Intrinsic red-noise PSD

$$\rho_k = S(f_k)\Delta f$$

$$\rightarrow \frac{A_{\rm gwb}^2}{12\pi^2 T_{\rm obs}} \left(\frac{f_k}{\rm yr^{-1}}\right)^{-\gamma} \rm yr^2$$

Lentati, Taylor et al. (2015)

- Simultaneously fit for stochastic GW background, correlated clock-noise, dipole process due to imprecise solar-system ephemerides, and intrinsic low-frequency pulsar noise.
- Investigated consistency of constraints with astrophysical predictions from Sesana (2013). Upper limit cuts out 5% of plausible amplitude distribution.
- Detailed analysis of constraints on possible cosmic-string network and primordial GWs

Shannon et al. (2015)

- Best published constraints to date.
- Used 4 pulsars, but limit dominated by J1909-3744 which is exceptionally well-timed with no measured low-frequency noise.
- Limit is in tension with our basic astrophysical predictions. Excludes 91 - 99.7% of range of basic predictions.

Arzoumanian et al. (2016)

- Detailed investigations of consistency of upper limits with Sesana (2013) and McWilliams, Ostriker, Pretorious (2014) predictions.
- Searched with a **generalized turnover model** to investigate “final-parsec” processes.
- Detailed analysis of constraints on possible cosmic-string network and primordial GWs

Parametrizing the GWB angular power

$$\Gamma_{ab} \propto (1+\delta_{ab}) \int \mathrm{d}^2\hat{\Omega}\, P(\hat{\Omega}) \left[F(\hat{\Omega})^+_a F(\hat{\Omega})^+_b + F(\hat{\Omega})^\times_a F(\hat{\Omega})^\times_b\right]$$

$$\boldsymbol{\Gamma} = \boldsymbol{R} \cdot \boldsymbol{P} \cdot \boldsymbol{R}^T$$

$$\boldsymbol{\Gamma} \rightarrow [N_{\rm psr} \times N_{\rm psr}]$$

$$\boldsymbol{P} \rightarrow \mathrm{diag}(2N_{\rm pix})$$

$$\boldsymbol{R} \rightarrow [N_{\rm psr} \times 2N_{\rm pix}]$$

- **R** = pulsar response matrix [fixed]
- **P** = power in each pixel [parametrize]

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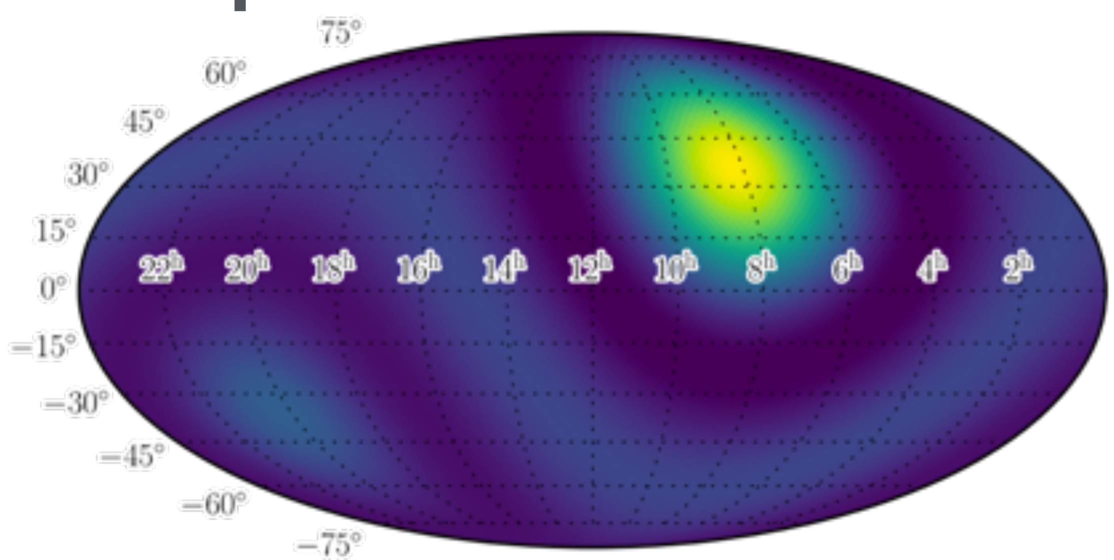
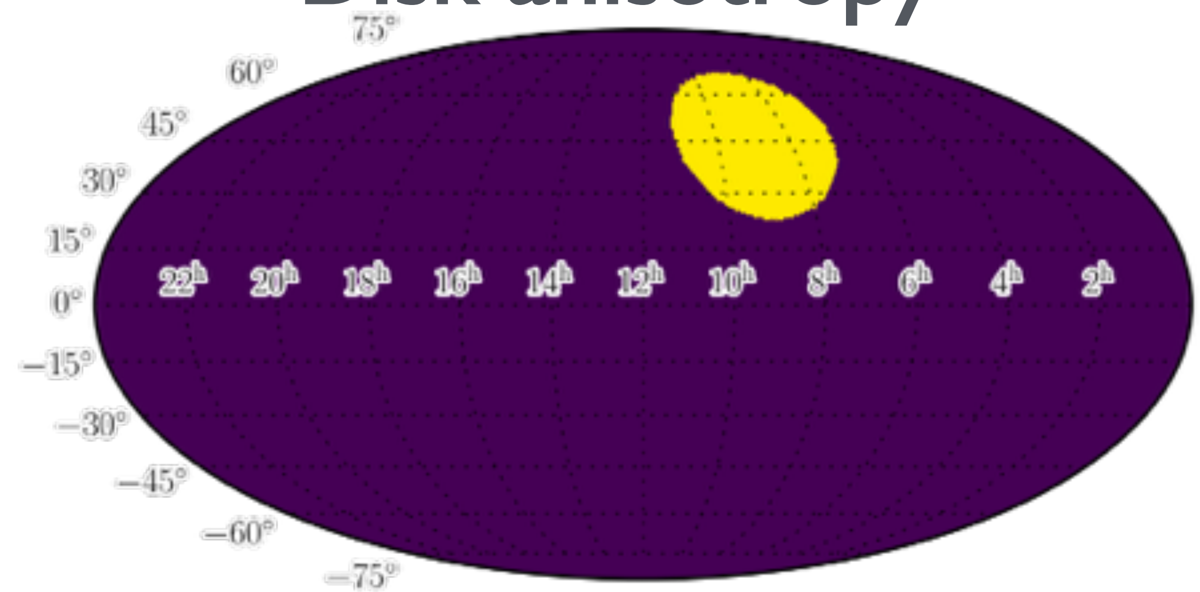
Mingarelli et al. (2013)
Taylor & Gair (2013)
Taylor & van Haasteren (in prep.)

Parametrizing the GWB angular power

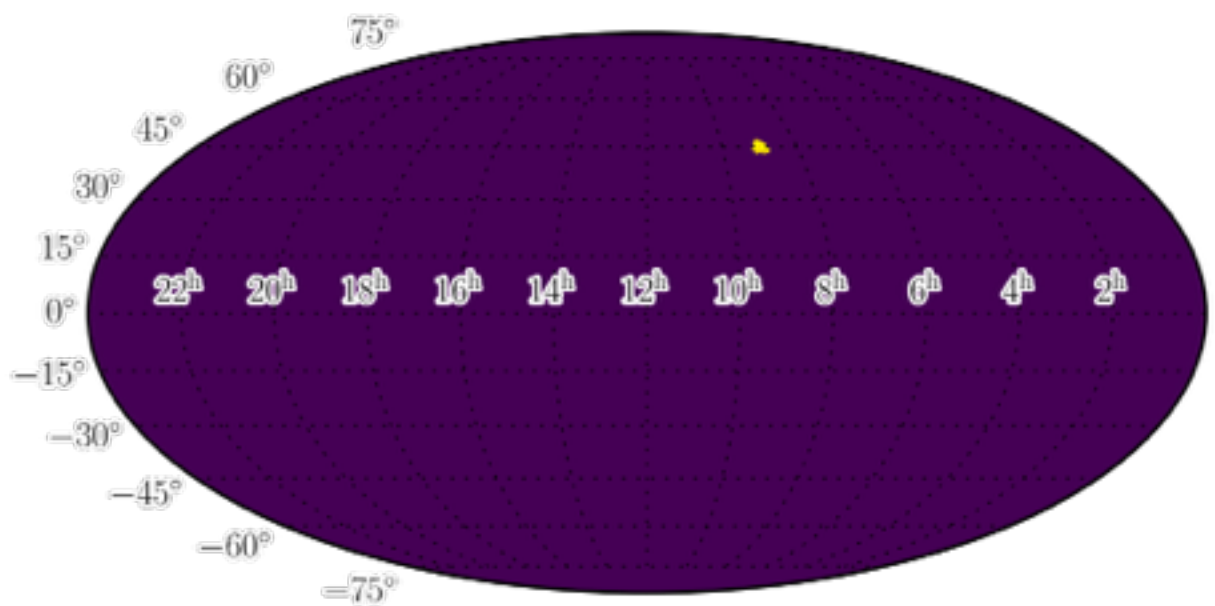
Spherical harmonics



Disk anisotropy



Point source



Detection

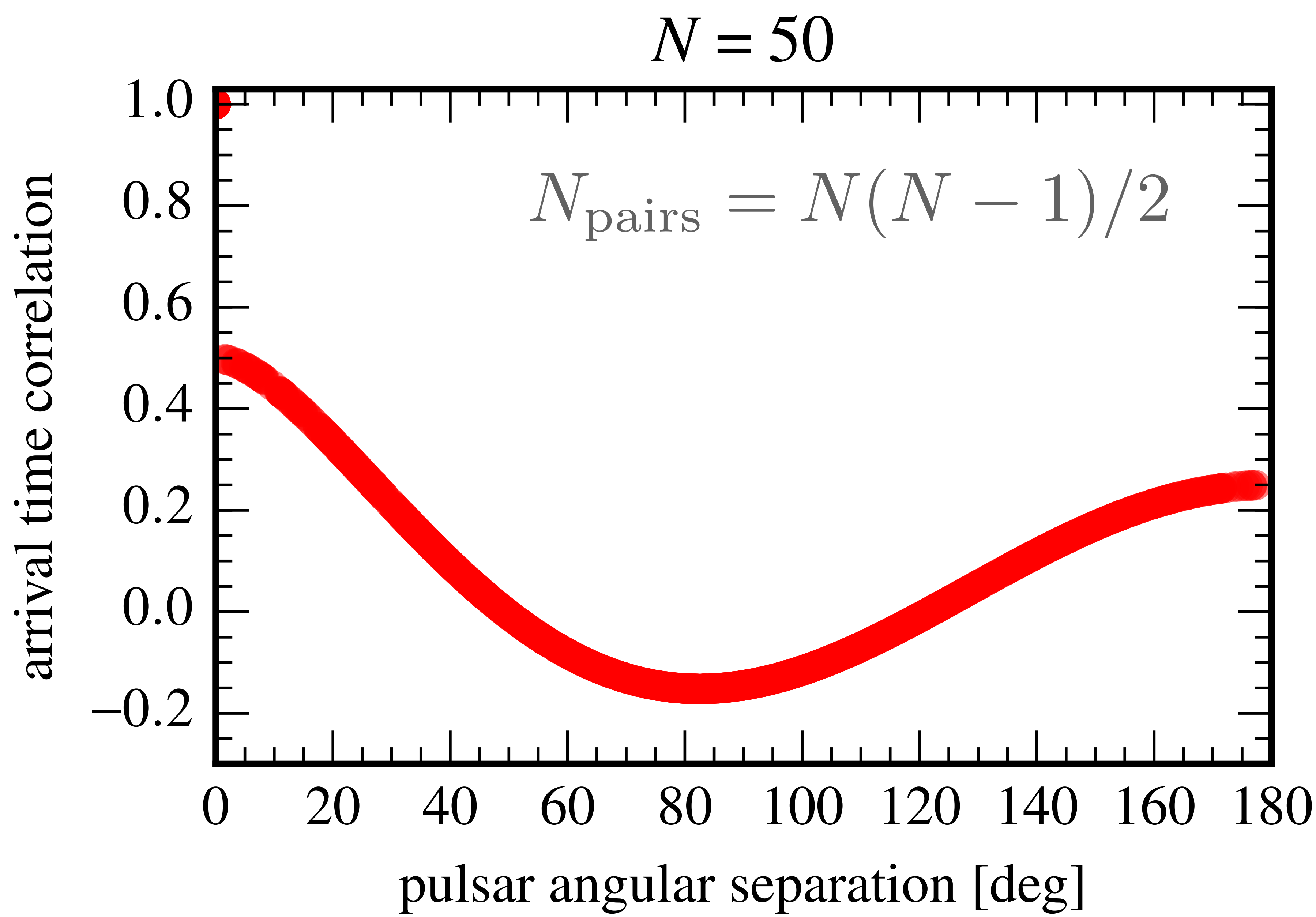
- Detection is a model-selection problem.
- We need to prove the presence of spatial correlations between pulsars.
- ***Compare Bayesian evidence for a model with Hellings and Downs correlations versus no correlations.***

$$\mathcal{P}_{12} = \frac{p(\mathcal{H}_1|\mathbf{d})}{p(\mathcal{H}_2|\mathbf{d})} = \frac{p(\mathbf{d}|\mathcal{H}_1)}{p(\mathbf{d}|\mathcal{H}_2)} \frac{p(\mathcal{H}_1)}{p(\mathcal{H}_2)}$$

Posterior odds ratio | Bayes factor | Prior odds ratio

- MultiNest
- Thermodynamic integration
- RJMCMC
- Savage-Dickey ratio
- **Product space**

Detection



$N = 50$
$N_{\rm pairs} = N(N-1)/2$
arrival time correlation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140
160
180
pulsar angular separation [deg]

Summary

- We can do single-source searches and stochastic background searches all together in our pipelines.
- Current limits are cutting into astrophysically interesting territory.
- We can probe the dynamics and environments of supermassive black-holes binaries.
- Within the next 10 years, we should have convincing signs of nanohertz GWs with pulsar-timing arrays.