ENSE803 Assessment 1 Algebraic Specifications

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1 The Rational Numbers

The set $\mathbb{Q}$ of rational numbers is defined by $\mathbb{Q} = \{(p, q) \mid p, q \in \mathbb{Z} \text{ and } q \neq 0\}$. Then, for two elements $(p_1, q_1), (p_2, q_2) \in \mathbb{Q}$ we can have

Definition of equality:

$$(p_1, q_1) = (p_2, q_2) \text{ if, and only if } p_1 q_2 = p_2 q_1$$

Definition of operations

- Addition(+): $(p_1, q_1) + (p_2, q_2) = (p_1 q_2 + p_2 q_1, q_1 q_2)$
- Additive inverse($-$): $-(p, q) = (-p, q)$
- Multiplication($\cdot$): $(p_1, q_1) \cdot (p_2, q_2) = (p_1 p_2, q_1 q_2)$
- Multiplicative inverse($()^{-1}$): $(p, q)^{-1} = (q, p)$ if $p \neq 0$

A function $\phi : \mathbb{Z} \to \mathbb{Q}$ is defined as $\phi(x) = (x, 1)$. Then we can have

- $\phi(x + y) = (x + y, 1) = (x \cdot 1 + y \cdot 1, 1 \cdot 1) = (x, 1) + (y, 1) = \phi(x) + \phi(y)$
- $\phi(-x) = (-x, 1) = -(x, 1) = -\phi(x)$
- $\phi(x \cdot y) = (x \cdot y, 1) = (x \cdot y, 1 \cdot 1) = (x, 1) \cdot (y, 1) = \phi(x) \cdot \phi(y)$

2 Isomorphic Algebras: Binary and Decimal