ENSE803 Assessment 1 Algebraic Specifications

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1 The Rational Numbers

The set $\mathbb{Q}$ of rational numbers is defined by $\mathbb{Q} = \{(p,q) \mid p,q \in \mathbb{Z} \text{ and } q \neq 0\}$. Then, for two elements $(p_1,q_1),(p_2,q_2) \in \mathbb{Q}$ we can have

Definition of equality:

$$(p_1,q_1) = (p_2,q_2) \text{ if, and only if } p_1q_2 = p_2q_1$$

Definition of operations

- Addition(+): $(p_1,q_1)+(p_2,q_2) = (p_1q_2 + p_2q_1, q_1q_2)$
- Additive inverse(−): $-(p,q) = (-p,q)$
- Multiplication(·): $(p_1,q_1)\cdot(p_2,q_2) = (p_1p_2, q_1q_2)$
- Multiplicative inverse($()^{-1}$): $(p,q)^{-1} = (q,p)$ if $p \neq 0$

A function $\phi : \mathbb{Z} \to \mathbb{Q}$ is defined as $\phi(x) = (x,1)$. Then we can have

- $\phi(x+y) = (x+y,1) = (x\cdot 1 + y\cdot 1, 1\cdot 1) = (x,1)+(y,1) = \phi(x)+\phi(y)$
- $\phi(-x) = (-x,1) = -(x,1) = -\phi(x)$
- $\phi(x\cdot y) = (x\cdot y,1) = (x\cdot y, 1\cdot 1) = (x,1)\cdot(y,1) = \phi(x)\cdot\phi(y)$