Mastering the game of Go with deep neural networks and tree search

Understanding Monte Carlo Tree Search (MCTS)

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- The algorithm introduced is easily generalizable to other games.
- It makes use of the existing Monte Carlo Tree Search (MCTS) algorithm and enhances it using heuristics learnt through deep learning.

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- Derive and explore vanilla MCTS
- Understand the Exploration-Exploitation tradeoff using Upper Confidence Trees (UCT)
- Describe MCTS as used in the paper

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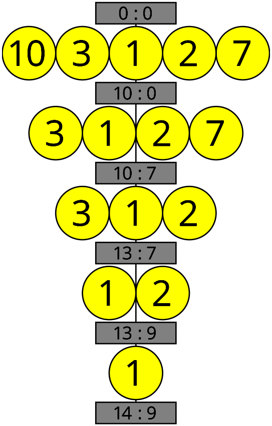
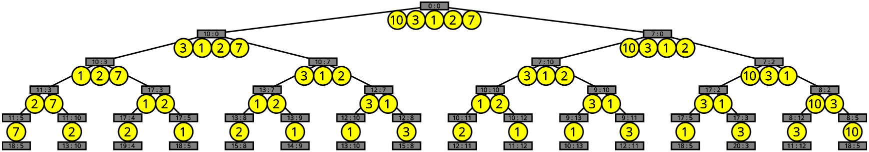
- Only one coin is picked on each turn from either the left or the right



The Game Tree



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$$\text{best score}|a_n, \ldots, a_1, s = \text{final score for player}|a_n, \ldots, a_1, s$$

Minimax Demo

See **examples/minimax**

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- Go has a branching factor of around 250 and depth of around 150.
- Even our game becomes infeasible if w start with a large number of coins.
- **However**, minimax requires no knowledge beyond the rules of the game!

What if we run minimax on a subset of games?

Good moves are much rarer than bad ones and are easy to miss when sampled randomly.

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- We have now defined the RHS in terms of a smaller LHS. We can solve for $P^*(a|s)$.

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- Now, the next time we play a few more games don't choose the first move randomly but choose it according to $P_e(\text{win}|a, s)$.
- Monte Carlo tells us that this process (repeated) guarantees that $P(a|s) \longrightarrow P^*(a|s)$.

Monte Carlo Tree Search: Example

See **examples/vanilla**

Monte Carlo Tree Search: Limitations

It can take a long time for $P(a|s) \longrightarrow P^*(a|s)$

Speeding up MCTS: Upper Confidence Trees

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$$\frac{1}{N_i}\sum_{j=1}^{N_i} x_{ij} + \sqrt{\frac{2\ln\sum_i N_i}{N_i}}$$

N_i = number of times machine i is played

x_{ij} = win or loss when playing machine i the jth time

Speeding up MCTS: Example

See **examples/uct**

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- Second, when a move is selected, UCT uses no prior knowledge of gameplay to make a better decision.
- AlphaGo uses a prior distribution of moves learnt by a deep learning model using reinforcement learning and human-expert moves.

Speeding up MCTS: AlphaGo Example

See **examples/alphago**

Thank You

https://github.com/nanonaren/PWL_AlphaGo