# Math 156 Project 3

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## Problem 1

Suppose we have $\mathbf{v}, \mathbf{x}$ as two vectors in the decision plane. Then

$$\mathbf{w}^T \mathbf{x} + w_0 = \mathbf{w}^T \mathbf{v} + w_0 = 0$$

by definition. So that

$$\mathbf{w}^T (\mathbf{v} - \mathbf{x}) = 0 \implies \mathbf{v} - \mathbf{x} \perp \mathbf{w}.$$

## Problem 2

### Part 1

$\mathbf{w}_1, \mathbf{w}_2$ correctly classifying $\mathbf{x}_1$ means that $\operatorname{sign}\left(\mathbf{w}_1^T \mathbf{x}_1\right) = \operatorname{sign}\left(\mathbf{w}_2^T \mathbf{x}_1\right) = 1$. Consider $y\left(\mathbf{x}_1, \lambda \mathbf{w}_1 + (1-\lambda)\mathbf{w}_2\right)$. We have

$$y\left(\mathbf{x}_1, \lambda \mathbf{w}_1 + (1-\lambda)\mathbf{w}_2\right) = \operatorname{sign}\left(\lambda \mathbf{w}_1^T \mathbf{x}_1 + (1-\lambda)\mathbf{w}_2^T \mathbf{x}_1\right).$$

Suppose that $\mathbf{w}_1^T \mathbf{x}_1 = a$, $\mathbf{w}_2^T \mathbf{x}_1 = b$ where by assumption, $a, b > 0$. Then we have

$$\lambda a + (1-\lambda) b > 0$$

since $\lambda \in [0,1]$. So

$$\operatorname{sign}\left(\lambda \mathbf{w}_1^T \mathbf{x}_1 + (1-\lambda)\mathbf{w}_2^T \mathbf{x}_1\right) = 1.$$

So convex combinations of $\mathbf{w}_i$ correctly classify $\mathbf{x}_1$.

### Part 2

## Problem 3

Let $\mathbf{v} = \sum_{i=1}^N \alpha_i \mathbf{x}_i$ be in the convex hull of $\{\mathbf{x}_i\}$. By linearity of linear discriminant functions over convex combinations, we have

$$\begin{aligned} y(\mathbf{v}) &= y\left(\sum_{i=1}^N \alpha_i \mathbf{x}_i\right) \\ &= \sum_{i=1}^N \alpha_i y(\mathbf{x}_i) \\ &> 0 \end{aligned}$$

by assumption.