On representing wave localisation due to 2D multiple scattering in operational wave models

Timothy D. Williams and Luke G. Bennetts

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1 Interpolation node-to-node

2 Interpolation node-to-element

N elements, nodes are $x_0, \ldots, x_N$

$$\tilde{f} = f_n \xi_0^{(n)} + f_{n+1} \xi_1^{(n)}, \tag{1a}$$

$$\xi_1^{(n)} = \frac{x - x_n}{x_{n+1} - x_n}, \tag{1b}$$

$$\xi_0^{(n)} = 1 - \xi_1^{(n)}. \tag{1c}$$

Want average value of $\tilde{f}$ over $y_m < y < y_{m+1}$:

$$\begin{aligned} F_m(y_{m+1} - y_m) &= \int_{y_m}^{y_{m+1}} \tilde{f}(x) \mathrm{d}x \\ &= \sum_{n=0}^{N-1} \left(I_0^{(mn)} f_n + I_1^{(mn)} f_{n+1} \right) \\ &= I_0^{(m0)} f_0 + I_1^{(m,N-1)} f_N \\ &\quad + \sum_{n=1}^{N-1} f_n \left(I_0^{(mn)} + I_1^{(m,n-1)} \right), \end{aligned} \tag{2}$$

where

$$\begin{aligned} I_1^{(mn)} &= H_{mn} \int_{L_{mn}}^{U_{mn}} \xi_j^{(n)} \mathrm{d}x \\ &= H_{mn} \left[\frac{(x - x_n)^2}{2(x_{n+1} - x_n)} \right]_{L_{mn}}^{U_{mn}}, \end{aligned} \tag{3a}$$

$$H_{mn} = H(x_{n+1} - y_m) H(y_{m+1} - x_n), \tag{3b}$$

$$L_{mn} = \max\{y_m, x_n\}, \tag{3c}$$

$$U_{mn} = \min\{y_{m+1}, x_{n+1}\}, \tag{3d}$$

$$I_0^{(mn)} = H_{mn} \Big(U_{mn} - L_{mn} - I_1^{(mn)} \Big), \tag{3e}$$

3 Interpolation element-to-element

4 Interpolation element-to-node