Machine learning and physical (Earth system) modelling

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NERSC

slides+notebook:*https://github.com/nansencenter/ml-crashcourse*

References

Ian Goodfellow, Yoshua Bengio, and Aaron Courville.
Deep Learning.
MIT Press, 2016.
http://www.deeplearningbook.org.

Jake VanderPlas.
Python Data Science Handbook: Essential Tools for Working with Data.
O'Reilly Media, Inc., 1st edition, 2016.

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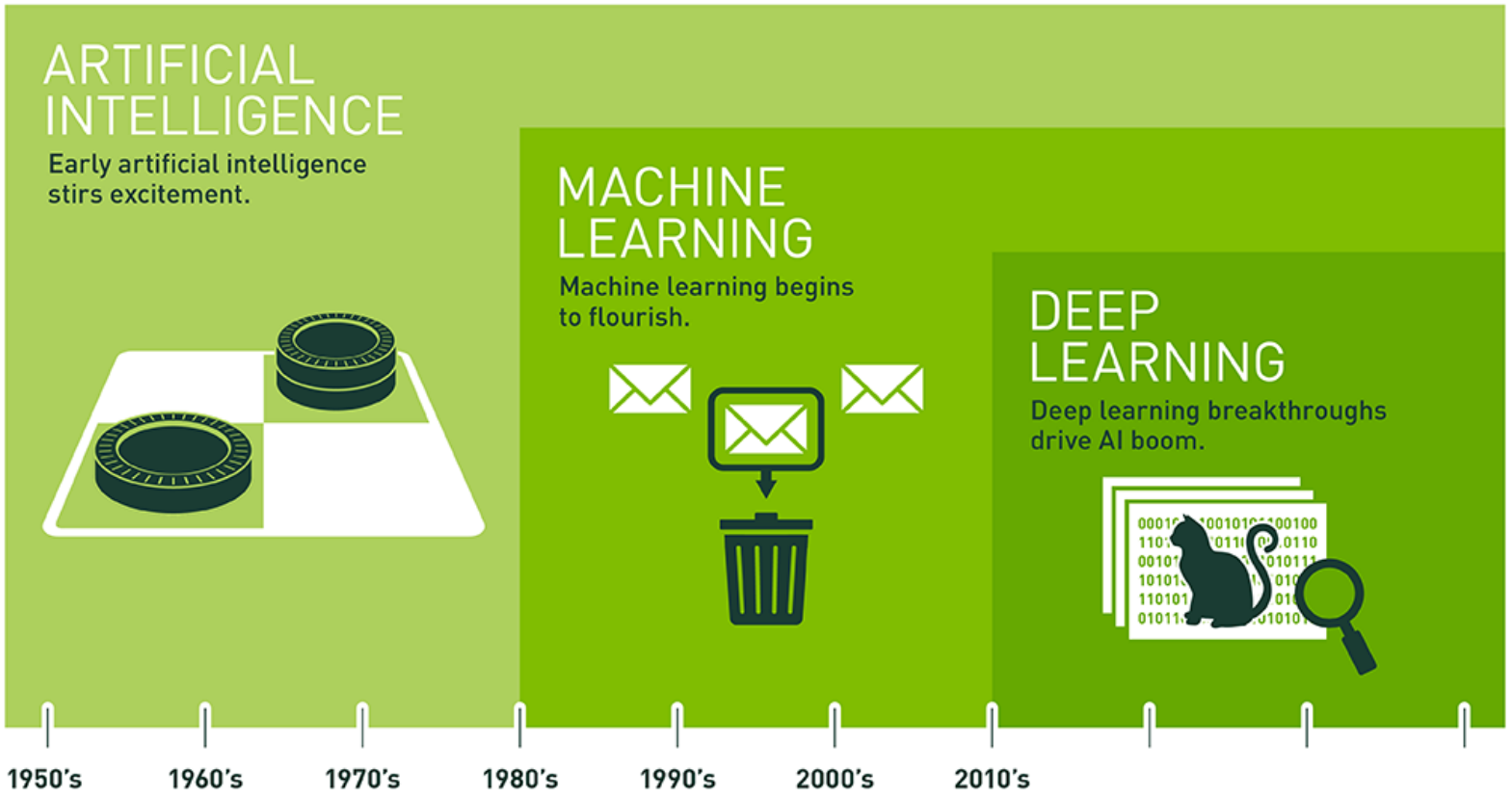
Introduction

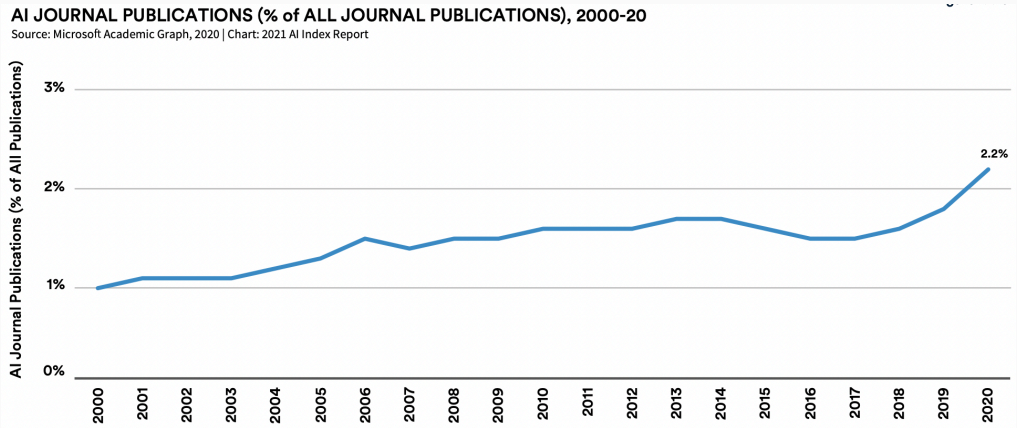
Scope of the lecture: Machine Learning

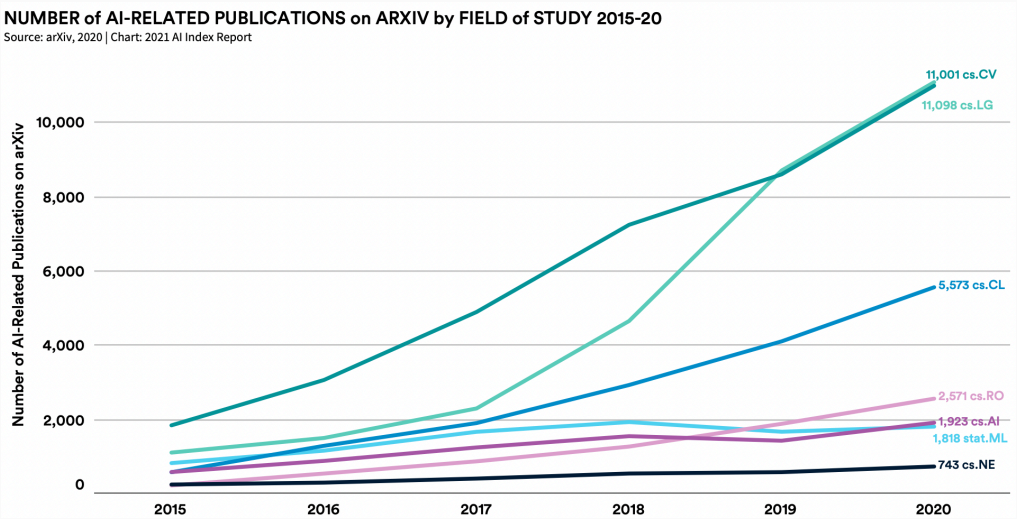


Since an early flush of optimism in the 1950s, smaller subsets of artificial intelligence – first machine learning, then deep learning, a subset of machine learning – have created ever larger disruptions.

Source: NVidia

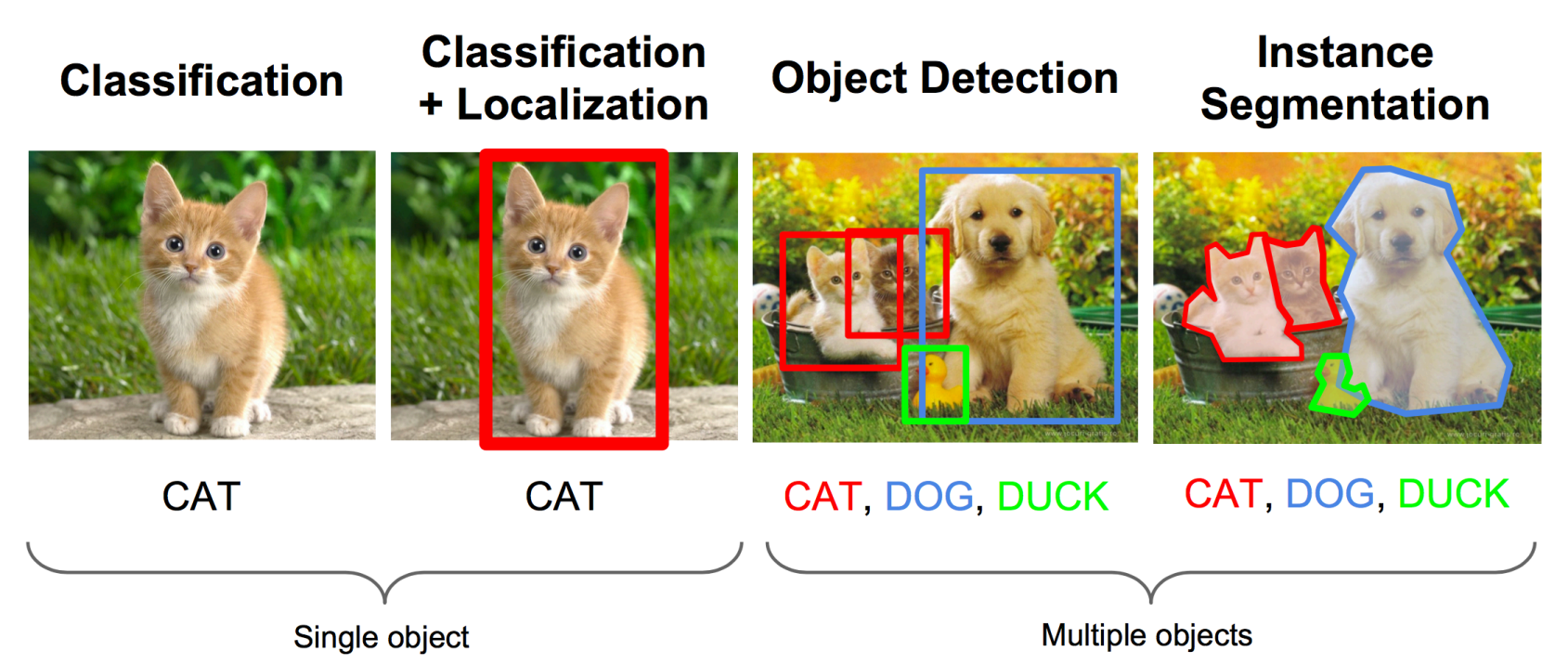
A (very) active field





Zhang et al., "The AI Index 2021 Annual Report"

Example 1: Computer Vision



Li, Karpathy and Johnson, 2016, Stanford CS231n course

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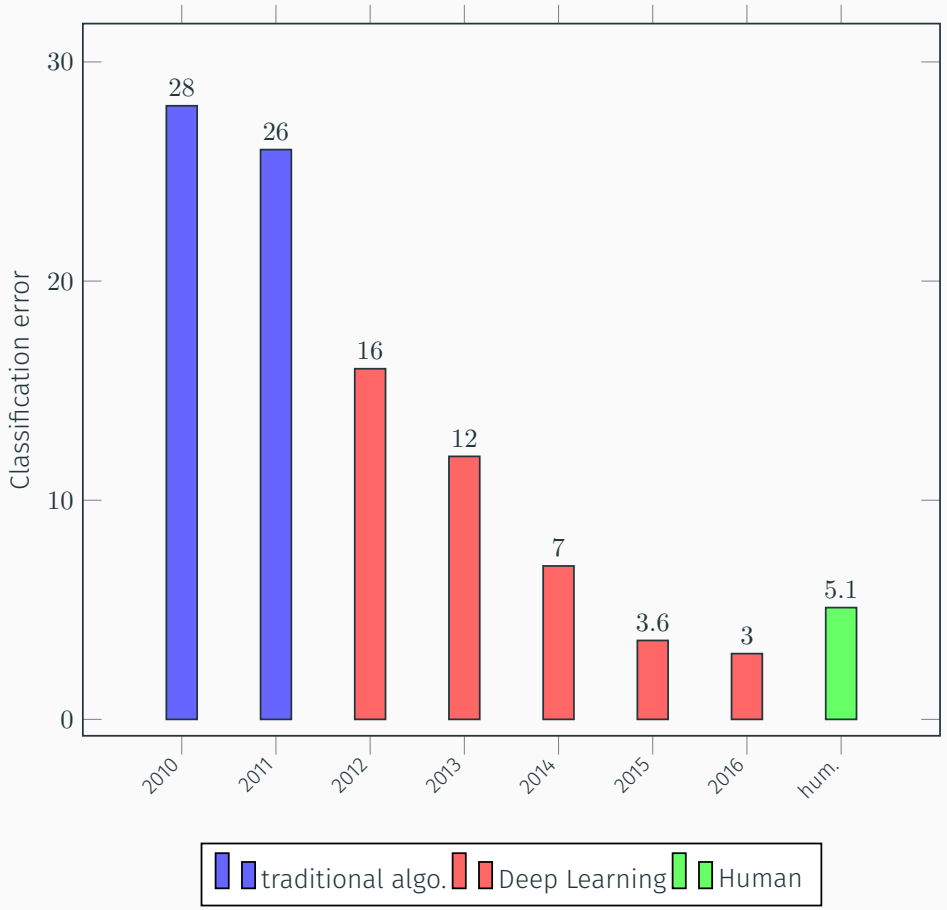
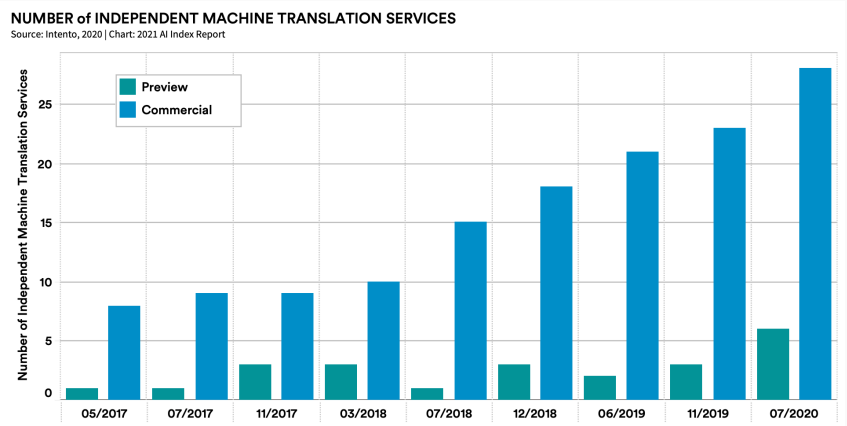


Deep learning architectures were based on Convolutional Neural Networks (CNN).

Example 2: Machine Translation

Objective : translate a text from a language to another.



Zhang et al., "The AI Index 2021 Annual Report"

- **Oct. 2013**: Pionneering scientic paper (Kalchbrenner, N., and Blunsom, P).
- **2016**: Neural machine translation outperform traditional approaches on public benchmarks
- **2017**: Major systems switch to neural machine translation (using deep recurrent neural networks)

Example 3: Playing Games

- **1997**: Deep Blue defeats Kasparov at Chess.
- **2016**: AlphaGo's victory again Lee Sedol at Go.
- **2017**: AphaGo Zero learns how to play Go only by playing against itself. It outperformed previous AlphaGo version (Reinforcement learning)
- **2017**: DeepStack beats professional human poker players.

Protein folding

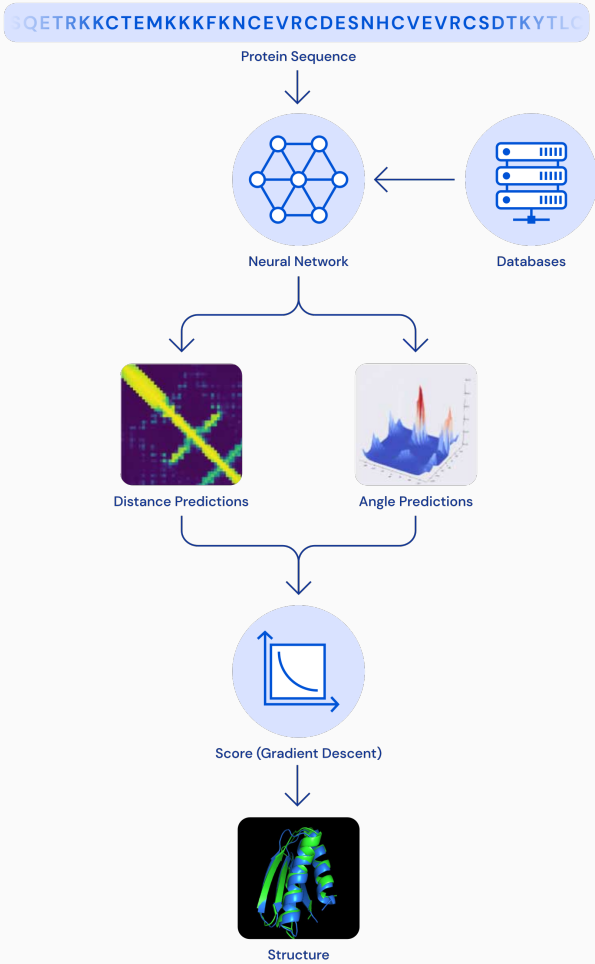


Diagram of Alpha Fold (source: Deepmind)

Senior et al. 2020

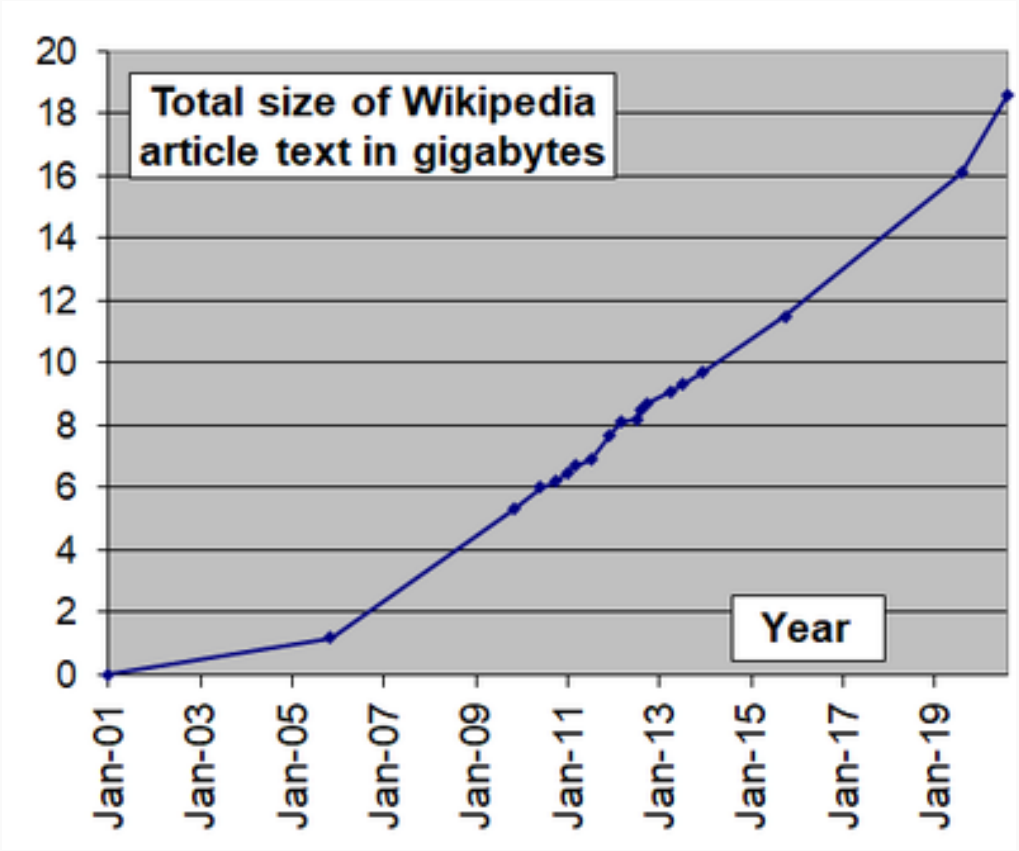
AI Art?

Edmond de Bellamy by Obvious(collective)

Generated using a Generative Adversarial Network.
Selling prince (Oct. 2018): $432,000

Reasons for these recent achievements?

- Increasing of the datasets in size and quality



source: Wikipedia

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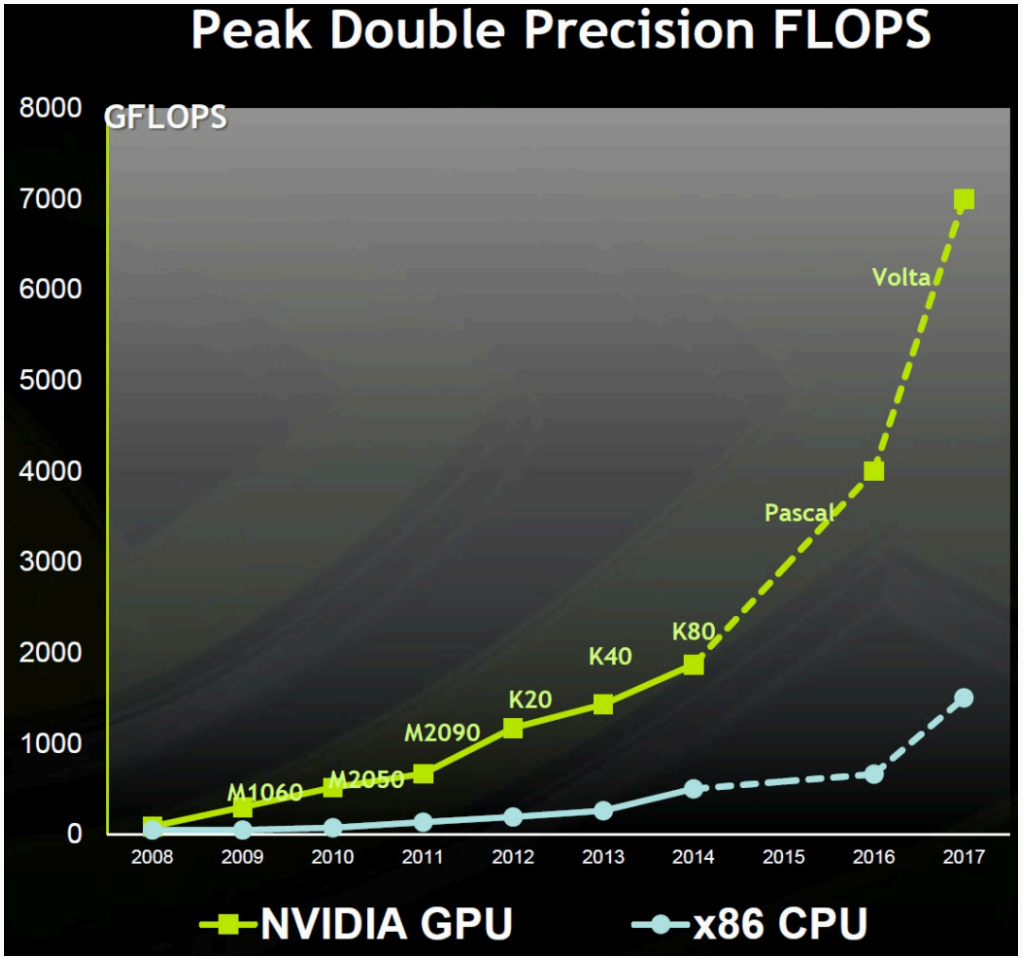

source: NVDIA

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- Scientific research on new algorithms (e.g adapted to image processing)

source: Deep Dream Generator

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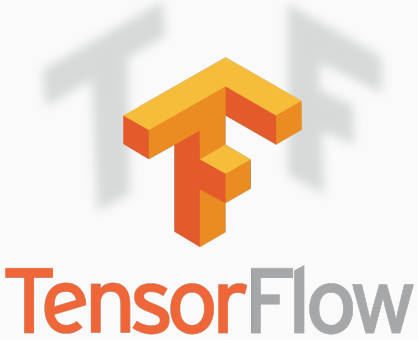
- Very efficient software (GPU, cloud computing, automatic differentiation, ...)



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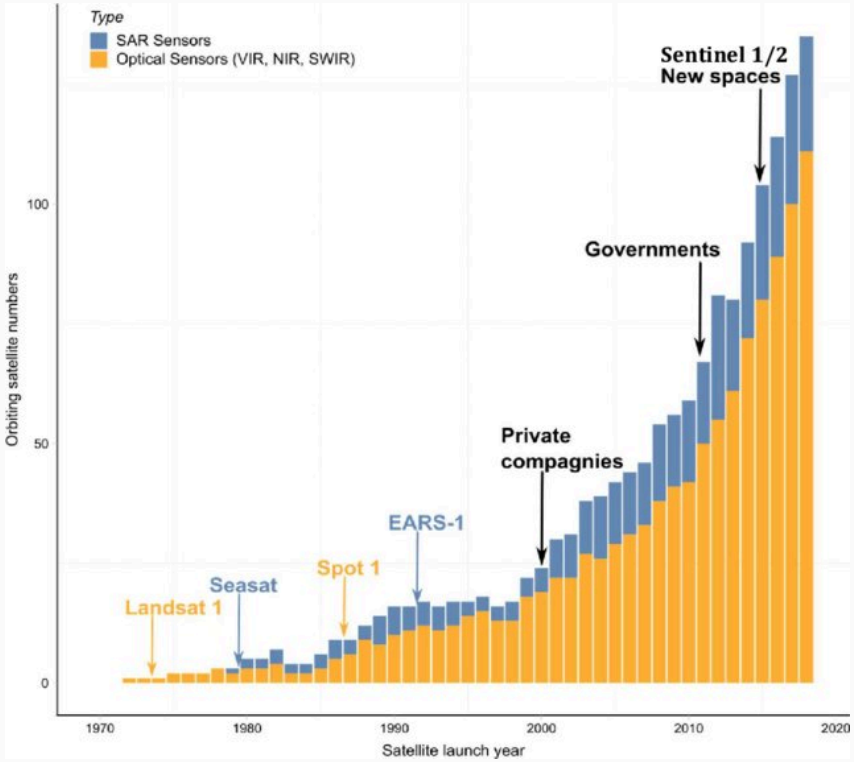
- Very efficient software (GPU, cloud computing, automatic differentiation, ...)
- Free software and open data culture.

Apply Machine-Learning to physical (Earth-system) modelling?

Why is it a good idea?

- A increasing number of geophysical data (one spatial mission: 24 TB/day)



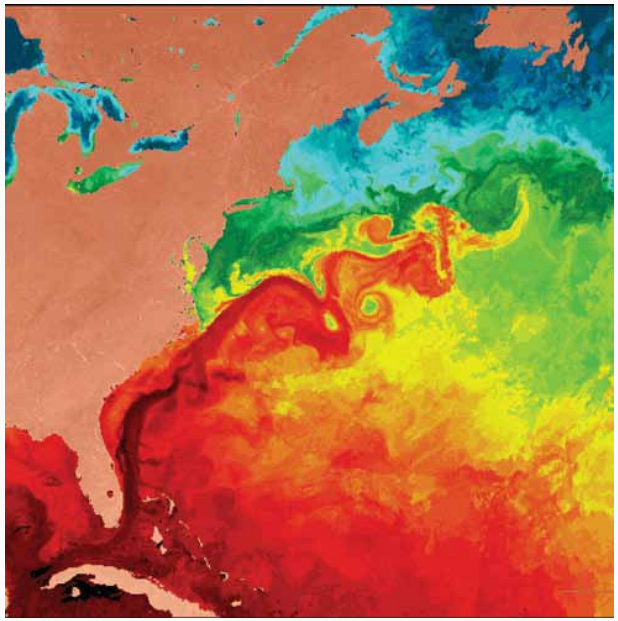
Earth System observavtion satellites

source: Tonneau et al. (2020)

Apply Machine-Learning to physical (Earth-system) modelling?

Why is it a good idea?

- A increasing number of geophysical data (one spatial mission: 24 TB/day)
- Data with highly significant spatial patterns



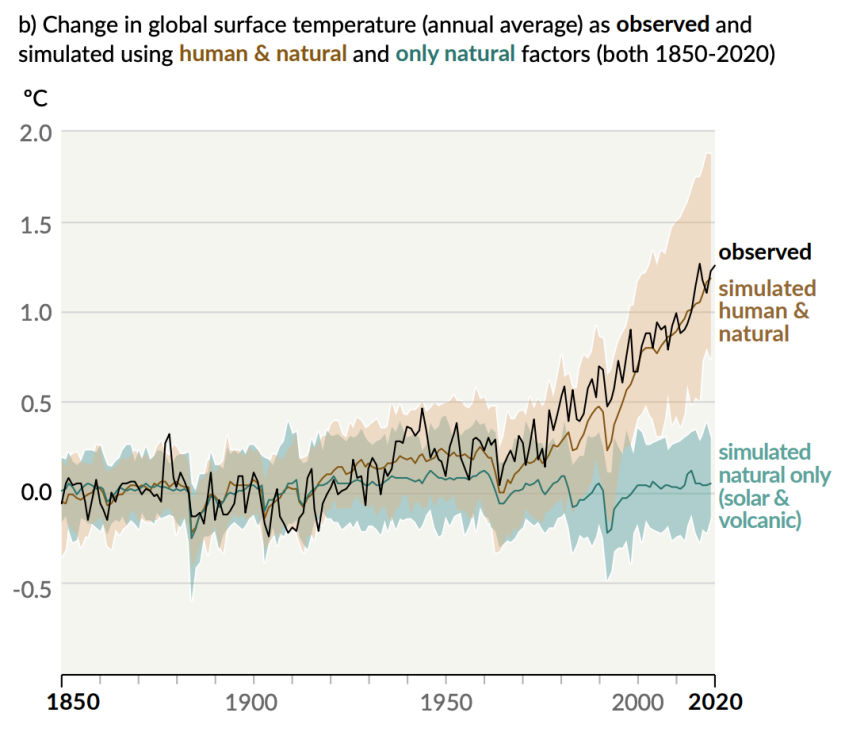
Sea Surface temperature of the gulf stream

source: Talley (2000)

Why is physical modelling specific?

NASDAQ Composite sock market index over the last 10 years

Figure 1: IPCC, AR6, WG1



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Mostly unknown dynamical processes

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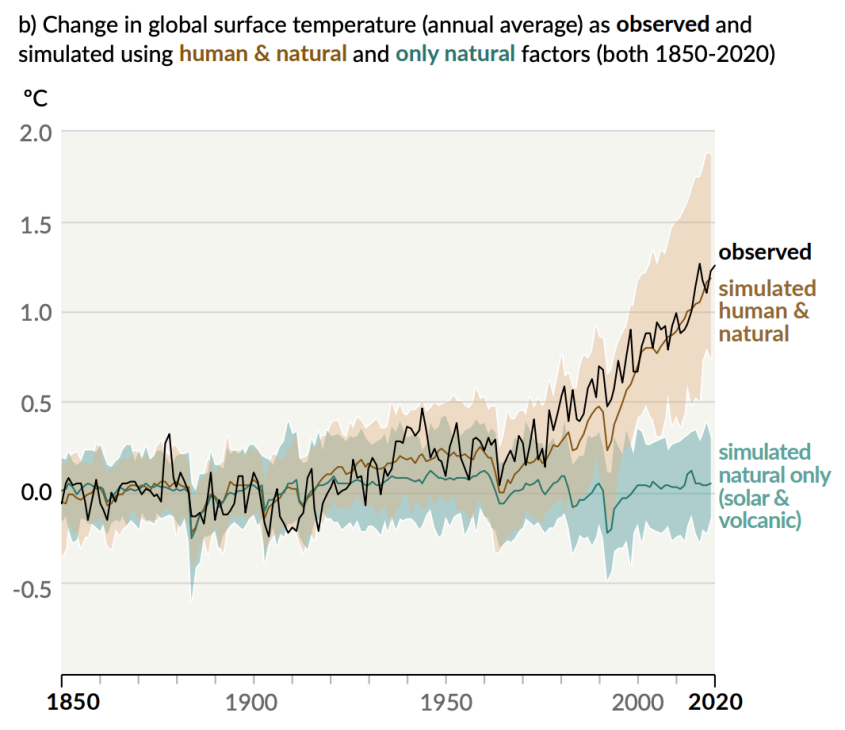

Mostly known dynamical processes (based on physical principles)

What about data assimilation?

Machine learning and data assimilation are closely linked.

Some references:

- Geer, A.J., 2021. Learning earth system models from observations: machine learning or data assimilation?. *Philosophical Transactions of the Royal Society A*, 379(2194)
- Brajard et al. 2019. Connections between data assimilation and machine learning to emulate a numerical model. *Proceedings of the 9th International Workshop on Climate informatics*
- Bocquet et al. 2019. Data assimilation as a learning tool to infer ordinary differential equation representations of dynamical models. *Nonlinear processes in geophysics.* 26(3).
- Abarbanel, H.D., Rozdeba, P.J. and Shirman, S., 2018. Machine learning: Deepest learning as statistical data assimilation problems. *Neural computation, 30(8).*

During this summer school:

Marc Bocquet talk on Friday Aug. 27th, 9:30 (CEST).

Generalities on Machine Learning

What is this about ?

Can we extract knowledge, make some predictions, determine a ”model” using this large amount of data ?

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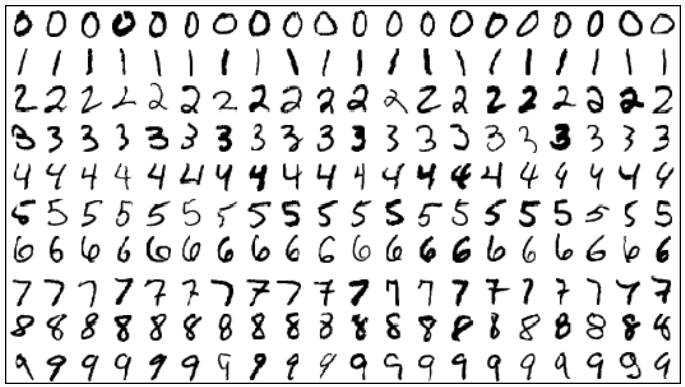

Base of images

$\longrightarrow$ Digite$\in \{0, \ldots, 9\}$

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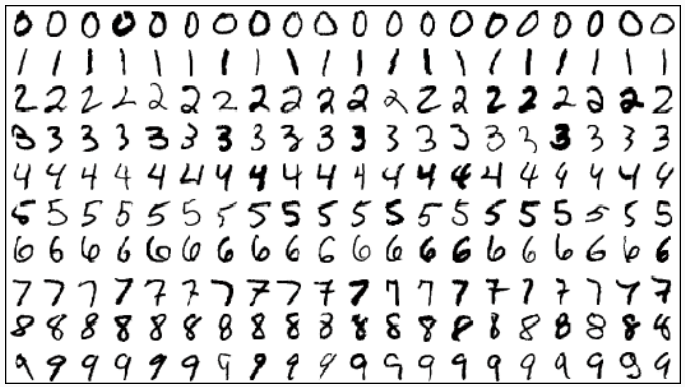

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- From high dimensional data (thousands to millions dimensions) to reduced dimensional data (less than 100)
- From disorganized data to comprehensive information
- Can we teach a machine how to do that ?

Two classes of Machine Learning problems

1. Regression: Determination of a quantitative variable from a set of data
   - The price of a building from various predictors (Surface, ...)
   - A physical value (Temperature, humidity, ...) in the future knowing the past
   - ...

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   - The price of a building from various predictors (Surface, ...)
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2. Classification: Determination of a class
   - A digit from a image
   - Identification of the content of an image
   - ...

Two types of objectives

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    - Playing a chess game.

Formally

A Machine

$$y = \mathcal{M}(x, \theta)$$

- x: input
- y: output
- $\mathcal{M}$: a model (named "machine")
- θ : parameters of the model $\mathcal{M}$.

Machine learning consists in optimizing θ using a set of data. This is the training process.

The Machine Learning recipe

A Machine

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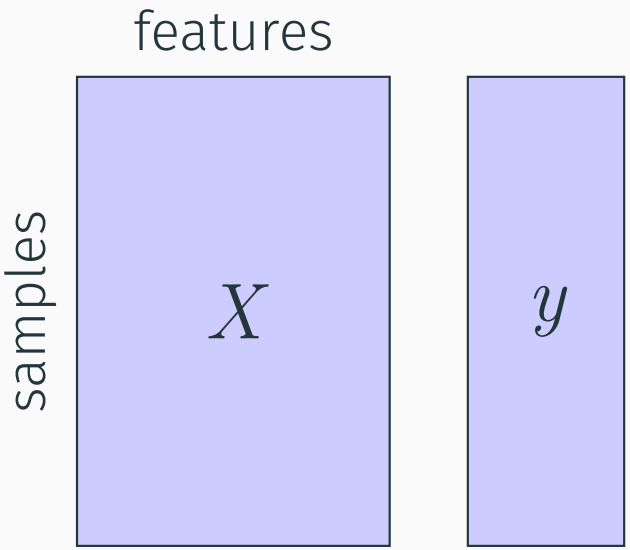
- A learning process
  - Estimation of θ

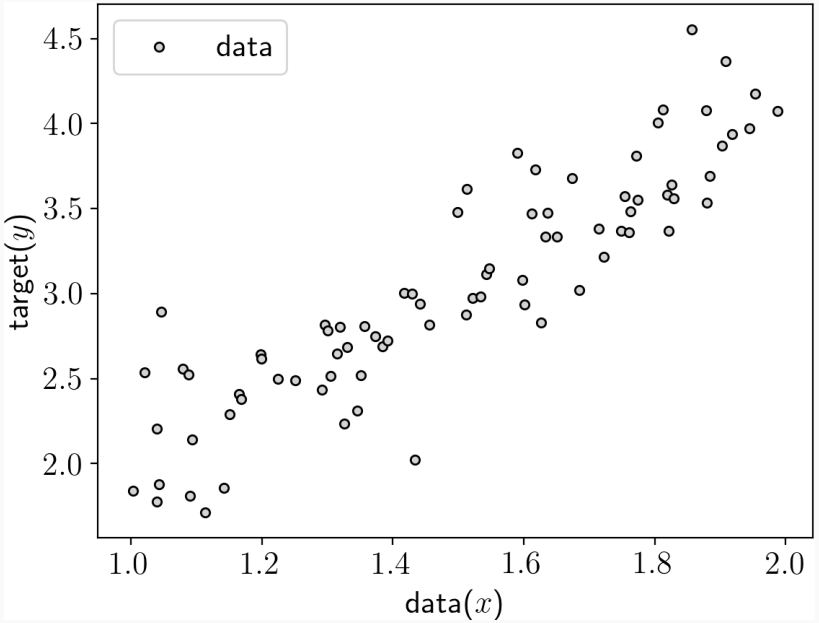
Multidimensional data

Generally, we have multidimensional data X and a one-dimensional target y.



An illustration

- Some Data



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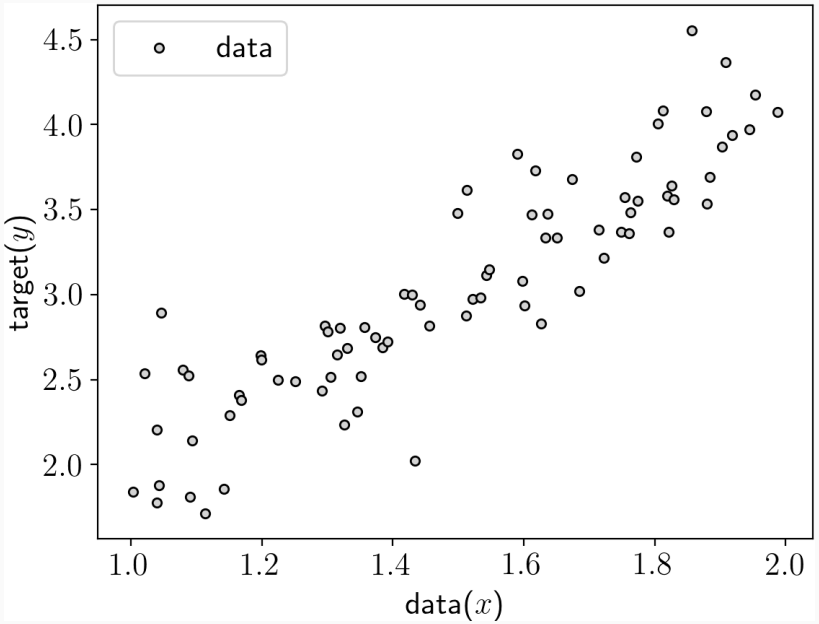

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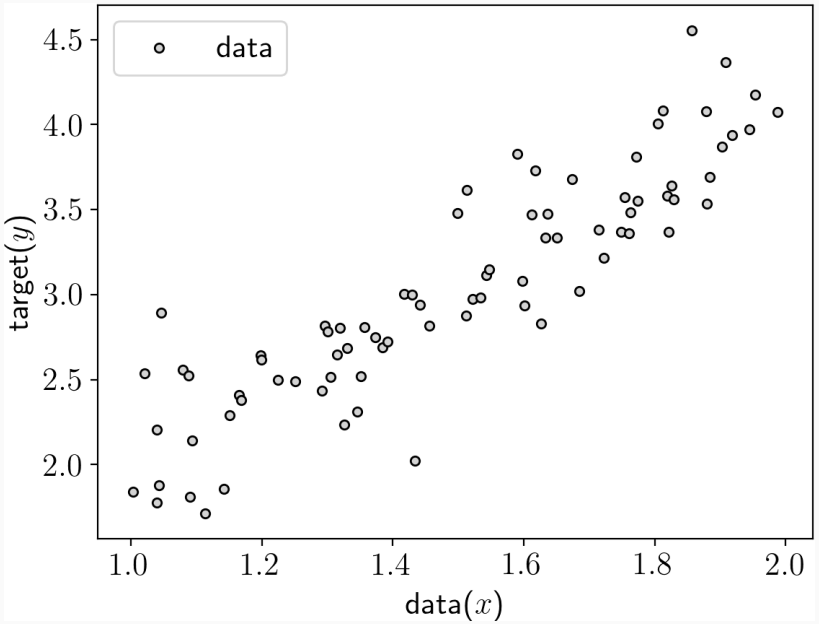

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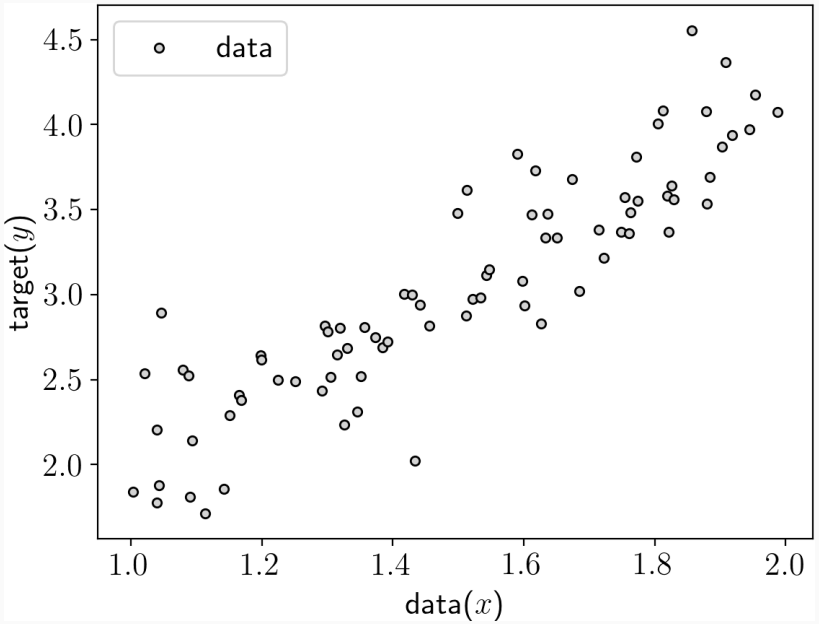

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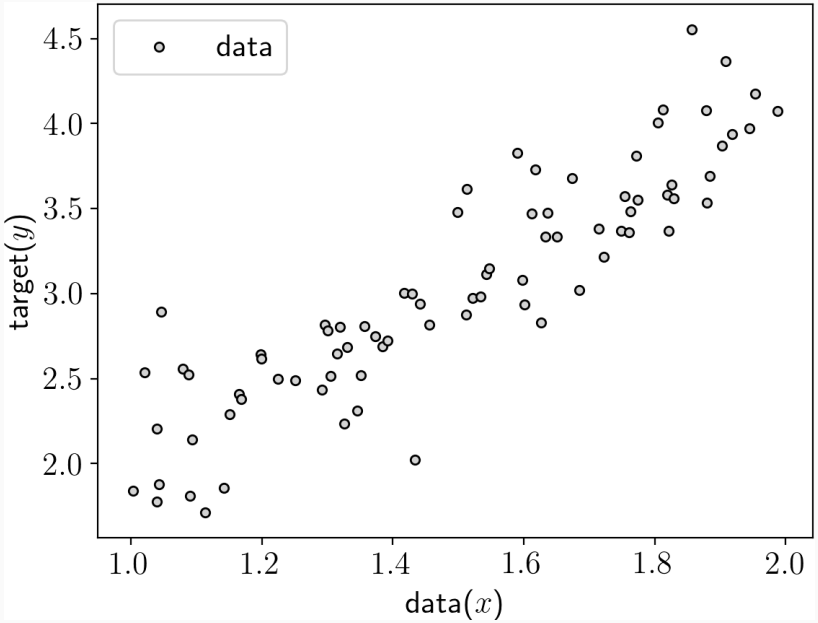

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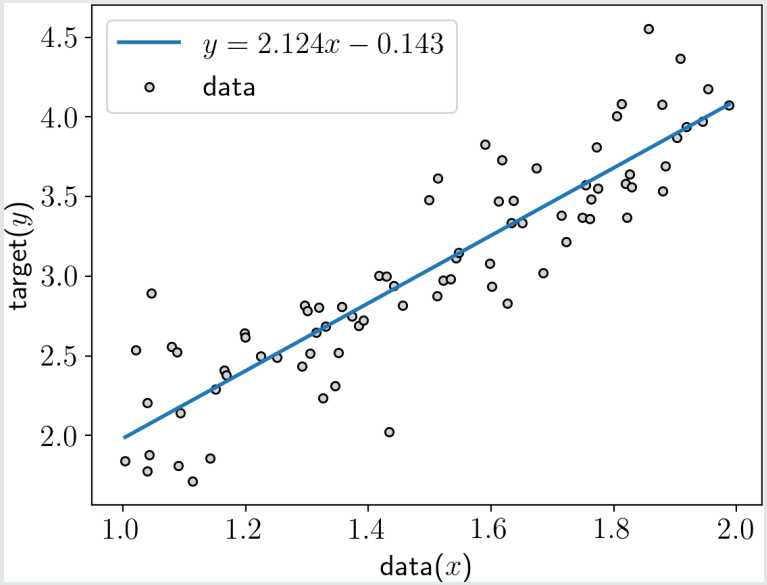
Result



Model selection/validation

Choice of the model

Polynomial regression

$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_d x^d = \sum_{i=0}^{d} \theta_i X^i$$

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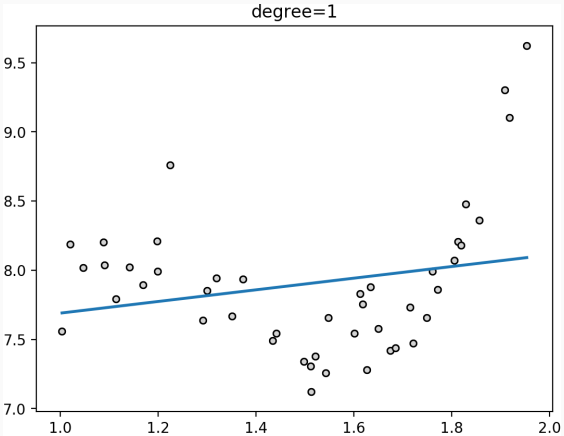

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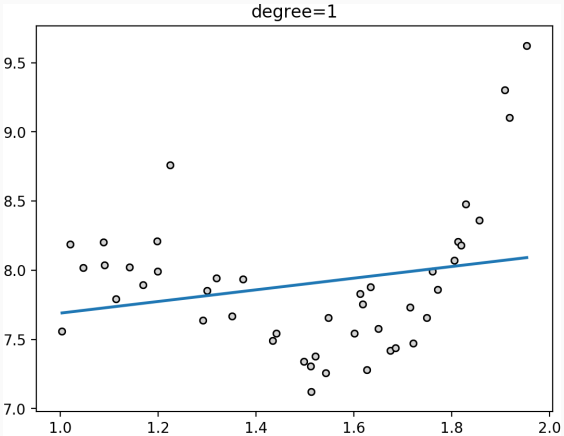
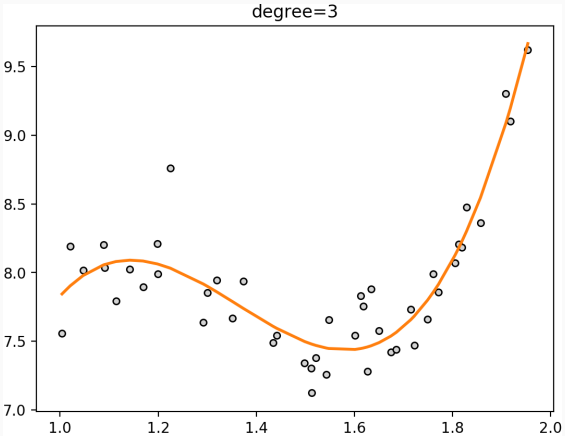


degree = 3



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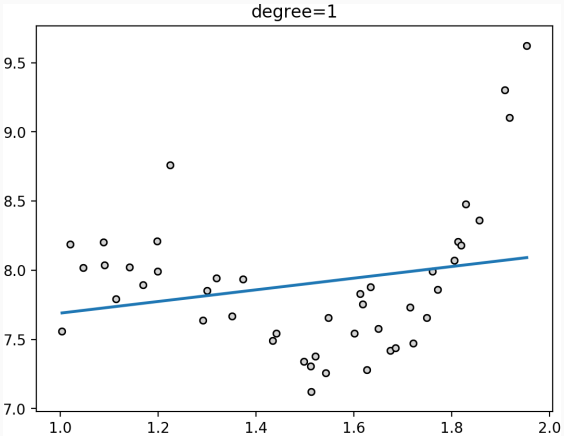

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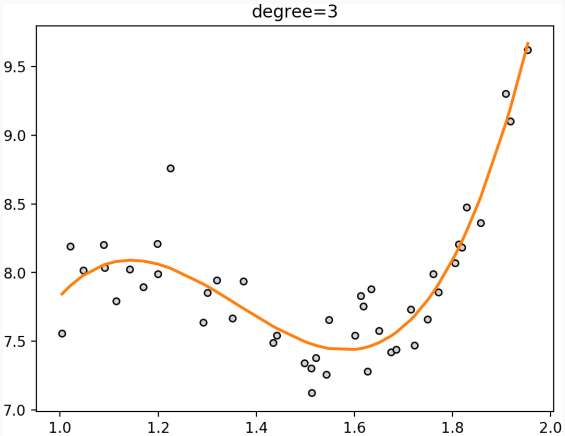
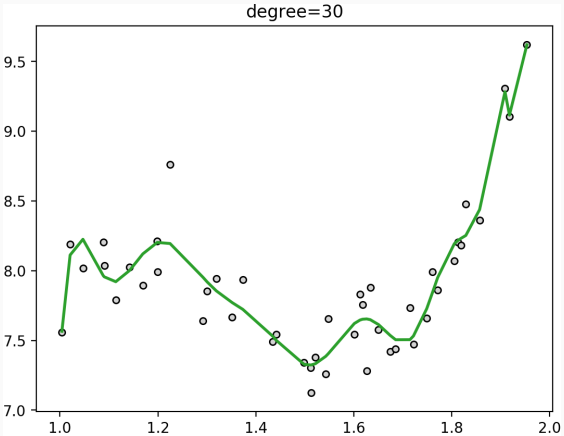


degree = 30



What is the best model?

Train/Validation split

The idea

Evaluate a score on a independent dataset

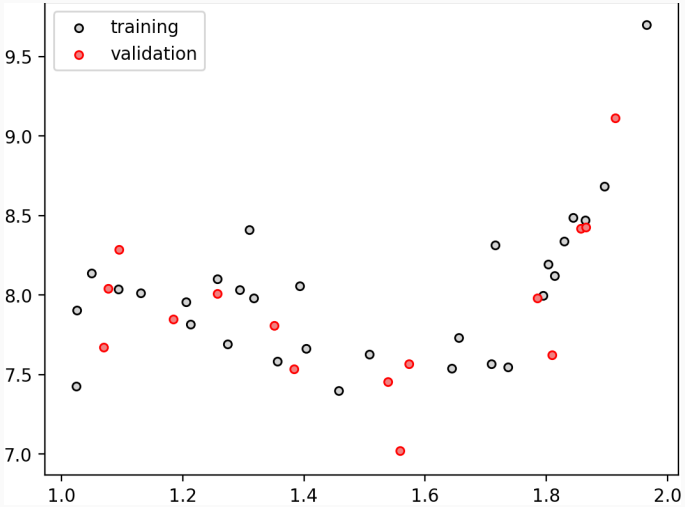
Train/Validation split

The idea

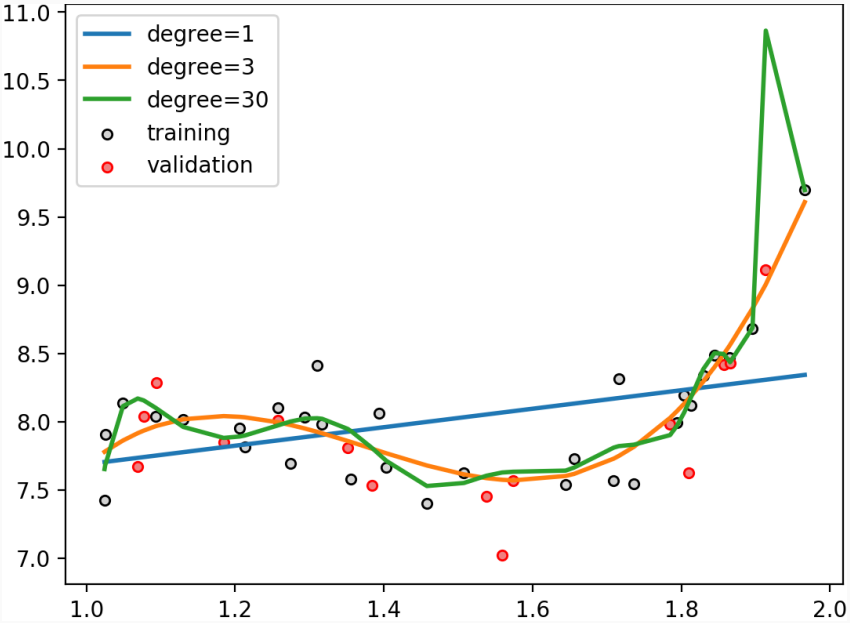
Evaluate a score on a independent dataset

In our example we can randomly divide (X, y) in two datasets:

- The training dataset X_{train}, y_{train} used to fit the model.
- The validation dataset X_{val}, y_{val} used to compute the score (e.g., correlation, mean-squared error)



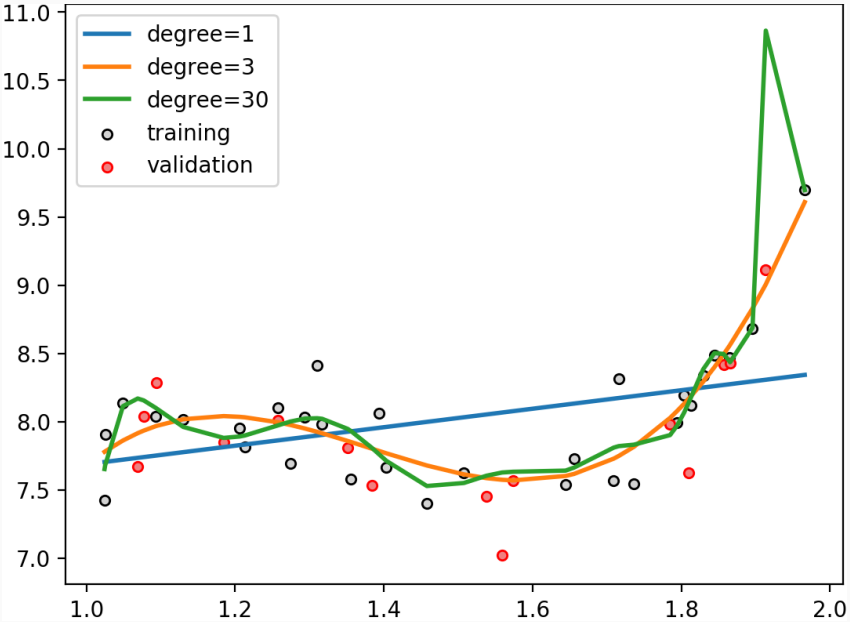
Choice of the model



Score: Mean Square Error (MSE)

Deg.	Train Score	Val. Score
1	0.17	0.23
3	0.045	0.062
30	0.035	0.27

Choice of the model



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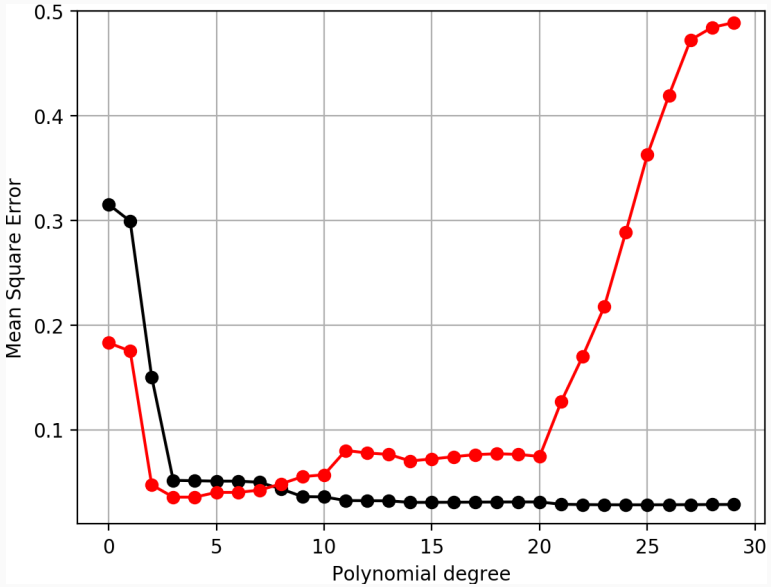
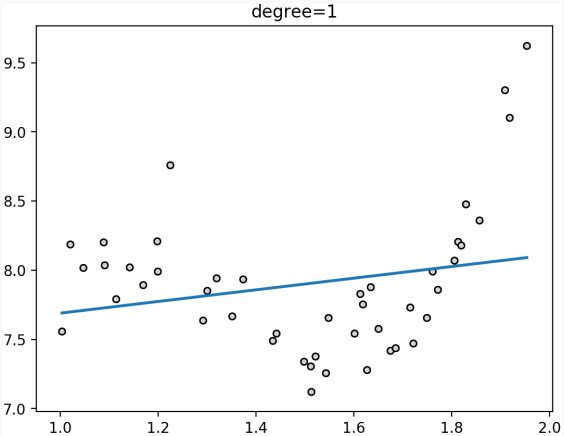


Choice of the model

Polynomial regression

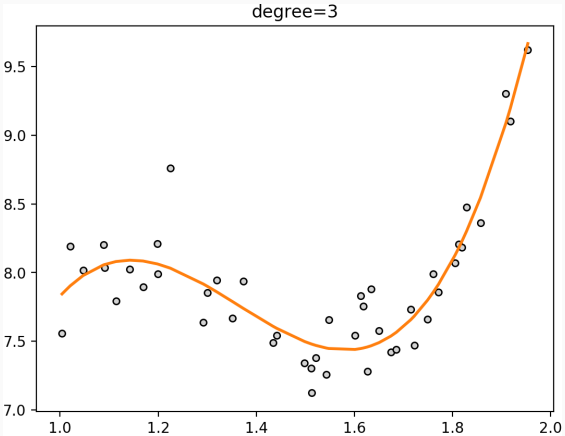
$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_d x^d = \sum_{i=0}^{d} \theta_i X^i$$

degree = 1 (linear)



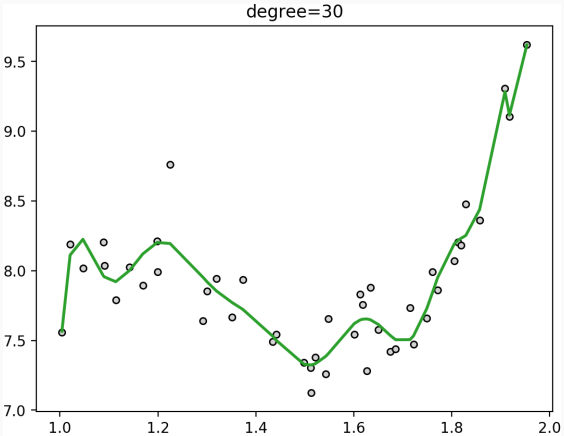
underfitting

degree = 3



good fit

degree = 30



overfitting

Train/Validation split

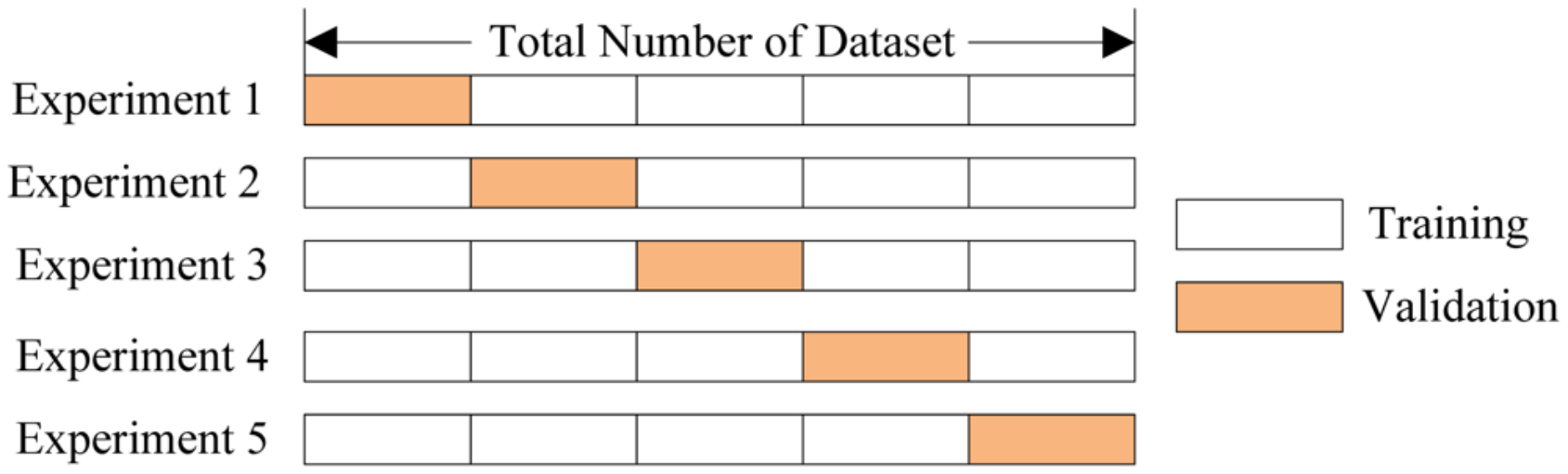
Drawbacks

- drastically reduce the number of samples which can be used for learning the model
- Results can depend on a particular random choice for the pair of (train, validation) sets.

More Robust: cross validation

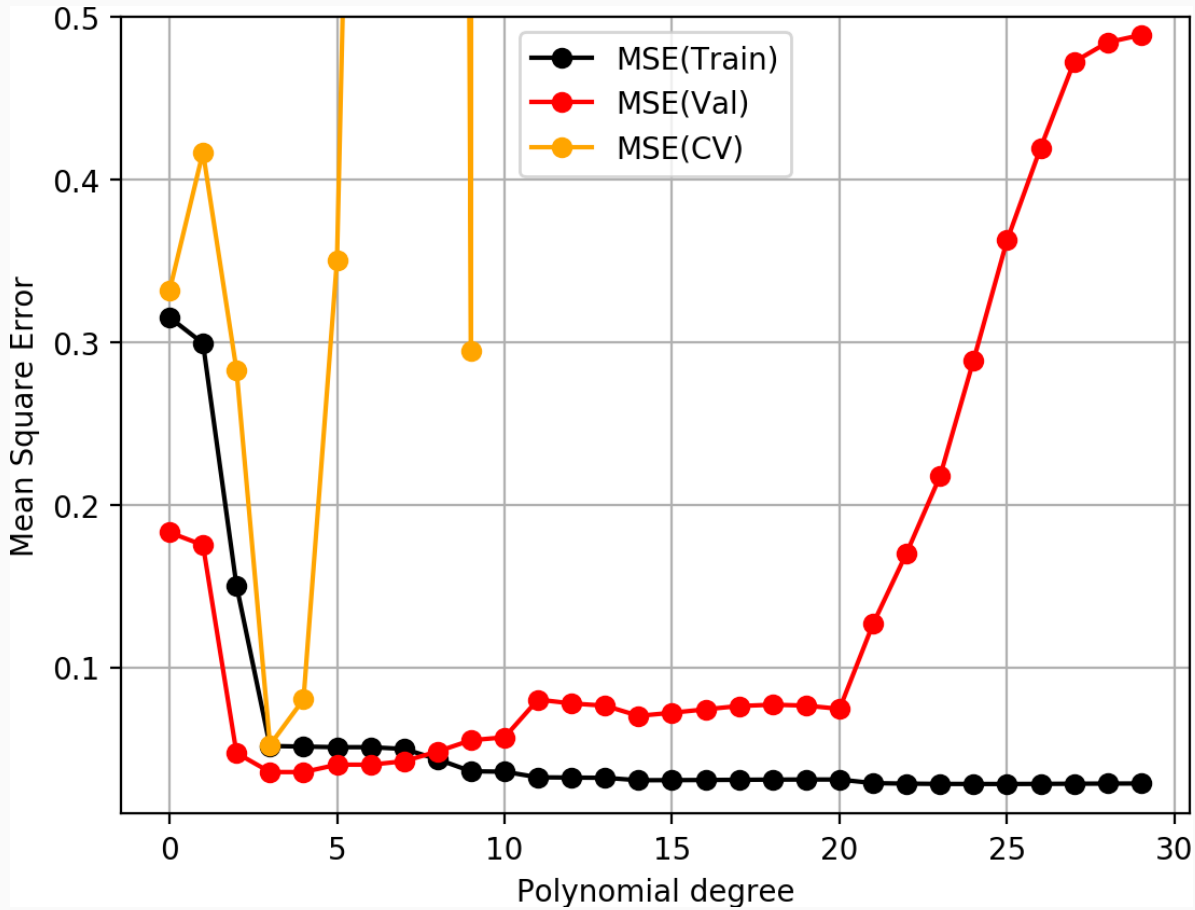
The idea

- Dividing the data in n folds,
- Learning n model (each time with a different training set),
- Compute the mean score over n validation set.



Cross-Validation

Fold	MSE
1	0.052
2	0.043
3	0.137
4	0.025
5	0.048
6	0.144
7	0.011
8	0.025
9	0.010
10	0.028
Mean	0.05



Wrapping up

1. When applying machine learning techniques there are hyperparameters to be determined (e.g., degree of the polynomial in polynomial regression).

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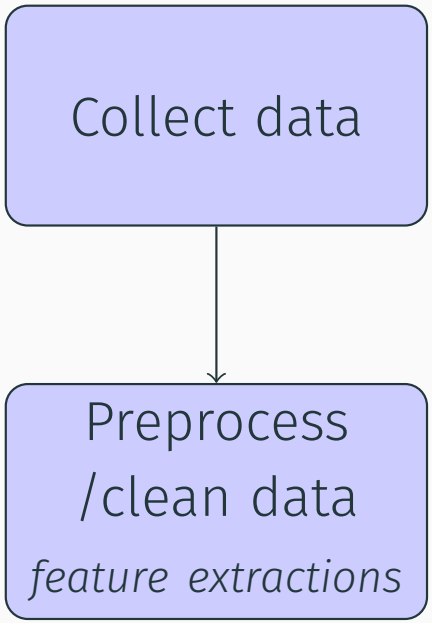
2. These hyperparameters can be determined by splitting the data into training/validation or by cross-validation.
3. But then... the validation set was used to determine the best machine learning process
4. To evaluate independanly the performance of our model, we should compute the score on a third independant dataset: The test dataset.

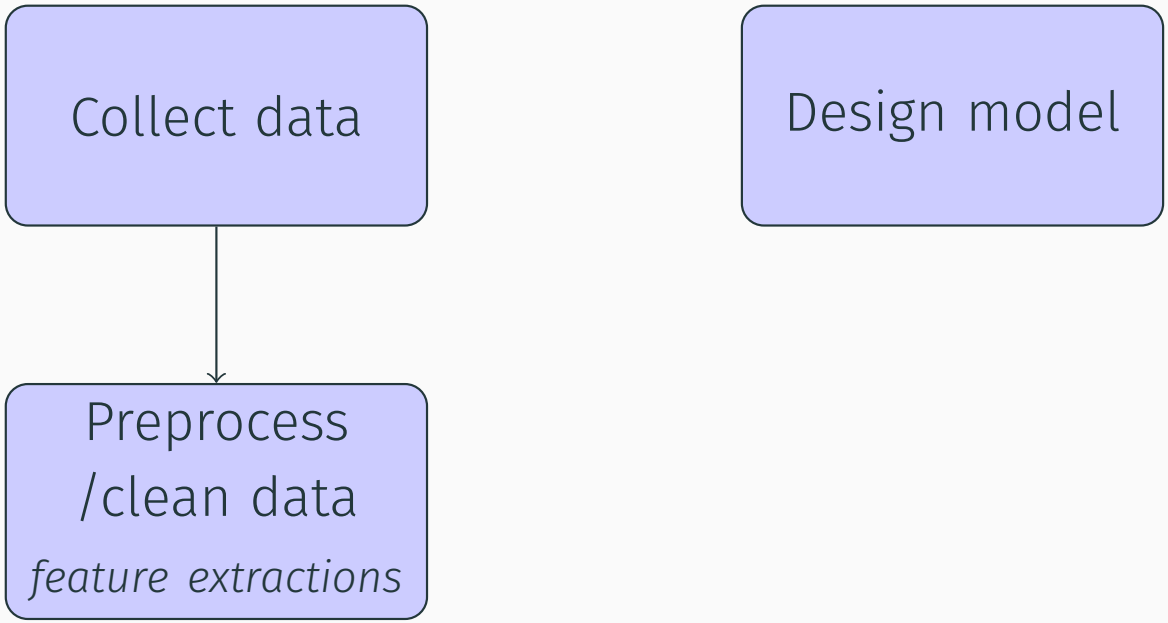
Steps of a machine learning process

Steps

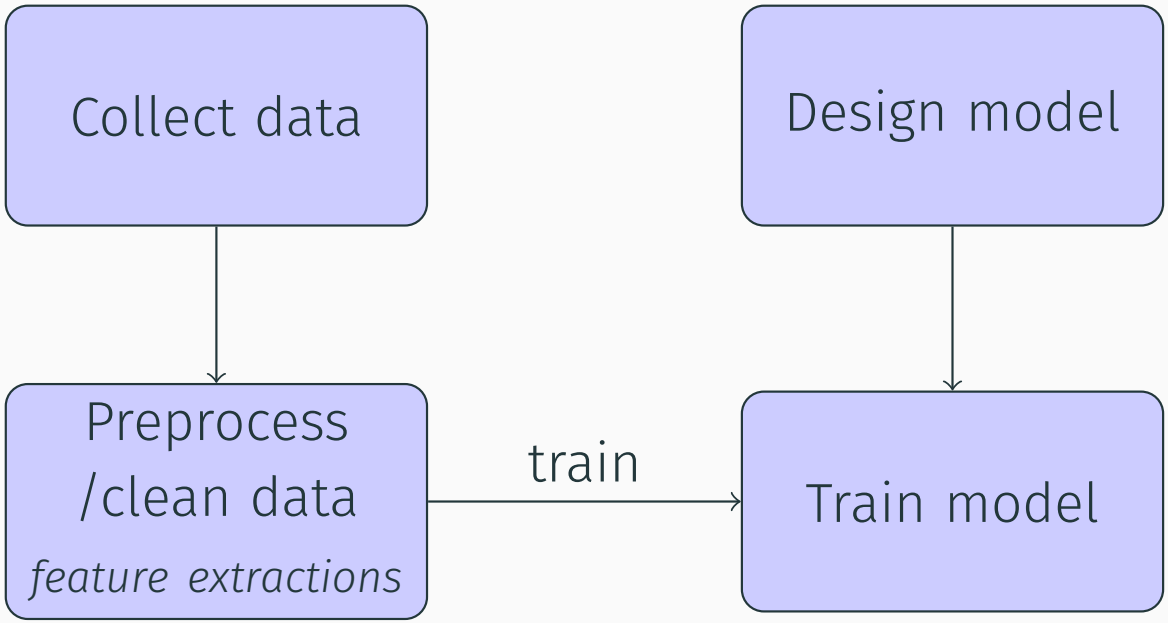
Steps



Steps


Collect data
Preprocess
/clean data
feature extractions
Design model

Steps


Collect data
Design model
Preprocess
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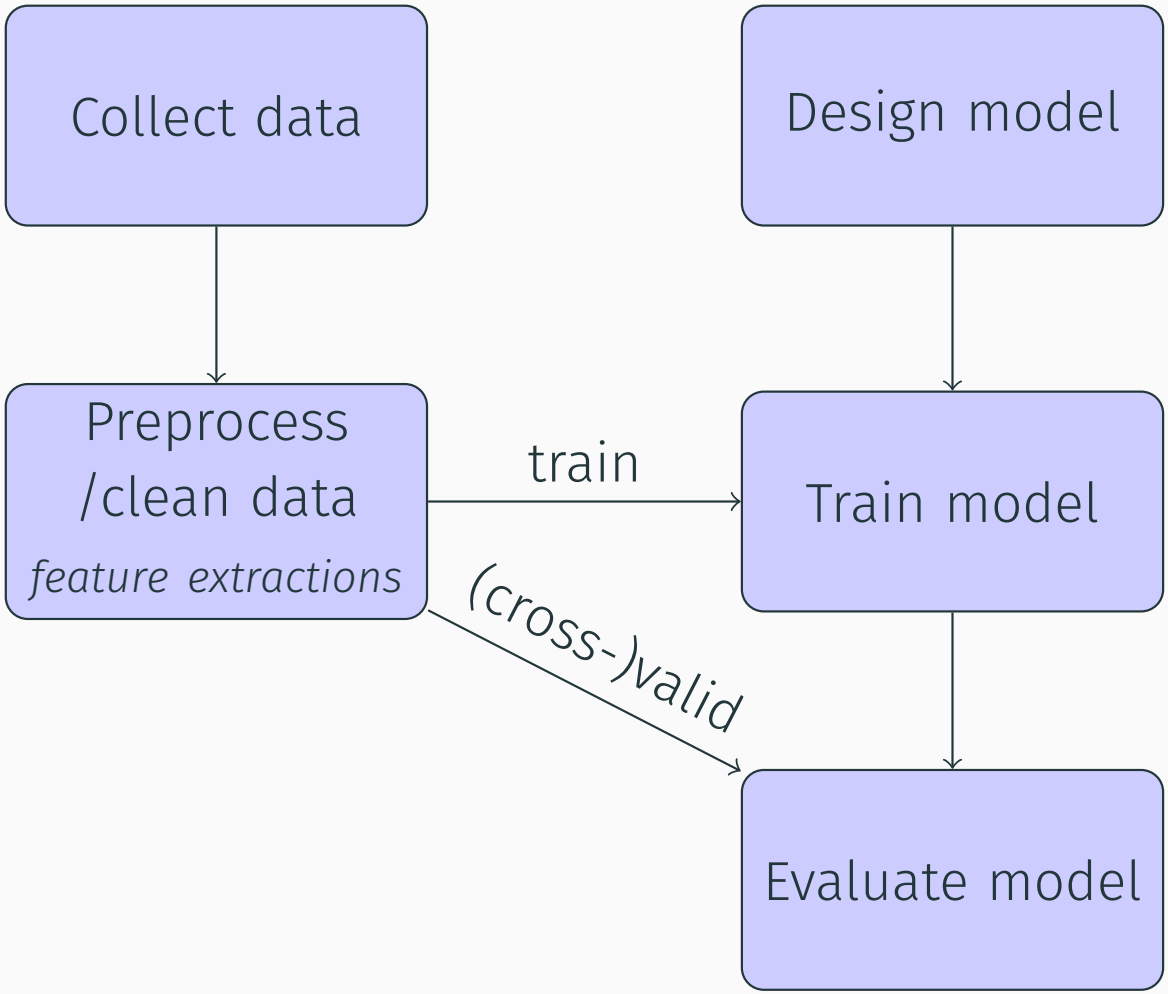
feature extractions
train
Train model

Steps


Collect data
Design model
Preprocess
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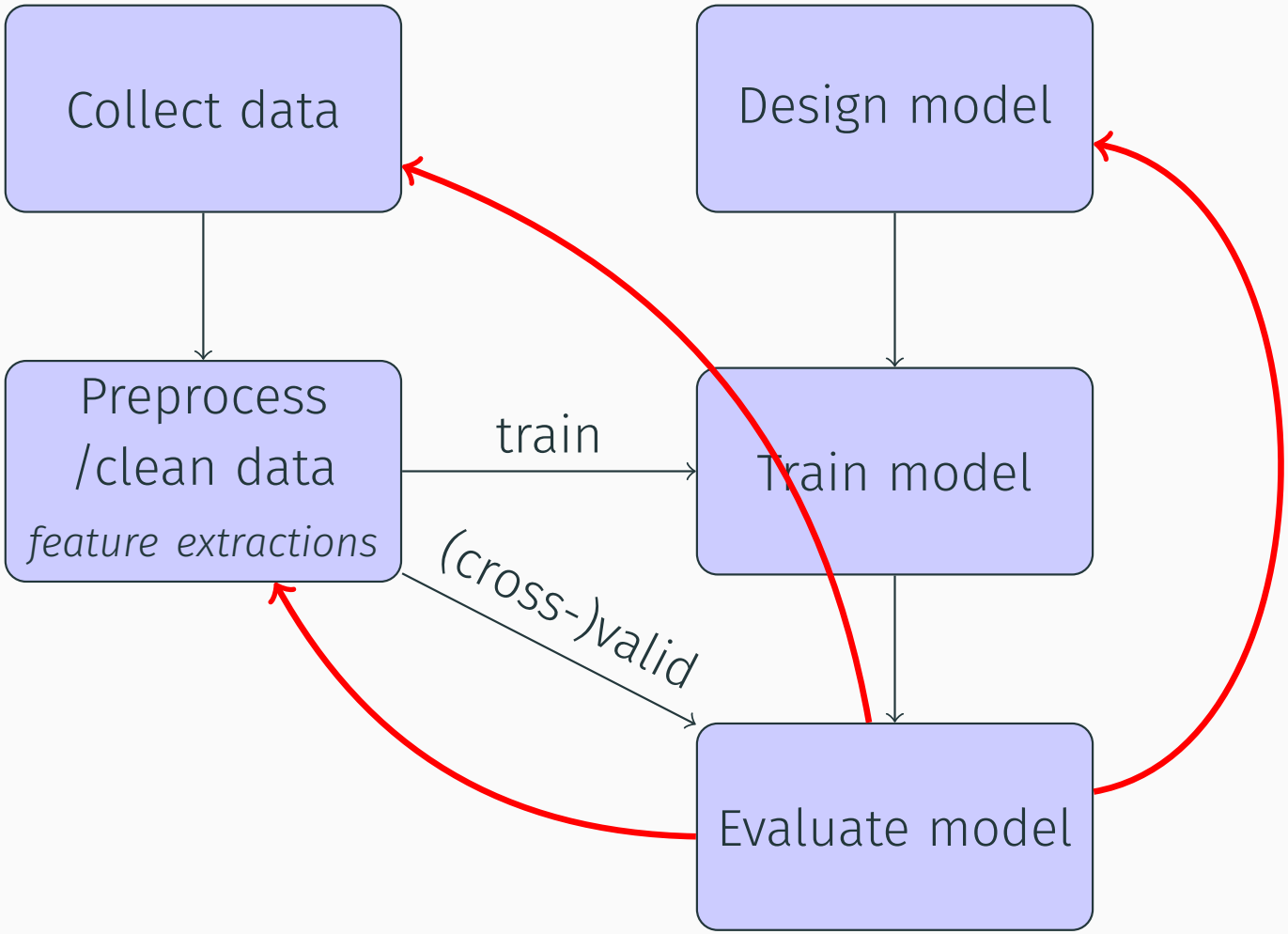
train
Train model
(cross-)valid
Evaluate model

Steps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Train model
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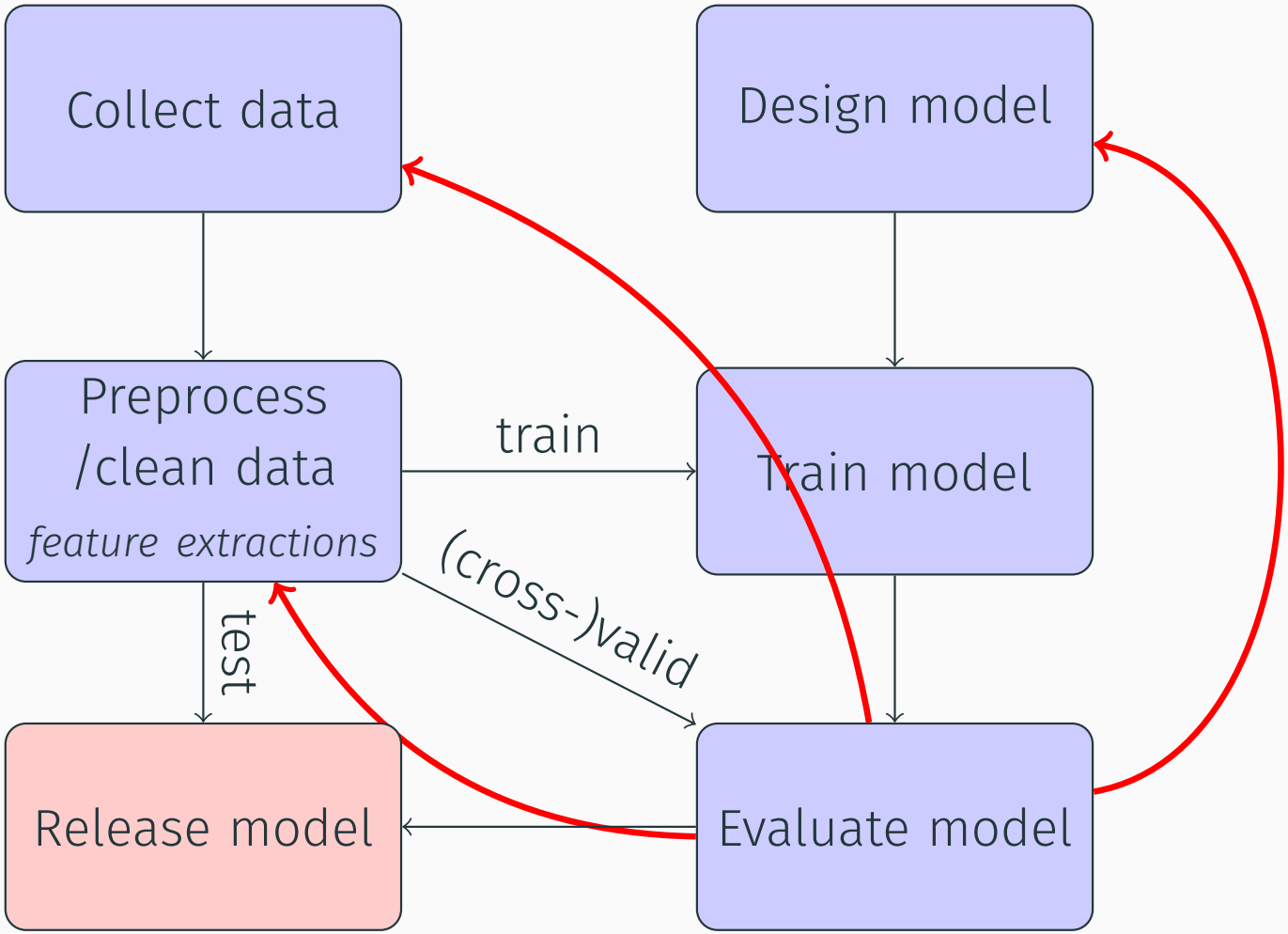
Evaluate model

Steps


Collect data
Design model
Preprocess
/clean data
feature extractions
train
Train model
(cross-)valid
test
Release model
Evaluate model

In summary

From one dataset, 3 sub-datasets have to be extracted:

- A training dataset
- A validation dataset

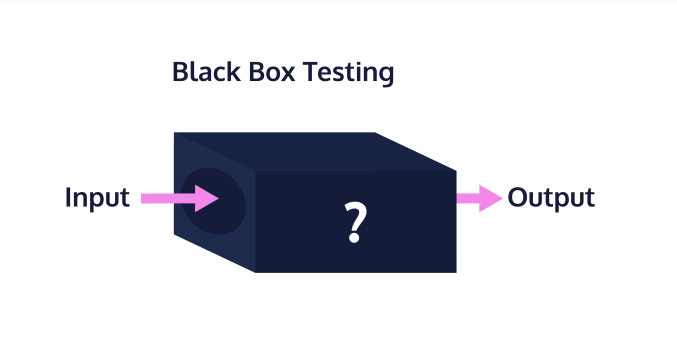
Can be done iteratively in a cross-validation procedure.
Some parameters of the model (e.g. polynomial order in a polynomial regression) were determined from the validation dataset.

- A test dataset (independent from the two other) to estimate the final performance of the model.

A black box?

The black box paradigm

The machine-learning based model is a black box. It gives some results but we don't understand how.



Do we need to understand the model?

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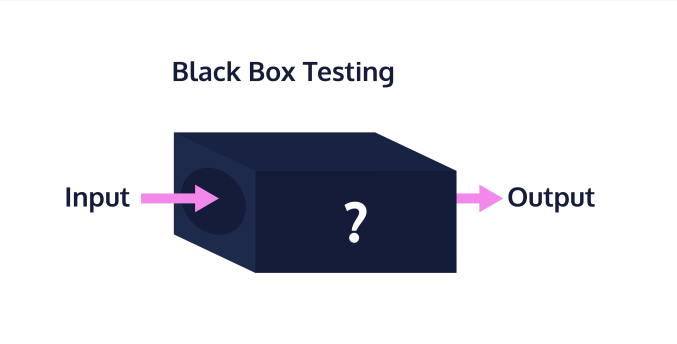


Do we need to understand the model?

"Every time I fire a linguist, the performance of our speech recognition system goes up."

F. Jelinek, 1988

Frederick Jelinek 1932-2010

Beyond the black box

Motivation

- Build models that can be trusted
- Use other source of knowledge (e.g. physical properties) when data are not sufficient.

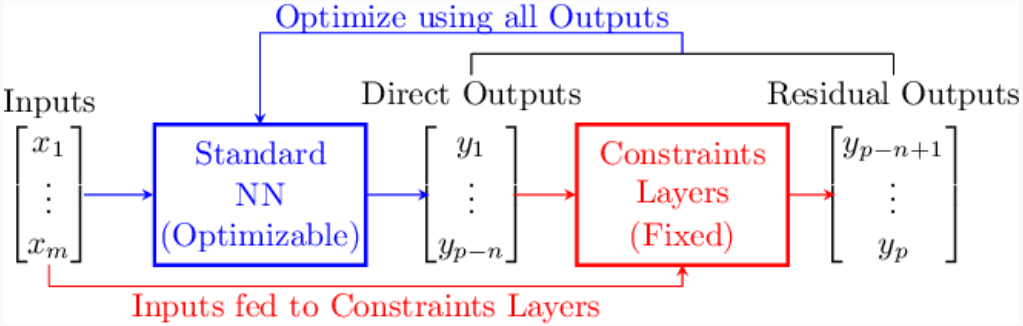
Two directions:

- Add physical constraints to ML models
- Explainable/Transparent ML.

Add physical constraints to ML models

- Simple example: enforce the positivity of some quantities (e.g. concentration)
- More complex: enforce conservation laws

  Beucler, T., Pritchard, M., Rasp, S., Ott, J., Baldi, P. and Gentine, P., 2021. Enforcing analytic constraints in neural networks emulating physical systems. *Physical Review Letters*, 126(9), p.098302.

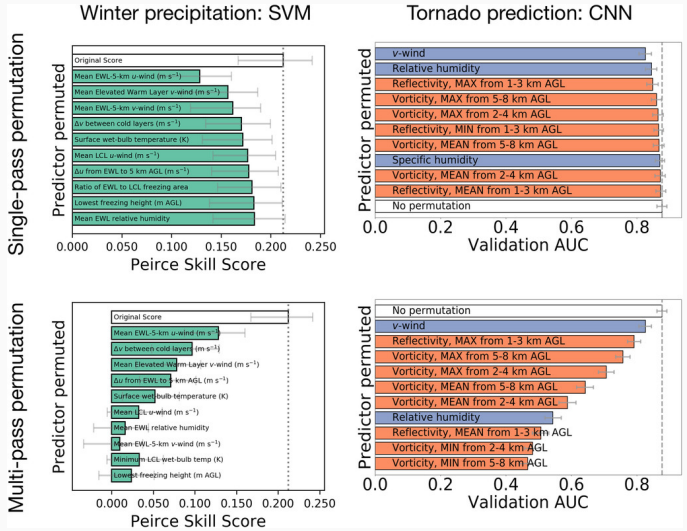


Beucler et al.

Explainable/Transparent ML.

The objective is to understand how the machine learning makes a prediction (e.g. which feature is important for the prediction).

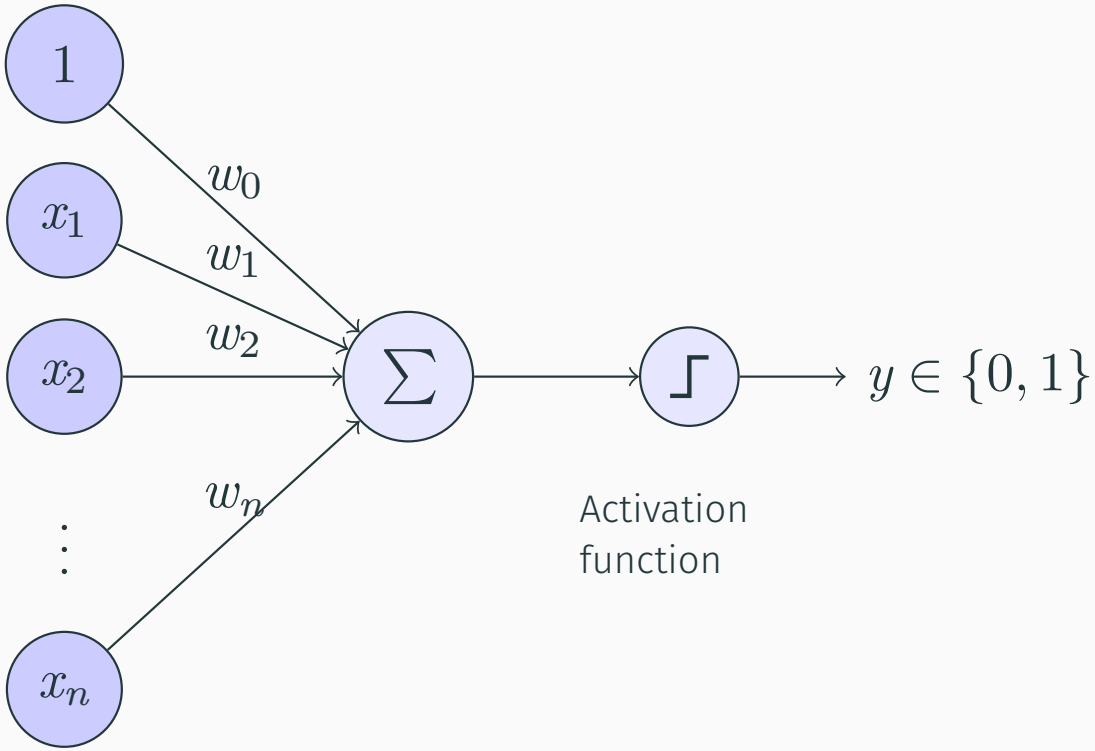
- McGovern, A., Lagerquist, R., Gagne, D.J., Jergensen, G.E., Elmore, K.L., Homeyer, C.R. and Smith, T., 2019. Making the black box more transparent: Understanding the physical implications of machine learning. *Bulletin of the American Meteorological Society, 100(11)*, pp.2175-2199.
- Sonnewald, M., Lguensat, R., Jones, D. C., Dueben, P. D., Brajard, J., and Balaji, V.,2021. Bridging observation, theory and numerical simulation of the ocean using Machine Learning. *Environ. Res. Lett.* 16 073008



McGovern et al.

Neural Networks

The perceptron : an artificial neuron



Rosenblatt, 1957

Computation

$$y = f(w_0 + w_1.x_1 + w_2.x_2 + \cdots + w_n.x_n) = f\left(w_0 + \sum_{i=1}^{n} w_i.x_i\right)$$

Some remarks

- Inputs x_i are the different features of the data

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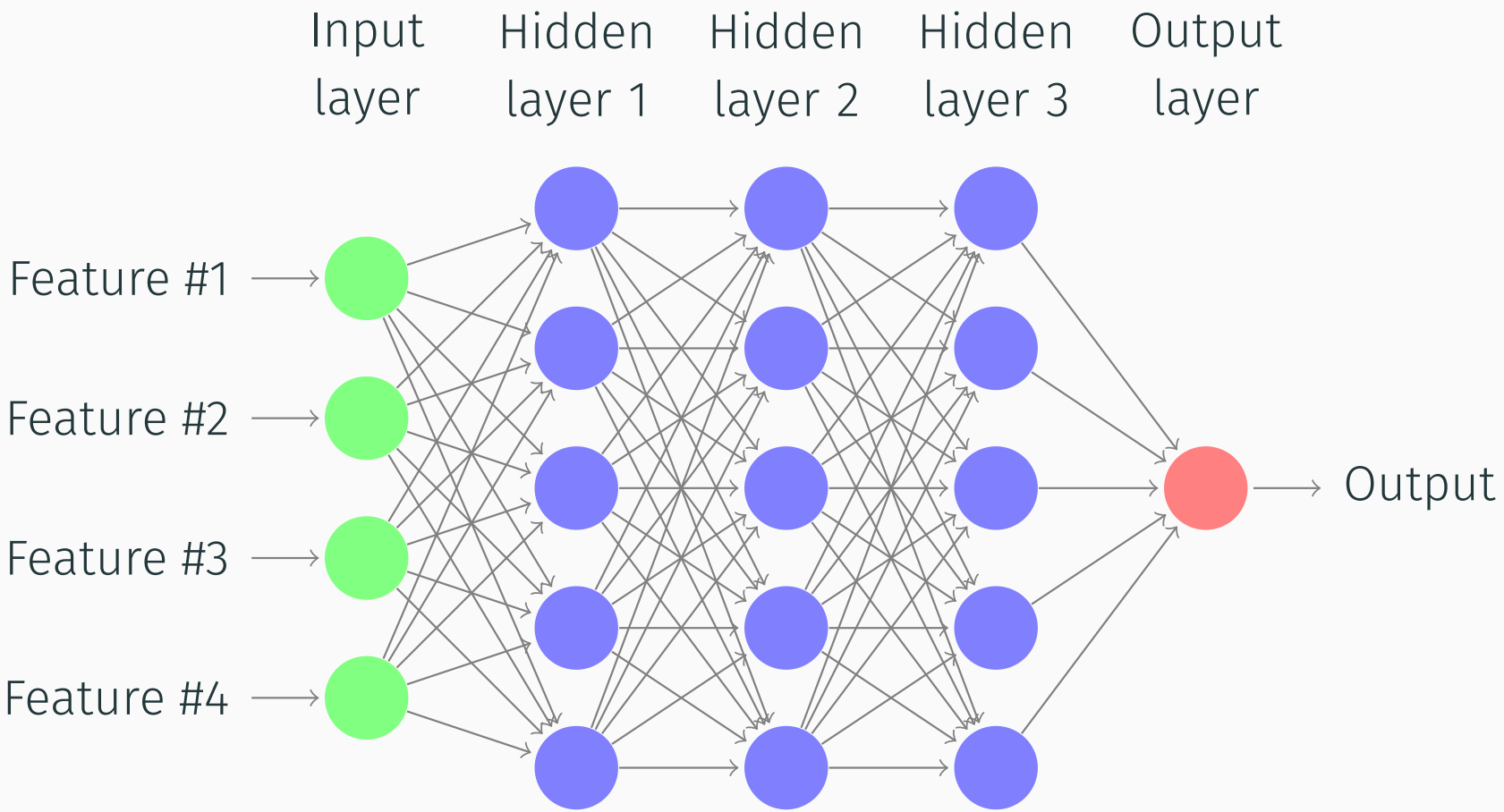
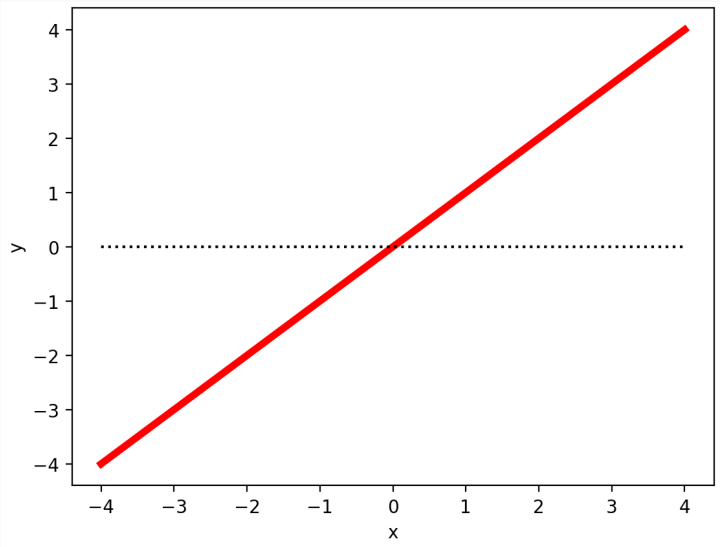
More complexe models are build by combining several perceptrons

Multi-layer perceptron (Densely connected layers)

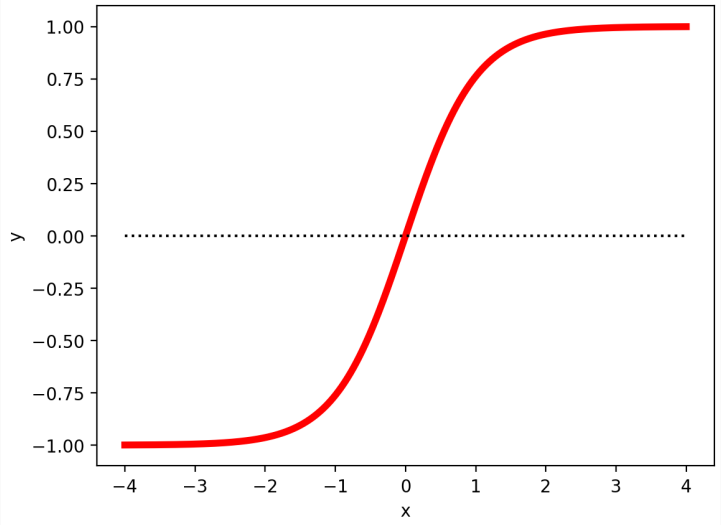


More usual activation functions

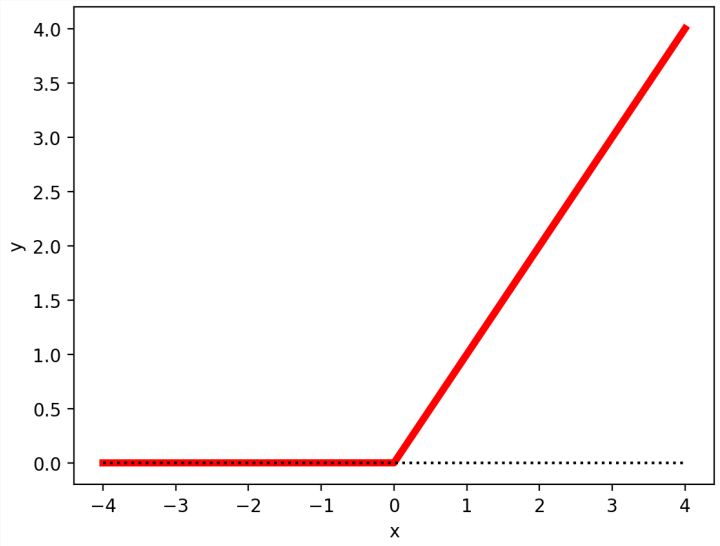
Linear



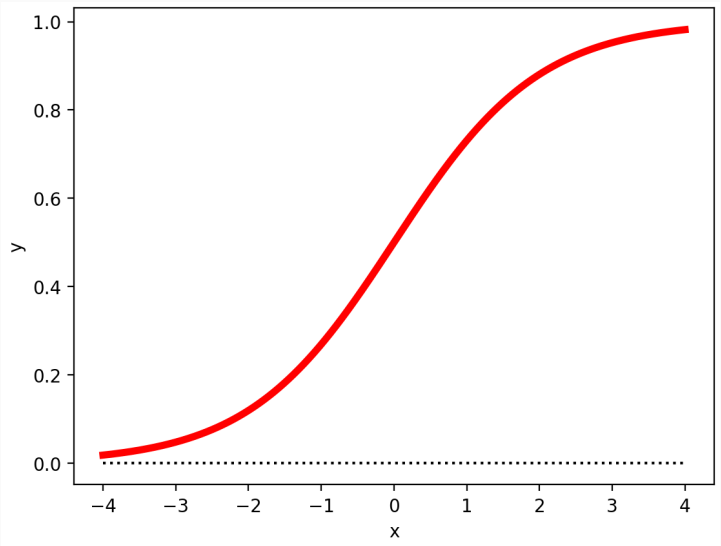
Hyperbolic tangent



ReLU

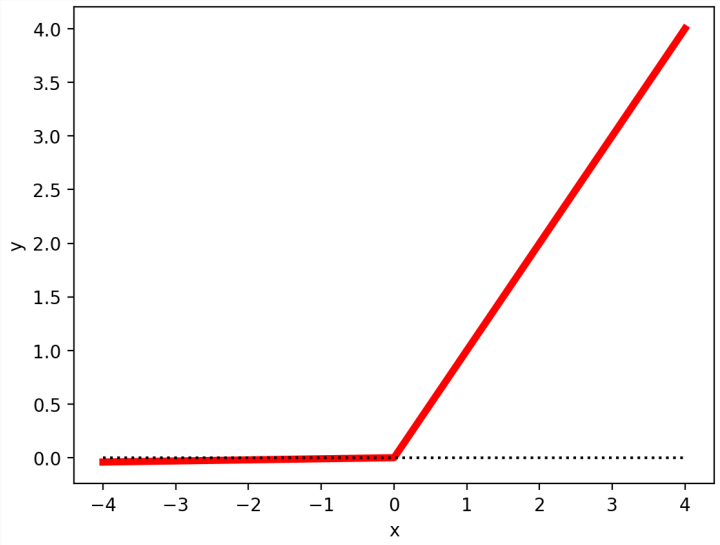


Sigmoid

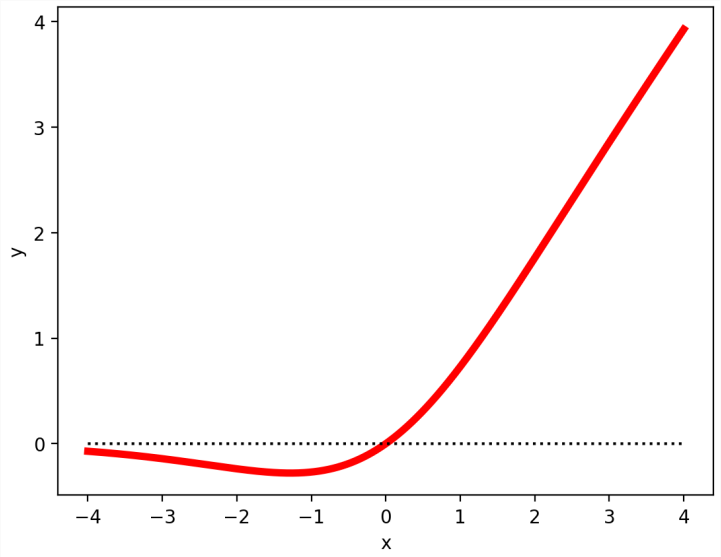


Other ”fancy” activation functions

Leaky-ReLU



Swish



Classification and regression loss

Regression

- Last layer:
  linear or hyperbolic tangent
- Loss function:

$$L(\hat{y}, y) = \sum_i (\hat{y}_i - y_i)^2$$

Classification and regression loss

Regression

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Classification

- Last layer:
  Soft-max

$$p_j = f_j(\mathbf{h}) = \frac{e^{h_j}}{\sum_k e^{h_k}}$$

- Loss function:
  Negative crossentropy

$$L(p, y) = -\sum_i \sum_j y_{i,j}. \log p_{i,j}$$

Probabilistic interpretation

Maximum likelihood estimator and loss

We can assume that the observation y follows a gaussian law:

$$p(y/x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp -\frac{(y-\mu(x))^2}{2\sigma^2},$$

where x is observed and $\mu(x)$ is a function of x.
Given a set of samples $(x_k, y_k)_{1:K}$, the negative log-likelihood is defined by

$$L = \sum_{k=1}^{K} \left(\frac{\log 2\pi\sigma^2}{2} + \frac{(y_k - \mu(x_k))^2}{2\sigma^2} \right)$$

Minimizing L is maximizing the probability of having the observations y_k given x_k.

Loss function of a neural net

First case: σ is constant

$\mu(x)$ is parametrized by a neural net $G_\mu(x, \boldsymbol{\theta}_\mu)$
The Maximum likelihood estimator is found by minizming:

$$L(\boldsymbol{\theta}_\mu) = \sum_{k=1}^{K} (y_k - G_\mu(x, \boldsymbol{\theta}_\mu))^2$$

which is exactly the regression loss already introduced.

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which is exactly the regression loss already introduced.

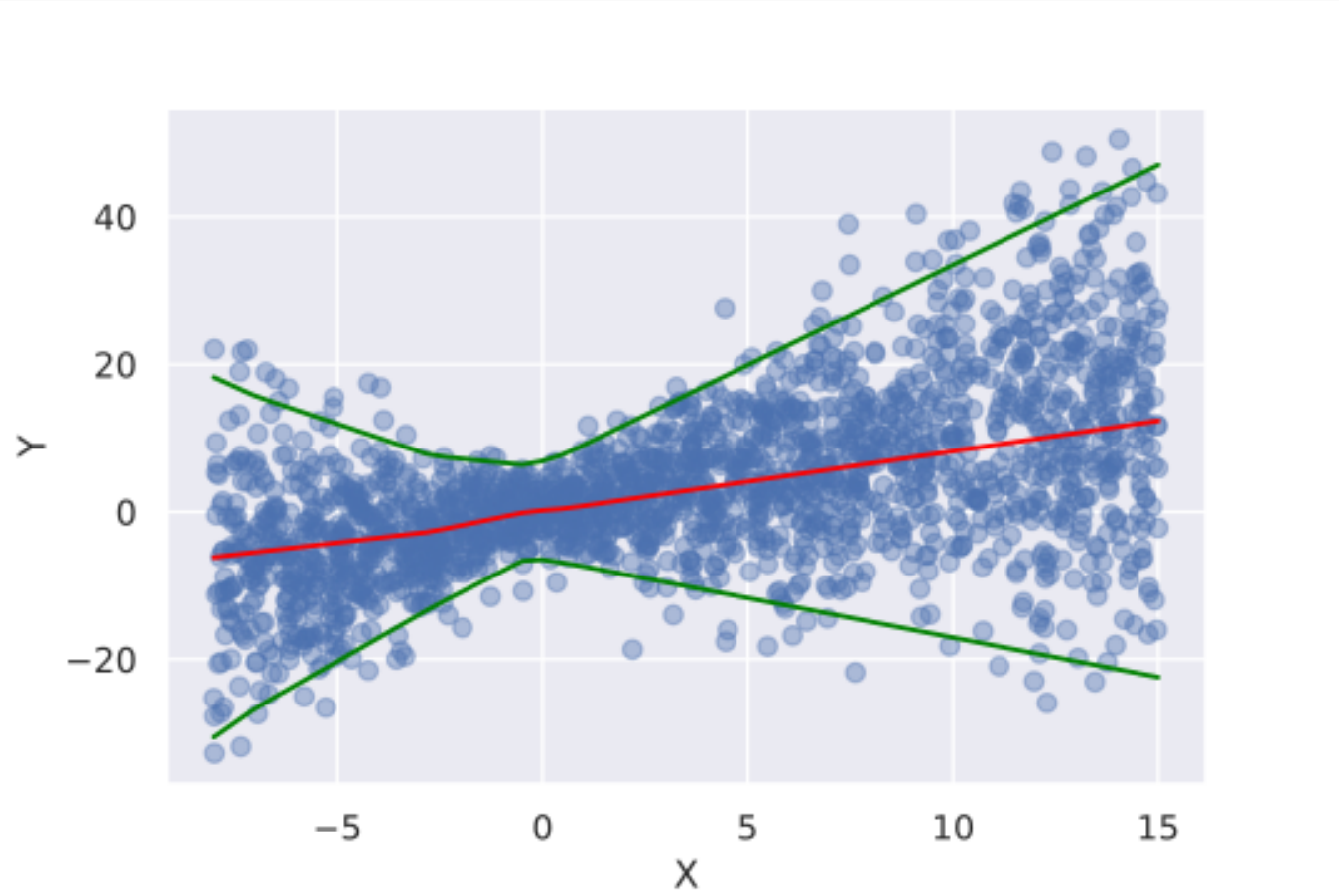
Second case: $\sigma(x)$ is a function of x.

In addition to $G_\mu(x, \boldsymbol{\theta}_\mu)$, $\sigma(x)$ is parametrized by $G_\sigma(x, \boldsymbol{\theta}_\sigma)$. The loss to minimize is then:

$$L(\boldsymbol{\theta}_\mu, \boldsymbol{\theta}_\sigma) = \sum_{k=1}^{K} \left(\frac{\log 2\pi\, G_\sigma(x_k, \boldsymbol{\theta}_\sigma)^2}{2} + \frac{(y_k - G_\mu(x_k, \boldsymbol{\theta}_\mu))^2}{2 G_\sigma(x_k, \boldsymbol{\theta}_\sigma)^2} \right)$$

The neural net $G = (G_\mu, G_\sigma)$ gives also the uncertainty of its estimation in the form of the standard deviation.

Illustration

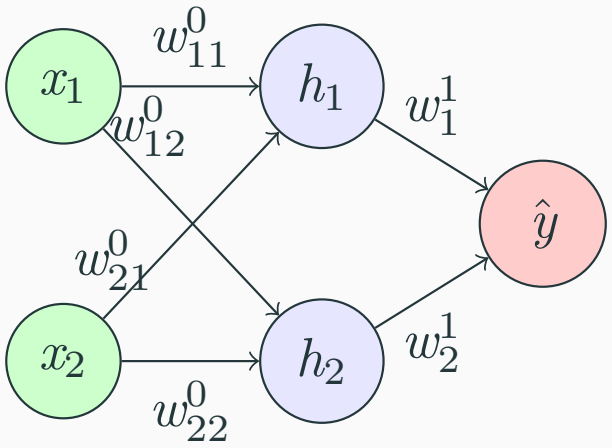


In red the estimation of the mean $G_\mu(x, \boldsymbol{\theta}_\mu)$

In green the confidence interval $G_\mu(x, \boldsymbol{\theta}_\mu) \pm \sigma(x_k, \boldsymbol{\theta}_\sigma)$

Gradient backpropagation

Training a neural-net: gradient backpropagation

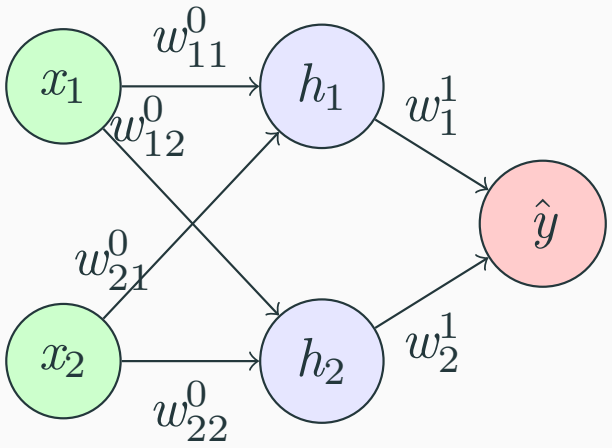


1. Given a couple (x, y)

Objective

Determination of the best set of weights $\mathbf{w}$ to minimize the Loss function $L(\mathbf{w}) = ||\hat{y}(\mathbf{w}) - y||^2$.
Calculation of $\partial L / \partial w$

Training a neural-net: gradient backpropagation



1. Given a couple (x, y)
2. Forward computation:
   $h_j = f_0(\sum_{i=1}^{2} w^0_{ij}.x_i)$
   $\hat{y} = f_1(\sum_{j=1}^{2} w^1_j.h_j)$

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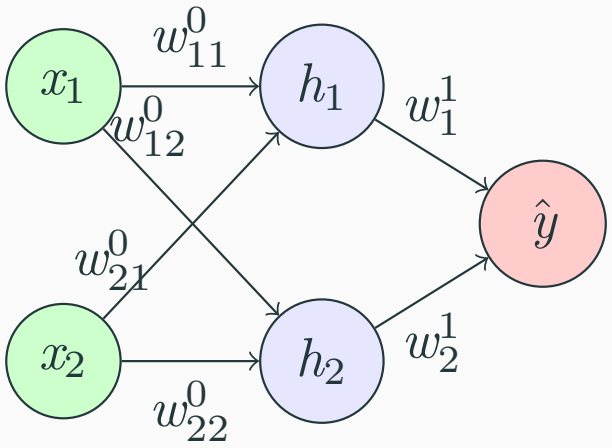

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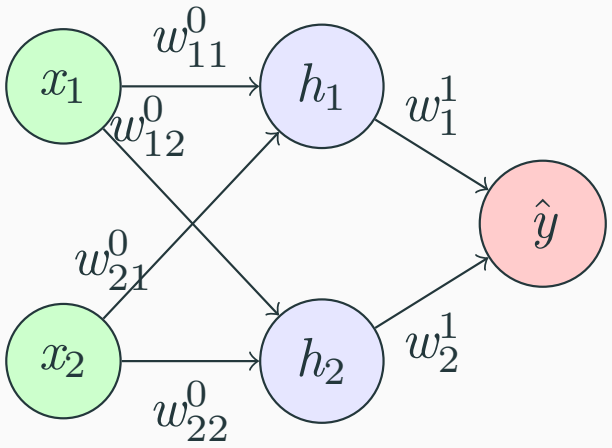

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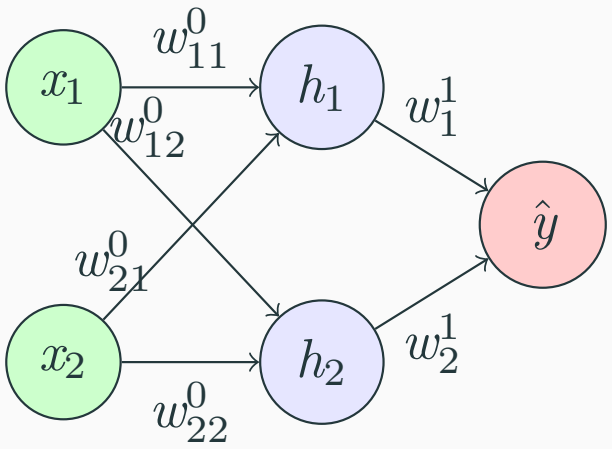
4. Gradient Backpropagation:

Training a neural-net: gradient backpropagation

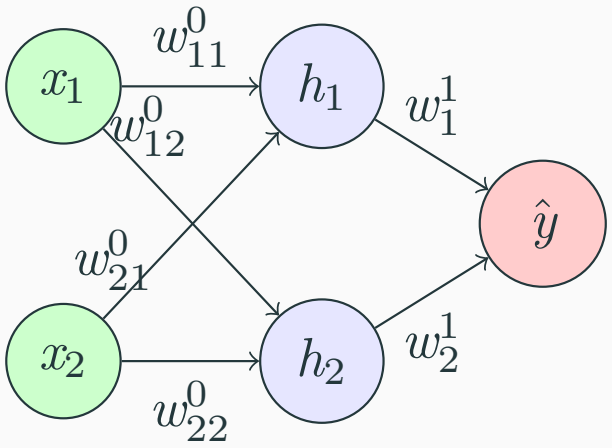


Objective

Determination of the best set of weights $\mathbf{w}$ to minimize the Loss function $L(\mathbf{w}) = ||\hat{y}(\mathbf{w}) - y||^2$.
Calculation of $\partial L/\partial w$

1. Given a couple (x, y)
2. Forward computation:
   $h_j = f_0(\sum_{i=1}^{2} w^0_{ij}.x_i)$
   $\hat{y} = f_1(\sum_{j=1}^{2} w^1_j.h_j)$
3. Compute the gradient of the loss:
   $\boxed{\partial L/\partial \hat{y}}$
4. Gradient Backpropagation:
   - Layer 1
     $\partial L/\partial w^1_j = \boxed{\partial L/\partial \hat{y}}.\partial f_1/\partial w^1_j$
     $\boxed{\partial L/\partial h_j} = \boxed{\partial L/\partial \hat{y}}.\partial f_1/\partial h_j$

Training a neural-net: gradient backpropagation



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     $\boxed{\partial L/\partial h_j} = \boxed{\partial L/\partial \hat{y}}.\partial f_1/\partial h_j$
   - Layer 0
     $\partial L/\partial w^0_{ij} = \boxed{\partial L/\partial h_j}.\partial f_1/\partial w^0_{ij}$

Optimizing a machine learning (gradient method)

Optimizing the loss

Several loss function (depending on the problem) can be defined.

For example, Mean Square Error:

Method

Find a minimum of L by adjusting the parameters (weights) $\mathbf{w}$ given the gradient of the loss with respect to the weights $\nabla_{\mathbf{w}} L$.

Batch Vs Stochastic training

Dataset: (X, Y) with N samples denoted $(\mathbf{x_i}, y_i)$

Batch gradient:

Require: Learning rate(s): ν_k
Require: Initial weights: $\mathbf{w}$
$k \leftarrow 1$
while stopping criterion not met **do**
Compute gradient:
$\mathbf{g} \leftarrow \frac{1}{N} \sum_i^N \nabla_{\mathbf{w}} L(f(\mathbf{x}_i, y_i))$
Update weights: $\mathbf{w} \leftarrow \mathbf{w} - \nu_k \mathbf{g}$
$k \leftarrow k + 1$
end while

1 Update / N forwards

Batch Vs Stochastic training

Dataset: (X, Y) with N samples denoted $(\mathbf{x_i}, y_i)$

Batch gradient:

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    $k \leftarrow k + 1$
  end while

1 Update / N forwards

Stochastic gradient:

Require: Learning rate(s): ν_k
Require: Initial weights: $\mathbf{w}$
  $k \leftarrow 1$
  while stopping criterion not met **do**
    Sample an example $(\mathbf{x}, y)$ from (X, Y)
    Compute gradient: $\mathbf{g} \leftarrow \nabla_{\mathbf{w}} L(f(\mathbf{x}, y))$
    Update weights: $\mathbf{w} \leftarrow \mathbf{w} - \nu_k \mathbf{g}$
    $k \leftarrow k + 1$
  end while

1 Update / 1 forward

Mini-Batch training

Dataset: (X, y) with N samples

Mini-Batch gradient:

Require: Learning rate(s): ν_k
Require: Initial weights: $\mathbf{w}$
  $k \leftarrow 1$
  while stopping criterion not met **do**
    Sample m examples $(\mathbf{x}_i, y_i)$ from (X, y)
    Compute gradient: $\mathbf{g} \leftarrow \frac{1}{m} \sum_i^m \nabla_{\mathbf{w}} L(f(\mathbf{x}_i, y_i)$
    Update weights: $\mathbf{w} \leftarrow \mathbf{w} - \nu_k \mathbf{g}$
    $k \leftarrow k + 1$
  end while

Mini-Batch training

Dataset: (X, y) with N samples

Mini-Batch gradient:

Require: Learning rate(s): ν_k
Require: Initial weights: $\mathbf{w}$
  $k \leftarrow 1$
  while stopping criterion not met **do**
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    Compute gradient: $\mathbf{g} \leftarrow \frac{1}{m} \sum_i^m \nabla_{\mathbf{w}} L(f(\mathbf{x}_i, y_i)$
    Update weights: $\mathbf{w} \leftarrow \mathbf{w} - \nu_k \mathbf{g}$
    $k \leftarrow k + 1$
  end while

1 Update / m forward

$m = 1$: Pure stochastic gradient.

$m = N$: Batch gradient

Convolutional Neural Networks

Convolutional neural net

X: an image

x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}
x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}
x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}
x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}
x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}

$$\begin{matrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{matrix}$$

$\mathbf{w}$

h: first feature

h_{11}	h_{12}	h_{13}	h_{14}
h_{21}	h_{22}	h_{23}	h_{24}
h_{31}	h_{32}	h_{33}	h_{34}
h_{41}	h_{42}	h_{43}	h_{44}

Perform a standard convolution

$$h_{i,j} = \sum_{k=1}^{3} \sum_{l=1}^{3} x_{i+k-1,j+l-1} \cdot w_{k,l}$$

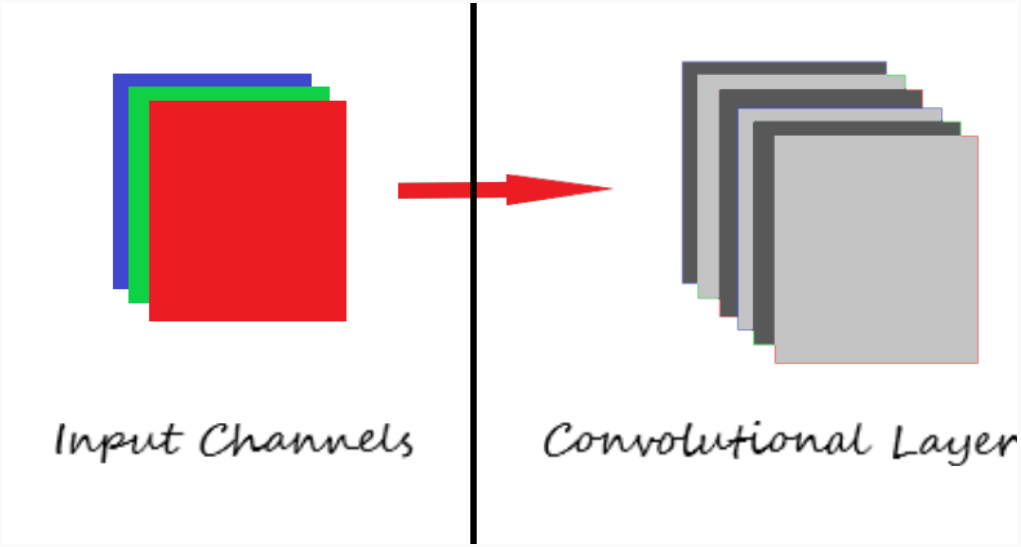
Main parameters of a convolutional layer

- Size of the filter K

Main parameters of a convolutional layer

- Size of the filter K
- Number of filters p

  A convolutional layer is composed of p convolutions (size of layer) extracting p features from the data.

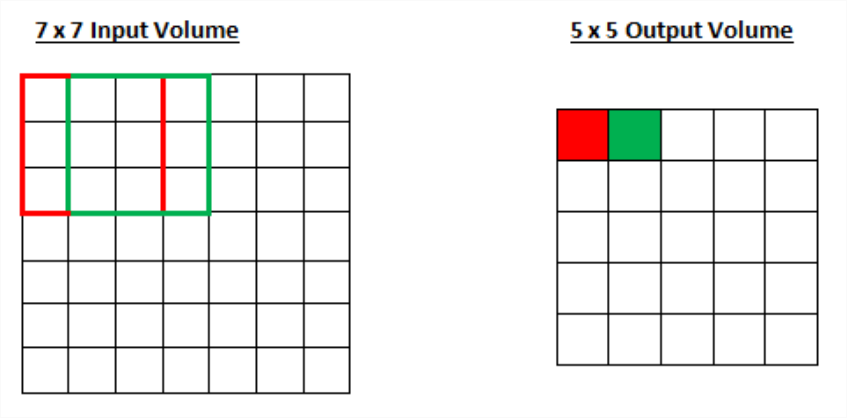


$O = \frac{W-K+2P}{S} + 1$, where O is the output size and W the input size.

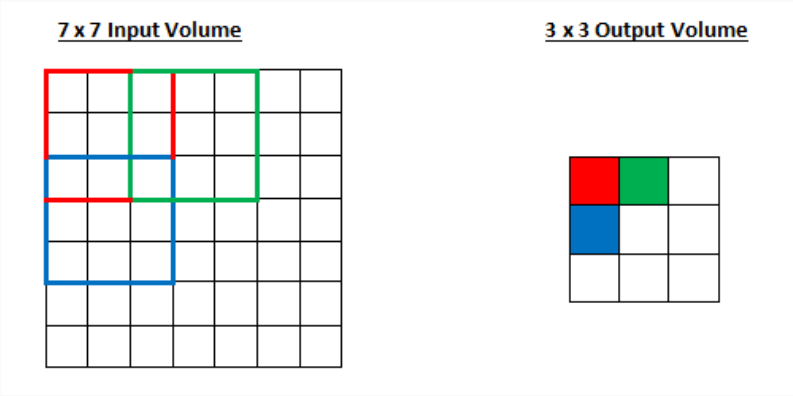
Main parameters of a convolutional layer

- Size of the filter K
- Number of filters p
- Strides S

$S = 1$



$S = 2$

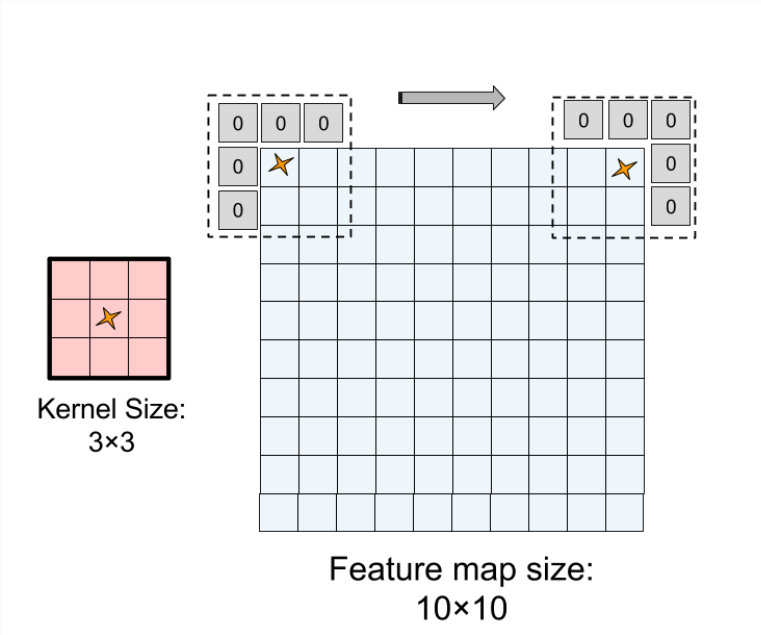


$O = \frac{W-K+2P}{S} + 1$, where O is the output size and W the input size.

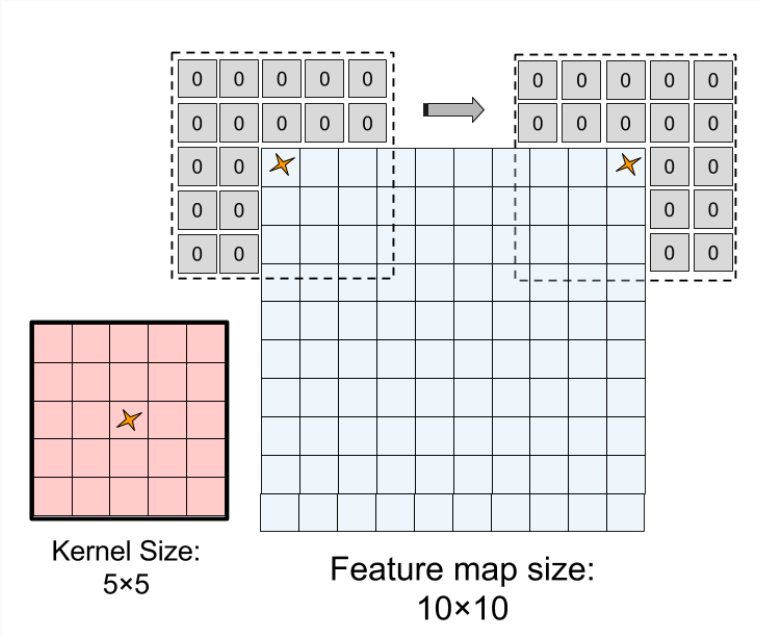
Main parameters of a convolutional layer

- Size of the filter K
- Number of filters p
- Strides S
- Padding P

$P = 1$



$P = 2$



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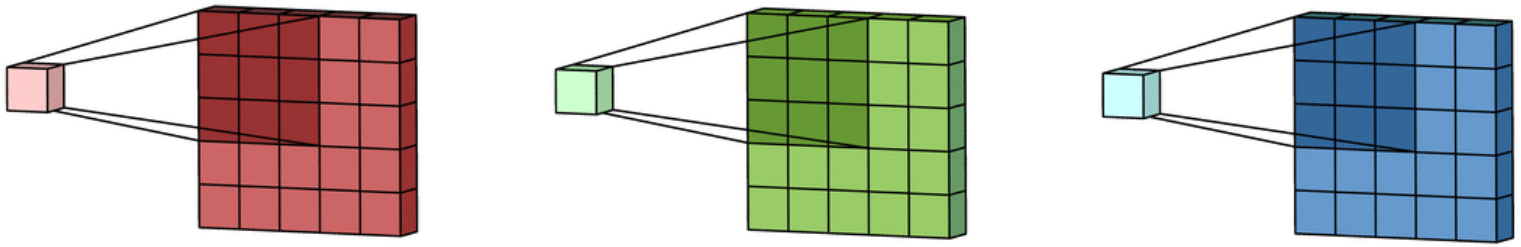
Summary of Convolutional layer steps

1. Convolution



Summary of Convolutional layer steps

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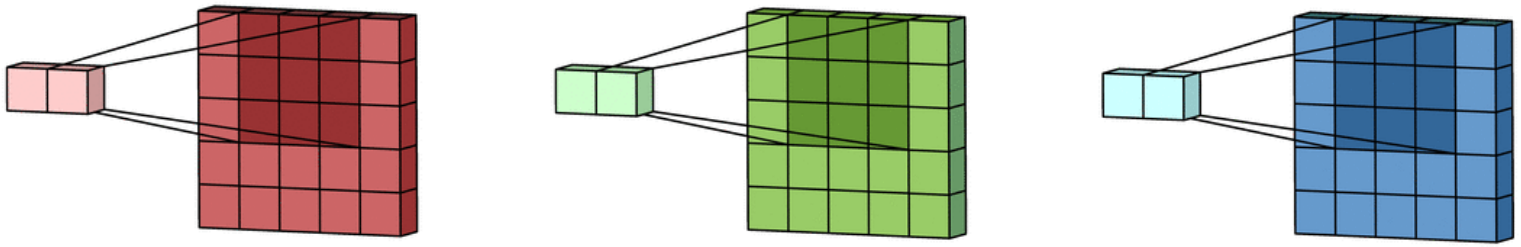

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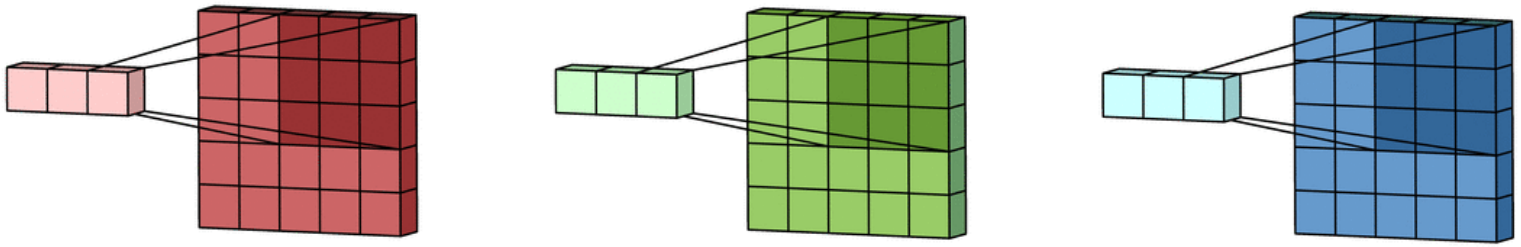

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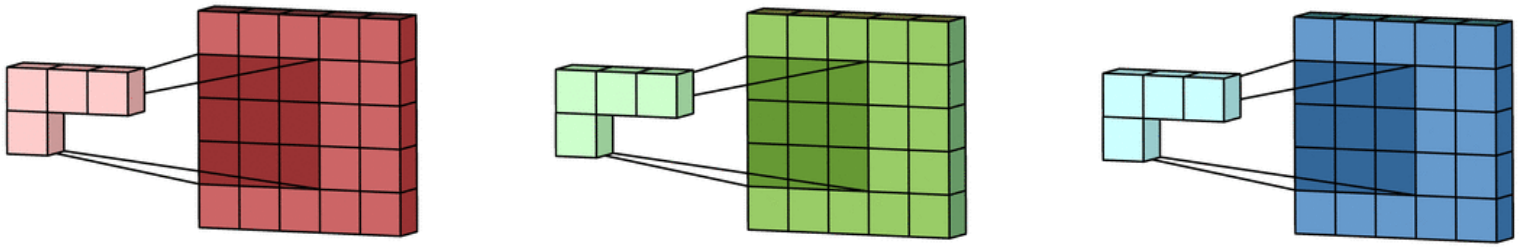

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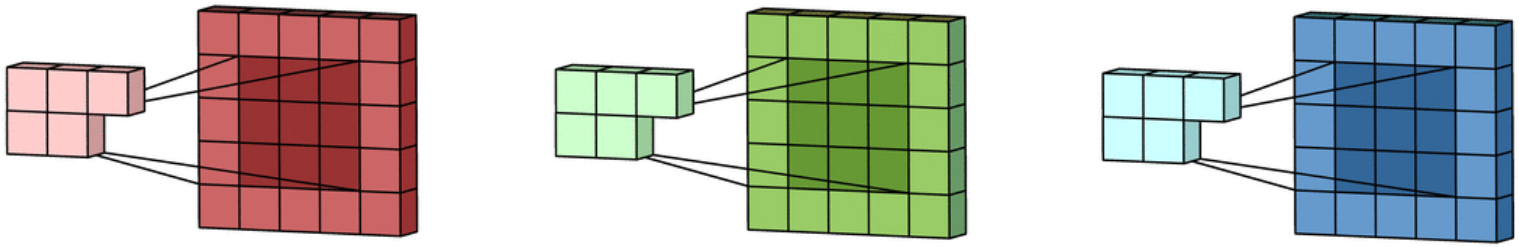

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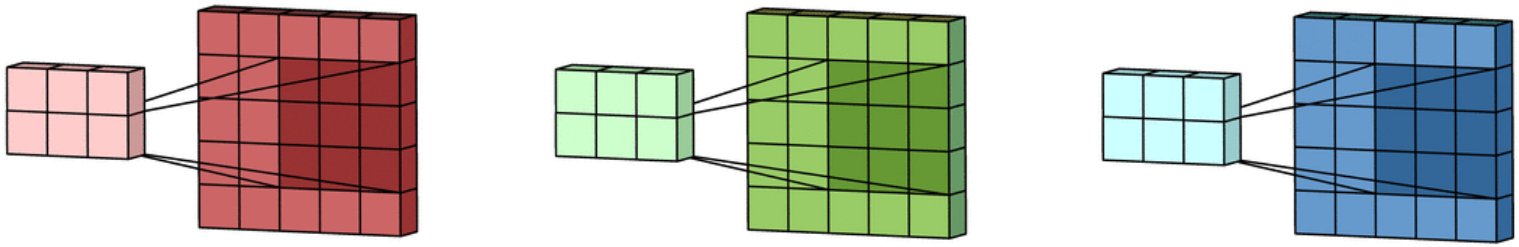

Summary of Convolutional layer steps

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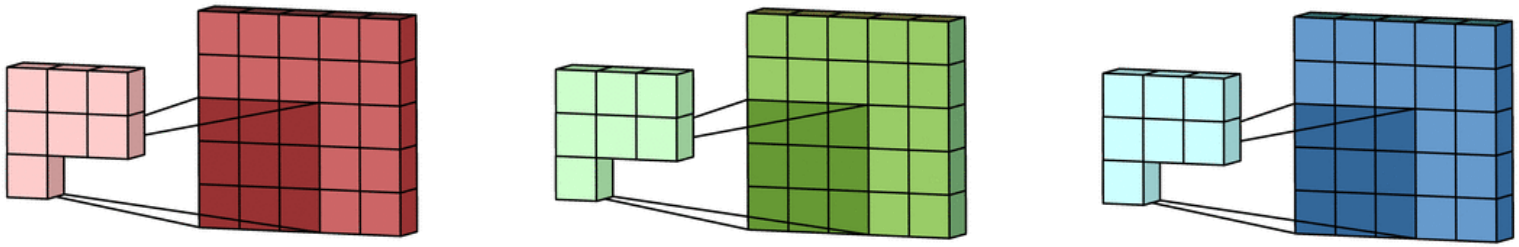

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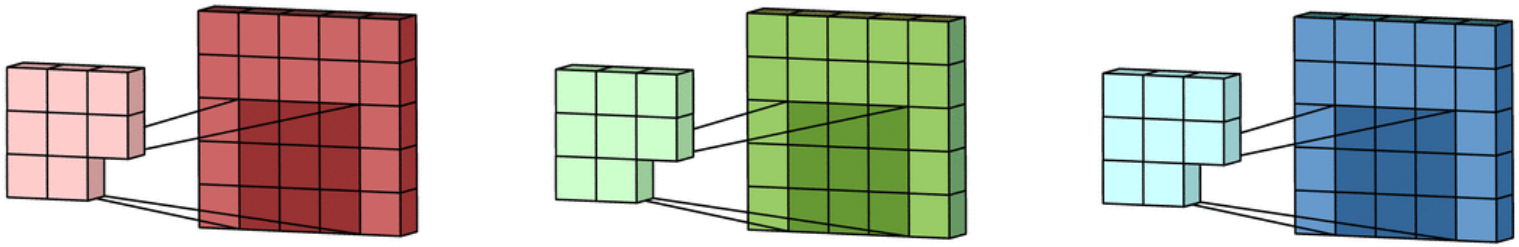

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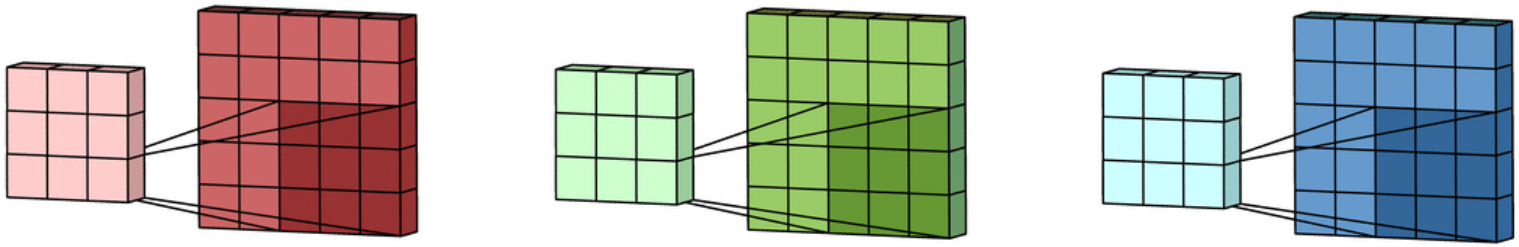

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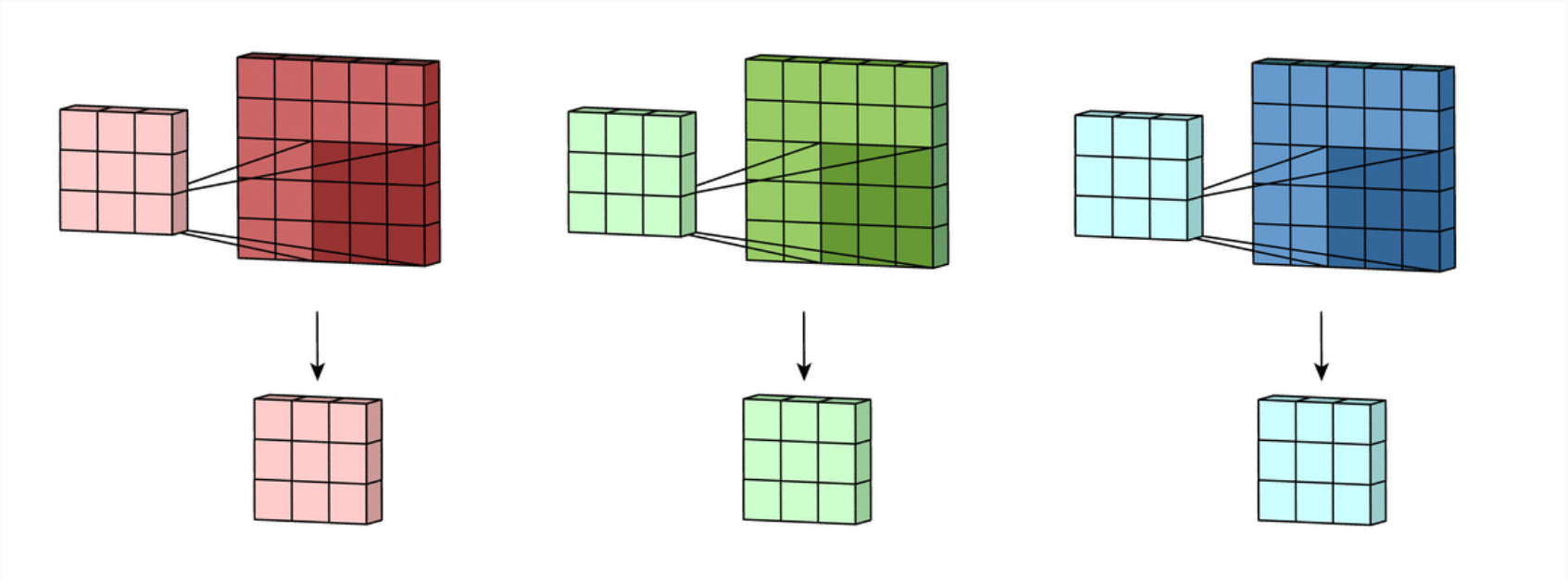

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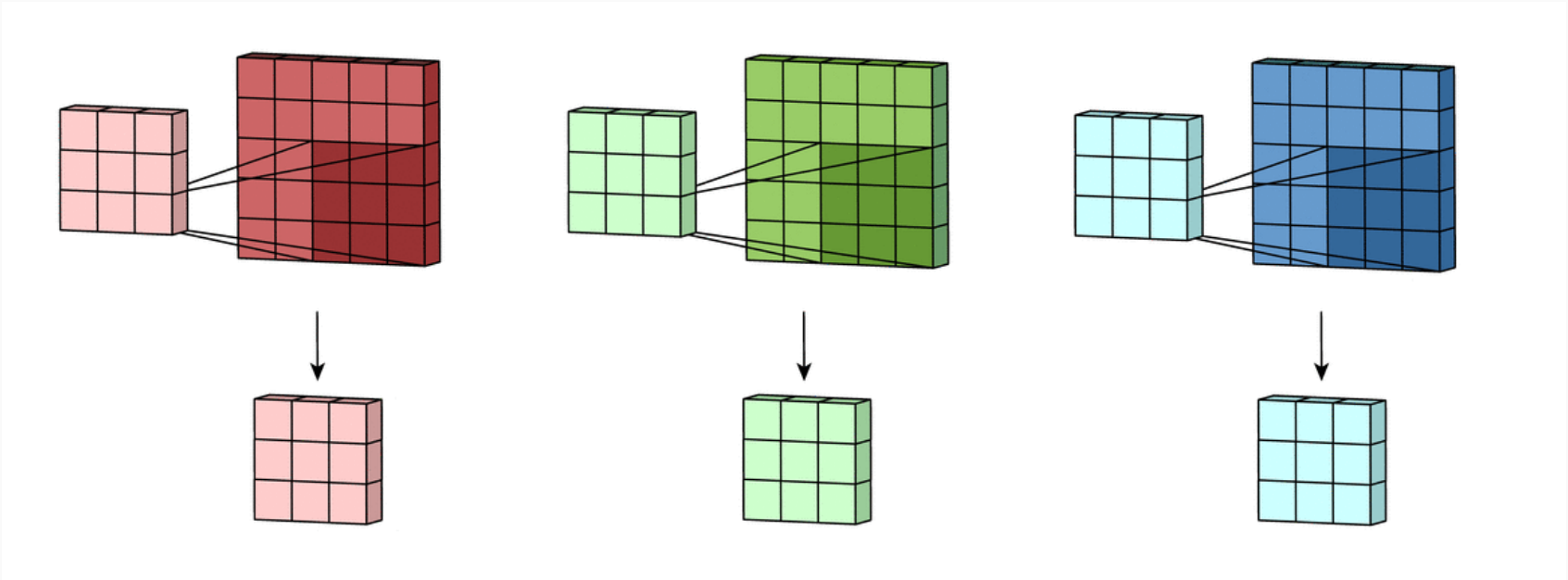

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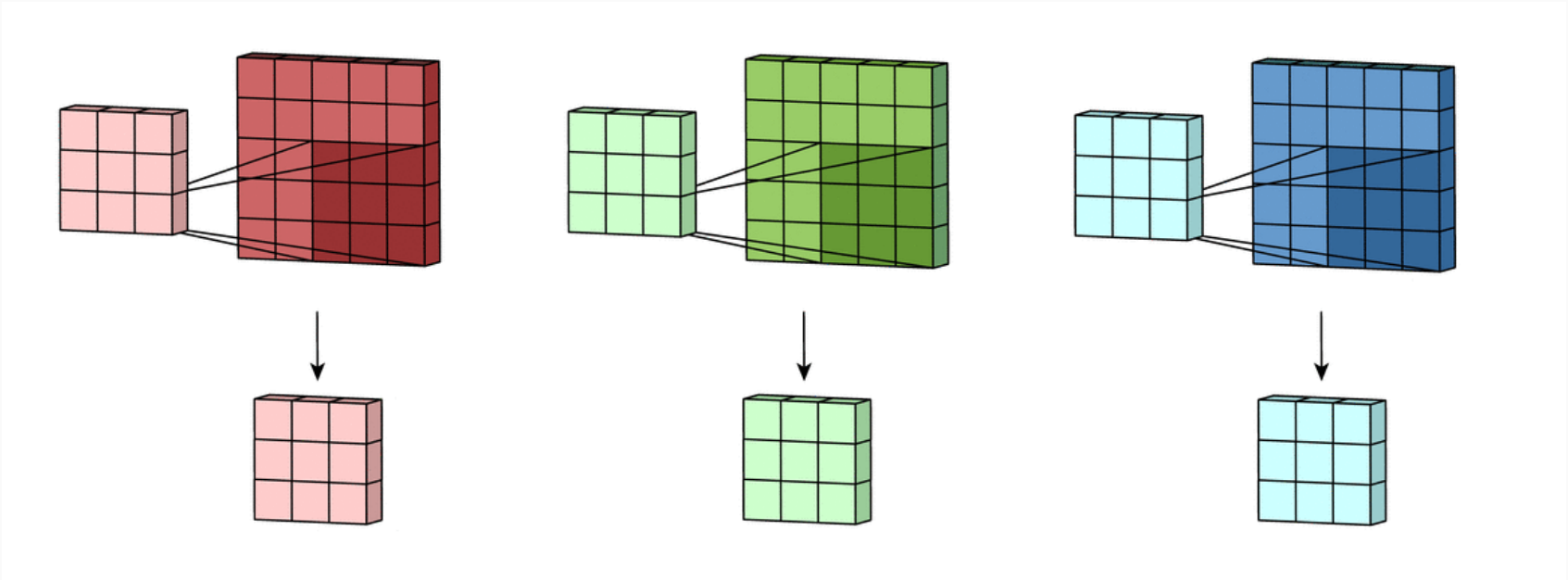
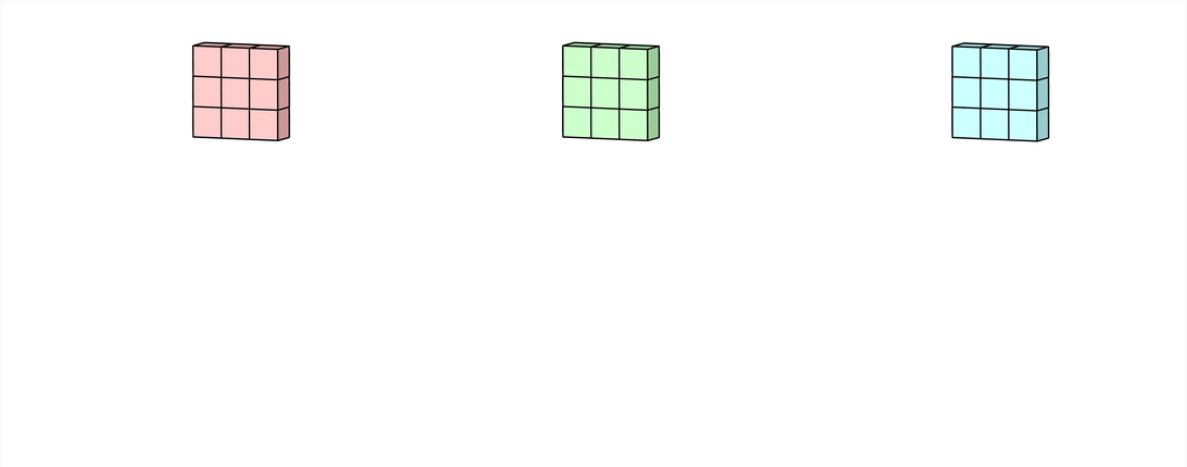


2. Addition



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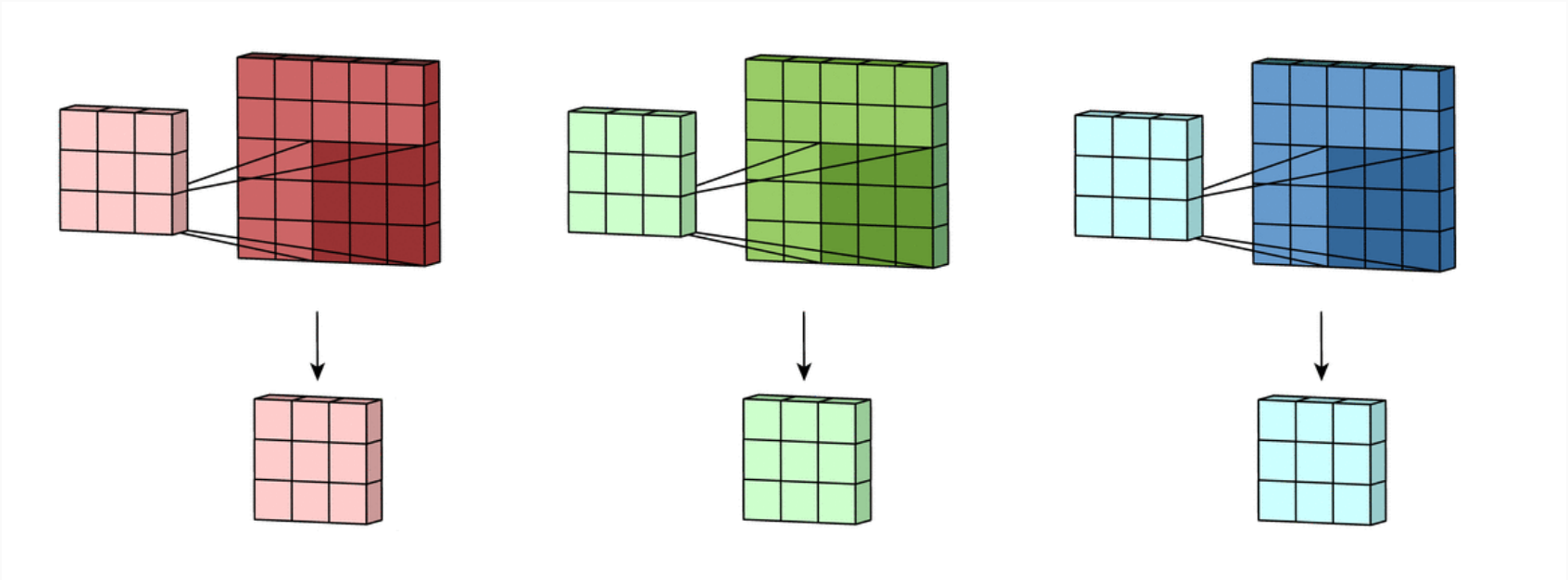

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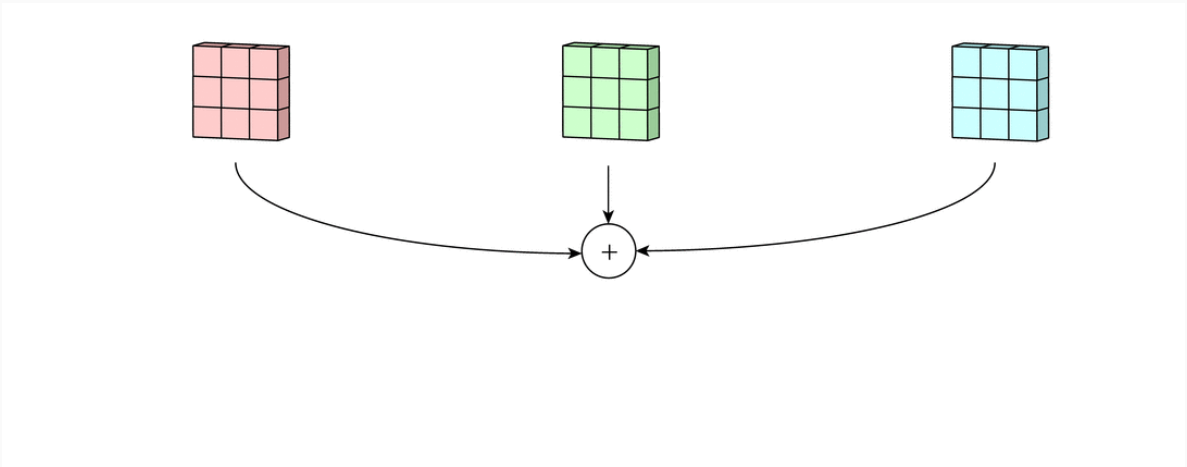

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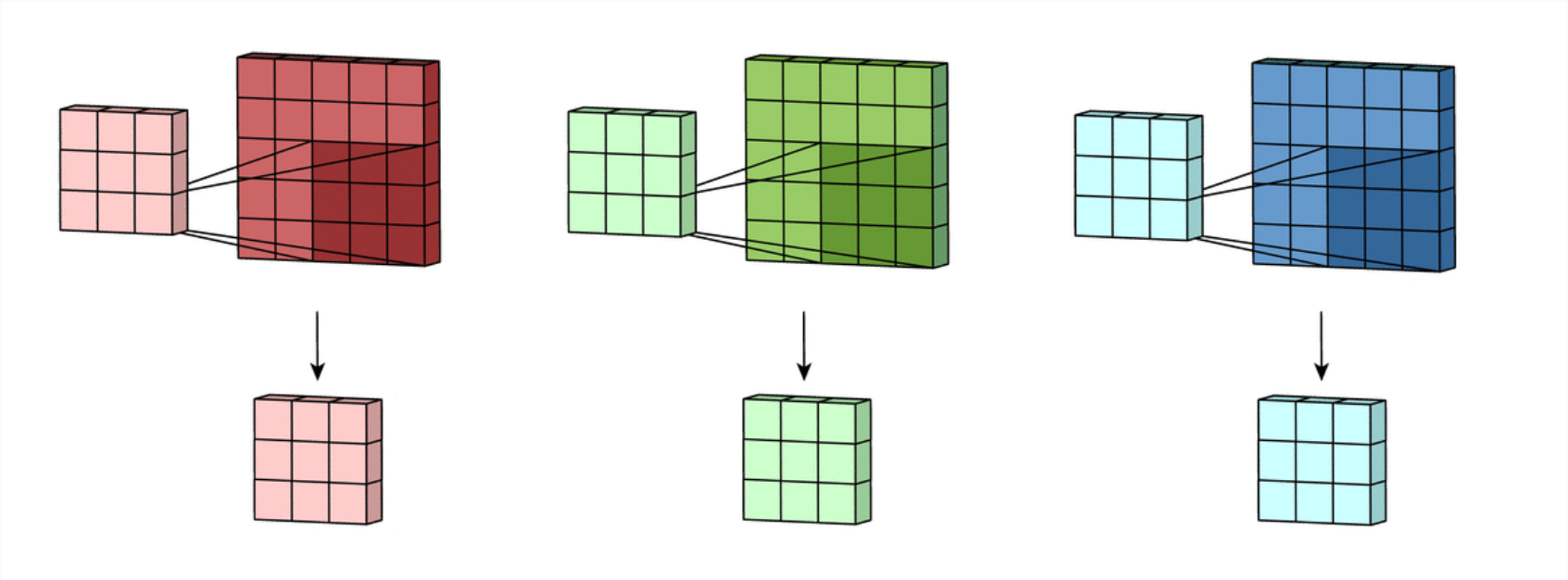

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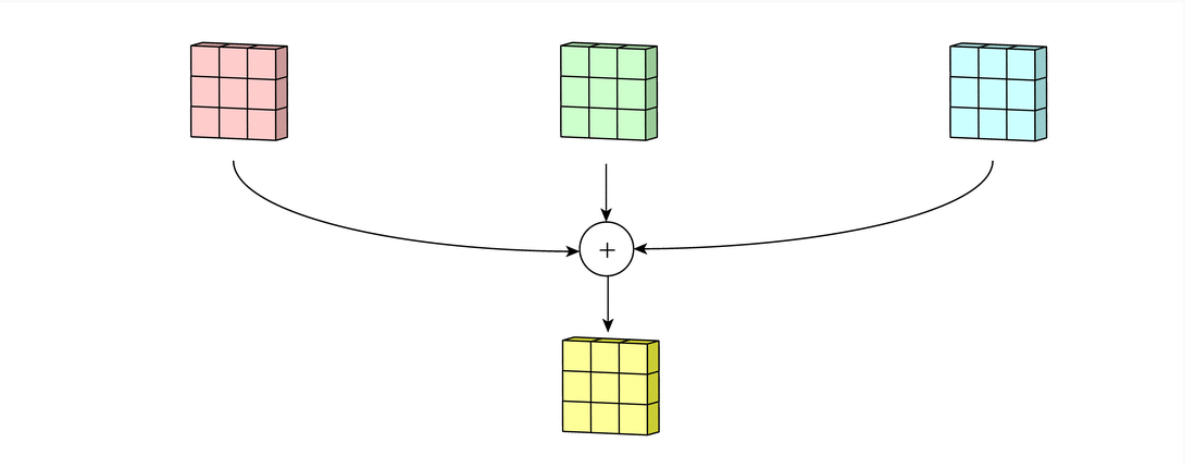

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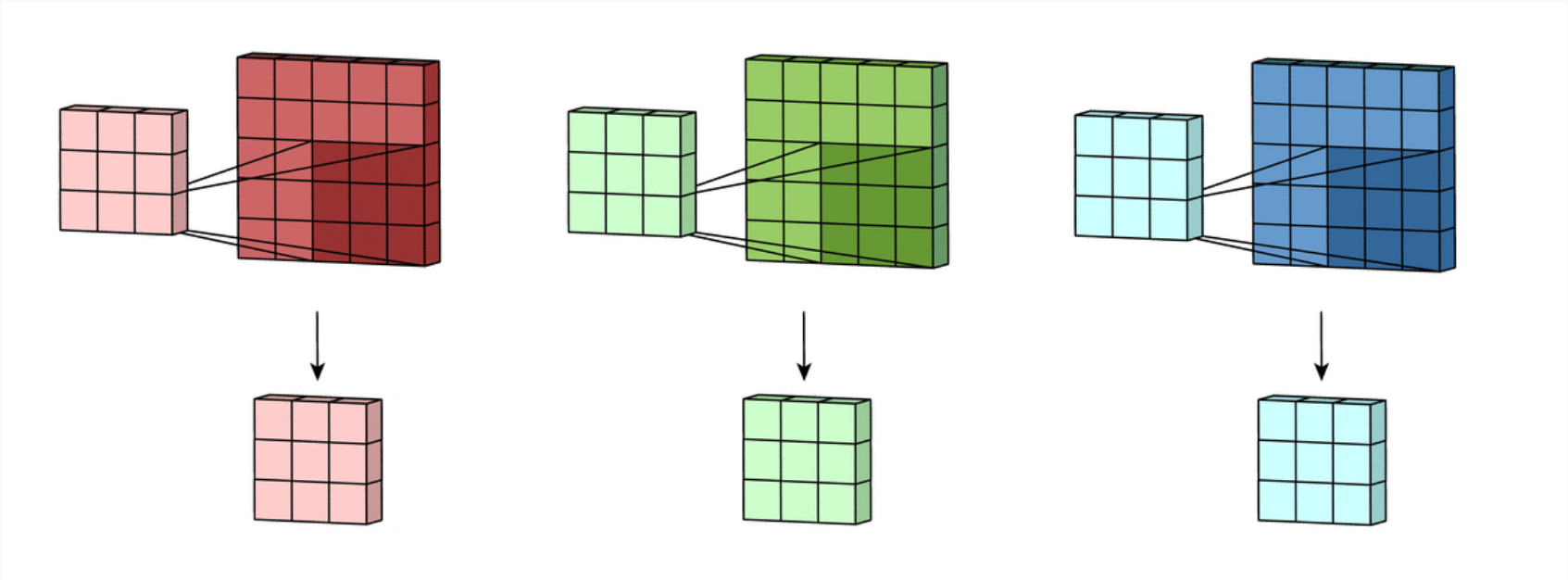
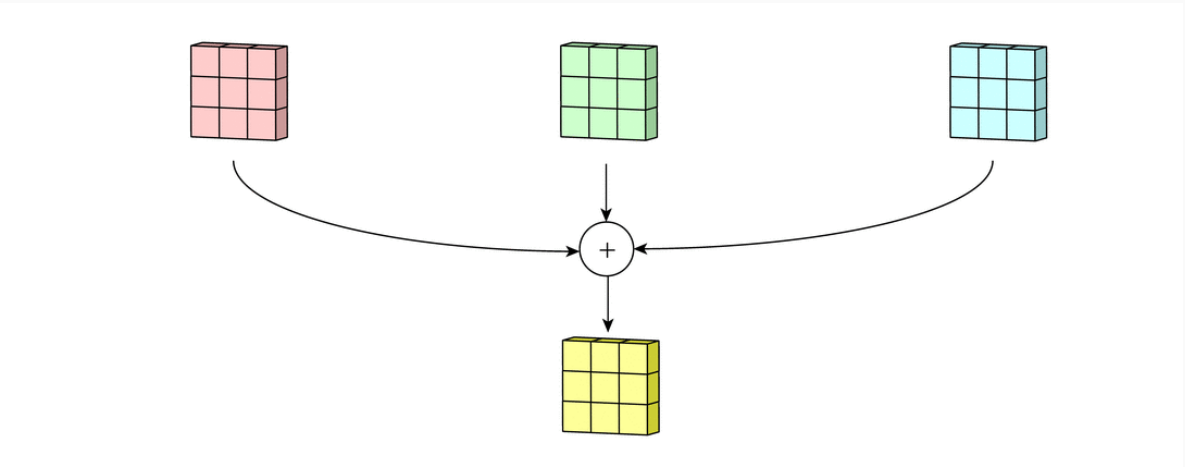


2. Addition



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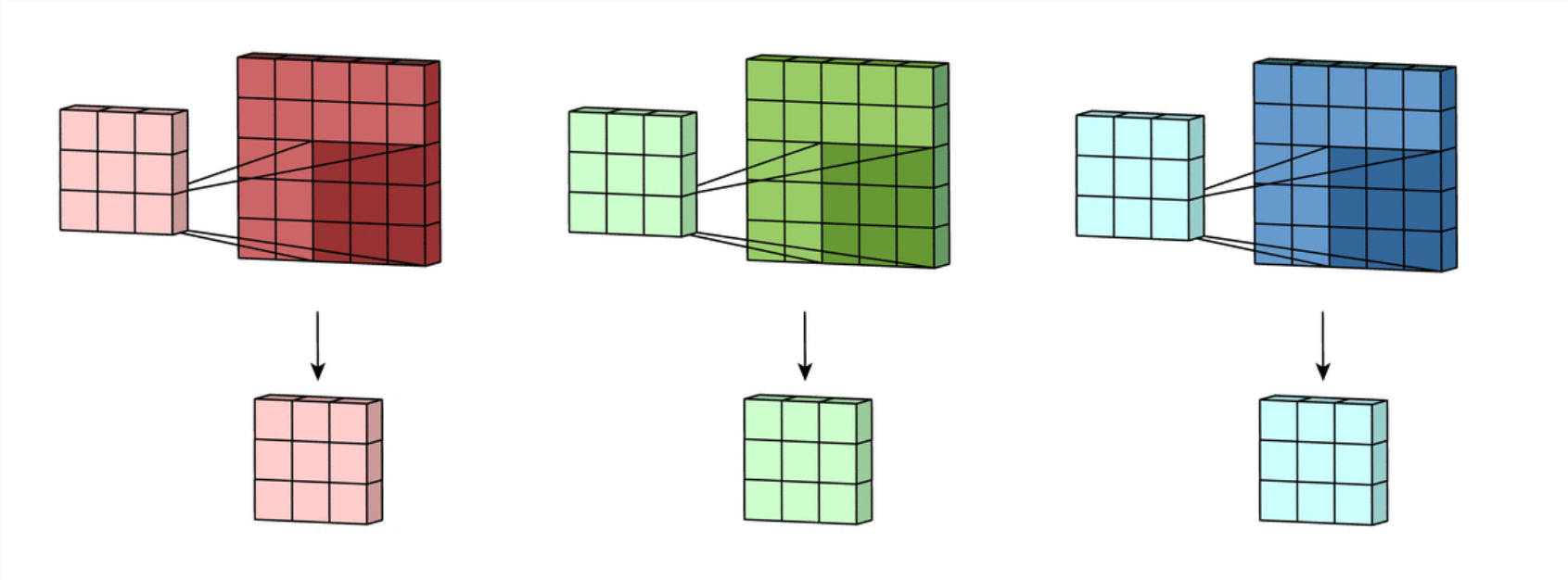
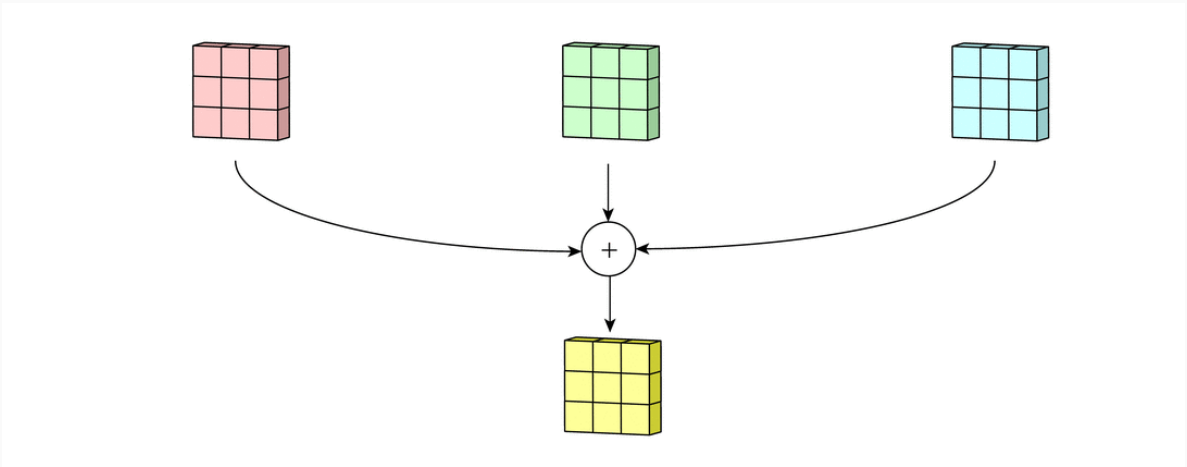
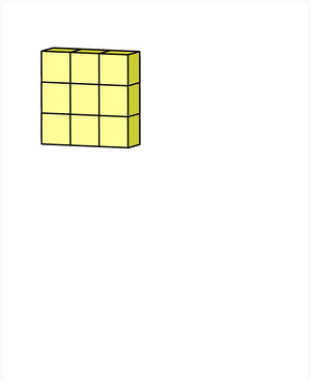


2. Addition



3. Bias



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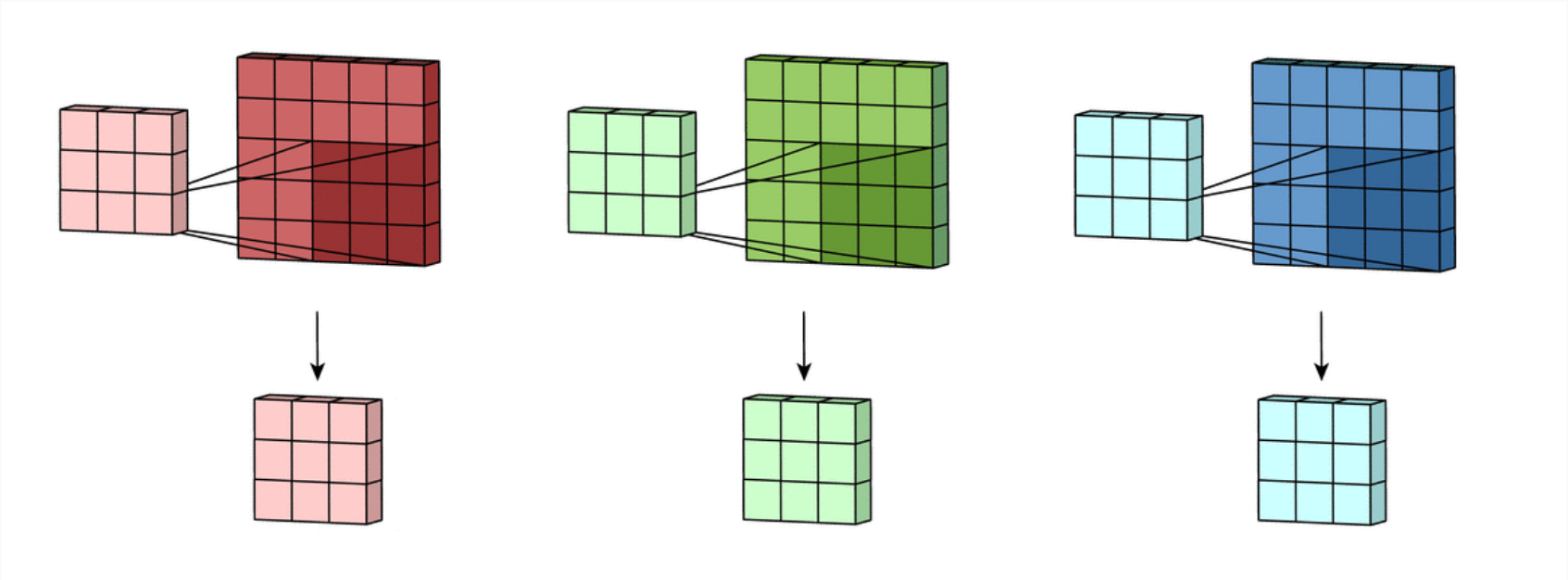
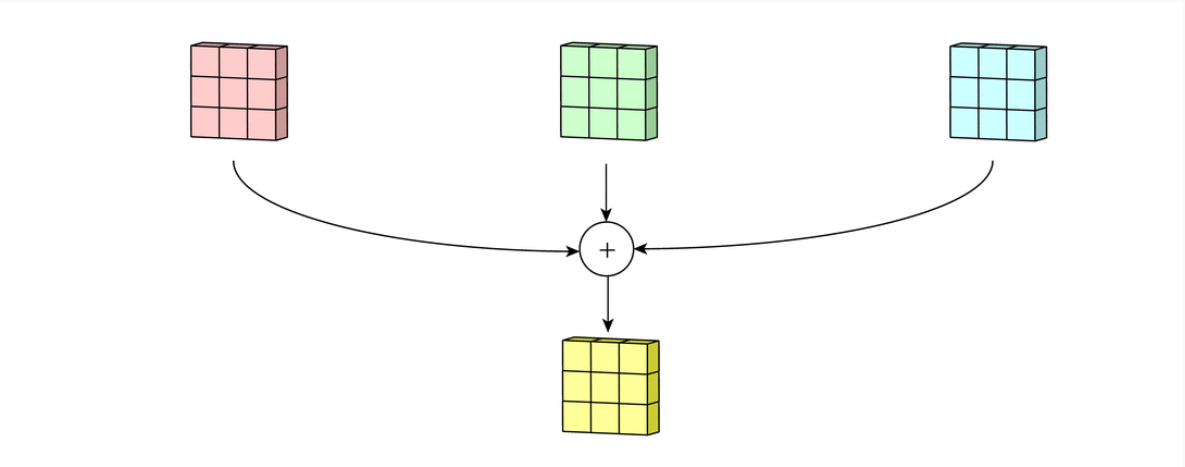
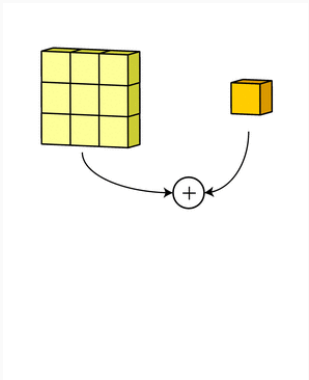


2. Addition



3. Bias



Summary of Convolutional layer steps

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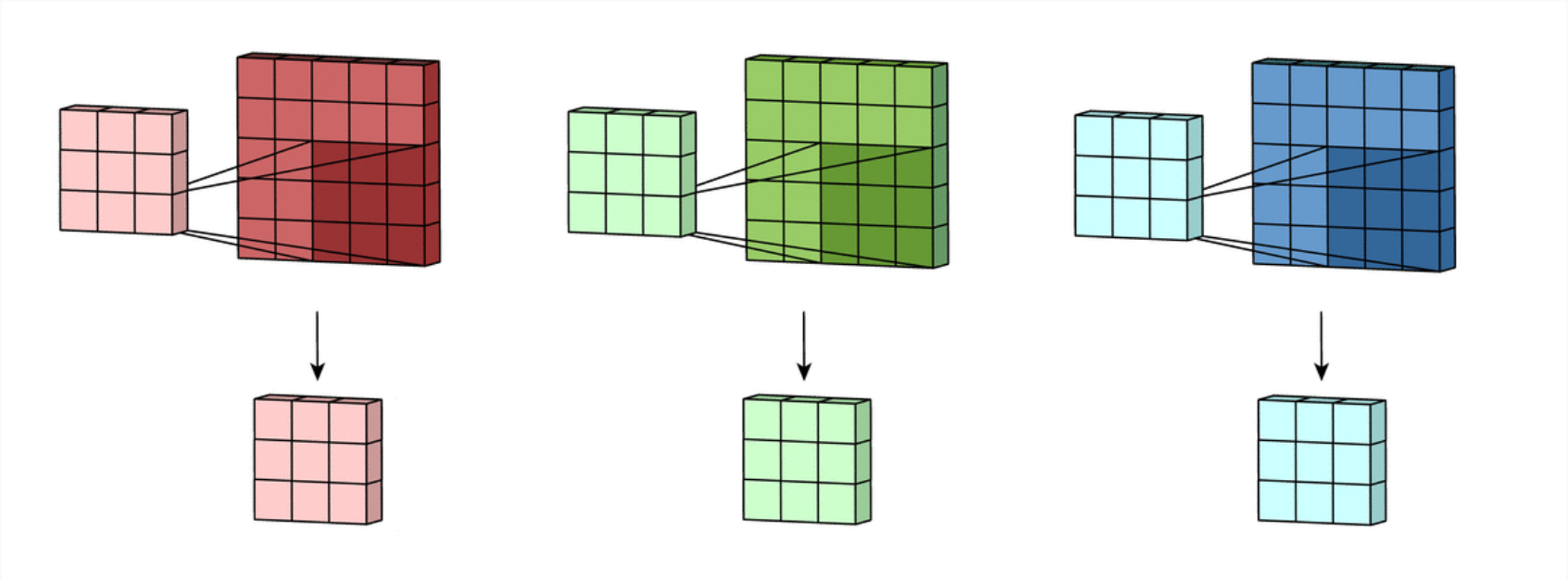
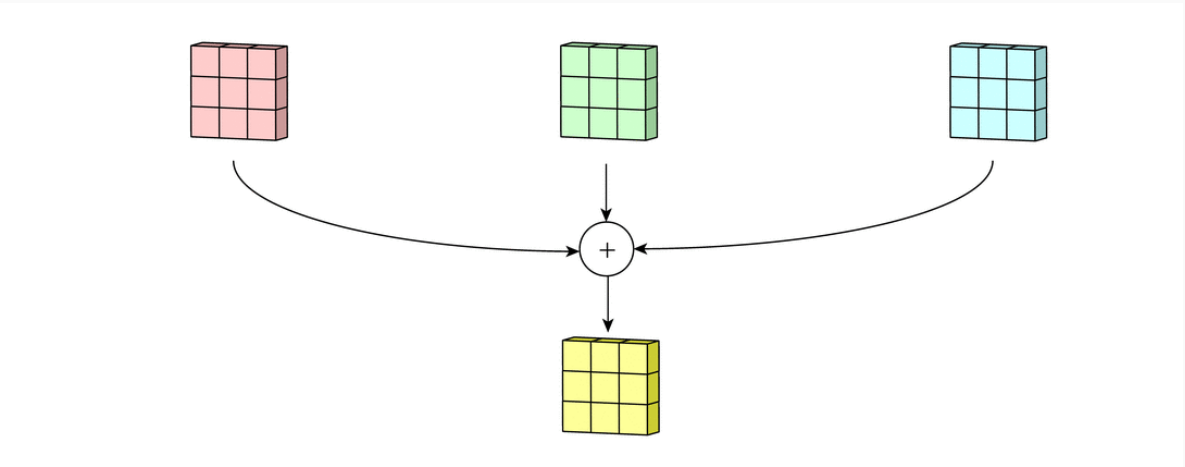


2. Addition



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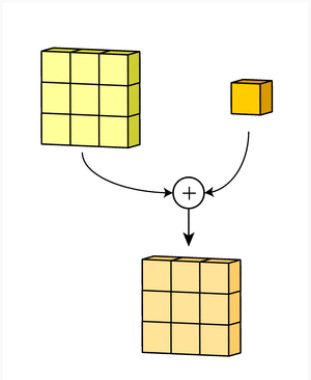
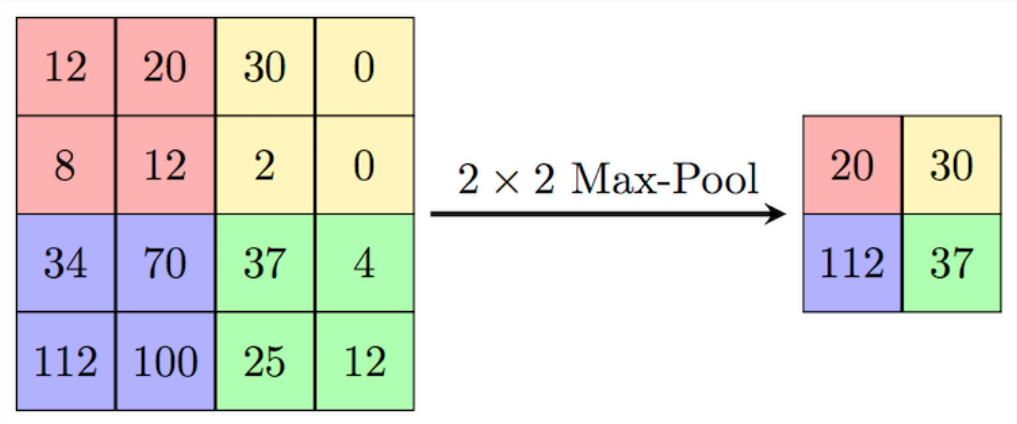


Remarks on Convolutional layers

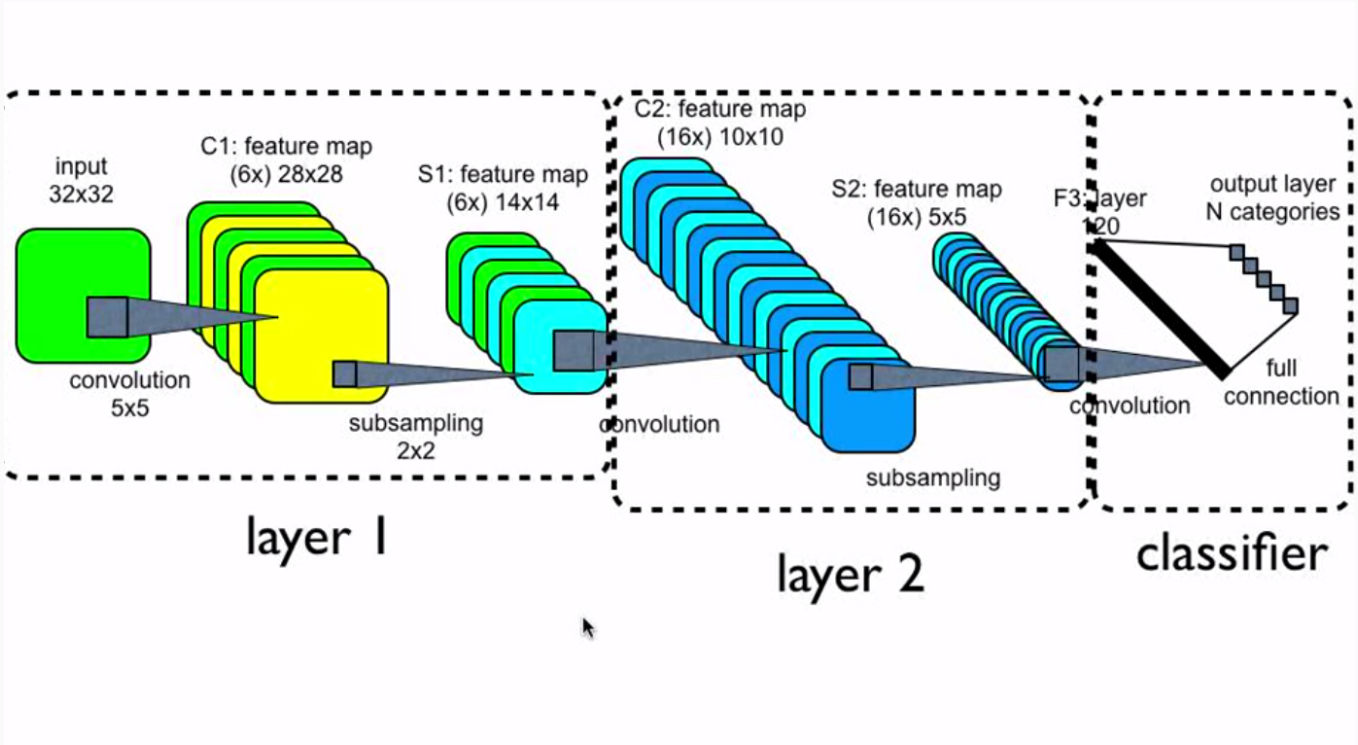
- Convolutional layers are acting locally on the image (But you can still use large scale information by adding more layers)
- Convolutions are invariant by translation (the weights do not depend on the location on the image).
- They can handle images of different sizes.

Max-Pooling

In order to reduce the size of the feature space (en to enhance the gradients), a common operation is to perform a max-pooling.



A traditionnal CNN architecture



Example of AlexNet

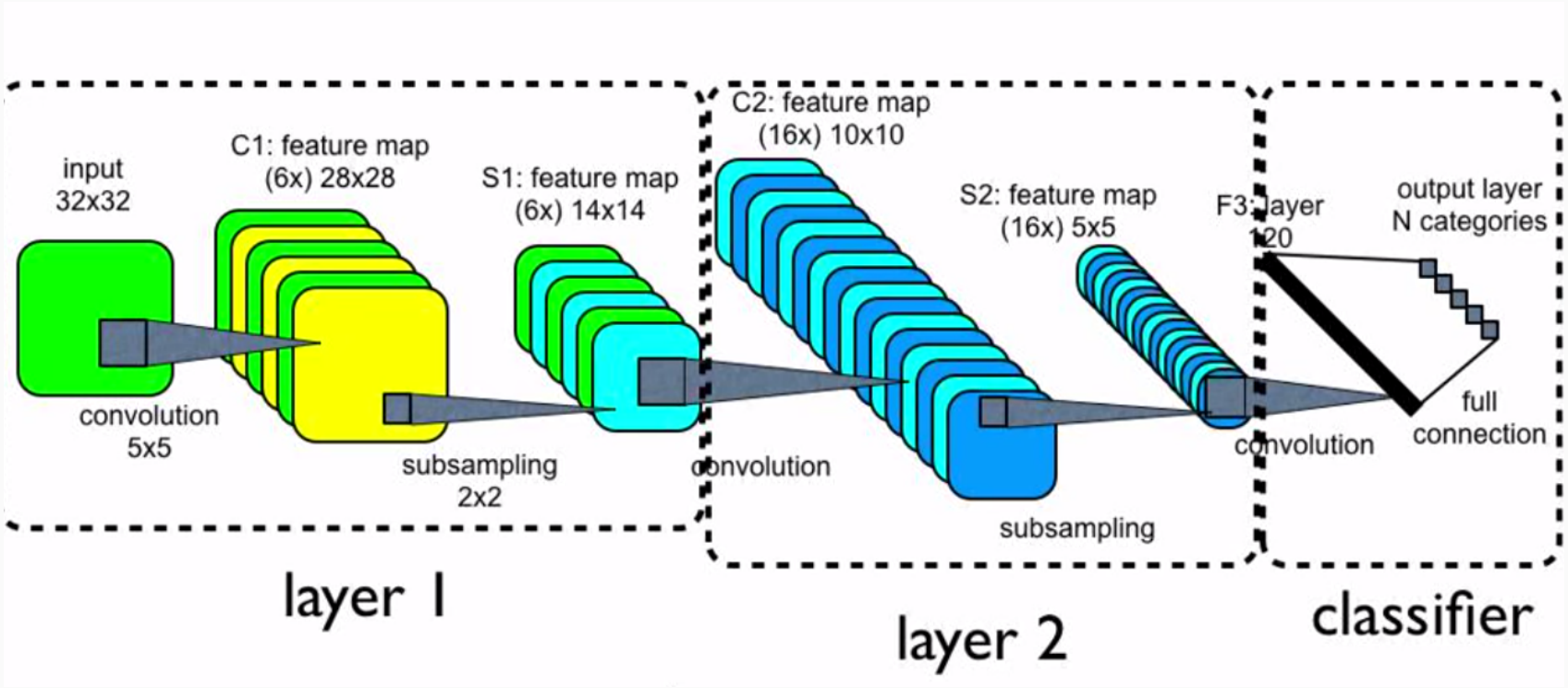
AlexNet is the first Deep architecture used on ImageNet challenge in 2012 and achieved an error of 15.3% (10% better than the previous best classifier). The paper was cited more than 34,000 times.

Alex Krizhevsky and Geoffrey E Hinton, *ImageNet Classification with Deep Convolutional Neural Networks*, Neural Information Processing Systems (2012), 1–9.

Layer		Feature Map	Size	Kernel Size	Stride	Activation
Input	Image	1	227x227x3	-	-	-
1	Convolution	96	55 x 55 x 96	11x11	4	relu
	Max Pooling	96	27 x 27 x 96	3x3	2	relu
2	Convolution	256	27 x 27 x 256	5x5	1	relu
	Max Pooling	256	13 x 13 x 256	3x3	2	relu
3	Convolution	384	13 x 13 x 384	3x3	1	relu
4	Convolution	384	13 x 13 x 384	3x3	1	relu
5	Convolution	256	13 x 13 x 256	3x3	1	relu
	Max Pooling	256	6 x 6 x 256	3x3	2	relu
6	FC	-	9216	-	-	relu
7	FC	-	4096	-	-	relu
8	FC	-	4096	-	-	relu
Output	FC	-	1000	-	-	Softmax

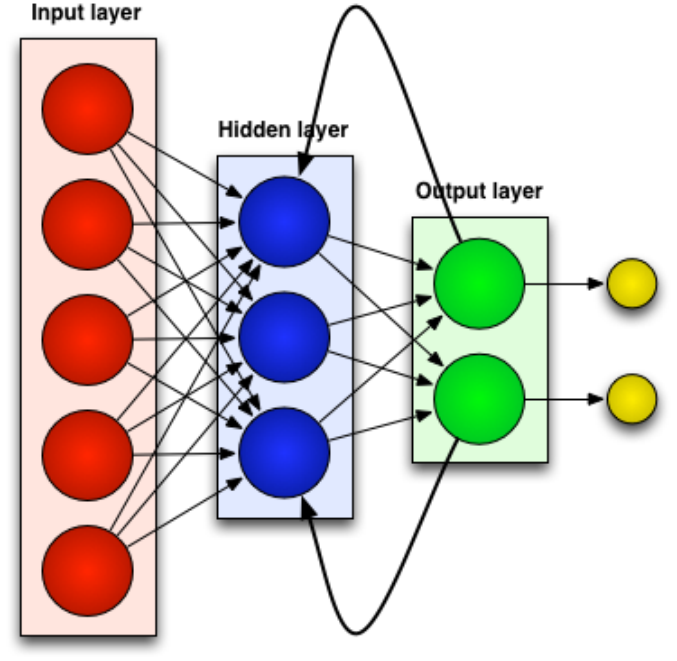
A quick typology of few neural nets

Convolutional neural networks



- Very efficient for computer vision (processing images with spatial patterns)
- Act locally on the input feature space.

Recurrent Neural Networks

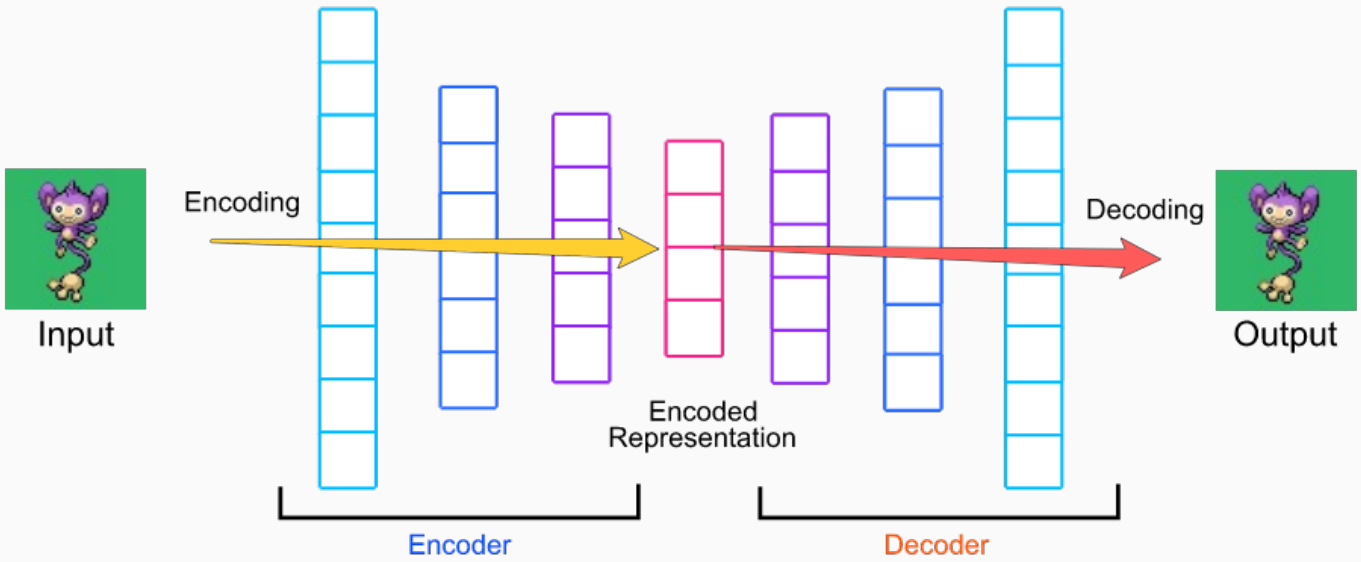


Some popular types of recurrent neural networks:

- Long short-term memory (LSTM)
- Gated Reccurent Unit (GRU)

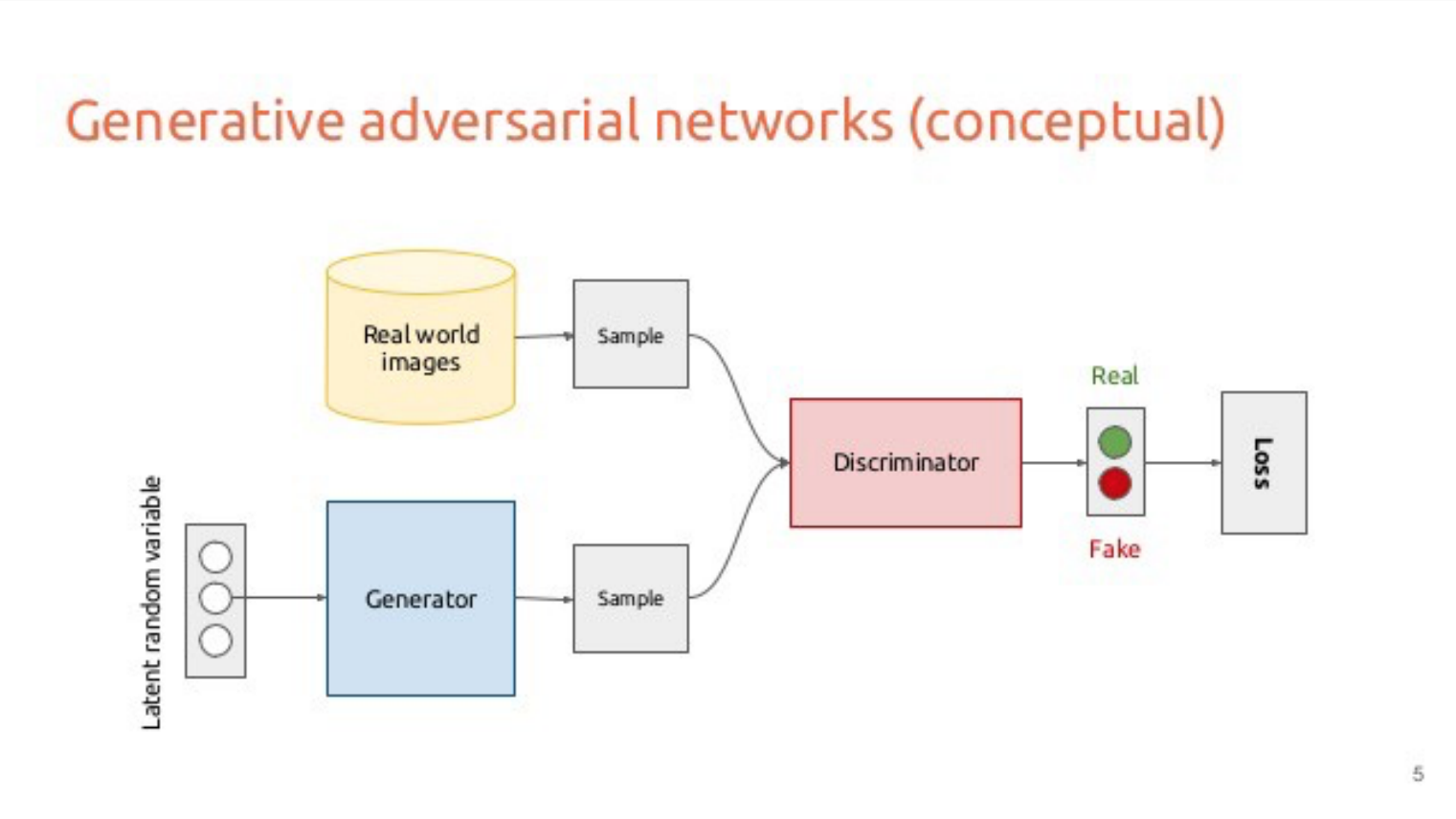
Used in machine translation and text processing

Autoencoders

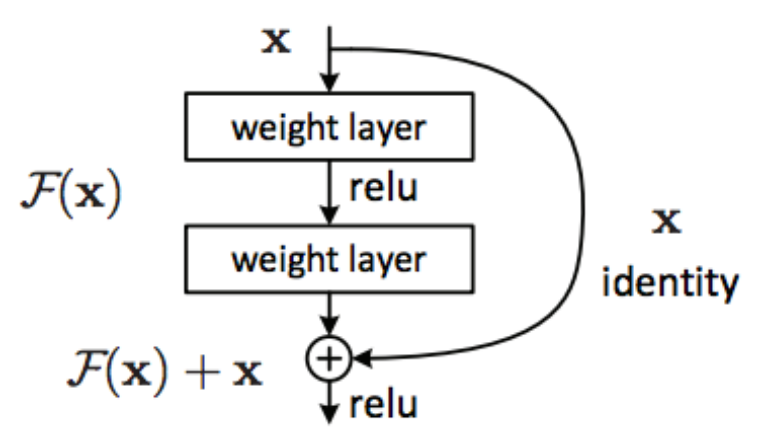


Used in image denoising, compressing, generation,...

Generative adversarial networks



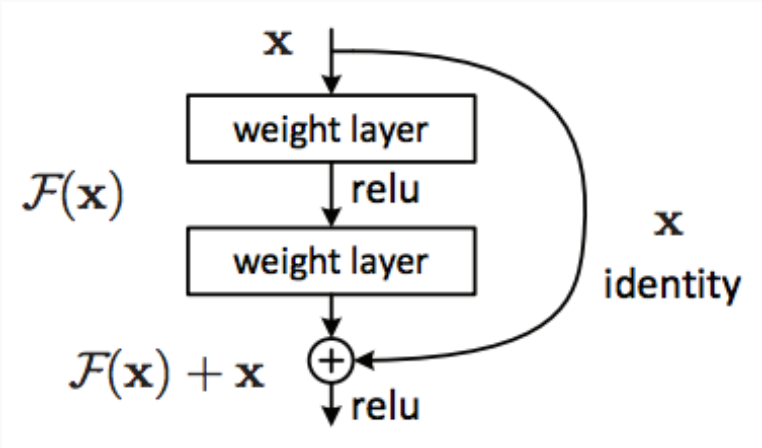
Residual Networks



x : input, y : output

$$y = x + \mathcal{F}(x)$$

Residual Networks



x : input, y : output

$$y = x + \mathcal{F}(x)$$