Estimation of amplified wind field by adding perturbation

Sukun Cheng, Yumeng Chen

13 May 2020

1 Wind perturbation

The perturbed wind velocity field is statistically amplified by adding the perturbation. That is, the spatial average of the perturbed wind velocity field is larger than that of the original wind field. See a proof in the next paragraph.

We define velocity vector $\boldsymbol{u}(x_i, y_j) = (u(x_i, y_j), v(x_i, y_j))$ as the horizontal surface wind velocity, and $\Delta\boldsymbol{u}(x_i, y_j) = (\Delta u(x_i, y_j), \Delta v(x_i, y_j))$ as the corresponding perturbations at the (i, j)-th grid point. Omitting the individual grid point, the expectation of the ensemble members are:

$$E[(u+\Delta u)^2 + (v+\Delta v)^2] = E(u^2) + E(v^2) + E(\Delta u^2) + E(\Delta v^2) + 2E(u\Delta v) + 2E(v\Delta u) >= E(u^2) + E(v^2) \quad (1)$$

where E indicates the expectation, then $E(u\Delta v) = E(v\Delta u) = 0$ as the perturbation has a zero mean in each dimension. While, the perturbations $E(\Delta u^2) + E(\Delta v^2)$ is positive.

It is required to adjust the magnitude of the perturbed wind field, which is used in calculating the wind-ice stress (air drag). We define a coefficient R as

$$\text{Air drag correction R} = \frac{1}{N_{grid}} \sum_{i,j} \frac{\sqrt{u^2+v^2}}{\frac{1}{N_e}\sum_{mem}\sqrt{(u+\Delta u)^2 + (v+\Delta v)^2}} \quad (2)$$

where N_{grid} is the total number of model nodes, with i,j are grid indices. N_e is ensemble size. That is, a ratio for a given location is defined as the original wind speed over the perturbed wind speed. R is the spatial average of the ratio.

In the wind-ice stress calculation:

$$\nabla \cdot (h\sigma) + \boldsymbol{\tau}_{\mathrm{a}} + \boldsymbol{\tau}_{\mathrm{w}} = 0 \quad (3)$$

$$\boldsymbol{\tau}_{\mathrm{a}} = \rho_{\mathrm{a}} c_{\mathrm{a}} |\mathbf{u}| (u, v) \quad (4)$$

$$|\boldsymbol{u} + \Delta\boldsymbol{u}|_{mem} \longmapsto R|\boldsymbol{u} + \Delta\boldsymbol{u}|_{mem} \quad (5)$$

To reduce the effect of wind perturbation due to the magnitude of $|\mathbf{u}|$, the correction should be:

$$\tau_{\mathrm{a},correction} = \rho_{\mathrm{a}} c_{\mathrm{a}} R |\mathbf{u} + \Delta\mathbf{u}| (u+\Delta u, v+\Delta v) = \rho_{\mathrm{a}} c_{\mathrm{a}} R |\mathbf{u} + \Delta\mathbf{u}| (\mathbf{u} + \Delta\mathbf{u}) \quad (6)$$

such that

$$E(\tau_{\mathrm{a},correction}) = E(\tau_{\mathrm{a}}) \quad (7)$$

where E() is the expectation over all ensemble members.

An ensemble of sequential perturbations with 40 members are generated using an offline FORTRAN code. The perturbations are nonphysical, but the domain size is set as the wind data that to be perturbed. The FORTRAN code is modified from the perturbation modules in the neXtSIM model, which is originally developed by Sakov Pavel. For each ensemble member, perturbations are generated sequentially. That is, each perturbation is generated based on a perturbation at previous time in the series. I understand that you mean here that perturbation are "time correlated". If so please give some little details about it. Sequential does not necessarily mean time-correlation The first perturbation is generated from random choice. The studied wind data set is ECMWF forecast from 09.2019 to 09.2020, which updates every 6 hours. The perturbations are saved in files for 300 GB.

We investigate the dependence of R on time in Fig.1. We also investigate the effect of different magnitudes of perturbations by multiplying an amplifier r to Δu and Δv in Eq. (2), i.e., $r\Delta u$ and $r\Delta v$, where $r = 0.5, 1, 2, ..., 5$. Fig.1 shows the variation of R over time using different strengths of perturbations. The original wind field is dominant when R is close to 1, where the magnitude of perturbation is much smaller than the original wind field, vice versa. The variation of R is more significant for larger r. It implies the wind is relatively weaker during 06.2020 to 08.2020.

Fig.2 shows the temporal average of R decreases monotonically as the strength of perturbation r increases. That is, stronger the perturbation, more amplified is the perturbed wind velocity field. It is superimposed with a red line by a linear curve fitting model, which formula is

$$R(r) = -0.05339 * r + 1.026. \tag{8}$$

Characteristics about goodness of fit include SSE: 0.0001217, R-square: 0.9979, Adjusted R-square: 0.9977, RMSE: 0.0039.

It is better to keep the consistency with the previous paper. Specifically, the variance of wind speed is 3 m/s. Applying the drag correction, I ran one member with and without perturbations for 1 month. Fig.3 shows the the spatial averages of the damage field over time from 03.09.2019 to 14.10.2019. The variable is overall in agreement in the perturbed run and in the unperturbed run.

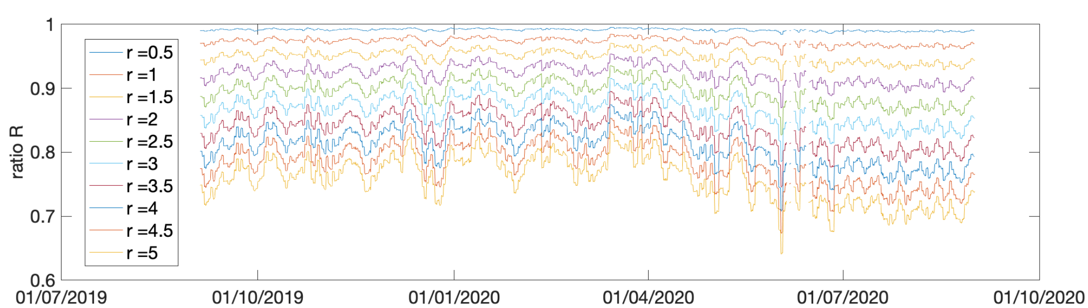

Figure 1: Evolution of R by perturbing the ECMWF forecast data set with different strength r

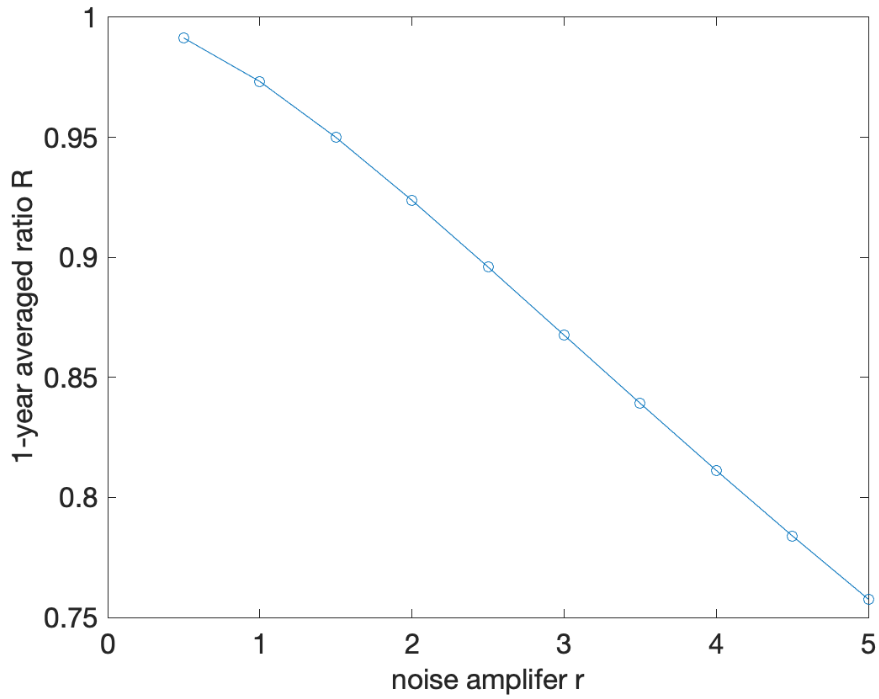

Figure 2: Temporal average of R against perturbation strength coefficient r

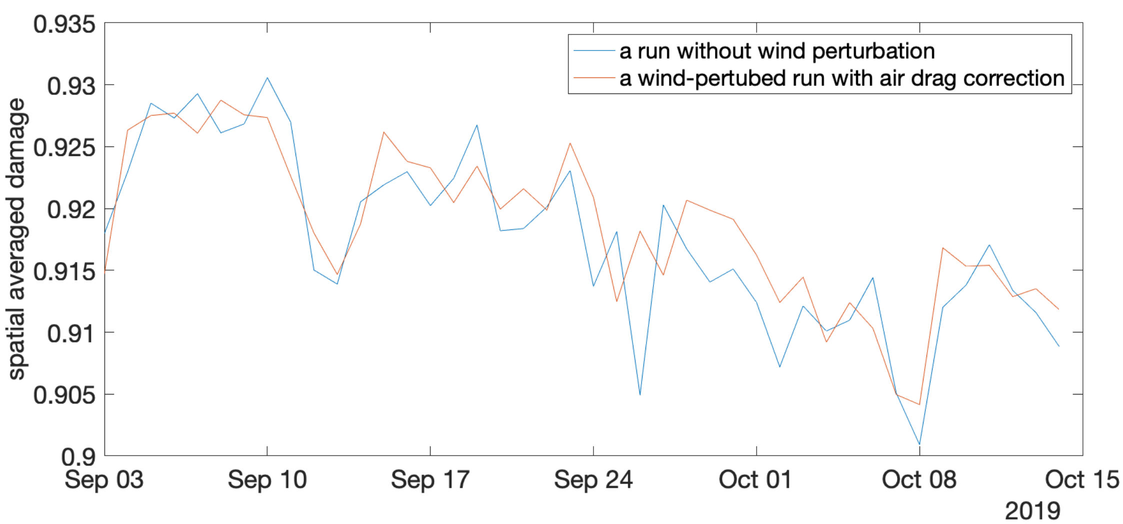

Figure 3: Comparison of evolution of spatial averaged damage between a run with and without perturbation for 1.5 months