# Odometery of mobile robotics by low-cost IMUs


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***Abstract*—this paper shows using microelectromechanical system (MEMS) low-cost Inertial Measurement Unit (IMU) to realize odometer for mobile vehicle/robotics, provide wheel's rotation angle and bogie's angle respect gravity. In addition, with the proposed algorithm we can calculate the yaw and roll angle information for the bogie by integrating the rotation gyro's output, significantly decreasing the angle drift. A test-bed was setting up for validating the algorithm, the results was analysis in details.**



***Keywords*— MEMS, IMU, EKF, AFS, mobile robotics,**


## I. Introduction

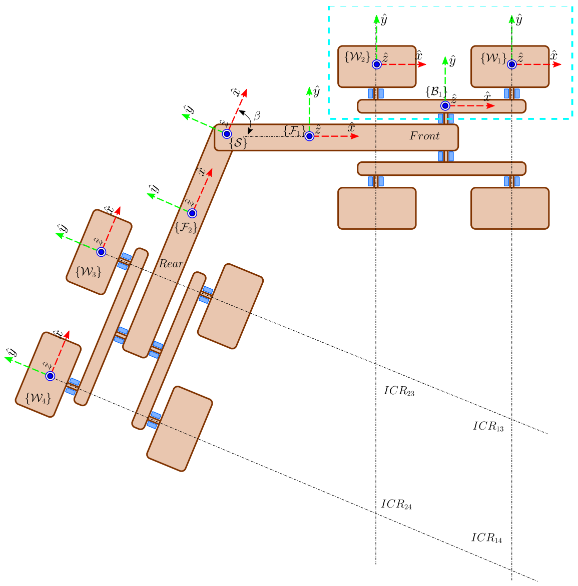


Fig. 1. Top view of the base for one type of heavy duty mobile machine

In mobile robotics, it is a common practice to attach a localization sensory system to the robot body, to measure the robots pose and sometimes fusing this information with dead reckoning of the robot's wheel odometery. However, this method might be not a suitable approximation of an Articulated-Frame-Steerable Vehicles (AFS) mobile robot even on a flat, and balanced driving surface. It becomes even worse in rough terrain motions. The pure rolling assumptions, which Nonholonomic dynamic constraints are built on it [1], cannot be valid without measurement and estimation of robot's internal dynamic parameters. For instance, it is impossible to assume zero side slippage of wheels for an AFS even inside a simple configuration space and it is necessary to have appropriate measurements for them.

Therefore, even if we temporarily neglect environmental effects, the velocities of rear and front parts cannot be independent variables for measurements and at the same time, each of them cannot express vehicle status solely, as showing in Fig.1. Based on this, it is clear that installation of an IMU, or a localization by simple dead-reckoning methods cannot be enough for determination of the robot's status.


Fig. 2. Wheels and bogie of a forest vihecle, the size and dimension are showing with unit of mm.

For solving the problem mentioned above, and the tradeoff between system cost and complexity, we proposal install one strap-down MEMS IMU on the rotation center for each wheel, and on each bogie. With the gravity as reference, using Extended Kalman Filter (EKF) fusing the measurements of accelerometers and the measurements of gyroscopes to estimate the pitch angle of bogie and rotation angle of wheel.

Since the wheel of vehicle is a nature rotation platform, and a triad rotation gyroscope with constant rotation speed along one axis can form a virtue gyroscope which can decrease the gyro's drift except the rotation axis [3][4]. We use this property to calculate the yaw and roll angle for boogies when the wheel of vehicle has stable angular rate or the rotation speed change small.

In current phase, we simplified our test-bed as one set of bogie-wheel pair, the prototype shows in Fig.2.

## II. THEORETICAL BACKGROUND

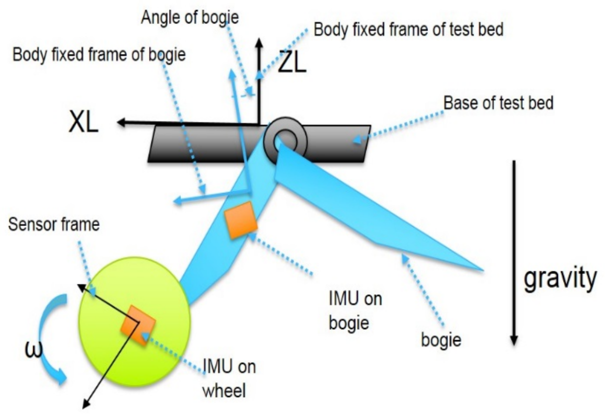


Fig. 3. The blue V shape repsentative the bogie, a IMU attached on it. The green cirle indicates the wheel, and a IMU put on its rotation center.

In Fig.1, two IMUs with triad accelerometers and triad gyroscopes installed on the test bed, one is attached on the bogie, and the other is on the rotation center of wheel, which rotate with the wheel. The bogie has one Degree of Freedom (DOF) to the test-bed, can move up and down with a range about 20 degrees. Currently the base of our test bed keep stationary, we define its axis ZL align with gravity, the positive direction toward up. All the frames in Fig.3 share the same y direction, toward to outward. The angle of bogie is the angle difference between the body fixed frame of bogie and test-bed, and the angle of wheel rotation is the angle difference of sensor frame and bogie's body fixed frame. The rotation positive direction of bogie and wheel is counter-clockwise.

We choose Extended Kalman Filter (EKF) for estimating these two angles in a 2D frame.

### A. *EKF for pitch angle estimation*

$\theta[k]$ is the angle in time step $k$, $\dot{\theta}$ is the angular velocity, and $b_g$ is the bias of angular velocity, the state of EKF is

$$x[k] = \begin{bmatrix} \theta[k] \\ \dot{\theta}[k] \\ b_g[k] \end{bmatrix} \quad (1)$$

The discrete process model is

$$x[k+1] = Ax[k] + w[k] \quad (2)$$

where the state transition matrix is

$$A = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$T$ is the sampling interval.
The process noise is

$$w[k] = T \begin{bmatrix} n_{\omega,k} \\ n_{\alpha,k} \\ \eta_{b_a} \end{bmatrix} \quad (4)$$

$n_{\omega,k}$, $n_{\alpha,k}$, and $\eta_{b_a}$ are Gaussian noise.
Assume the noises are not correlated, the matrix of process noise is

$$Q = E(ww^T) = T^2 \begin{bmatrix} \sigma_\omega^2 & 0 & 0 \\ 0 & \sigma_a^2 & 0 \\ 0 & 0 & \sigma_{ba}^2 \end{bmatrix} \quad (5)$$

We use three of IMU's outputs as observation, the accelerometer's measures in x and z direction, and gyro's in y direction, take the gravity as reference for the pitch angle estimation.

The observation model is

$$h = \begin{bmatrix} 9.81\cos\theta + az \\ -9.81\sin\theta + ax \\ \dot{\theta} + b_g \end{bmatrix} + \begin{bmatrix} n_z \\ n_x \\ n_\theta \end{bmatrix} \quad (6)$$

$n_z$, $n_x$, and $n_\theta$ are Gaussian noise. $az$, $ax$ representative the motion acceleration in z and x direction of sensor frame, please notice we assume gravity mainly project into x-z plane of sensor frame. Take $az$ and $ax$ into noise vector, then the observation matrix is

$$H_k = \begin{bmatrix} -9.81\sin\theta_k & 0 & 0 \\ 9.81\cos\theta_k & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad (7)$$

Please notice, we can get the roll angle, $\theta_{roll_b}$, from Section II B and the acceleration in x-z plane is

$$a_{xz} = 9.81\cos|\theta_{roll_b}| \quad (8)$$

However, currently our test-bed can't provide roll rotation, we just use $9.81m/s^2$ in (6) and (7).

### B. *Caculate yaw and roll with rotation gyroscope*

In Fig.1, donate the measurements of gyro on the wheel as $w_x$ ,$w_z$, in axis x and z, respectively. They can write as true angular rates plus long-term bias and noise.

$$w_x = w_{xt} + b_x + n_x \qquad (9)$$

$$w_z = w_{zt} + b_z + n_z \qquad (10)$$

Ignore the noise terms, transfer these two outputs into body fixed frame of bogie and integrate them with time, get the yaw and roll angle of bogie.

$$\theta_{yaw_b} = \int_0^{t_{end}} [(w_{x_t} + b_x)\cos\emptyset - (w_{z_t} + b_z)\sin\emptyset]dt$$
(11)

$$\theta_{roll_b} = \int_0^{t_{end}} [(w_{z_t} + b_z)\cos\emptyset_k + (w_{x_t} + b_x)\sin\emptyset_k]dt$$
(12)

Where $\emptyset_k$ is the rotation angle of the wheel, it is the difference between the sensor frame respect the gravity expressed in inertial frame, and

$$\emptyset_k = \int_0^{t_k} r\,dt \qquad (13)$$

In (13), $r$ is the rotation speed of the wheel. If assume $r$ as constant, combine equation (11) and (12) we notice that the integration term that contain the long-term bias part will disappear after one complete circle.

We can estimate the bogie's angle, and wheel' angle respect to test-bed's frame or gravity as mentioned above, then get the angle of $\emptyset_k$. The whole solution based on pure IMU, no external information needed.

## III. TEST ENVIRONMENT AND RESULTS

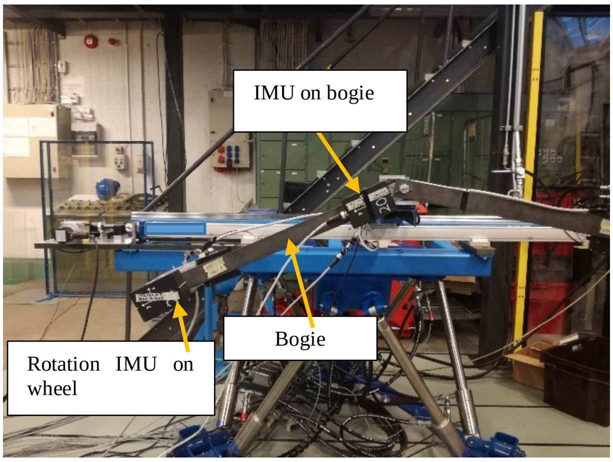


Fig. 4. The test-bed

### A. *Test enviroments*

The test-bed as showing in Fig.2 contains the following measurement units:

- One BMI160 IMU sensor wrap into a box with wireless communication function mounted on the tip of the bogie. It can drive by a motor to simulate wheel's rotation.
- Another same type IMU attached close the joint of bogie and the base of test-bed, and the joint also has a motor to control its rotation.
- The two rotation joints have incremental encoders to provide the angle information as sound reference for the angle estimation.
- The sensors units sending data to a dSpace DS1005 for real-time data acquisition and control the motors, the sampling time is 400 Hz.

### B. *pitch angle of bogie*

The test process last about 12 minutes:

- The first minute keep the IMUs and bogie stationary.
- From the second minute, the rotation IMU start rotation with a constant speed of 170 degree/s.
- The bogie start and motion of oscillation from 3 minute to the test end, the amplitude is about 8 degrees, and the period is xxx second, to simulate the vehicle's motion on an uneven ground.

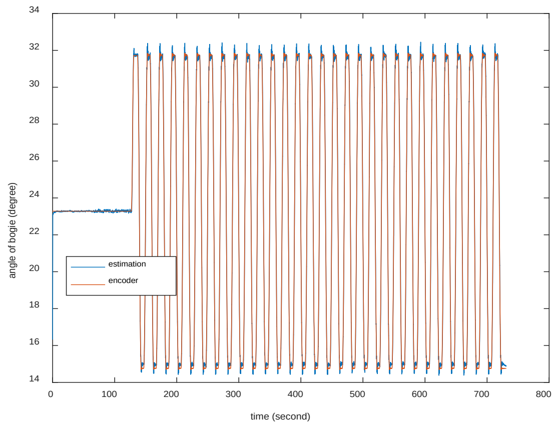


Fig. 5. Bogie's motion and estimation results. The red line is from encoder, and blue line is the estimated output from EKF.

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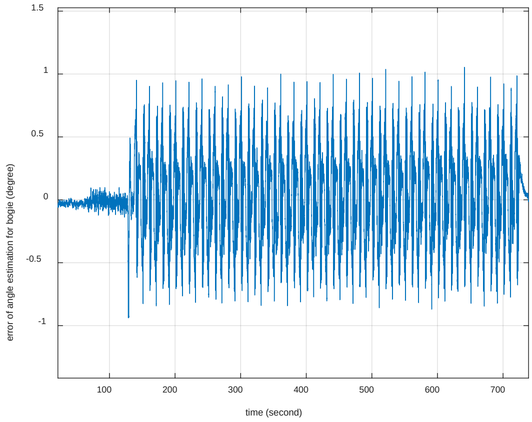


Fig. 6. The error of bogie's motion from estimation.

RMS=0.6727

### C. *Rotation angle of wheel*

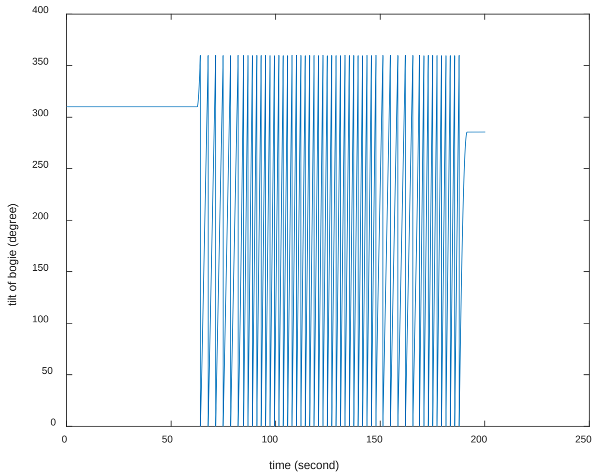


Fig. 7. Wheel rotation of test, first minute keep stationary, then start rotation with a speed of 100 $degree/s$ for about 20 seconds, then 170 $degree/s$ for about 40 seconds, then 100 $degree/s$ for another 20 seconds, then 170 degree/s for the last 20 seconds.

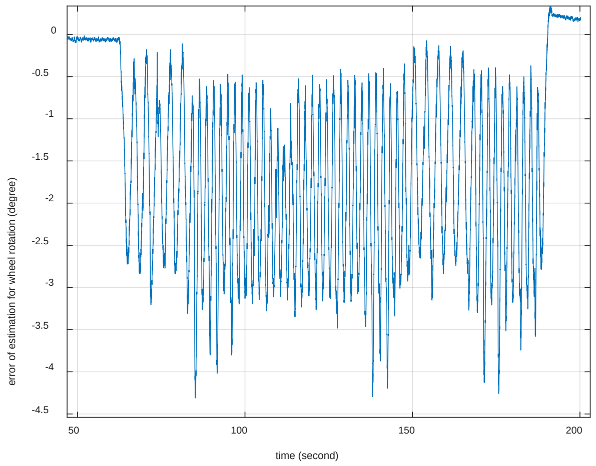


Fig. 8. The error for wheel rotation test.

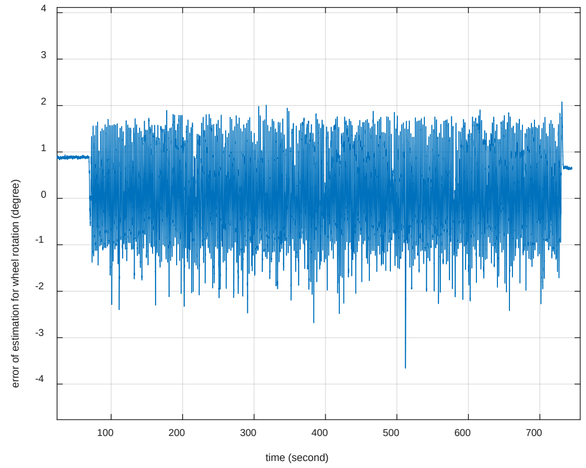


Fig. 9. Error of wheel rotation test, both wheel and bogie have motion. The bogie start oscilation as in Fig.5, and wheel rotate with a speed of 100 degree after 60 seconds.

Rms=1.886

### D. *Caculate yaw and roll of bogie*

The test process are same as in section III B, but the measures $degree/s$ integrate the yaw and roll angle of wheel. Since our test-bed has no yaw and roll motion, we simply use zero as reference, just validate the idea that the rotation gyroscope can decrease the drift of angle integration in the other two axis except the rotation axis.

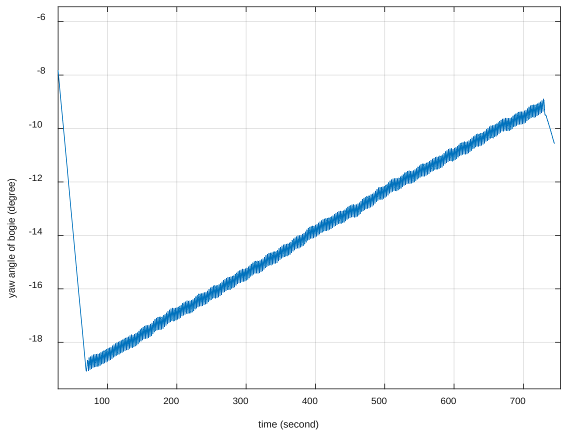


Fig. 10. Yaw angle.

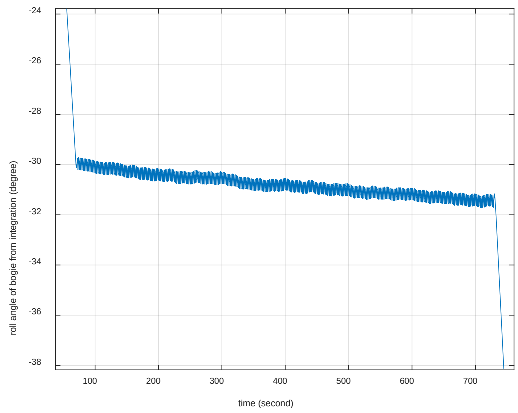


Fig. 11. Roll angle

The values of yaw and roll angle are showing in Fig.10 and Fig.11, respectively. No surprise, when the wheel has no rotation during the first one minute, the angles' drift significantly, since we use low-cost IMUs. Once the wheel start rotating, most part of the long-term bias in the gyro's x, y-axis has been eliminated, the angles' drift for yaw and roll become much smaller.

Please notice, the wheel has a rotation speed of 170 $degree/s$ respect to the bogie's body fixed frame after about 65 second, but after about 125 seconds, the bogie start an oscillation, this introduce an extra velocity to the wheel respect to inertial frame, violate the assumption the wheel hold constant rotation speed in equation (13). Since the bogie's maximum angular speed is less than 12 $degree/s$, less one order than the wheel's, the error introduce by this speed changes of rotation is not significant, but still shows in Fig.10 and Fig.11 after 125 seconds.

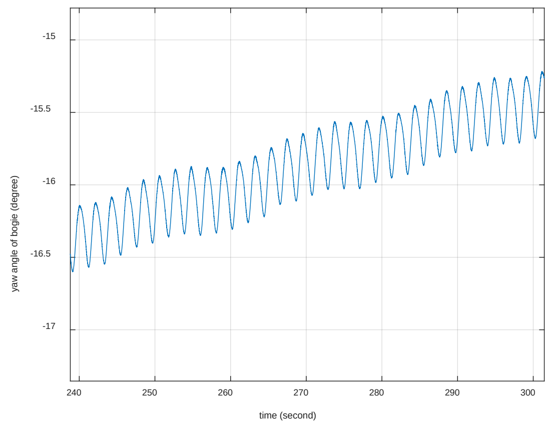


Fig. 12. Enlarge part of Fig.10.

From Fig.12, and equation (11),(12), we notice that the integration angle for yaw and roll should be some sinesoid, the amplitude depends on the long-term bias in gyroscopes in x and z axis. In section IV A, we show after dedue this long-term bias from row data of gyro's, the amplitude of this sinusoid wave decreased.

## IV. DISCUSSION

### *A. error correction*

In [4] and [5], a calibration method for MEMS IMUs are given. For our implementation, if the IMU rotate around y-axis, ignore the Earth rotation, a simple error model for gyro is

$$\begin{bmatrix} \delta\omega_x \\ \delta\omega_y \\ \delta\omega_z \end{bmatrix} = \begin{bmatrix} k_{xy}\omega + d_x \\ S_y\omega + d_y \\ k_{zy}\omega + d_z \end{bmatrix} \qquad (14)$$

Where $d_x$, $d_y$ and $d_z$ are the gyro biases in sensor frame, $[\delta\omega_x \quad \delta\omega_y \quad \delta\omega_z]^T$ is the error of gyro in sensor frame. $k_{xy}$ is the factor of installation error for x-axis relate to y-axis, which means if the y-axis has a rotation speed of $\omega$, because of installation inaccuracy, it will project into x-axis an angular speed with the factor. $k_{zy}$ is the factor of installation error for z-axis relate to y-axis. $S_y$ is the scale factor in y-axis, indicate nonlinearity of the gyro in this axis, and $\omega$ is the rotation speed of y-axis.
Since we can use gravity as reference to correct gyro's error in y-axis, as showing in section III B, C, here we do not discuss the $S_y$ and $d_y$.
For the test in section III B, the system in the first 65 seconds are stationary, take the mean values of the first 5 seconds from gyros raw output in x and z axis, compare with zero, get the bias $d_x$, and $d_y$. During the test of 65 second to 125 second, only the wheel has a rotation with a speed of 170 $degree/s$, take the mean value of gyro's output in other two axis during this period as $\delta\omega_x$ and $\delta\omega_z$, with the bias $d_x$, and $d_y$ from the first 5 seconds period, using equation (14), we get $k_{xy}$ and $k_{zy}$. In

each time step $k$, use equation (15) and (16) give the gyro error in x and z axis.

$$\delta\omega_x[k] = k_{xy}\omega_{est}[k] + d_x \quad (15)$$
$$\delta\omega_z[k] = k_{zy}\omega_{est}[k] + d_z \quad (16)$$

$\omega_{est}[k]$ is the estimated rotation speed in y-axis. After correcting the gyro's error given by (15) and (16), apply (11) and (12) to get the yaw and roll angle again, the result show in Fig.13.

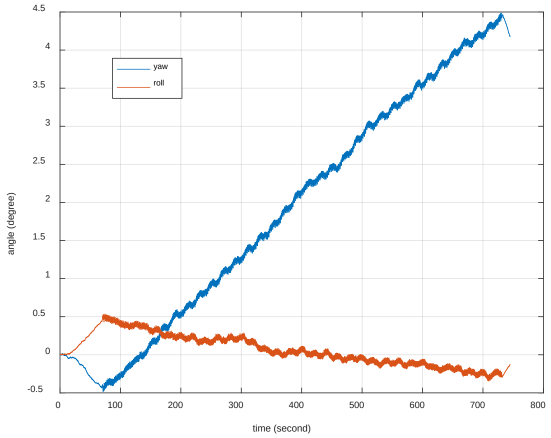


Fig. 13. After the correction of error for gyro in x and z axis, the drift in the integratio of yaw and roll angle became smaller.

Fig.13 show that with the correction for gyro in x and z-axis in sensor from, the drift in yaw and roll angle calculation decreased. Compare Fig.10 and Fig.11, during about 11 minutes the yaw angle drift from about 10 degree to 5 degree, and roll angle's drift from 2 degree to about 1 degree. This is because the fact that our estimation is inaccuracy for the rotation angle of wheel, if the bias or error in gyro's x and z axis, it cannot eliminate completely, and will accumulate as the integration going on. Additionally, the amplitude of this sinusoid wave is shorten to 0.07 degree in Fig.13; in Fig.12, it is about 0.4 degree before the correction.

### *B. Gyro's error in short period*

In Fig.14, the blue part indicate the raw output of gyro's x-axis when the wheel rotate around y-axis with a speed of 170 $degree/s$, and the red line is the rotation phase of the wheel dived by 360 for convenient showing. We can notice that the gyro's output in x-axis has a short-term bias or error as it rotate in the gravity field. This means for the low-cost IMUs we use in the test, gyros' output correlate with acceleration. This short-term bias cannot be eliminate with (11) and (12) when integrate the yaw and roll angle. Our next step will try to estimate this bias with EKF.

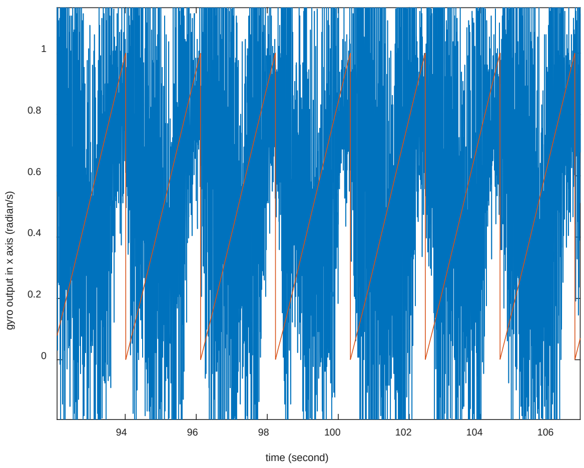


Fig. 14. Example of a figure caption. (*figure caption*)

## V. CONCLUSION

We use low-cost IMUs build an odometer for mobile robotics, the maximum rotation angle for wheel is within 5 degree. Additionally, this odometer can also provide useful angle information for yaw and roll angle during several minutes.


### ACKNOWLEDGMENT *(Heading 5)*

The


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