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ORIGINAL ARTICLE

Multipath mitigation via component analysis methods for GPS dynamic deformation monitoring

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Abstract Multipath is one of the main error sources in high-precision global positioning system (GPS) dynamic deformation monitoring, as it is difficult to be mitigated by differencing between observations. In addition, since a specific frequency threshold value between multipath and deformation signals may not exist, multipath is usually inseparable from the low-frequency vibration signal using conventional frequency-domain filter methods. However, the multipath repeats in two sidereal days when the surroundings of a GPS antenna remain unchanged. This characteristic can be exploited to model and thus mitigate multipath effectively in dynamic deformation monitoring. Unfortunately, a major issue is that the degree of repeatability decreases as the interval between first day and subsequent days increases. To overcome this problem, we develop a new sidereal filtering referred to as reference EMD-ICA (EMD-ICA-R), where empirical mode decomposition (EMD) and independent component analysis (ICA) are jointly used to model multipath and renew the reference multipath. For the successful implementation of the EMD-ICA-R, an a priori denoised multipath signal is needed as a reference. We further propose to use the principal component analysis (PCA) method to extract more accurate reference multipath signal and form a combined PCA-EMD-ICA-R approach. Simulation experiments with a motion simulation platform were conducted, and the testing results indicate that the proposed methods can mitigate the multipath by around 67 % when a reliable reference multipath signal is extracted from a static situation. Furthermore, simulation experiments with different deformation signals added into the coordinate time series of three consecutive days show that the two proposed methods are also effective in a dynamic situation. Since wavelet filtering is used to denoise the reference multipath signals in the new approaches, simulation experiments with several wavelet filters are tested, and the results indicate that the PCA-EMD-ICA-R approach can work well with various wavelet filters.

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Keywords GPS · Multipath mitigation · Independent component analysis (ICA) · Empirical mode decomposition (EMD) · Principal component analysis (PCA)

Abbreviations

EMD	Empirical mode decomposition
ICA	Independent component analysis
ICA-R	ICA with reference
PCA	Principal component analysis

Introduction

The global positioning system (GPS) has been widely used in dynamic deformation monitoring of engineering structures in recent years. However, GPS observations are always contaminated by various errors that affect the accuracy and reliability of positioning. In short-baseline GPS measurements for deformation monitoring, various error sources, such as satellite and receiver clock errors, can be eliminated by double-difference processing. Ionospheric and tropospheric delays can also be precisely modeled. Nevertheless, multipath effects occur when satellite signals arrive at a GPS antenna via different paths due to the reflections from nearby objects, such as water surfaces, trees, buildings, vehicles and so on. Multipath is a dominant error source in GPS dynamic deformation monitoring because it cannot be effectively eliminated by differential positioning. Currently, multipath can be mitigated using three processing strategies: (1) receiver antenna design (Schupler et al. 1994); (2) receiver signal filtering (Van Dierendonck et al. 1992; Townsend and Fenton 1994; Townsend et al. 1995); and 3) observation data processing (Axelrad et al. 1996; Ge et al. 2000; Souza and Monico 2004; Zheng et al. 2005).

When the frequencies of multipath are different from deformation signals, the frequency-domain filter, one of the above observation data processing strategies, can be used to extract or eliminate multipath directly. Han and Rizos (2000) utilize the finite impulse response (FIR) filter to extract or eliminate the multipath to obtain deformation from coordinate time series. However, the limitation of such a filter is that deformation signals falling in the same frequency band will be filtered out by the filter (Ge et al. 2000). Souza and Monico (2004) and Satirapod and Rizos (2005) used a wavelet filter technique to separate deformation signals from multipath under the assumption that the frequencies of deformation signals are known and different from the multipath frequencies. However, the characteristics of multipath are very complicated. If there were frequency aliasing between multipath and deformation signals, it is very difficult to determine the specific frequency threshold value required in those filters (Kijewski-Correa 2003).

Another widely used multipath mitigation strategy is sidereal filtering which is immune from the effect of frequency aliasing. When the environment around a GPS antenna remains unchanged, multipath has an intrinsic repeatability characteristic due to the approximate repetition of satellite geometry in the cycle of a sidereal day (i.e., 23 h 56 m 04 s). Subsequently, Choi et al. (2004), Axelrad et al. (2005) and Ragheb et al. (2007) suggested that the repetition time was about 10 s earlier than a true sidereal day, and Agnew and Larson (2007) showed that the repeat time was on average 246 s less than a solar day. Sidereal filtering is implemented as follows: (1) assuming there is no displacement in the coordinate time series during the first day and it can be low-pass filtered to remove high-frequency noise unrelated to the satellite geometry; (2) the denoised coordinate time series are regarded as multipath models and shifted by a sidereal day or a known satellite orbital period; and (3) the shifted multipath models are differenced from the coordinate time series of subsequent days. This strategy is effective to mitigate multipath if the following three issues can be resolved in real applications. The first one is how to obtain the coordinate time series only including multipath and noise on first day for modeling multipath. The second issue is how to accurately extract the multipath models from the coordinate time series. The third one is the repeatability of multipath of two consecutive days is high, but it decreases as the interval between the first day and subsequent days increases (Zhong et al. 2005). Most current research focuses on the second issue, by using filtering techniques such as adaptive filter (Dodson et al. 2001), Vondrak filters (Zheng et al. 2005) and wavelet filters (Zhong et al. 2008). Most of the filter-based approaches can fix the second issue effectively, but are powerless to address the first and third issues. In order to tackle all three issues, a new sidereal filtering approach based on component analysis is proposed. The new approach combines independent component analysis (ICA), principal component analysis (PCA) and empirical mode decomposition (EMD).

Independent component analysis, as proposed by Hérault and Jutten (1986), transforms a multivariate random signal into a series of signals whose components are mutually independent in the statistical sense, regardless of the original sources of the signal. Some effective algorithms, such as fast ICA, kernel ICA and ICA with reference (ICA-R), have been developed (Hyvärinen 1999; Hyvärinen and Oja 2000; Lu and Rajapakse 2006) and have been widely applied to various fields. However, ICA will not work when the number of mixing components (channels) is larger than or equal to the number of sources. In contrast, empirical mode decomposition (EMD) is a signal-analysis tool that is able to decompose a signal into multidimensional time series in a natural way, without prior knowledge about the signal of interest that is embedded in the data series. In conjunction with EMD, a combination method called EMD-ICA-R is proposed to mitigate multipath. In the EMD-ICA-R method, the first day multipath signal is input as the reference signal, and the desired output signal is used as the multipath model of second day that can be employed as the reference signal in the next sidereal filter in the case of the repeatability of multipath decreasing as the interval between the first day and subsequent days increases. To extract the reference multipath signal more precisely, we further use the principal component analysis (PCA), which is a mathematical procedure that uses an orthogonal transformation to extract the main uncorrelated features from a set of observations of possibly correlated variables, in conjunction with the EMD-ICA-R algorithms to form the PCA-EMD-ICA-R method.

The fundamentals of ICA, ICA-R, EMD and PCA are briefly introduced in the following section. Then the two proposed methods, EMD-ICA-R and PCA-EMD-ICA-R, are described in detail and assessed by several experimental tests with a motion simulation platform, which was developed to simulate various types of 2D motions that are typical for large civil engineering structures, in either the horizontal or the vertical plane.

Independent component analysis

Independent component analysis was originally developed for recovering mutually independent but unknown source signals from their mixtures based on higher-order statistics (Hyvärinen and Oja 2000). The standard ICA model could be written as

$$\mathbf{x} = \mathbf{A}\mathbf{s} = \sum_{i=1}^{n} \boldsymbol{a}_i s_i \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_m]^T$ is regarded as the observed mixtures, $\mathbf{s} = [s_1, s_2, \cdots, s_n]^T$ represents the unknown sources, and $\mathbf{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n]$ is the unknown mixing matrix of size $m \times n$. With the assumptions that $m \geq n$ (usually $m = n$) and that sources $\mathbf{s}$ are mutually independent, the ICA could optimally estimate a demixing matrix, say $\mathbf{B}$, to separate the original signals $\mathbf{s}$ based on some rules of optimization and learning methods. Then the best approximation vector of $\mathbf{s}$ can be derived from

$$\mathbf{y} = \mathbf{B}\mathbf{x} \tag{2}$$

where $\mathbf{y} = [y_1, y_2, \cdots, y_n]^T$, which is the best approximation vector of $\mathbf{s}$. Generally, the process of the ICA algorithm can be divided into two steps: First, whiten the observed mixtures to get the whitening signals $\mathbf{z}$, $\mathbf{z} = \mathbf{V}\mathbf{x}$, where $\mathbf{V}$ is the whitening matrix, and $E(\mathbf{z}\mathbf{z}^{\mathrm{T}}) = \mathbf{I}$ ($\mathbf{I}$ is a unit matrix); second, get a rotation matrix $\mathbf{W}$, such that $\mathbf{y} = \mathbf{W}\mathbf{z}$, by the specific independence optimization rule. In the ICA algorithm, the sources $\mathbf{s}$ consist of one Gaussian source at most, but the separated components $\mathbf{y}$ are uncertain in amplitude and order.

Single-channel ICA (Davies and James 2007) is an efficient algorithm that expands single-channel data into multichannel and transforms them to the same-dimensional independent components by standard ICA. If the vector $\boldsymbol{a}_i$ becomes a coefficient a_i in the mixing matrix $\mathbf{A}$, then the standard ICA transforms into single-channel ICA. Therefore, the single-channel ICA model can be written as:

$$x = \mathbf{A}\mathbf{s} = \sum_{i=1}^{n} a_i s_i \tag{3}$$

Under this situation, the main issue is the expansion of one signal x into multichannel observation in the single-channel ICA.

ICA-R is a kind of constrained ICA which can extract the desired source signals when the reference signals $\mathbf{r} = [r_1, r_2, \cdots, r_l]$ with a priori information of the source signals are available. In the ICA-R algorithm, the separated signals $\mathbf{y} = [y_1, y_2, \cdots, y_l]^T$ are given by $\mathbf{y} = \mathbf{B}\mathbf{x}$, where $\mathbf{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_l]^T$. When only one reference signal r is given, the desired component y could be given by $y = \boldsymbol{b}^T\mathbf{x}$. This special ICA-R is called one-unit ICA-R, and its constrained functions are as follows (Lu and Rajapakse 2006):

$$\text{Maximize} \quad J(y) \approx \rho[E\{G(y)\} - E\{G(v)\}]^2 \tag{4}$$

$$\begin{aligned}&\text{Subject to}\\ &g(\boldsymbol{b}) = \theta(r, y) - \xi \leq 0, \;\; h(\boldsymbol{b}) = E\{y^2\} - 1 = 0\end{aligned} \tag{5}$$

where $J(y)$ is the negentropy of y, ρ is a positive constant, v is Gaussian variable with same variance as y, $G(\cdot)$ is the quadratic function, $\theta(r, y)$ is a principle of proximity given by $E\{(y - r)^2\}$, and ξ is a threshold parameter with $\theta(r, y) - \xi \leq 0$ only when y reaches the global optimum solution. The inequality constraint $g(\boldsymbol{b})$ can be transformed into an equation $g(\boldsymbol{b}) + Z^2 = 0$. By explicitly manipulating the optimum Z^{2*}, an augmented Lagrange function could be given by

$$\begin{aligned}L_{\boldsymbol{b}} = J(y) - \frac{1}{2\gamma}[\max^2\{0, \mu + \gamma g(\boldsymbol{b})\} - \mu^2] - \lambda h(\boldsymbol{b})\\ - 0.5\gamma||h(\boldsymbol{b})||^2\end{aligned} \tag{6}$$

where u and λ are the Lagrange multipliers; γ is the learning rate.

With some rules of optimization and learning algorithms such as Newton-like learning algorithm, the demixing matrix could be derived as

$$\boldsymbol{b}_{k+1} = \boldsymbol{b}_k - \eta \mathbf{R}_{\mathbf{xx}}^{-1} L'_{\boldsymbol{b}_k} / \delta(\boldsymbol{b}_k) \tag{7}$$

where k indicates iterations, η is the learning rate, $\mathbf{R}_{\mathbf{xx}}$ is the covariance matrix of signals $\mathbf{x}$, and $L'_{\boldsymbol{b}_k}$ is the first-order derivative of Lagrange function $L_{\boldsymbol{b}_k}$ with respect to $\boldsymbol{b}_k$. $L'_{\boldsymbol{b}_k}$ and $\delta(\boldsymbol{b}_k)$ are given respectively by

$$L'_{\boldsymbol{b}_k} = \tilde{\rho}\, E\{\mathbf{x}G'_y(y)\} - 0.5\mu E\{\mathbf{x}g'_y(\boldsymbol{b}_k)\} - \lambda E\{\mathbf{x}y\} \tag{8}$$

$$\delta(\boldsymbol{b}_k) = \tilde{\rho}\, E\{G''_y(y)\} - 0.5\mu E\{g''_y(\boldsymbol{b}_k)\} - \lambda \tag{9}$$

where $\tilde{\rho} = \rho[E\{G(y)\} - E\{G(v)\}]$, $G'_y(y)$ and $g'_y(\boldsymbol{b}_k)$ are the first-order derivatives of $G_y(y)$ and $g_y(\boldsymbol{b}_k)$ with respect to y; $G''_y(y)$ and $g''_y(\boldsymbol{b}_k)$ are the second-order derivatives of $G_y(y)$ and $g_y(\boldsymbol{b}_k)$ with respect to y. The Lagrange multipliers μ and λ are learned respectively by

$$\mu_{k+1} = \max\{0, \mu_k + \gamma g(\boldsymbol{b}_k)\} \tag{10}$$

$$\lambda_{k+1} = \lambda_k + \gamma h(\boldsymbol{b}_k) \tag{11}$$

More mathematical details on ICA and ICA-R can be found in Hyvärinen et al. (2000) and Lu et al. (2006).

Empirical mode decomposition

The method of EMD is designed to decompose a single-channel signal $x(t)$ into a number of 'intrinsic mode function' (IMF) components and a final residual (Res) through a sifting process (Huang et al. 1998) as

$$x(t) = \sum_{i=1}^{R} IMF_i(t) + Res(t) \tag{12}$$

An IMF is a function that satisfies two conditions: (1) In the whole data set, the number of extremes and the number of zero crossings must either be equal to or differ at most by one, and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

The process of EMD begins with the identification of the local minima and maxima of a time series $x(t)$. First, identify all local minima and maxima and connect them with cubic splines to form an upper envelope $e_u(t)$ and a lower envelope $e_l(t)$, respectively. The local mean can be given by:

$$m_1(t) = \frac{e_u(t) + e_l(t)}{2} \tag{13}$$

So, the first component $h_1(t)$ can be obtained in the following equation:

$$h_1(t) = x(t) - m_1(t) \tag{14}$$

In the subsequent sifting process, regard $h_1(t)$ as the time series, and get

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t) \tag{15}$$

The above sifting process stops until the component $h_{1k}(t)$ is an IMF, such as

$$\mathrm{IMF}_1(t) = h_{1k}(t) \tag{16}$$

Once an IMF has been determined, it is subtracted from the original time series, and the process is repeated until the residual is less than a chosen threshold.

The constituent IMFs preserve not only the frequencies but also the length of the signals. As the decomposition is based on the direct extraction of the energy associated with various intrinsic timescales of signals, mode mixing during the sifting process would be possible. EMD makes it possible to address the nonstationary, nonlinear multipath issues. Because the IMFs are formed by explicitly fitting the envelope of a time series, we can expect that IMFs derived from coordinate time series closely reflect underlying multipath error.

Principal component analysis

Another component analysis that will be applied in this study is PCA, which is a well-known algorithm for feature extraction by transforming the high-dimensional input vectors into low-dimensional ones whose components are uncorrelated. PCA can project the high-dimensional vectors into other axes that are orthogonal to the statistically determined variances (Moore 1981). Assuming $\mathbf{R_{xx}}$ is the covariance matrix of signals $\mathbf{x}$, PCA finds the eigenvalue $\mathbf{D}$ from the following equation:

$$\mathbf{EDE}^T = \mathbf{R_{xx}} \tag{17}$$

where $\mathbf{E}$ is the eigenvector. The diagonal values of $\mathbf{D}$ are sorted in a descending order, such that $\mathbf{D} = diag([\delta_1^2, \delta_2^2, \cdots, \delta_m^2])$, where $\delta_1^2 \geq \delta_2^2 \geq \cdots \geq \delta_m^2$. Then the principal component $\mathbf{p}$ could be obtained by

$$\mathbf{p} = \mathbf{E^T x} \tag{18}$$

For the above reasons, the first principal component contains the largest variance and presents the main feature of the signals.

Development of a combined component analysis model

In this study, coordinate time series from short-baseline GPS deformation monitoring are the observation signals, while multipath and displacement series are the desired components. Single-channel ICA is required since only one channel observation datum is available in a deformation monitoring application. In order to decompose single-channel coordinate series into multichannel observation input of the ICA model, EMD is applied because of its direct extraction of the energy and stationarity. Instead of standard ICA, ICA-R is used in the single-channel ICA to extract more accurate desired signals. The use of a reliable reference signal is the key to know whether ICA-R can be successfully used to decompose signals. The mathematical feature of PCA is that the first principal component (PC) is usually the most common mode part of input multichannel signals of PCA. Thus, when several days' coordinate series are the input of PCA, the first PC is most likely to be the multipath signal of these days due to their high correlation. Then, the multipath signal extracted by the PCA can be used as the reference signal to the combined EMD and ICA-R method. Using PCA, ICA-R and EMD, we develop two combined methods, i.e., EMD-ICA-R and PCA-EMD-ICA-R, which will be discussed below in detail.

EMD-ICA-R method

As discussed above, we jointly use the EMD and ICA-R (EMD-ICA-R) method to identify and remove multipath in GPS coordinates for GPS dynamic deformation monitoring.

The implementation of the EMD-ICA-R algorithm can be divided into five steps:

(1) Denoise the raw coordinate time series of the first day, which are assumed to be a mixture of the multipath signal and other noises, to get multipath by wavelet filter analysis, and use the multipath as the reference signal r of ICA-R for the next day.

(2) Expand the raw coordinate time series on the next day to R intrinsic mode functions (IMFs) series and one Res (final residual) by the EMD method and get the virtual observations $\mathbf{x}_{(R+1)\times L}$, given by $\mathbf{x}_{(R+1)\times L} = [IMF_1, IMF_2, \cdots, IMF_R, Res]^T$, where L is the length of the IMF series. Then the corresponding multipath y can be obtained from $\mathbf{x}_{(R+1)\times L}$ by using ICA-R.

(3) Derive an amplitude scale factor a, given by $a = std(r)/std(y)$, where $std(\cdot)$ denotes the standard deviation estimator.

(4) Subtract the multipath model s, which is recovered by $s = ay$, from the raw coordinate time series to get the multipath-free solution.

(5) Regard the obtained multipath error model s as the reference signal for the next day sequentially and repeat steps (2)–(5).

From the above steps, the EMD-ICA-R method can not only separate the multipath using the first day's coordinate time series as a reference, but also output a reference multipath signal for use in the following day's analysis. It means that the EMD-ICA-R method can avoid the decrease in multipath repeatability as the interval between first day and subsequent days increases. This is a very important improvement to the traditional sidereal filtering method.

PCA-EMD-ICA-R method

In real deformation monitoring applications, the multipath signal determined from a single day may be contaminated by other error sources. To obtain a more accurate multipath signal as the reference, the first consecutive 3 days' coordinate time series were used, and the PCA algorithm is invoked to extract a reference multipath signal. After introducing the PCA, we rename the EMD-ICA-R method as PCA-EMD-ICA-R. In this method, PCA is used to transform those 3 days' multipath errors into three principal components, while the first principal component can be considered as the reference signal and the other components are uninteresting signals with noise. The corresponding processing steps are provided below:

(1) Choose coordinate time series of the first consecutive 3 days as the inputs for PCA if it is possible to assume that on those 3 days there is no deformation, and obtain the reference signal through denoising the first principal component by the wavelet filter.

(2) The following steps are the same as steps (2)–(5) in the EMD-ICA-R method.

Experiments and analysis of results

In order to test our proposed methods, data from an experiment with a motion simulation platform were used. This experiment was conducted on the roof of the Hong Kong Polytechnic University's teaching building Core E, which is about 20 m high. The location was chosen because it was clearly subject to multipath effects from the smooth reflective surfaces of nearby higher buildings. A Topcon GPS L1/L2 dual-frequency receiver GB1000 with a Topcon CR3 choke antenna was used as a base station, and a Leica 500 GPS L1 single-frequency receiver with a Leica AX1202 lightweight antenna was installed on the motion simulation platform as a rover station (see Fig. 1). The baseline between the base station and rover station was about 4 m. Due to this short baseline, satellite clock errors, receiver clock errors, satellite orbit error, ionospheric delay, tropospheric delay as well as other common errors are removed by double differencing. The

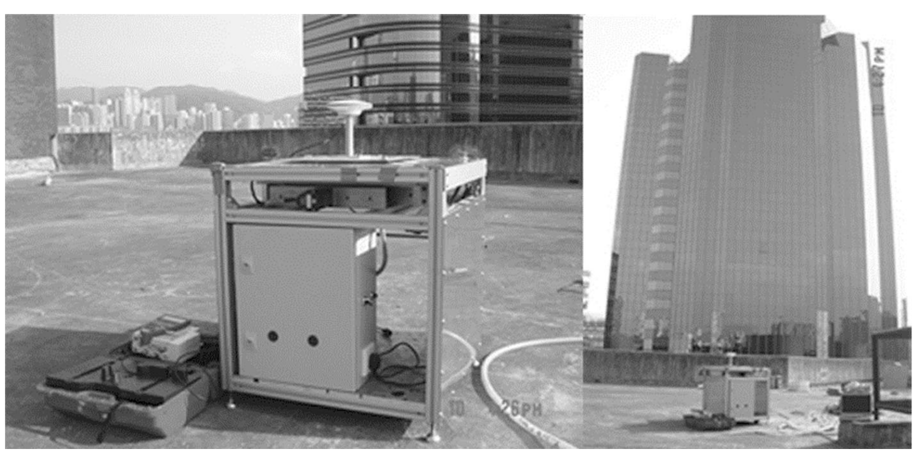


Fig. 1 GPS receiver setup on a motion simulation platform

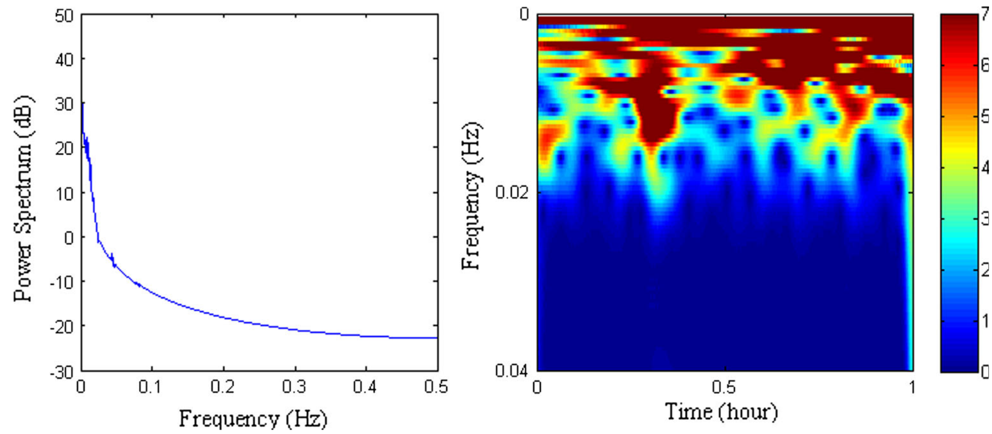


Fig. 2 Spectral analysis results of raw coordinate time series in north direction. Frequency spectrogram of maximum entropy (*left*) and time–frequency of diagram by wavelet analysis (*right*)

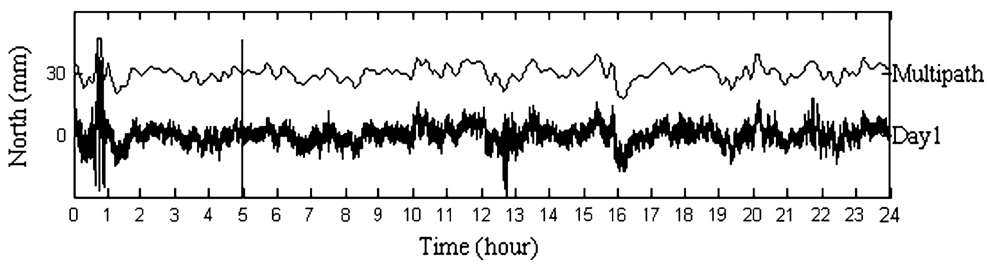


Fig. 3 Raw coordinate time series in north direction of Day 1 and its low-frequency multipath error by wavelet-based denoising. Note that to show all signals such as raw coordinate time series and multipath errors in one figure, each adjacent signal is shifted by a constant (same for Figs. 4, 5, 7 and 8)

test began on August 4, 2006, and lasted half a month. Except for a break on the 7th and 8th days caused by the power supply, continuous data were collected.

For Day 1 to Day 3, the platform with the GPS antenna was kept static. In-house computer software developed by The Hong Kong Polytechnic University, namely GPS Structure Monitoring (GPSSM), was then used to process the GPS data using double differencing with fixed ambiguities in a post-processing kinematic mode. The position of the rover antenna (with a data cutoff angle of 15 °) was finally solved in the coordinate system of the platform. The raw coordinate time series of these 3 days only consist of multipath and noise as the platform is static, and all other error sources have been removed by double-differencing processing. The spectral analysis results of the raw coordinate time series on Day 1 in the north direction are shown in Fig. 2. It can be seen that the frequency of raw coordinate time series is mostly below 0.02 Hz. The low-frequency multipath in the north direction of Days 1–3 is derived following step (1) of EMD-ICA-R method with wavelet-based denoising (Figs. 3 and 4), where nine levels of decomposition are performed with 8th Daubechies (db8) mother wavelet to make the coordinate time series smooth in a low-frequency domain. Note that in this study we only present graphics for the north direction. Our analysis indicated that the component analysis methods presented here performed equally well on east and up components (see Table 1), so we present only north component graphics to conserve space.

From Day 4 to Day 15, the platform generated a simulated deformation signal that consists of 0.15 and 0.015 Hz (7- and 67-s periods) sinusoidal signals both with amplitude of 5 mm; 0.015 Hz is included in the range of the multipath signal frequency, so there exits frequency aliasing between the deformation signal and multipath error. All the observations have an advance of about 246 s to take into account the GPS satellite orbital displacements from the solar day. Figure 4 shows raw coordinate time series in the north direction from Day 2 to Day 15, and Fig. 5 indicates their multipath errors after removing the known platform input signals and denoising noise by wavelet filter. Thus, the time series shown in Fig. 5 are the low-frequency multipath errors on each day. This study will compare the various multipath extraction methods against the results in Fig. 5.

Figure 6 shows the comparison of RMS of the differenced coordinate time series between the known platform input signals and the denoised coordinate time series in the north, east and up directions after applying the multipath corrections by the traditional sidereal filtering, EMD-ICA-R and PCA-EMD-ICA-R methods. In the traditional sidereal

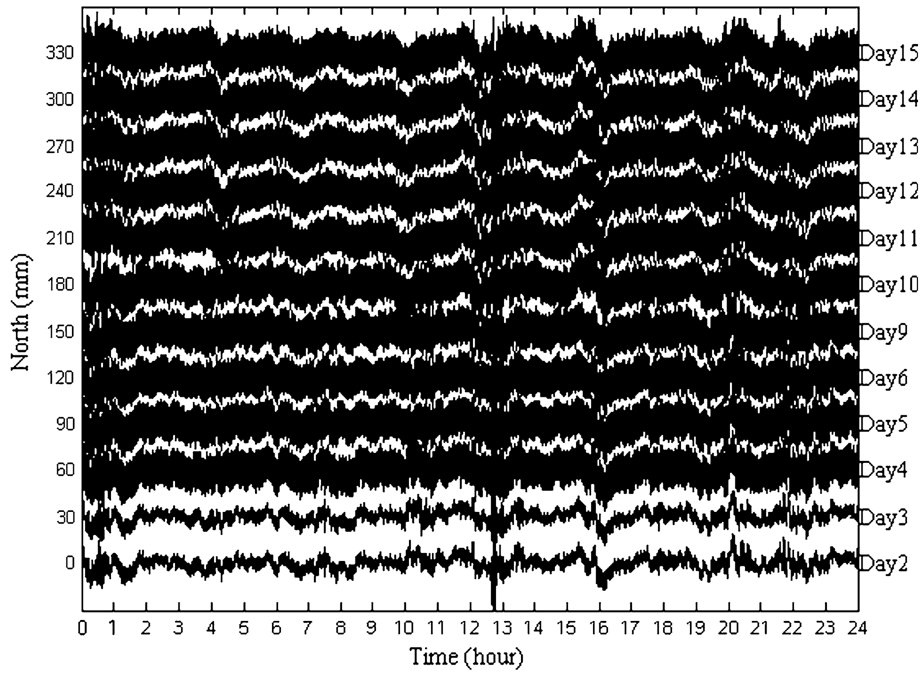


Fig. 4 Raw coordinate time series for north direction, where data from Day 4 to Day 15 bear simulated deformation signals

Table 1 Mean RMS values of the coordinate time series and percentage improvement in accuracy in each direction after multipath corrections using three methods (unit: mm)

Direction	Before	Trad	Impt (%)	EMD-ICA-R	Impt (%)	PCA-EMD-ICA-R	Impt (%)
North	3.48	2.88	17.3	1.31	62.4	1.24	64.5
East	3.10	2.10	32.4	1.04	66.6	1.02	67.3
Up	9.91	7.49	24.4	4.21	57.5	3.54	64.3

Trad, traditional sidereal filtering using wavelet filter; Impt, percentage improvement in accuracy

filtering, the first day's multipath models are established using wavelet filter and are removed from coordinate time series of consequent 2–15 days. As indicated in Fig. 6, the accuracy of the corrected coordinate time series of the initial days is relatively high (low RMS) but will decrease quickly on subsequent days. Table 1 shows the average RMS values and the percentage improvement of the differenced coordinate time series in north, east and up directions after applying each of the different multipath corrections.

We then applied EMD-ICA-R to the raw coordinate time series of Fig. 4 to generate each day's multipath models (see Fig. 7). Each day's multipath models are similar, showing low-frequency multipath as in Fig. 5, but not completely identical. Figure 6 indicates that RMS values after multipath corrections using EMD-ICA-R keep almost unchanged and significantly smaller than the ones using the traditional sidereal filtering. The average RMS values in north, east and up directions after multipath corrections using EMD-ICA-R are about 1.31, 1.04 and 4.21 mm, respectively. When applying PCA-EMD-ICA-R, the reference multipath signal obtained from first 3 days' coordinate time series and also each day's multipath models from Day 2 to Day 15 were established (see Fig. 8). Figure 8 shows that multipath models from PCA-EMD-ICA-R method are also similar to the low-frequency multipath errors on each day in Fig. 5 but smoother than the ones extracted by EMD-ICA-R, which is primarily due to the use of 3 days' average multipath as reference signal. The RMS values after applying the multipath corrections using PCA-EMD-ICA-R method are also shown in Fig. 6, and the corresponding average RMS values in the north, east and up directions are about 1.24, 1.02 and 3.54 mm, respectively (see Table 1). It can be seen from the RMS values showed in Fig. 6 and Table 1 that PCA-EMD-ICA-R method can reduce the effects of multipath more significantly than the EMD-ICA-R method.

The results of the experiment show that the two new component analysis methods can reduce the multipath error by not less than 57 %, and the derived multipath repeatability will not decrease over time. As presented in the component methods, we always use the desired output multipath models as a reference signal for next day. It means that the multipath models can update themselves day to day and take advantage of high repeatability of two adjacent days' multipath.

Experiment with simulated deformation signals in the coordinate time series of Days 1–3

In the above, the platform was static for Days 1–3, namely without deformation signals. However, it is very difficult to

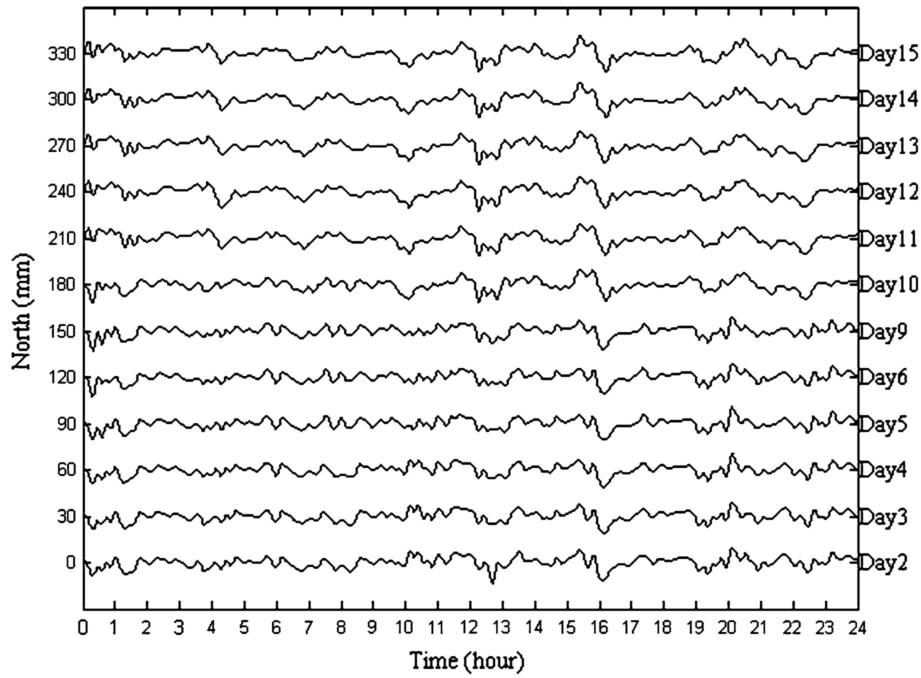


Fig. 5 Low-frequency multipath in north direction from Day 2 to Day 15

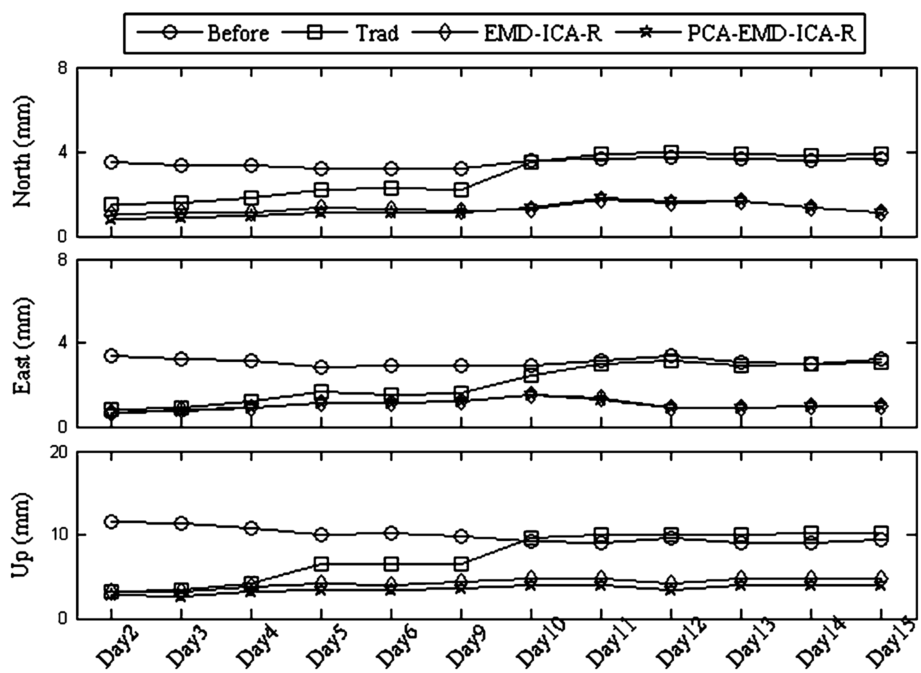


Fig. 6 Comparison of RMS of the coordinate time series after correction of three methods in each direction. Note: Trad, traditional sidereal filtering using wavelet filter

keep the antenna absolutely static in real large structure deformation monitoring. In order to test the practicability of the two newly proposed methods, we add some simulated different deformation signals into the coordinate time series of first 3 days for a further experiment. Three simulated sinusoidal signals with amplitude of 5 mm are added to coordinate time series as deformation signals as follows:

$$\mathbf{v} = \begin{bmatrix} 5\sin(2\pi t/20000) \\ 5\sin(2\pi(t+3600)/12500) \\ 5\sin(2\pi(t+7200)/10000) \end{bmatrix}$$

Figure 9 shows the RMS values of the coordinate time series in the north, east and up directions after applying the multipath corrections using traditional sidereal filtering

Fig. 7 Multipath models in north direction extracted by EMD-ICA-R method

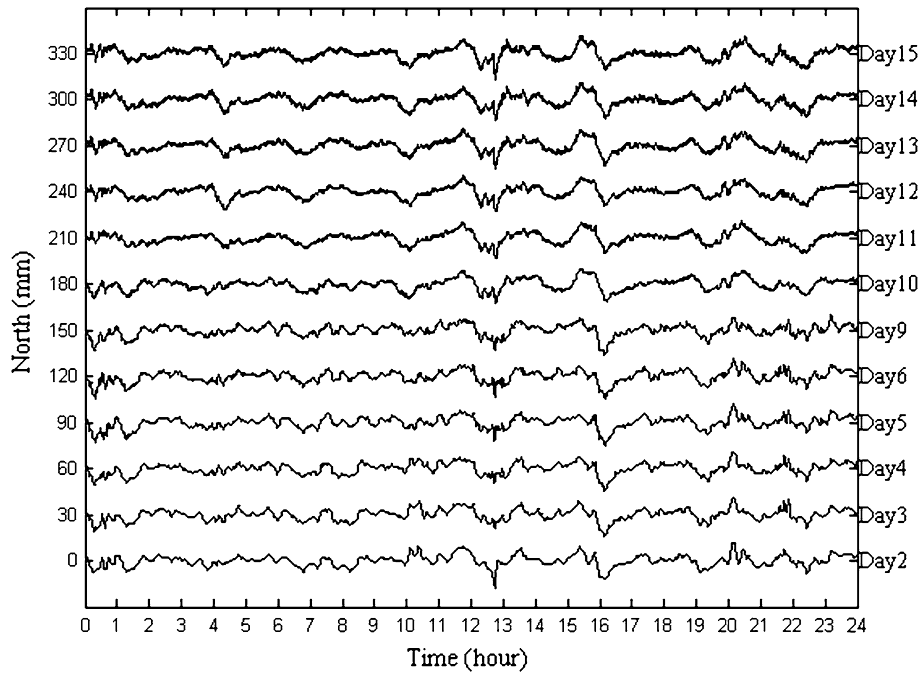
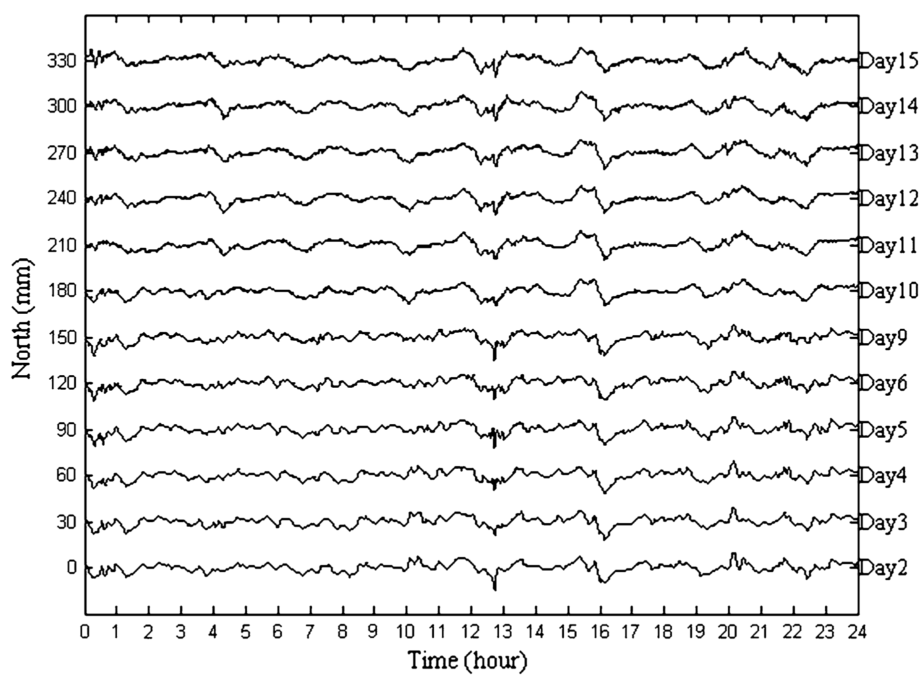


Fig. 8 Multipath models in north direction extracted by PCA-EMD-ICA-R method



method, EMD-ICA-R and PCA-EMD-ICA-R methods following the procedure of the static case. The corresponding average RMS values in the north, east and up directions are shown in Table 2. The results indicate that two newly proposed methods still can mitigate the multipath errors effectively even when there are some deformation signals in the coordinate time series on Days 1–3. More encouragingly, PCA-EMD-ICA-R can reduce the multipath error by around 45–63 %.

Experiment with various wavelet filters

In the two newly proposed approaches, a wavelet filter is used to denoise the coordinate time series with an accurate reference multipath signal. In order to test whether the new approaches will suffer from the choice of mother wavelet and decomposition levels, the experiment without deformation signals in the first 3 days was repeated using 8th Daubechies (db8) and 8th Symlets (sym8) wavelets with different

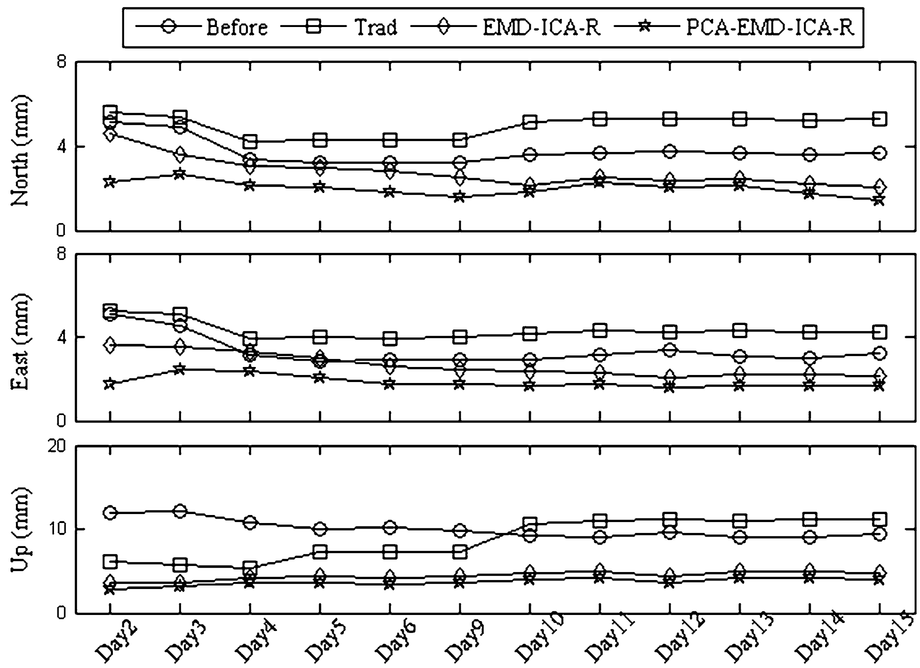


Fig. 9 Comparison of RMS of the coordinate time series after applying three methods in each direction when deformation signals were introduced in the first 3 days. *Note*: Trad, traditional sidereal filtering using wavelet filter

Table 2 Mean RMS values of the coordinate time series and percentage improvement in accuracy in each direction after applying three methods when deformation signals from Day 1 to Day 3 (unit: mm)

Direction	Before	Trad	Impt (%)	EMD-ICA-R	Impt (%)	PCA-EMD-ICA-R	Impt (%)
North	3.75	4.95	−32.0	2.76	26.4	1.98	47.2
East	3.36	4.30	−28.0	2.64	21.4	1.85	44.9
Up	10.02	8.74	12.8	4.41	56.0	3.71	63.0

Trad, traditional sidereal filtering using wavelet filter; Impt, percentage improvement in accuracy

decomposition levels for reference multipath signal. Table 3 shows the mean RMS values of the differenced coordinate time series and the improvements in accuracy after applying corrections using PCA-EMD-ICA-R method with various wavelet filters. The results indicate that PCA-EMD-ICA-R method can work well with various wavelet filters and slightly be affected by decomposition levels.

Conclusions

We proposed two sidereal filtering approaches for multipath mitigation. One is EMD-ICA-R, a combination of

Table 3 Mean RMS values of the coordinate time series and percentage improvement in accuracy in each direction before and after applying the multipath corrections using PCA-EMD-ICA-R with variant wavelet filters (unit: mm)

Direction	db8, 7 levels			db8, 8 levels			db8, 9 levels			db8, 10 levels		
	Before	After	Impt (%)	Before	After	Impt (%)	Before	After	Impt (%)	Before	After	Impt (%)
North	3.90	1.65	57.7	3.71	1.35	63.6	3.48	1.24	64.5	3.17	1.54	51.4
East	3.47	1.48	57.3	3.32	1.22	63.3	3.10	1.02	67.3	2.93	1.12	61.8
Up	11.02	4.36	60.4	10.53	3.71	64.8	9.91	3.54	64.3	9.28	4.28	53.9
Direction	sym8, 7 levels			sym8, 8 levels			sym8, 9 levels			sym8, 10 levels		
	Before	After	Impt (%)	Before	After	Impt (%)	Before	After	Impt (%)	Before	After	Impt (%)
North	3.90	1.65	57.7	3.71	1.36	63.3	3.47	1.24	64.3	3.19	1.51	52.7
East	3.48	1.48	57.5	3.32	1.22	63.3	3.10	1.01	67.4	2.92	1.12	61.6
Up	11.02	4.36	60.4	10.54	3.72	64.7	9.92	3.54	64.3	9.28	4.27	54.0

Impt, percentage improvement in accuracy

EMD and ICA-R, which is more appropriate for mitigating multipath than the traditional sidereal filtering method. Another is PCA-EMD-ICA-R, a combination of PCA, EMD and ICA-R, which can be regarded as an upgraded EMD-ICA-R method. The PCA-EMD-ICA-R method is more effective than the EMD-ICA-R method because more accurate reference multipath signal can be extracted using the PCA with 3 days' coordinate time series. Both EMD-ICA-R and PCA-EMD-ICA-R methods have the advantage that the multipath models can update themselves day to day and take advantage of the high repeatability of two adjacent days' multipath. This means that the new approaches can fix the sidereal filtering problem that the repeatability of multipath decreases as the interval between the first day and subsequent days increases. The results of the experiment with a static situation for Days 1–3 show that the two new component analysis methods can reduce the multipath errors by around 67 % when the reference multipath signal can be extracted from a static situation. Furthermore, the results of another experiment with simulated deformation signals in the coordinate time series of Days 1–3 show that the PCA-EMD-ICA-R method can still mitigate the multipath errors significantly when the static situation during the first day is not available.

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