Системы второго порядка (у/з)

№1 ① $\begin{cases} \dot{x} = y \\ \dot{y} = -2x - 3y \end{cases}$

$A = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$ $\quad \tau = -3$, $\Delta = 2$ $\Rightarrow$ устойчивый узел.

$\lambda^2 - \tau\lambda + \Delta = 0$

$\lambda^2 + 3\lambda + 2 = 0 \quad \Rightarrow D = 9 - 8 = 1$

$\lambda_1 = \frac{-3+1}{2} = -1$

$\lambda_2 = \frac{-3-1}{2} = -2$

$\begin{pmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

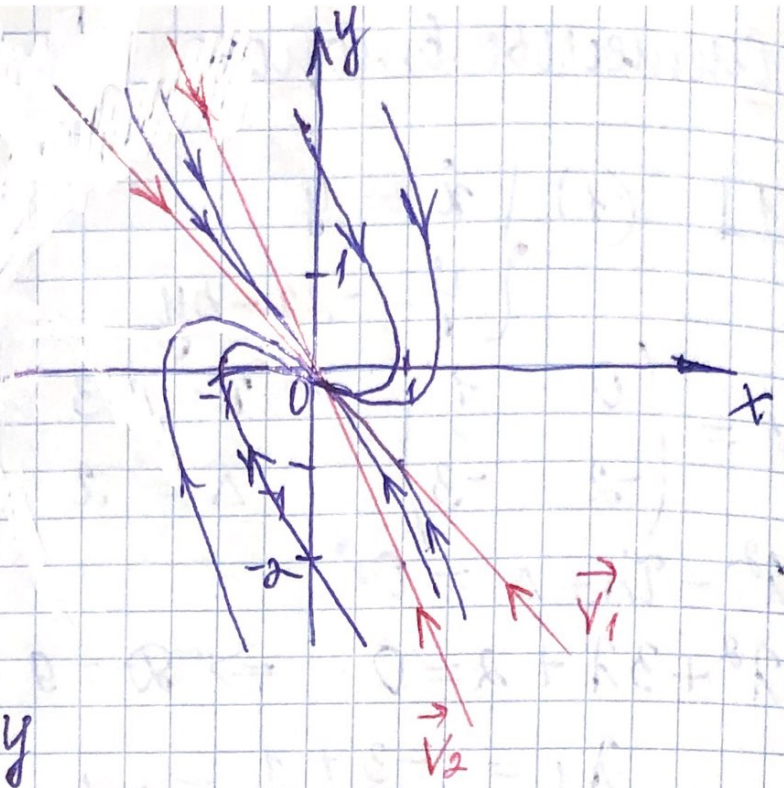
$\lambda_1 = -1$: $\begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} v_1^{(1)} \\ v_2^{(1)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_1^{(1)} = -v_2^{(1)}$

$(1; -1)$

$\lambda_2 = -2$: $\begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} v_1^{(2)} \\ v_2^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_2^{(2)} = -2 v_2^{(2)}$

$(1; -2)$

$\vec{v_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$; $\vec{v_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ – собств. вектора



② $\begin{cases} \dot{x} = 3x - 4y \\ \dot{y} = x - y \end{cases}$

$A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

$\tau = 2$

$\Delta = -3 + 4 = 1$

$\tau^2 = 4\Delta \Rightarrow$ неустойчив вырожден. узел.

$\lambda^2 - 2\lambda + 1 = 0$

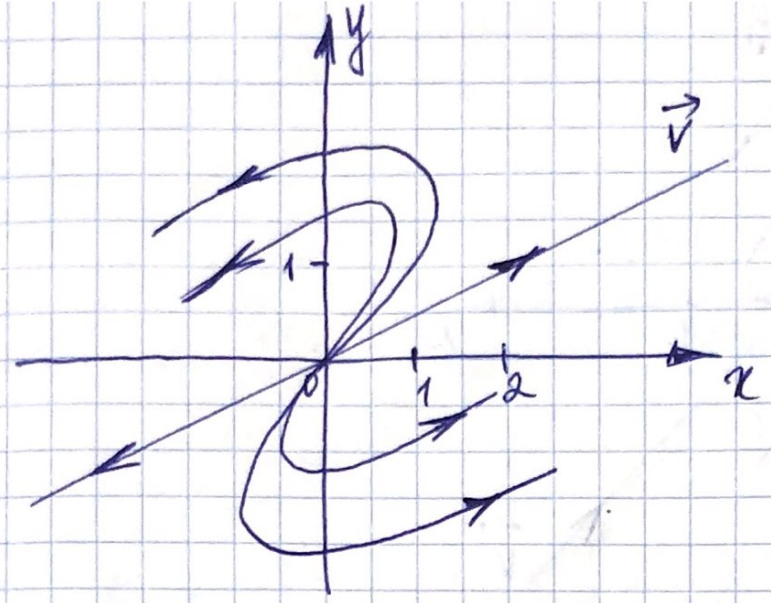
$(\lambda - 1)^2 = 0 \Rightarrow \lambda_1 = \lambda_2 = 1$

$\begin{pmatrix} -3-\lambda & -4 \\ 1 & -1-\lambda \end{pmatrix} \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -4 & -4 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$v^{(1)} = 2v^{(2)}$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \vec{v}$



③ $\begin{cases} \dot{x} = y \\ \dot{y} = -x - 2y \end{cases}$

$A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$

$\tau = -2$

$\Delta = 0 + 1 = 1$ $\Rightarrow$ устойчив. вырожд. узел

$\tau^2 = 4\Delta$

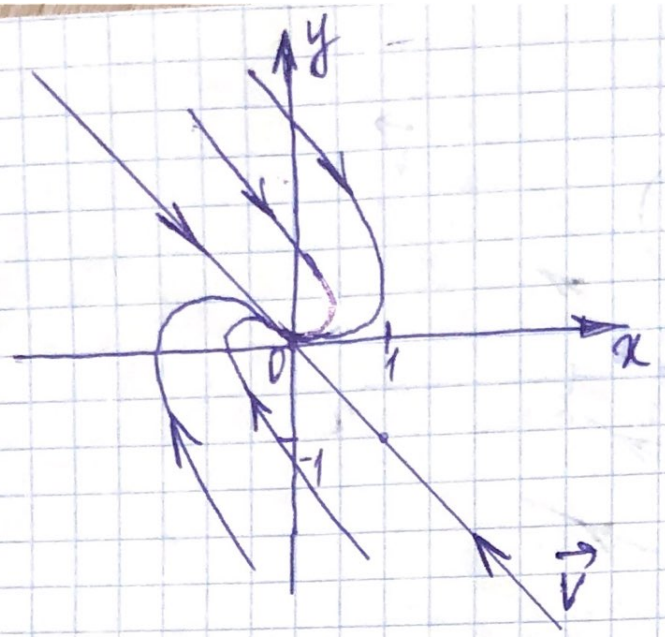
$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda+1)^2 = 0 \Rightarrow \lambda_1 = \lambda_2 = -1$

$\begin{pmatrix} -\lambda & 1 \\ -1 & -2-\lambda \end{pmatrix} \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \quad v^{(1)} = -v^{(2)}$

$\vec{v} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$



④ $\begin{cases} \dot{x} = 4x - 3y \\ \dot{y} = 8x - 6y \end{cases}$

$A = \begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix}$

$\tau = -2$

$\Delta = -24 + 24 = 0$

⇓

нестрогих. стац. точка

$\lambda^2 + 2\lambda + 0 = 0$

$\lambda(\lambda + 2) = 0 \Rightarrow \lambda_1 = 0$

$\lambda_2 = -2$

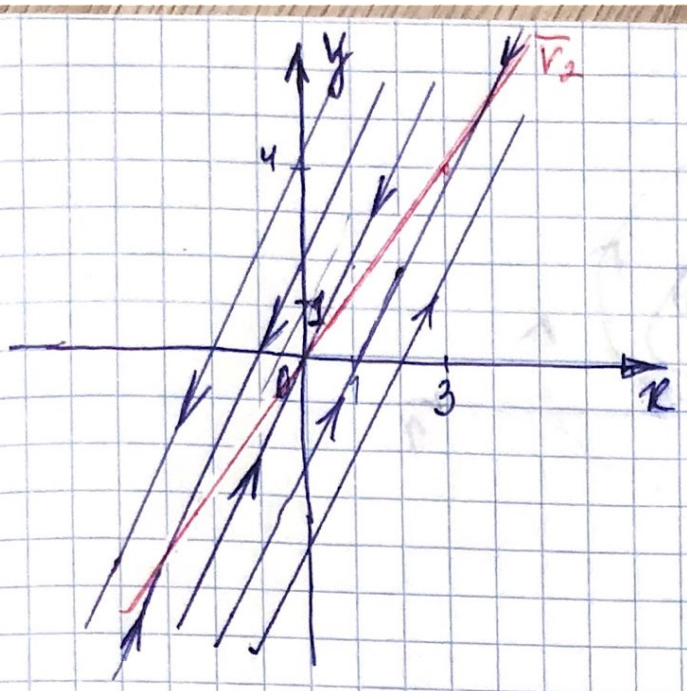
$\lambda_2 = -2: \begin{pmatrix} 6 & -3 \\ 8 & -4 \end{pmatrix} \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 6v^{(1)} = 3v^{(2)}$

$v^{(2)} = 2v^{(1)} \Rightarrow \vec{V}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda_1 = 0: \begin{pmatrix} 4 & -3 \\ 8 & -6 \end{pmatrix} \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 4v^{(1)} = 3v^{(2)}$

$v^{(1)} = \frac{3}{4} v^{(2)}$

$\vec{V}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$



семейство параллельных прямых

$y = ax + b$

$\frac{dy}{dx} = a$

$\frac{dy}{dx} = \frac{8x - 6y}{4x - 3y} = 2 = a$

$\boxed{y = 2x + b}$, b – const

№2 1) $\begin{cases} \dot{x} = x - y \\ \dot{y} = x^2 - 4 \end{cases}$

Стац. точки: $\begin{cases} x - y = 0 \Rightarrow x = y \\ x^2 - 4 = 0 \Rightarrow x = \pm 2 \end{cases}$

$(x_1^*, y_1^*) = (2; 2)$

$(x_2^*, y_2^*) = (-2; -2)$

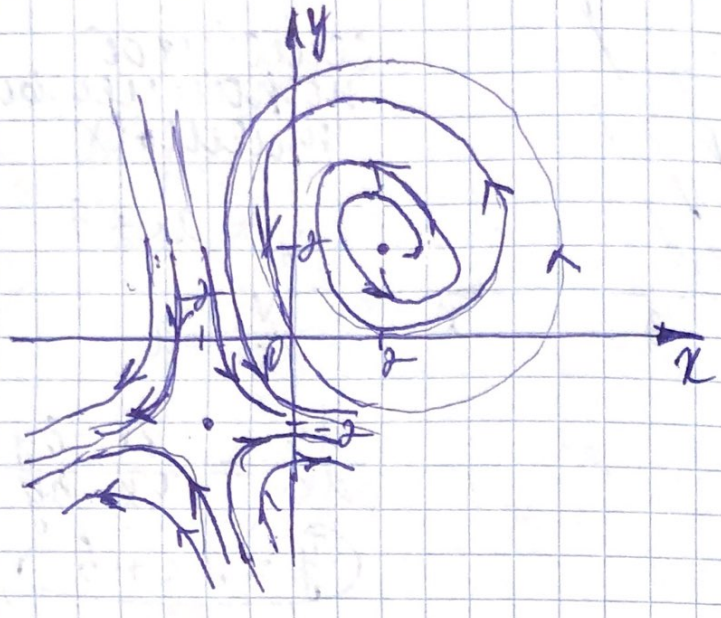
$$J_1 = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix}\Bigg|_{(x_1^*, y_1^*)} = \begin{pmatrix} 1 & -1 \\ 2x & 0 \end{pmatrix}\Bigg|_{(2;2)} = \begin{pmatrix} 1 & -1 \\ 4 & 0 \end{pmatrix}$$

$\tau = 1$; $\Delta = 4$

$\Rightarrow$ неустойчив. фокус

$$J_2 = \begin{pmatrix} 1 & -1 \\ 2x & 0 \end{pmatrix}\Bigg|_{(-2;-2)} = \begin{pmatrix} 1 & -1 \\ -4 & 0 \end{pmatrix}$$

$\left.\begin{matrix} \tau = 1 \\ \Delta = -4 \end{matrix}\right| \Rightarrow$ седло



② $\begin{cases} \dot{x} = \sin y \\ \dot{y} = \cos x \end{cases}$

стац. точки: $\begin{cases} \sin y = 0 \\ \cos x = 0 \end{cases} \Rightarrow \begin{array}{l} y = 0 \\ x = \frac{\pi}{2} \end{array}$

$(x^*, y^*) = \left(\frac{\pi}{2}; 0\right)$

$$J = \begin{pmatrix} 0 & \cos y \\ -\sin x & 0 \end{pmatrix}\Bigg|_{(x^*, y^*)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$\tau = 0$

$\Delta = 0 + 1 = 1$

центр ⇐