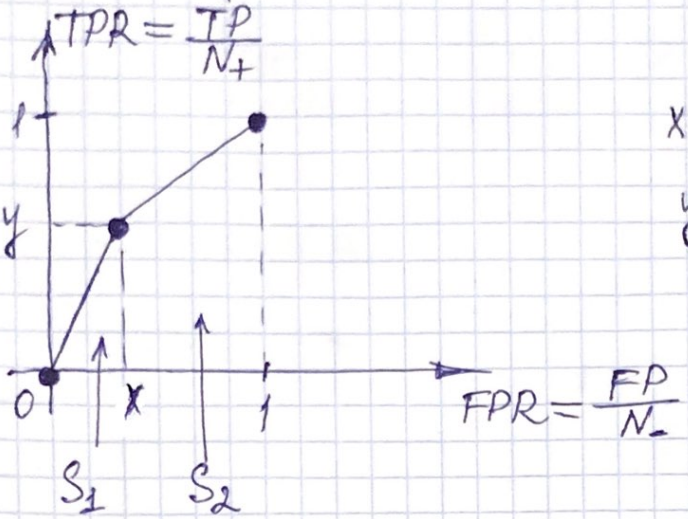
Задание №1

1) ROC-кривая



$x = \frac{FP}{N_-}$

$y = \frac{TP}{N_+}$

2) $AUC\text{-}ROC = S_1 + S_2$

$Acc = \frac{TP+TN}{N} \Rightarrow TP + TN = Acc \cdot N$

$TN = N \cdot Acc - TP$

$Re = \frac{TP}{TP+FN} = \frac{TP}{N_+} = y \Rightarrow TP = Re \cdot N_+$

$x = \frac{FP}{N_-} = \frac{N_- - TN}{N_-} = 1 - \frac{TN}{N_-} = 1 - \frac{N \cdot Acc - TP}{N_-} =$

$= 1 - \frac{N \cdot Acc - Re \cdot N_+}{N_-}$

$S_1 = \frac{1}{2} x \cdot y = \frac{1}{2}\left(1 - \frac{N \cdot Acc - N_+ Re}{N_-}\right) Re$

$S_2 = \frac{1}{2}(y+1)(1-x) = \frac{1}{2}(Re+1)\left(1 - 1 + \frac{N \cdot Acc - N_+ Re}{N_-}\right) =$

$= \frac{1}{2}(Re+1) \cdot \frac{N Acc - N_+ Re}{N_-} = \frac{1}{2} Re \cdot \frac{N Acc - N_+ Re}{N_-} + \frac{1}{2} \cdot \frac{N \cdot Acc - N_+ Re}{N_-}$

$AUC\text{-}ROC = \frac{1}{2} Re \left(1 - \frac{N \cdot Acc - N_+ Re}{N_-} + \frac{N \cdot Acc - N_+ Re}{N_-}\right) +$

$+ \frac{1}{2} \cdot \frac{N Acc - N_+ Re}{N_-} = \frac{1}{2} Re + \frac{1}{2}\left(\frac{N \cdot Acc - N_+ Re}{N_-}\right) =$

$= \frac{1}{2}\left(Re + \frac{N Acc - N_+ Re}{N_-}\right) = \frac{1}{2 N_-}\left(Re(N_- - N_+) + N \cdot Acc\right)$