$\boxed{\text{Задача 1}}$ Смещение константного алгоритма

$x \in R^d$, $x_i \in U[0,1]$, $i = 1, 2, \dots, d.$, $X = (x_i, y_i)_{i=1}^N$

$E[y|x] = x^T x$, $a(x) = const$

$$Q = \frac{1}{N}\sum_{i=1}^{N}(y_i - a)^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i - const)^2 \longrightarrow \min_{const}$$

$$a(x) = const = \frac{\sum_{i=1}^{N} y_i}{N}$$

$$p(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases}$$

$$E(x) = \int_0^1 x \cdot 1 \, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$D(x) = \int_0^1 \left(x - \frac{1}{2}\right)^2 dx = \left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{x}{4}\right)\Big|_0^1 = \frac{1}{12}$$

$$Bias = \left(E(y) - E(a)\right)^2$$ — для одного объекта

$$E[y|x] = x^T x = \sum_{j=1}^{d} x_j^2$$

$$E[a] = E\left[\frac{\sum_{i=1}^{N} y_i}{N}\right] = \frac{\sum_{i=1}^{N} E[y_i]}{N} = E\left[\sum_{j=1}^{d} x_j^2\right] = \sum_{j=1}^{d} E(x_j^2) =$$

$$= \sum_{j=1}^{d}\left(D(x_j) + (D(x_j))^2\right) = \sum_{j=1}^{d}\left(\frac{1}{12} + \frac{1}{4}\right) = \sum_{j=1}^{d}\frac{1}{3} = \frac{d}{3}$$

$$Bias(x) = \left(\sum_{j=1}^{d} x_j^2 - \frac{d}{3}\right)^2$$

Для нахождение смещение всего алгоритма, необходимо усреднить:

$$Bias = \left(E\left(\sum_{j=1}^{d} x_j^2\right) - \frac{d}{3}\right)^2 = \left(\frac{d}{4} - \frac{d}{3}\right)^2 = \frac{d^2}{144}$$

$$\boxed{\text{Ответ: } Bias = \frac{d^2}{144}}$$