Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page (http://vision.stanford.edu/teaching/cs231n/assignments.html)](http://vision.stanford.edu/teaching/cs231n/assignments.html) on the course website.

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [1]:
```
# Run some setup code for this notebook.

import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-i
python
%load_ext autoreload
%autoreload 2
```

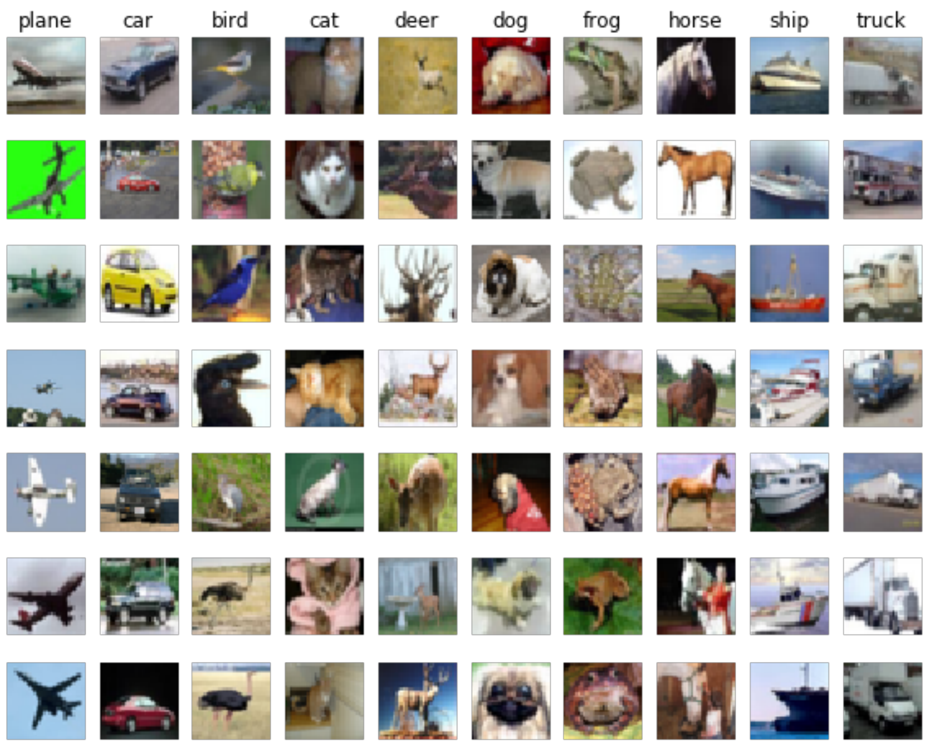
CIFAR-10 Data Loading and Preprocessing

In [2]:
```
# Load the raw CIFAR-10 data.
cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

In [3]:
```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', '
ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



In [4]:
```
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

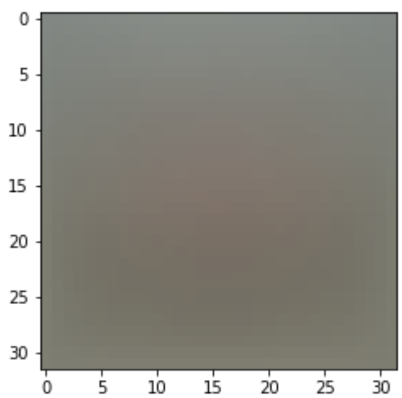
In [5]:
```
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

In [7]:
```
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the m
ean image
plt.show()
```

```
[ 130.64189796  135.98173469  132.47391837  130.05569388  135.34804082
  131.75402041  130.96055102  136.14328571  132.47636735  131.48467347]
```



In [8]:
```
# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

In [9]:
```
# third: append the bias dimension of ones (i.e. bias trick) so that our SV
M
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
```

```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

SVM Classifier

Your code for this section will all be written inside **cs231n/classifiers/linear_svm.py**.

As you can see, we have prefilled the function `compute_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

In [54]:
```
# Evaluate the naive implementation of the loss we provided for you:
from cs231n.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))
```

```
loss: 9.202859
```

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

In [79]:
```
# Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions
, and
# compare them with your analytically computed gradient. The numbers should
match
# almost exactly along all dimensions.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

```
numerical: -24.092120 analytic: -24.092120, relative error: 7.550954e-12
numerical: 7.529526 analytic: 7.529526, relative error: 4.703042e-13
numerical: -8.918813 analytic: -8.918813, relative error: 3.383046e-11
numerical: 5.565055 analytic: 5.565055, relative error: 6.561682e-12
numerical: -3.608664 analytic: -3.608664, relative error: 9.093804e-11
numerical: 10.627653 analytic: 10.627653, relative error: 4.644962e-11
numerical: -2.627004 analytic: -2.627004, relative error: 5.490596e-11
numerical: 8.915052 analytic: 8.915052, relative error: 1.449909e-11
numerical: 20.554319 analytic: 20.554319, relative error: 1.813724e-11
numerical: 18.340247 analytic: 18.340247, relative error: 2.353922e-11
numerical: 11.856638 analytic: 11.856638, relative error: 2.815582e-11
numerical: -22.011697 analytic: -22.011697, relative error: 8.336613e-12
numerical: 14.873594 analytic: 14.873594, relative error: 2.159896e-12
numerical: 7.400634 analytic: 7.400634, relative error: 3.708265e-11
numerical: 1.767468 analytic: 1.767468, relative error: 2.189601e-10
numerical: 20.518994 analytic: 20.518994, relative error: 1.824924e-12
numerical: 23.962368 analytic: 23.962368, relative error: 7.776575e-12
numerical: -5.598447 analytic: -5.598447, relative error: 3.810914e-11
numerical: -10.390464 analytic: -10.390464, relative error: 3.654251e-11
numerical: 15.408389 analytic: 15.408389, relative error: 2.661217e-11
```

Inline Question 1:

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? *Hint: the SVM loss function is not strictly speaking differentiable*

Your Answer: *fill this in.*

In [80]:
```
# Next implement the function svm_loss_vectorized; for now only compute the
loss;
# we will implement the gradient in a moment.
tic = time.time()
loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs231n.classifiers.linear_svm import svm_loss_vectorized
tic = time.time()
loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# The losses should match but your vectorized implementation should be much
faster.
print('difference: %f' % (loss_naive - loss_vectorized))
```

```
Naive loss: 9.202859e+00 computed in 0.144402s
Vectorized loss: 9.202859e+00 computed in 0.011787s
difference: -0.000000
```

In [82]:
```
# Complete the implementation of svm_loss_vectorized, and compute the gradi
ent
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match,
but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss and gradient: computed in %fs' % (toc - tic))

tic = time.time()
_, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, s
o
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('difference: %f' % difference)
```

```
Naive loss and gradient: computed in 0.129456s
Vectorized loss and gradient: computed in 0.011257s
difference: 0.000000
```

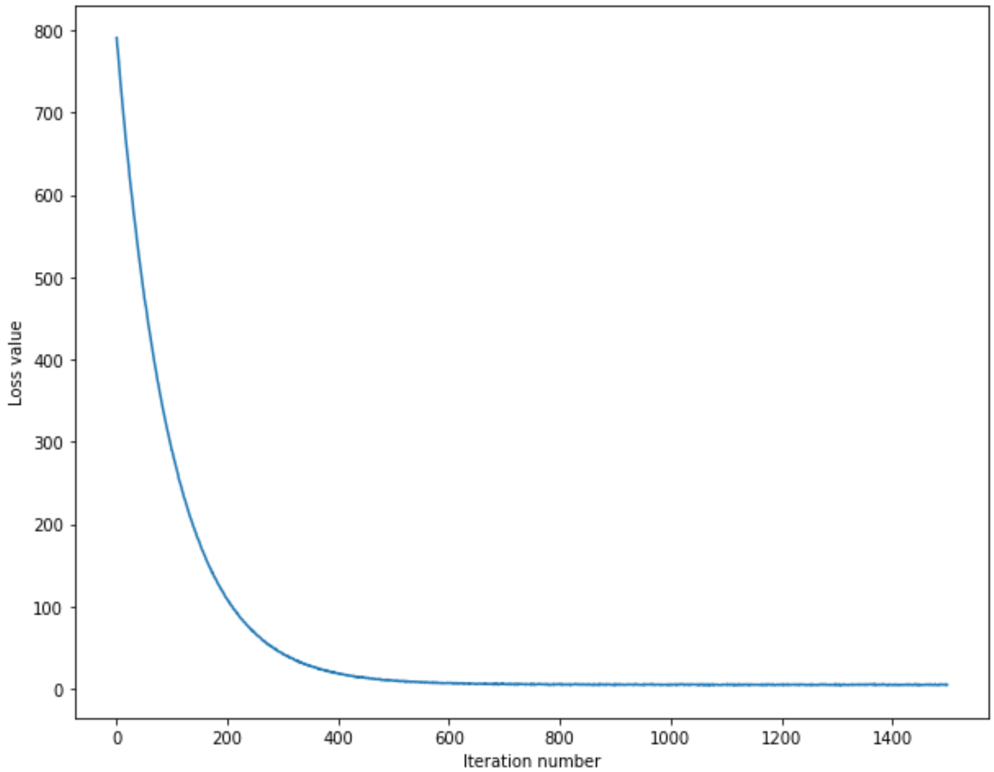
Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

In [88]:
```
# In the file linear_classifier.py, implement SGD in the function
# LinearClassifier.train() and then run it with the code below.
from cs231n.classifiers import LinearSVM
svm = LinearSVM()
tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 790.737796
iteration 100 / 1500: loss 289.712916
iteration 200 / 1500: loss 108.688994
iteration 300 / 1500: loss 43.347752
iteration 400 / 1500: loss 18.812025
iteration 500 / 1500: loss 9.998929
iteration 600 / 1500: loss 7.314452
iteration 700 / 1500: loss 5.737963
iteration 800 / 1500: loss 5.501456
iteration 900 / 1500: loss 4.761576
iteration 1000 / 1500: loss 5.100302
iteration 1100 / 1500: loss 5.164558
iteration 1200 / 1500: loss 5.347295
iteration 1300 / 1500: loss 4.882258
iteration 1400 / 1500: loss 5.226292
That took 11.754984s
```

In [89]:
```
# A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```



In [90]:
```
# Write the LinearSVM.predict function and evaluate the performance on both
the
# training and validation set
y_train_pred = svm.predict(X_train)
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = svm.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

```
training accuracy: 0.371857
validation accuracy: 0.381000
```

In [104]:
```
# Use the validation set to tune hyperparameters (regularization strength a
nd
# learning rate). You should experiment with different ranges for the learn
ing
# rates and regularization strengths; if you are careful you should be able
to
# get a classification accuracy of about 0.4 on the validation set.
learning_rates = [1e-7,1e-6,1e-5, 1e-4, 1e-3]
regularization_strengths = [1e3,8e4, 2.5e4, 5e4, 8e4]

# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the frac
tion
# of data points that are correctly classified.
results = {}
best_val = -1   # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation
rate.

############################################################################
#####
# TODO:
#
# Write code that chooses the best hyperparameters by tuning on the validat
ion #
# set. For each combination of hyperparameters, train a linear SVM on the
#
# training set, compute its accuracy on the training and validation sets, a
nd  #
# store these numbers in the results dictionary. In addition, store the bes
t   #
# validation accuracy in best_val and the LinearSVM object that achieves th
is  #
# accuracy in best_svm.
#
#
#
# Hint: You should use a small value for num_iters as you develop your
#
# validation code so that the SVMs don't take much time to train; once you
are #
# confident that your validation code works, you should rerun the validatio
n   #
# code with a larger value for num_iters.
#
############################################################################
#####
for alpha_ in learning_rates:
    for beta_ in regularization_strengths:
        print('----------------------------')
        print('Current alpha: %f, beta: %f' % (alpha_, beta_))
        svm = LinearSVM()
        loss_hist = svm.train(X_train, y_train, learning_rate=alpha_, reg=b
eta_,
                              num_iters=1500, verbose=False)
        y_val_pred = svm.predict(X_val)
        val_pred_acc = np.mean(y_val_pred == y_val)
        y_train_pred = svm.predict(X_train)
        train_pred_acc = np.mean(y_train_pred == y_train)
        results[(alpha_,beta_)]=(train_pred_acc, val_pred_acc)
        if val_pred_acc > best_val:
            best_val=val_pred_acc
```

```
----------------------------
Current alpha: 0.000000, beta: 1000.000000
----------------------------
Current alpha: 0.000000, beta: 80000.000000
----------------------------
Current alpha: 0.000000, beta: 25000.000000
----------------------------
Current alpha: 0.000000, beta: 50000.000000
----------------------------
Current alpha: 0.000000, beta: 80000.000000
----------------------------
Current alpha: 0.000001, beta: 1000.000000
----------------------------
Current alpha: 0.000001, beta: 80000.000000
----------------------------
Current alpha: 0.000001, beta: 25000.000000
----------------------------
Current alpha: 0.000001, beta: 50000.000000
----------------------------
Current alpha: 0.000001, beta: 80000.000000
----------------------------
Current alpha: 0.000010, beta: 1000.000000
----------------------------
Current alpha: 0.000010, beta: 80000.000000
----------------------------
Current alpha: 0.000010, beta: 25000.000000
----------------------------
Current alpha: 0.000010, beta: 50000.000000
----------------------------
Current alpha: 0.000010, beta: 80000.000000
----------------------------
Current alpha: 0.000100, beta: 1000.000000
----------------------------

/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_svm.py:87: Ru
ntimeWarning: overflow encountered in double_scalars
  loss+=reg*np.sum(W*W)
/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_svm.py:87: Ru
ntimeWarning: overflow encountered in multiply
  loss+=reg*np.sum(W*W)
/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_svm.py:109: R
untimeWarning: overflow encountered in multiply
  dW+=2*reg*W

Current alpha: 0.000100, beta: 80000.000000
----------------------------

/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_svm.py:104: R
untimeWarning: invalid value encountered in greater
  mask[margin>0]=1
/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_classifier.py
:72: RuntimeWarning: invalid value encountered in subtract
  self.W-=learning_rate*grad
/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_svm.py:83: Ru
ntimeWarning: overflow encountered in subtract
  margin=scores-yscore+1;
```

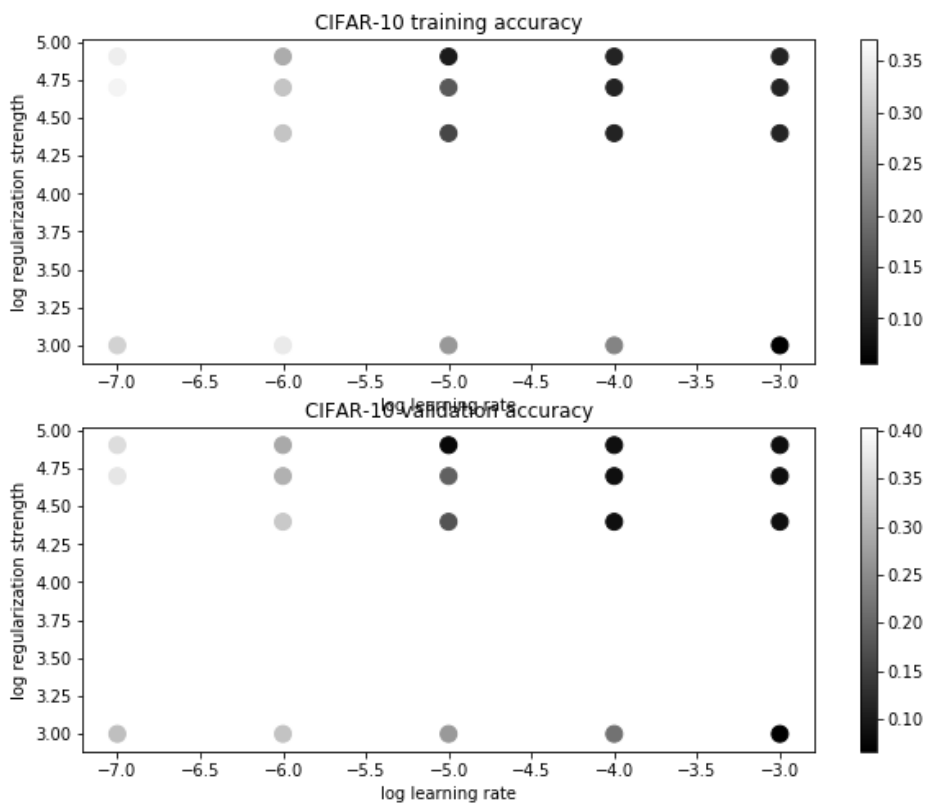
```
Current alpha: 0.000100, beta: 25000.000000
-----------------------------
Current alpha: 0.000100, beta: 50000.000000
-----------------------------
Current alpha: 0.000100, beta: 80000.000000
-----------------------------
Current alpha: 0.001000, beta: 1000.000000
-----------------------------
Current alpha: 0.001000, beta: 80000.000000
-----------------------------
Current alpha: 0.001000, beta: 25000.000000
-----------------------------
Current alpha: 0.001000, beta: 50000.000000
-----------------------------
Current alpha: 0.001000, beta: 80000.000000
lr 1.000000e-07 reg 1.000000e+03 train accuracy: 0.315143 val accuracy: 0.3
18000
lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.371000 val accuracy: 0.4
03000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.356735 val accuracy: 0.3
69000
lr 1.000000e-07 reg 8.000000e+04 train accuracy: 0.349408 val accuracy: 0.3
58000
lr 1.000000e-06 reg 1.000000e+03 train accuracy: 0.344286 val accuracy: 0.3
23000
lr 1.000000e-06 reg 2.500000e+04 train accuracy: 0.298286 val accuracy: 0.3
31000
lr 1.000000e-06 reg 5.000000e+04 train accuracy: 0.297592 val accuracy: 0.3
00000
lr 1.000000e-06 reg 8.000000e+04 train accuracy: 0.268694 val accuracy: 0.2
89000
lr 1.000000e-05 reg 1.000000e+03 train accuracy: 0.243551 val accuracy: 0.2
68000
lr 1.000000e-05 reg 2.500000e+04 train accuracy: 0.143816 val accuracy: 0.1
76000
lr 1.000000e-05 reg 5.000000e+04 train accuracy: 0.166429 val accuracy: 0.1
95000
lr 1.000000e-05 reg 8.000000e+04 train accuracy: 0.090163 val accuracy: 0.0
77000
lr 1.000000e-04 reg 1.000000e+03 train accuracy: 0.219367 val accuracy: 0.2
17000
lr 1.000000e-04 reg 2.500000e+04 train accuracy: 0.100265 val accuracy: 0.0
87000
lr 1.000000e-04 reg 5.000000e+04 train accuracy: 0.100265 val accuracy: 0.0
87000
lr 1.000000e-04 reg 8.000000e+04 train accuracy: 0.100265 val accuracy: 0.0
87000
lr 1.000000e-03 reg 1.000000e+03 train accuracy: 0.056837 val accuracy: 0.0
66000
lr 1.000000e-03 reg 2.500000e+04 train accuracy: 0.100265 val accuracy: 0.0
87000
lr 1.000000e-03 reg 5.000000e+04 train accuracy: 0.100265 val accuracy: 0.0
87000
lr 1.000000e-03 reg 8.000000e+04 train accuracy: 0.100265 val accuracy: 0.0
87000
best validation accuracy achieved during cross-validation: 0.403000

/home/nan/StanfordCS231/assignment1/cs231n/classifiers/linear_svm.py:83: Ru
ntimeWarning: invalid value encountered in subtract
  margin=scores-yscore+1;
```

In [105]:
```
# Visualize the cross-validation results
import math
x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')


# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()
```



In [106]:
```
# Evaluate the best svm on test set
y_test_pred = best_svm.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('linear SVM on raw pixels final test set accuracy: %f' % test_accurac
y)
```

```
linear SVM on raw pixels final test set accuracy: 0.380000
```

In [107]:
```
# Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, th
ese may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', '
ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```

Inline question 2:

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look they way that they do.

Your answer: *fill this in*