

RAO-BLACKWELLISED PARTICLE FILTERING FOR BLIND SYSTEM IDENTIFICATION

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ABSTRACT

This paper develops a Rao-Blackwellised particle filtering algorithm for blind system identification. The state space model under consideration uses a time-varying autoregressive (AR) model for the sources, and a time-varying finite impulse response (FIR) model for the channel. The multi-sensor measurements result from the convolution of the sources with the channels in the presence of additive noise. A numerical approximation to the optimal Bayesian solution for the nonlinear sequential state estimation problem is implemented using sequential Monte Carlo (SMC) methods. The Rao-Blackwellisation technique is applied to improve the efficiency of the particle filter by marginalizing out the AR and FIR coefficients from the joint posterior distribution. Simulation results are given to verify the performance of the proposed method.

1. INTRODUCTION

The blind system identification problem arises in many fields, including speech processing, communications, biomedical signal processing, and seismology. The problem addressed in this paper is to recover the unknown input sources, as well the unknown time-varying AR and FIR coefficients and unknown source and measurement noise variances, from the observed signals of a multiple-input multiple-output (MIMO) system.

An important application of blind system identification techniques is the recovery of speech signals in a reverberant audio environment. The reverberation of speech signals can cause significant degradation in the perceptual quality of speech for hands-free telephony, hearing aids, and other audio applications. A typical acoustic impulse response (AIR) in an audio enclosure is “tailed” with coefficients that decay smoothly towards zero, making the channel identification problem ill-conditioned. The proposed algorithm will use a Rao-Blackwellised particle filter (RBPF) to directly estimate the sources by marginalizing out the AR and FIR coefficients. This offers a more computationally stable method for source recovery. A previous Bayesian approach to speech dereverberation using a filterbank implementation was presented in [1].

The use of SMC methods for nonlinear/non-Gaussian problems in signal processing was prompted by the introduction of the resampling step into the sequential importance sampling (SIS) procedure [2]. Recent advances in computational power has led to application in a wide variety of technical fields, including speech processing [3], wireless communications [4], and target tracking. A particle filtering approach can result in significant computational

complexity, however, they lend themselves well to a parallel implementation.

2. STATE SPACE MODEL

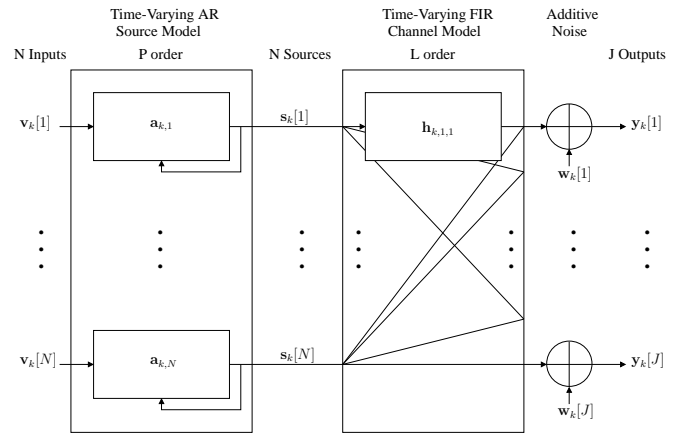


Fig. 1. Graphical representation of state space model

The state space model under consideration is shown graphically in Figure 1. The n_{th} source $s_k[n]$ is assumed to evolve according to the following P -order time-varying autoregressive (TVAR) model:

$$\mathbf{s}_k[n] = \mathbf{a}_{k,n}^T \mathbf{s}_{P,k-1,n} + \mathbf{v}_{k-1}[n]. \quad (1)$$

The source vector $\mathbf{s}_{P,k-1,n} \in \mathbb{R}^{P \times 1}$ is the concatenation of the most recent P samples at time $k-1$ for the n_{th} source, and the vector $\mathbf{a}_{k,n} \in \mathbb{R}^{P \times 1}$ contains the corresponding AR coefficients. The source noise $\mathbf{v}_{k-1}[n] \in \mathbb{R}$ is assumed to be white Gaussian distributed with mean zero and unknown variance $\sigma_{v,n}^2$. The source noise variances are assumed to be independent between sources. The following two matrix representations for the dynamics of the N sources will be used in the development of the algorithm:

$$\mathbf{s}_k = \mathbf{A}_k \mathbf{s}_{P,k-1} + \mathbf{v}_{k-1} \quad (2)$$

$$= \mathbf{S}_{k-1} \mathbf{a}_k + \mathbf{v}_{k-1}. \quad (3)$$

The quantities \mathbf{S}_{k-1} and $\mathbf{s}_{P,k-1}$ are formed from the source samples in the set of $\mathbf{s}_{P,k-1,n}$, $n = 1, 2, \dots, N$, and \mathbf{A}_k , \mathbf{a}_k are formed from the AR coefficients contained in the set of $\mathbf{a}_{k,n}$. Appropriate definitions are used in order to satisfy equation (1) for $n = 1, 2, \dots, N$, but the details are omitted for lack of space.

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The time-varying AR coefficient vector \mathbf{a}_k is itself assumed to evolve according to a first-order AR model as follows:

$$\mathbf{a}_k = a_a \mathbf{a}_{k-1} + \mathbf{v}_{a,k-1}, \quad (4)$$

with $0 < a_a < 1$ assumed known and the noise vector $\mathbf{v}_{a,k-1}$ Gaussian distributed with zero mean and known covariance Σ_a .

The measurement equation for the j_{th} sensor is assumed to evolve according to the convolution of the sources with time-varying FIR channels in the presence of additive noise as follows:

$$\mathbf{y}_k[j] = \mathbf{h}_{k,j}^T \mathbf{s}_{L,k} + \mathbf{w}_k[j]. \quad (5)$$

The source vector $\mathbf{s}_{L,k} \in \mathbb{R}^{N \times 1}$ is the concatenation of the most recent L source vectors $\mathbf{s}_{k-\ell}$, $\ell = 0, 1, \dots, L-1$ at time k , and the channel vector $\mathbf{h}_{k,j} \in \mathbb{R}^{N \times 1}$ is formed from the N FIR filters $\mathbf{h}_{k,j,n}$ of length L from the n_{th} source to the j_{th} sensor. The measurement noise $\mathbf{w}_k[j] \in \mathbb{R}$ is assumed to be white Gaussian distributed with mean zero and unknown variance $\sigma_{w,j}^2$. The measurement noise variances are assumed to be independent between sensors. The two matrix representations used for the measurements at the J sensors are

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s}_{L,k} + \mathbf{w}_k \quad (6)$$

$$= \mathbf{T}_k \mathbf{h}_k + \mathbf{w}_k. \quad (7)$$

The source matrix \mathbf{T}_k is formed from $\mathbf{s}_{L,k}$, and $\mathbf{H}_k, \mathbf{h}_k$ are formed from the FIR coefficients contained in the set of $\mathbf{h}_{k,j}$ in order to satisfy (5) for $j = 1, 2, \dots, J$.

The time-varying FIR coefficient vector \mathbf{h}_k is assumed to evolve according to a first-order AR model as follows:

$$\mathbf{h}_k = a_h \mathbf{h}_{k-1} + \mathbf{v}_{h,k-1}, \quad (8)$$

with $0 < a_h < 1$ assumed known and the noise vector $\mathbf{v}_{h,k-1}$ Gaussian distributed with zero mean and known covariance Σ_h .

3. SEQUENTIAL MONTE CARLO METHODS

A Bayesian approach to the sequential state estimation problem is to recursively compute the posterior distribution of the states, denoted in general as $\mathbf{x}_{1:k}$, given the measurements $\mathbf{y}_{1:k}$. When the state model is linear-Gaussian, the optimal Bayesian solution for the recursion can be found analytically using the Kalman filter. The resulting posterior distribution in that case is Gaussian, and therefore can be represented completely by its mean and covariance. The given state space model is nonlinear since both the source and channel are unknown, so that SMC methods [5] are required. SMC methods numerically approximate the posterior distribution using a set of particles \mathbf{x}_k^i and importance weights w_k^i for $i = 1, 2, \dots, N_p$:

$$p(\mathbf{x}_{1:k} | \mathbf{y}_{1:k}) \approx \sum_{i=1}^{N_p} w_k^i \delta(\mathbf{x}_{1:k} - \mathbf{x}_{1:k}^i) \quad (9)$$

According to conventional sequential importance sampling (SIS) theory [9], a recursion for the importance weights w_k^i is given by

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{y}_k | \mathbf{x}_k^i) p(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i | \mathbf{x}_{1:k-1}^i, \mathbf{y}_{1:k})}. \quad (10)$$

The derivation assumes the state \mathbf{x}_k is first-order Markov and the measurement \mathbf{y}_k is only dependent on the current state and the

measurement noise. The importance weights are normalized such that $\sum_i w_k^i = 1$.

In practice, SIS algorithms suffer from the problem of importance weight degeneracy, in which after a few iterations of the recursion only one particle has a significant normalized importance weighting. The resampling step introduced in [2] reduces the weight degeneracy by duplicating particles with large weights and removing particles with small weights after the weight update in (10).

An undesired consequence of resampling is that particles with high importance weights can be selected numerous times. One method of reintroducing statistical diversity after the resampling procedure is the use of a Markov Chain Monte Carlo (MCMC) step [5].

4. RAO-BLACKWELLISED PARTICLE FILTERING

The Rao-Blackwellisation (RB) strategy [9] is applied to exploit the analytical structure in the proposed state space model. The RB technique marginalizes out conditionally linear-Gaussian state variables from the joint posterior distribution in order to reduce the state dimension for the particle filtering algorithm. This strategy can be shown to reduce the variance of the state estimates obtained using the particle filter [9]. The intuition behind this results is that a particle filter is now only used to estimate the truly nonlinear/non-Gaussian states, while the remaining conditional linear-Gaussian states can be estimated analytically using the optimal Kalman filter [6].

It can be seen from the proposed state space model that conditional on the sources $\mathbf{s}_{1:k}$ (which form \mathbf{T}_k) and the measurement noise covariance Σ_w , equations (7)-(8) for the FIR coefficients \mathbf{h}_k form a linear-Gaussian subsystem. Similarly, the pair of equations (3)-(4) for the AR coefficients \mathbf{a}_k conditioned on the sources and the source noise covariance Σ_v also form a linear-Gaussian subsystem. The joint posterior distribution for the sources, FIR and AR coefficients is factorized using Bayes' rule to exploit this structure:

$$p(\mathbf{s}_{1:k}, \mathbf{a}_{1:k}, \mathbf{h}_{1:k} | \mathbf{y}_{1:k}) = p(\mathbf{s}_{1:k} | \mathbf{y}_{1:k}) p(\mathbf{a}_{1:k} | \mathbf{s}_{1:k}, \mathbf{y}_{1:k}) p(\mathbf{h}_{1:k} | \mathbf{s}_{1:k}, \mathbf{y}_{1:k}) \quad (11)$$

The dependence on the noise variances Σ_v and Σ_w are not shown explicitly since maximum a posteriori (MAP) estimates can be developed separately assuming non-informative inverse Gamma variance priors:

$$\hat{\sigma}_{v,n}^2[k]^i = \frac{\frac{1}{2} \sum_{k=1}^k (\mathbf{s}_{\tilde{k}}[n]^i - \mathbf{s}_{P,\tilde{k}-1,n}^{(i)T} \mathbf{a}_{k,n}^i)^2}{\frac{k}{2} + 1}, \quad (12)$$

$$\hat{\sigma}_{w,j}^2[k]^i = \frac{\frac{1}{2} \sum_{k=1}^k (\mathbf{y}_{\tilde{k}}[j]^i - \mathbf{s}_{L,\tilde{k}}^{(i)T} \mathbf{h}_{k,j}^i)^2}{\frac{k}{2} + 1}. \quad (13)$$

The filtered distributions $p(\mathbf{a}_k | \mathbf{s}_{1:k}, \mathbf{y}_{1:k})$ and $p(\mathbf{h}_k | \mathbf{s}_{1:k}, \mathbf{y}_{1:k})$ are computed recursively in parallel for the decoupled conditionally linear-Gaussian problems using the standard Kalman filter:

$$p(\mathbf{a}_k | \mathbf{s}_{1:k}, \mathbf{y}_{1:k}) = \mathcal{N}(\hat{\mathbf{a}}_{k|k}^i, \Phi_{a,k|k}^i), \quad (14)$$

$$p(\mathbf{h}_k | \mathbf{s}_{1:k}, \mathbf{y}_{1:k}) = \mathcal{N}(\hat{\mathbf{h}}_{k|k}^i, \Phi_{h,k|k}^i). \quad (15)$$

The quantities $\hat{\mathbf{a}}_{k|k}, \hat{\mathbf{h}}_{k|k}$ are the filtered means and $\Phi_{a,k|k}, \Phi_{h,k|k}$ are the filtered covariances from the Kalman recursions for the AR and FIR coefficients.

The marginalized posterior distribution $p(\mathbf{s}_{1:k}|\mathbf{y}_{1:k})$ is obtained using the Rao-Blackwellisation strategy for marginalizing out the conditionally linear-Gaussian AR and FIR coefficients. The resulting nonlinear estimation problem for the sources \mathbf{s}_k is implemented using the particle filter. The development of the SIS method assumes the state transition model for \mathbf{s}_k is first-order Markov and the measurement model is dependent on only the current state. The state equation (2) and the measurement equation (6) do not satisfy this requirement in general since they are dependent on P and L source samples, respectively. To satisfy the requirements of an SIS implementation, the new state variable $\mathbf{s}_{M,k} \in \mathbb{R}^{MN \times 1}$ is introduced, where $M = \max(P, L)$ is the maximum of the orders of the AR and FIR models. The state transition equation is then:

$$\mathbf{s}_{M,k} = \tilde{\mathbf{A}}_k \mathbf{s}_{M,k-1} + \tilde{\mathbf{v}}_{k-1}, \quad (16)$$

where, using $\mathbf{0}_{a,b}$ to denote a matrix of zeros of dimension $a \times b$ if $a, b > 0$ and empty otherwise,

$$\tilde{\mathbf{A}}_k = \begin{bmatrix} [\mathbf{0}_{(M-1)N,N}, \mathbf{I}_{(M-1)N}] \\ [\mathbf{0}_{N,(L-P)N}, \mathbf{A}_k] \end{bmatrix}, \quad (17)$$

$$\tilde{\mathbf{v}}_{k-1} = [\mathbf{0}_{1,(M-1)N}, \mathbf{v}_{k-1}^T]^T. \quad (18)$$

The corresponding measurement equation is:

$$\mathbf{y}_k = \tilde{\mathbf{H}}_k \mathbf{s}_{M,k} + \mathbf{w}_k, \quad (19)$$

where

$$\tilde{\mathbf{H}}_k = [\mathbf{0}_{J,(P-L)N}, \mathbf{H}_k]. \quad (20)$$

Using this formulation in terms of $\mathbf{s}_{M,k}$, we now develop the importance function and weight update used in the particle filter for estimation of the sources. An approximation to the optimal importance function is used to generate the particles. The optimal importance function is defined as the function which minimizes the variance of the true importance weights [9], and is shown to be

$$q(\mathbf{s}_{M,k}|\mathbf{s}_{M,1:k-1}, \mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{s}_{M,k})p(\mathbf{s}_{M,k}|\mathbf{s}_{M,k-1}). \quad (21)$$

From the form of (16), only the quantity \mathbf{s}_k of $\mathbf{s}_{M,k}$ is stochastic, while the remaining blocks are shifted quantities from the previous state $\mathbf{s}_{M,k-1}$. Thus, it is only required to consider generating particles for the current source vector \mathbf{s}_k from an importance density of the form:

$$q(\mathbf{s}_k|\mathbf{s}_{M,1:k-1}, \mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{s}_{M,k})p(\mathbf{s}_k|\mathbf{s}_{M,k-1}), \quad (22)$$

where $p(\mathbf{y}_k|\mathbf{s}_{M,k})$ is the marginalized likelihood and $p(\mathbf{s}_k|\mathbf{s}_{M,k-1})$ is the marginalized prior. These distributions are determined by marginalizing over the FIR and AR coefficients, and are found to be [6]:

$$p(\mathbf{s}_k|\mathbf{s}_{M,k-1}) = \mathcal{N}(\mathbf{S}_{k-1}\hat{\mathbf{a}}_{k|k-1}, \mathbf{R}_k), \quad (23)$$

$$p(\mathbf{y}_k|\mathbf{s}_{M,k}) = \mathcal{N}(\mathbf{T}_k\hat{\mathbf{h}}_{k|k-1}, \mathbf{Q}_k), \quad (24)$$

where

$$\mathbf{R}_k = \mathbf{S}_{k-1}\Phi_{a,k|k-1}\mathbf{S}_{k-1}^T + \Sigma_v, \quad (25)$$

$$\mathbf{Q}_k = \mathbf{T}_k\Phi_{h,k|k-1}\mathbf{T}_k^T + \Sigma_w, \quad (26)$$

and $\hat{\mathbf{a}}_{k|k-1}, \hat{\mathbf{h}}_{k|k-1}$ are the predicted means and $\Phi_{a,k|k-1}, \Phi_{h,k|k-1}$ are the predicted covariances from the Kalman filter recursions. Even though the optimal importance function for \mathbf{s}_k in (22) is the

product of the two Gaussian distributions (23),(24), it is not Gaussian itself since the covariance term \mathbf{Q}_k has a dependence on the variable of interest \mathbf{s}_k (through \mathbf{T}_k). In order to derive a Gaussian importance function that has the necessary feature of being easy to sample from, the state-dependent covariance \mathbf{Q}_k is approximated by $\hat{\mathbf{Q}}_k$ with \mathbf{s}_k replaced by its predicted value from the transition prior:

$$\hat{\mathbf{s}}_{k|k-1} = \mathbf{S}_{k-1}\hat{\mathbf{a}}_{k|k-1} \quad (27)$$

To factorize the two distributions into an equivalent Gaussian distribution for \mathbf{s}_k , the variable \mathbf{s}_k is isolated from the matrix \mathbf{T}_k in the mean of (24) using the equivalent forms of the measurement equation in (6) and (7):

$$\begin{aligned} \mathbf{T}_k\hat{\mathbf{h}}_{k|k-1} &= \sum_{\ell=0}^{L-1} \hat{\mathbf{H}}_{k|k-1,\ell}\mathbf{s}_{k-\ell} \\ &= \hat{\mathbf{H}}_{k|k-1,0}\mathbf{s}_k + \hat{\mathbf{y}}_{k|k-1} \end{aligned} \quad (28)$$

where the predicted matrices of FIR coefficients at lag ℓ from the current time $\hat{\mathbf{H}}_{k|k-1,\ell}$ are formed from $\hat{\mathbf{h}}_{k|k-1}$, and the predicted measurement $\hat{\mathbf{y}}_{k|k-1}$ is defined as the summation excluding $\hat{\mathbf{H}}_{k|k-1,0}\mathbf{s}_k$. The resulting importance function is then Gaussian with mean $\boldsymbol{\mu}_o$ and covariance Σ_o given by:

$$\boldsymbol{\mu}_o = \hat{\mathbf{s}}_{k|k-1} + \mathbf{W}_k(\mathbf{y}_k - \hat{\mathbf{H}}_{k|k-1,0}\hat{\mathbf{s}}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1}) \quad (29)$$

$$\Sigma_o = \mathbf{R}_k - \mathbf{W}_k\hat{\mathbf{H}}_{k|k-1,0}\mathbf{R}_k, \quad (30)$$

which is in the form of a Kalman update on the predicted particles with gain given by:

$$\mathbf{W}_k = \mathbf{R}_k\hat{\mathbf{H}}_{k|k-1,0}^T[\hat{\mathbf{H}}_{k|k-1,0}\mathbf{R}_k\hat{\mathbf{H}}_{k|k-1,0}^T + \hat{\mathbf{Q}}_k]^{-1}. \quad (31)$$

The corresponding weight update from (10) then simplifies to:

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{y}_k|\mathbf{s}_{M,k}^i)}{\hat{p}(\mathbf{y}_k|\mathbf{s}_{M,k}^i)} \hat{p}(\mathbf{y}_k|\mathbf{s}_{M,k-1}^i), \quad (32)$$

where

$$\hat{p}(\mathbf{y}_k|\mathbf{s}_{M,k}) = \mathcal{N}(\mathbf{T}_k\hat{\mathbf{h}}_{k|k-1}, \hat{\mathbf{Q}}_k) \quad (33)$$

The steps of the proposed RBPF algorithm are summarized:

RBPF Algorithm Summary

Generation of Particles: Draw particles from the approximation to the optimal importance function in (29),(30).

Importance Weight Update: Update the importance weights (32), and normalize to sum to unity.

Resampling Step: Resample the source particles, and the corresponding Kalman filter means and covariances, and noise variance MAP estimates using the systematic resampling method [7] when the approximate effective sample size [8] falls below $N_p/3$.

MCMC Diversity Step: Restore particle diversity by applying a Metropolis-Hastings (MH) algorithm using the approximation to the optimal importance function as the proposal distribution.

Kalman filtering for AR,FIR coefficients: Update the filtered (and predicted) mean and covariance of the distributions (14),(15) for the pair of conditionally linear-Gaussian systems using the standard Kalman filter recursions.

MAP estimation for noise variances: Compute the MAP estimates of the source (12) and measurement (13) noise variances.

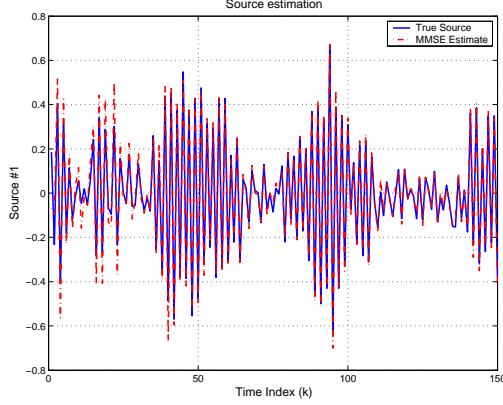


Fig. 2. RBPf source estimation

5. SIMULATION RESULTS

The dynamical state space model was simulated with $N = 1$ source and $J = 2$ sensors. The initial $P = 4$ order AR coefficient vector \mathbf{a}_0 was generated from a low-pass Butterworth filter with normalized cutoff frequency $w_n = 0.25$. The time-varying state \mathbf{a}_k was then generated from the AR model in (4) using $a_a = 0.9999$ and $\Sigma_a = 0.001\mathbf{I}_P$. The initial $L = 6$ order FIR channel vectors $\mathbf{h}_{0,j,n}$ were produced from independent draws from a zero-mean Gaussian distribution with exponentially decaying covariance matrix using $W = 0.15$:

$$\Sigma_{h,0} = \text{diag} \left([e^{-\frac{L-1}{WL}}, e^{-\frac{L-2}{WL}}, \dots, e^{-\frac{0}{WL}}] \right). \quad (34)$$

The time-varying \mathbf{h}_k was then generated from (8) using $a_h = 0.9999$ and $\Sigma_h = (1 - a_h^2)\Sigma_{h,0}$. The noise variance parameters were $\sigma_v^2 = 0.01$, $\sigma_w^2 = 0.005$. The average signal-to-noise (SNR) ratio computed numerically over the Monte Carlo runs was 17.3 dB. The number of particles was $N_p = 50$.

The performance is measured using the mean square error (MSE) averaged over the time steps and Monte Carlo runs:

$$\text{MSE} = 10 \log_{10} \left(\frac{1}{N_t} \sum_{t=1}^{N_t} \left(\frac{1}{K} \sum_{k=1}^K \frac{\|\mathbf{s}_k^t - \hat{\mathbf{s}}_k^t\|_2^2}{N} \right) \right), \quad (35)$$

where \mathbf{s}_k^t is the true source from the t_{th} Monte Carlo trial, $\hat{\mathbf{s}}_k^t$ is the minimum mean square error (MMSE) estimate, $N_t = 50$ is the number of Monte Carlo simulations, $K = 500$ is the number of time steps, and N is the dimension of the state \mathbf{s}_k . Performance measures for the MMSE estimates of \mathbf{h}_k , \mathbf{a}_k , σ_v^2 , and σ_w^2 also follow the form of (35). The MSE values are shown in Table 1.

Table 1. MSE simulation results

Variable	\mathbf{s}_k	\mathbf{h}_k	\mathbf{a}_k	σ_v^2	σ_w^2
MSE	-23.08	-15.83	-12.09	-46.31	-72.40

Figure 2 compares the true source with the MMSE estimate from one Monte Carlo run. The learning curves of $\text{MSE}(k)$ (computed without averaging over time) are presented in Figure 3, for the AR/FIR coefficients and noise variances.

In conclusion, an efficient Rao-Blackwellised particle filtering algorithm has been presented to directly recover the source for

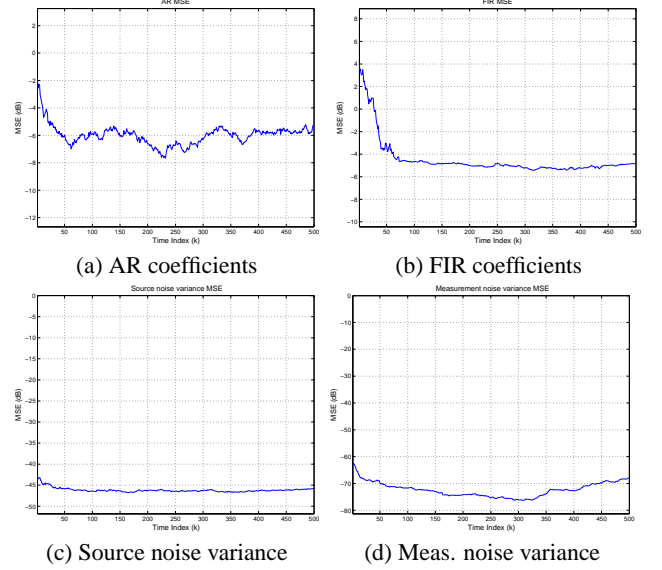


Fig. 3. MSE learning curves

the blind system identification problem. Simulation results have shown the effectiveness of the method.

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