TEMPORALLY CONSTRAINED SCA WITH APPLICATIONS TO EEG DATA

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ABSTRACT

In this paper we propose an iterative algorithm for solving the problem of extracting a sparse source signal when a reference signal for the desired source signal is available. In the proposed algorithm, a nonconvex objective function is used for measuring the diversity (antisparsity) of the desired source signal. The nonconvex function is locally replaced by a quadratic convex function. This results in a simple iterative algorithm. The proposed algorithm has two different versions, depending on the measure of closeness between the extracted source signal and the reference signal. The proposed algorithm has useful applications to EEG/MEG signal processing. This is demonstrated by an example in which eye blink artifacts are automatically removed from a real EEG data.

Indexing Terms: *blind source separation, sparse component analysis, independent component analysis, EEG/MEG.*

I. INTRODUCTION

The blind source separation (BSS) problem is defined as the problem of reconstructing n *unknown* source signals from m linear measurements when the mixing matrix is *unknown*. The relation between the measured signals and the original source signals can be expressed mathematically as

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{S}, \tag{1}$$

where $\boldsymbol{X} \in \mathbb{R}^{m\times T}$ is a matrix of measured signals, $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ is an *unknown* mixing matrix, $\boldsymbol{S} \in \mathbb{R}^{n\times T}$ is a matrix of it unknown source signals, m is the number of observations, n is the number of sources, and T is the number of samples. In this paper we consider the case $m = n$.

Over the last two decades, the BSS problem has been solved using the independent component analysis technique (ICA). Generally speaking, there are three different approaches for solving the BSS problem via ICA. The first approach is based on estimating the hidden sources simultaneously. This is usually done by finding a separating matrix $\boldsymbol{B}$ such that the estimated sources $\boldsymbol{Y} = \boldsymbol{B}\boldsymbol{X}$ are mutually independent [1]. The second approach is based on sequentially extracting the source signals one after the other, a technique known in the literature as blind signal extraction (BSE) [2]. The most popular algorithm that follows this strategy is called FastICA [3]. FastICA is based on finding a separating vector $\boldsymbol{b}$ by maximizing the *non Gaussianity* of the separated signal $\boldsymbol{y} = \boldsymbol{b}^T\boldsymbol{X}$. Therefore, the algorithm will converge (theoretically) to the most non-Gaussian source signal. However, when one desires a specific source, there is no guarantee that the first extracted source signal corresponds to the desired source signal. To overcome this difficulty, a third approach has been developed for extracting a specific source signal. This approach is based on incorporating any available *a priori* temporal information about the desired source signal as additional constraints into the FastICA algorithm. Following this technique, a third approach, known as constrained ICA (cICA), is developed in the literature for solving the BSS problem [4], [5].

In the field of biomedical signal analysis it is common that some prior information is available about the temporal characteristics of the desired signal. The prior information can then be used for constructing a reference signal, which in turn can be used as a temporal constraint in the cICA algorithm. For example, for the case of rejecting ocular artifacts from an EEG/MEG data, the ocular artifact most likely contaminates the frontal electrodes. Accordingly, a reference signal in this case can be easily derived from one of the frontal electrodes as the time samples that exceed a certain threshold [5]. Another approach for generating a reference signal for extracting periodic signals, e.g., the event related potential (ERP) signal, is to use a sequence of pulses which have the same periodic frequency as the desired signal. This approach was utilized in [4] for estimating the different components of the ERP signal from real EEG data. In all these case, the reference signal must be selected carefully to have enough information to guide the algorithm towards the desired signal. However, this does not mean that the reference signal is a replica of the desired signal.

In this paper we consider the case of using a reference signal to extract a *sparse* source signal. We have shown in [6] that a sparse source signal can be extracted with high accuracy by utilizing an objective function that measures the *diversity* (the number of nonzero samples) of the separated signal rather than maximizing its non-Gaussianity, as in FastICA. Therefore, in this paper we propose an algorithm which is based on utilizing an objective function that explicitly measures the diversity of the extracted source signal.

A previous approach for utilizing an objective function that explicitly measures the diversity of the extracted source signal was proposed in [7]. The objective function used in [7] is a smooth approximation of the ℓ_1–norm. However, it was shown in [8] that there is a class of nonconvex objective functions that perform better than the ℓ_1–norm in estimating sparse signals. Therefore, the proposed algorithm is based on utilizing a nonconvex objective function for measuring the diversity of the extracted source signal. Moreover, the nonconvex objective function is minimized by following the approach proposed in [8].

This paper is organized as follows. The least squares solution is derived in Section II. The proposed algorithm is presented in Section III. Section IV shows two numerical examples for assessing the performance of the proposed algorithm. Finally, conclusions are given in Section V.

II. LEAST SQUARES APPROACH

In this section we consider the least squares approach for estimating a desired source signal, which is close to a reference signal $\bar{\boldsymbol{r}}_t$ that conveys temporal information about the desired source signal. In least squares, a separating (row)[1] vector $\bar{\boldsymbol{b}}_{ls} \in \mathbb{R}^n$ is estimated by minimizing the following objective function

$$\bar{\boldsymbol{b}}_{ls} = \arg\min_{\bar{\boldsymbol{b}}} \mathcal{J}_{ls}\left(\bar{\boldsymbol{b}}\right) \triangleq ||\bar{\boldsymbol{r}}_t - \bar{\boldsymbol{b}}\boldsymbol{X}||^2_{\ell_2} \quad (2)$$

for which the corresponding extracted signal is given by

$$\bar{\boldsymbol{y}}_{ls} = \bar{\boldsymbol{b}}_{ls}\boldsymbol{X} = \bar{\boldsymbol{r}}_t\boldsymbol{X}^T\left(\boldsymbol{X}\boldsymbol{X}^T\right)^{-1}\boldsymbol{X}. \quad (3)$$

It is clear that the least squares solution does not exploit the sparsity of the desired source signal. In the next section, we propose an iterative algorithm that is based on minimizing an objective function which explicitly measures the diversity and the closeness of the solution vector to the reference vector $\bar{\boldsymbol{r}}_t$.

III. TEMPORALLY CONSTRAINED SCA (TCSCA) ALGORITHM

In this section we consider the case of finding a separating vector $\bar{\boldsymbol{b}}$ such that the extracted source signal $\bar{\boldsymbol{y}} = \bar{\boldsymbol{b}}\boldsymbol{X}$ is sparse and close to a reference signal $\bar{\boldsymbol{r}}_t$. The separating vector $\bar{\boldsymbol{b}}$ can be obtained by solving the following optimization problem

$$\begin{aligned} \bar{\boldsymbol{b}}_1 \;&=\; \arg\min_{\bar{\boldsymbol{b}}} g(\bar{\boldsymbol{b}}\boldsymbol{X}) + \gamma\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X};\bar{\boldsymbol{r}}_t) \\ &\quad\;\text{subject to}\quad ||\bar{\boldsymbol{b}}||_{\ell_2} = 1, \end{aligned} \quad (4)$$

where $\gamma \geq 0$ is a regularization parameter, $\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X};\bar{\boldsymbol{r}}_t)$ is a regularizing function that measures the closeness between the separated signal $\bar{\boldsymbol{y}} = \bar{\boldsymbol{b}}\boldsymbol{X}$ and the reference signal $\bar{\boldsymbol{r}}_t$, and $g(\bar{\boldsymbol{y}})$ is a function that measures the diversity (antisparsity) of the separated vector $\bar{\boldsymbol{y}}$. In this paper we consider a class of objective functions of the form $g(\bar{\boldsymbol{y}}) = \sum_{t=1}^{T} g_c(y[t])$, where $g_c(\cdot)$ is a symmetric and monotonically increasing concave function on the nonnegative orthant $\mathcal{O}_1$ [8]. In [8] we proposed replacing $g\left(\bar{\boldsymbol{y}}\right)$ by the quadratic convex function $f(\bar{\boldsymbol{y}})$ defined as

$$\begin{aligned} f(\bar{\boldsymbol{y}}) \;&=\; g(\bar{\boldsymbol{y}}_0) + \nabla g(\bar{\boldsymbol{y}}_0)(\bar{\boldsymbol{y}} - \bar{\boldsymbol{y}}_0)^T \\ &-\; 0.5(\bar{\boldsymbol{y}} - \bar{\boldsymbol{y}}_0)\nabla^2 g(\bar{\boldsymbol{y}}_0)(\bar{\boldsymbol{y}} - \bar{\boldsymbol{y}}_0)^T \end{aligned} \quad (5)$$

where $\bar{\boldsymbol{y}} \in \mathbb{R}^T$, $g(\bar{\boldsymbol{y}}) = \sum_i g_c(y[i])$ and $\nabla g(\bar{\boldsymbol{y}}_0)$, and $\nabla^2 g(\bar{\boldsymbol{y}}_0)$ are the gradient and Hessian of $g(\bar{\boldsymbol{y}})$ at $\bar{\boldsymbol{y}} = \bar{\boldsymbol{y}}_0$, respectively.

For the case of $g_c(y[i]) = \log\left(|y[i]|\right)$, it is straightforward to show that $f(\bar{\boldsymbol{y}})$ at the kth iteration is reduced to

$$f(\bar{\boldsymbol{y}}) = ||\bar{\boldsymbol{y}}\boldsymbol{W}_k||^2_{\ell_2} + C, \quad (6)$$

where C is a constant that does not depend on $\bar{\boldsymbol{y}}$, and $\boldsymbol{W}_k$ is a diagonal matrix, whose tth diagonal element is given by

$$W_k[t,t] = \frac{1}{|y_k[t]|}, \quad t = 1,\ldots,T. \quad (7)$$

The proposed algorithm is based on minimizing the objective function $g(\bar{\boldsymbol{y}})$ by iteratively minimizing $f(\bar{\boldsymbol{y}})$. Therefore, given an initial estimate of the separating vector $\bar{\boldsymbol{b}}_k$, a new estimate can be obtained by solving the following optimization problem

$$\begin{aligned} \bar{\boldsymbol{b}}_{k+1} \;&=\; \arg\min_{\bar{\boldsymbol{b}}} ||\bar{\boldsymbol{b}}\boldsymbol{X}\boldsymbol{W}_k||^2_{\ell_2} + \gamma\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X};\bar{\boldsymbol{r}}_t) \\ &\quad\;\text{subject to}\quad ||\bar{\boldsymbol{b}}||_{\ell_2} = 1. \end{aligned} \quad (8)$$

Note that the constraint $||\bar{\boldsymbol{b}}||_{\ell_2} = 1$ is introduced to prevent $||\bar{\boldsymbol{b}}||$ from growing without limit.

III-A. First approach: the correlation coefficient as a measure of closeness

In the first approach, the correlation coefficient between the desired signal and the reference signal is used as a measure of closeness between these two signals. Accordingly, in this case, $\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X};\bar{\boldsymbol{r}}_t)$ in (8) has the form $\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X};\bar{\boldsymbol{r}}_t) = -\bar{\boldsymbol{y}}\bar{\boldsymbol{r}}_t^T = -\bar{\boldsymbol{b}}\boldsymbol{X}\bar{\boldsymbol{r}}_t^T$. Substituting this expression into (8) we get

$$\begin{aligned} \bar{\boldsymbol{b}}_{t1} \;&=\; \arg\min_{\bar{\boldsymbol{b}}} \mathcal{J}_{t1}\left(\bar{\boldsymbol{b}}\right) \triangleq ||\bar{\boldsymbol{b}}\boldsymbol{X}\boldsymbol{W}_k||^2_{\ell_2} - \gamma\bar{\boldsymbol{b}}\boldsymbol{X}\bar{\boldsymbol{r}}_t^T \\ &\quad\;\text{subject to}\quad ||\bar{\boldsymbol{b}}||_{\ell_2} = 1. \end{aligned} \quad (9)$$

This optimization problem can be solved by minimizing $\mathcal{J}_{t1}\left(\bar{\boldsymbol{b}}\right)$ with respect to $\bar{\boldsymbol{b}}$, then projecting the solution vector onto the surface of the unit sphere. Following these two steps, the expression of $\bar{\boldsymbol{b}}_{t1}$ is given by

$$\bar{\boldsymbol{b}}^+_{t1} = \gamma\bar{\boldsymbol{r}}_t\boldsymbol{X}^T\left(\boldsymbol{X}\boldsymbol{W}_k^2\boldsymbol{X}^T\right)^{-1}. \quad (10)$$

$$\bar{\boldsymbol{b}}_{t1}(k+1) = \frac{\bar{\boldsymbol{b}}^+_{t1}}{||\bar{\boldsymbol{b}}^+_{t1}||_{\ell_2}}. \quad (11)$$

Note that, due to the normalization step (11), the value of $\bar{\boldsymbol{b}}_{t1}$ in this case does not depend on the value of the regularization parameter γ.

[1] A row vector is indicated by an overbar.

Table I. Temporally Constrained SCA

$[\bar{\boldsymbol{y}}, \bar{\boldsymbol{b}}]$ = tcSCA($\boldsymbol{X}$, $\bar{\boldsymbol{r}}_t$, α)

Select a small threshold parameter ϵ, and calculate $\bar{\boldsymbol{y}}_0 = \bar{\boldsymbol{r}}_t$.

1) For $k = 0, 1, \ldots$, repeat until convergence:
   - Calculate $W_k[t,t] = \frac{1}{|y_k[t]|}, \quad t = 1, \ldots, T.$
   - if $\alpha = 1$, then calculate the value of γ using the L–curve method (see text for more details).
   - Calculate the separating vector:
     $\bar{\boldsymbol{b}}^{+}_{k+1} = \bar{\boldsymbol{r}}_t \boldsymbol{X}^T \left(\boldsymbol{X}\left(\boldsymbol{W}_k^2 + \alpha\gamma\boldsymbol{I}\right)\boldsymbol{X}^T\right)^{-1}$
     $\bar{\boldsymbol{b}}_{k+1} = \frac{\bar{\boldsymbol{b}}^{+}_{k+1}}{||\bar{\boldsymbol{b}}^{+}_{k+1}||_{\ell_2}}.$
   - Get a new estimate of the desired signal: $\bar{\boldsymbol{y}}_{k+1} = \bar{\boldsymbol{b}}_{k+1}\boldsymbol{X}$.
   - if $||\bar{\boldsymbol{y}}_{k+1} - \bar{\boldsymbol{y}}_k||_{\ell_2}/||\bar{\boldsymbol{y}}_{k+1}||_{\ell_2} < \epsilon$, stop.
2) Output $\bar{\boldsymbol{y}}_{k+1}$ and $\bar{\boldsymbol{b}}_{k+1}$ as the solutions.

End

III-B. Second approach: MSE as a measure of closeness

In the second approach, the MSE between the desired source signal and the reference signal is used as a measure of closeness between the two signals. Accordingly, in this case, $\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X}; \bar{\boldsymbol{r}}_t)$ in (8) has the form $\mathcal{J}_r(\bar{\boldsymbol{b}}\boldsymbol{X}; \bar{\boldsymbol{r}}_t) = ||\bar{\boldsymbol{b}}\boldsymbol{X} - \bar{\boldsymbol{r}}_t||^2_{\ell_2}$. Substituting this expression into (8) we get

$$\bar{\boldsymbol{b}}_{t2} = \arg\min_{\bar{\boldsymbol{b}}} \mathcal{J}_{t2}\left(\bar{\boldsymbol{b}}\right) \triangleq ||\bar{\boldsymbol{b}}\boldsymbol{X}\boldsymbol{W}_k||^2_{\ell_2} + \gamma||\bar{\boldsymbol{b}}\boldsymbol{X} - \bar{\boldsymbol{r}}_t||^2_{\ell_2} \quad \text{subject to} \quad ||\bar{\boldsymbol{b}}||_{\ell_2} = 1. \tag{12}$$

This optimization problem can also be solved by minimizing $\mathcal{J}_{t2}\left(\bar{\boldsymbol{b}}\right)$ with respect to $\bar{\boldsymbol{b}}$, then projecting the solution vector onto the surface of the unit sphere. Following these two steps, the expression of $\bar{\boldsymbol{b}}_{t1}$ is given by

$$\bar{\boldsymbol{b}}^{+}_{t2} = \gamma\bar{\boldsymbol{r}}_t\boldsymbol{X}^T\left(\boldsymbol{X}\left(\boldsymbol{W}_k^2 + \gamma\boldsymbol{I}\right)\boldsymbol{X}^T\right)^{-1}. \tag{13}$$

$$\bar{\boldsymbol{b}}_{t2}(k+1) = \frac{\bar{\boldsymbol{b}}^{+}_{t2}}{||\bar{\boldsymbol{b}}^{+}_{t2}||_{\ell_2}}. \tag{14}$$

where $\boldsymbol{I}$ is the identity matrix. A method for calculating the optimum value of γ is presented later in this subsection. In both approaches, a new estimate of the desired signal is calculated and the diagonal elements of $\boldsymbol{W}_k$ are updated using (7). The process is repeated until convergence.

Calculating the value of the regularization parameter γ

The regularization parameter γ can be calculated by first converting the objective function $\mathcal{J}_{t2}\left(\bar{\boldsymbol{b}}\right)$ defined in (12) into a "standard form" for Tikhonov regularization, then calculating the regularization parameter using one of the standard techniques, e.g., the L–curve method [9]. The objective function $\mathcal{J}_{t2}\left(\bar{\boldsymbol{b}}\right)$ can be converted into standard form by defining $\bar{\boldsymbol{d}} = \bar{\boldsymbol{b}}\boldsymbol{X}\boldsymbol{W}_k$ and $\boldsymbol{C}_0 = \boldsymbol{W}_k^{-1}\boldsymbol{X}^T\left(\boldsymbol{X}\boldsymbol{X}^T\right)^{-1}\boldsymbol{X}$, then expressing $\mathcal{J}_{t2}\left(\bar{\boldsymbol{b}}\right)$ as

$$\mathcal{J}_{t2}\left(\bar{\boldsymbol{d}}\right) = ||\bar{\boldsymbol{d}}||^2_{\ell_2} + \gamma||\bar{\boldsymbol{d}}\boldsymbol{C}_0 - \bar{\boldsymbol{r}}_t||^2_{\ell_2}. \tag{15}$$

There are many techniques that have been proposed in the literature for determining an optimum value for γ, e.g., the L–curve and the generalized cross validation methods. See [9] and the references therein for more details. In this paper the L–curve method, which is implemented in the Regularization Toolbox [2], is used for calculating the regularization parameter.

The proposed algorithm is summarized in Table I. The input parameter α is a flag indicating which method is used as a measure of closeness, i.e., $\alpha = 1$ for the MSE case, while $\alpha = 0$ for the correlation coefficient case.

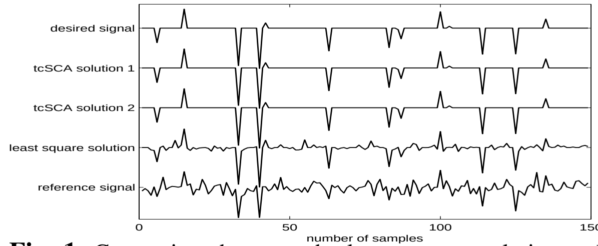


Fig. 1. Comparison between the least squares solution and the solutions produced by the tcSCA algorithm. The signals from top to bottom are the desired signal, the estimated signal using the tcSCA algorithm with $\alpha = 0$, the estimated signal using the tcSCA algorithm with $\alpha = 1$, the estimated signal using the least squares approach, and the reference signal, respectively.

IV. SIMULATION RESULTS

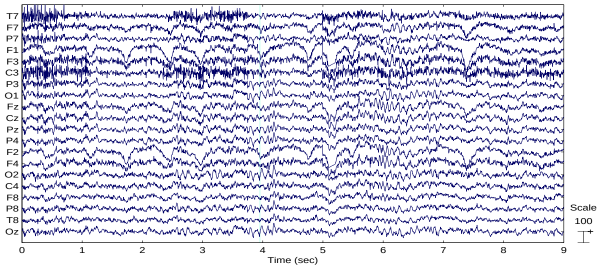
In this section we provide two examples to demonstrate the ability of the proposed algorithm to extract a sparse source signal that is close to a given reference signal. **Example 1:** In this example we provide a comparison between the least squares solution and that of the proposed algorithms. The data for this example were constructed by multiplying a randomly generated (10×150) sparse source matrix $\boldsymbol{S}$ by a square mixing matrix $\boldsymbol{A}$ randomly generated from an *iid normal distribution with zero mean and unit variance. The reference signal is constructed by adding a zero mean white Gaussian random noise to the desired source signal. The amplitude of the noise is adjusted to produce a 2dB SNR.*

As shown in this figure, and for the two cases $\alpha = 0$ and $\alpha = 1$, the desired signal is correctly estimated using the tcSCA algorithm, while the least–squares approach produced a non–sparse signal. As a measure of performance, the average normalized MSE (NMSE) between the desired signal and the extracted signal is calculated for each approach, where the NMSE between two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as

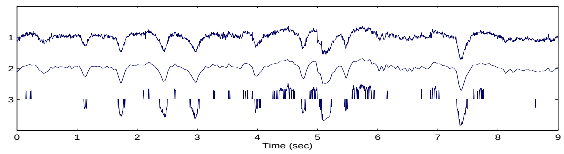
$$NMSE = \left|\left|\frac{\boldsymbol{x}}{||\boldsymbol{x}||_2} - \frac{\boldsymbol{y}}{||\boldsymbol{y}||_2}\right|\right|_2. \tag{16}$$

The average NMSE is calculated over 1000 different runs, where in each run new source and mixing matrices are generated. The first row in the generated source matrix is considered as the desired signal, and the corresponding reference signal is generated by adding a zero mean white Gaussian random noise to the desired signal. The amplitude of the noise is adjusted to produce a 2dB SNR. The value of the average NMSE for the least–squares approach, the tcSCA with $\alpha = 0$, and the tcSCA with $\alpha = 1$ are 0.2016, 3.67×10^{-4}, *and* 3.66×10^{-4}, *respectively. Clearly the value of the NMSE for the tcSCA approach is much less than that of the least–square approach.*

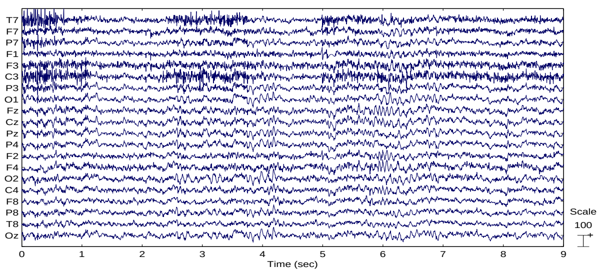
[2]This is a free software available at http://www2.imm.dtu.dk/~pch/Regutools/



(a)



(b)



(c)

Fig. 2. Automatic removal of eye blink artifact from real EEG data using the tcSCA algorithm. (a) Original EEG data, (b) The signals from top to bottom correspond to the extracted eye blink artifact, the denoised eye blink artifact, and the reference signal, respectively, (c) Clean version of the EEG data shown in (a).

***Example 2**: In this example the proposed example is used for automatic extraction of eye blink artifact from real EEG data. Therefore, in this example we use a reference signal which conveys temporal information about the eye blink artifact. This reference signal is extracted from one of the frontal electrodes, say F1, by setting to zero all samples with absolute value less than a threshold of 50 μv. This reference signal is shown by the third waveform in Figure 2(b).*

After running the tcSCA algorithm on the EEG data shown in Figure 2(a) the algorithm converged successfully to the desired artifact signal. The extracted signal is shown by the first waveform in Figure 2(b). The second waveform in the figure represents a denoised version of the estimated artifact, where the denoising step is performed using wavelet analysis. Finally, the clean EEG data is obtained by removing the denoised signals from the original EEG data using the technique mentioned in [2]. The cleaned EEG data obtained after running the tcSCA algorithm is shown in Figure 2(c). Clearly the eye blink artifact is removed successfully from the EEG data.

V. CONCLUSION

In this paper we proposed an iterative algorithm for solving the problem of extracting a sparse source signal when a reference signal for the desired source signal is available. In the proposed algorithm, a nonconvex objective function was used for measuring the diversity (antisparsity) of the desired source signal. The nonconvex was locally replaced by a quadratic convex function. This results in a simple iterative algorithm. The proposed algorithm has two different versions, depending on the measure of closeness between the extracted source signal and the reference signal. Eye blink artifacts were successfully and automatically removed from real EEG data using the proposed algorithm.

VI. REFERENCES

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