

THE DECOMPOSITION OF LARGE PROBLEMS USING SINGLE-SIDED SUBBANDING

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ABSTRACT

In this paper, we show that an NPR M -channel filter bank with a *diagonal* system inserted between the analysis and synthesis filterbanks, with appropriately chosen single-sided analysis and synthesis filters, may be used to decompose an arbitrary FIR system of order L into M FIR complex subband components each of order L/K , where K is the downsampling rate. This decomposition is at the expense of using complex arithmetic for the subband processing.

The proposed filter bank structure has application in the identification and equalization of long channels, (such as those that occur in reverberative environments) where existing algorithms may be intractable. By reducing the order of the problem in the subbands, such problems become computationally feasible. The improved performance and reduced computational requirements afforded by the proposed method are verified using the acoustic echo cancellation (AEC) problem as an example.

1. INTRODUCTION

In this paper, we show that a non-trivial (*diagonal*) MIMO system $\mathbf{C}(z)$ whose polynomial elements are of small degree, can be inserted between the analysis and synthesis filterbanks in such a way that the overall filterbank structure approximates an arbitrary FIR LTI system of interest, whose order is large. The scope of applications of filterbank theory [1], beyond the classical uses in data compression, etc., is therefore significantly expanded.

This proposed subband decomposition has application to blind and non-blind identification and equalization of long channels, such as those that occur in reverberative environments. These applications include the blind signal separation problem in the convolutive mixing case. Since downsampling by a factor of K makes the required processing rates in the subbands slower by a factor of K , a subbanded implementation of these problems will exhibit a reduction in complexity of $O(K)$ to $O(K^3)$, depending on the complexity of the algorithm being used. Such improvements can render computationally intractable problems feasible. It is also well established [2] that a decomposition of a large problem can result in improved statistical performance over the original configuration.

We show that aliasing in the subbands can be (approximately) eliminated through the use of filterbanks with single-sided fre-

quency responses. We then show the elimination of aliasing in the subbands is sufficient for the system $\mathbf{C}(z)$ to be *diagonal*.

Even though the idea of filterbanks with single-sided frequency responses is not new, e.g., [3], the novel contribution of this paper is the extension of classical filterbank theory to the diagonal decomposition of a broad class of problems into smaller, more tractable pieces. Further, the proposed decomposition is simplified over previous approaches [4][5] because it is diagonal, and therefore does not require interactions from other subbands. This simplicity is achieved at the expense of complex arithmetic in the subbands. However, as is shown later, the proposed structure supports downsampling rates K up to $2M$. This can result in significant computational savings over other subbanded schemes, despite the penalty for complex arithmetic.

2. GENERAL FILTERBANK STRUCTURE

In this section and the one following, we develop the complex subband decomposition of an arbitrary causal FIR LTI system $S(z)$ using the M -channel filter bank shown in Figure 1, where a diagonal MIMO system $\mathbf{C}(z)$ has been inserted between the analysis and synthesis filter banks. Note that, for the general case, as in [4][5], the system $\mathbf{C}(z)$ must be considered dense, as opposed to diagonal.

To begin, we define a set of M overlapping frequency bands Ω_m^M over the range $0 \leq \omega \leq \pi$ such that

$$\Omega_m^M = \left\{ \omega : \frac{\pi m}{M} - \epsilon \leq \omega < \frac{\pi(m+1)}{M} + \epsilon \right\}, m = 0, 1, \dots, M-1, \quad (1)$$

where ϵ is a transition bandwidth corresponding to a practical analysis filter response. Without loss of generality, we have defined the Ω_m^M only for positive values of ω ; they can also be defined only for negative values of ω .

The salient features of the analysis filters of Figure 1 are as follows. They have *single-sided* frequency responses $H_m(z)$, $m = 0 \dots, M-1$, with the response specification

$$H_m(z)|_{z=e^{j\omega}} \approx \begin{cases} \text{finite response,} & \omega \in \Omega_m^M \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

These filters are generated according to

$$H_m(\omega) = H_p\left(\omega - \frac{\pi}{M}\left(m + \frac{1}{2}\right)\right) \quad (3)$$

where $e^{j\omega}$ has been substituted for z and $H_p(\omega)$ is a prototype low-pass filter response, symmetrically spaced about $\omega = 0$, with

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cutoff frequency $\omega = \frac{\pi}{2M}$. The synthesis filters $F_m(z)$ have similar characteristics.

It is trivial to show that this filterbank corresponds to a $\frac{2M}{K}$ -times oversampled generalized DFT filterbank [6] with only the first M subband signals retained. For real input signals, the remaining M subband signals are complex conjugates of the first M signals and are therefore redundant.

The analysis and synthesis filters are designed to satisfy the following properties:

1.

$$F_m(z)H_m(zW_K^k)|_{z=e^{j\omega}} \approx 0, \begin{cases} k = 1, \dots, K-1 \\ m = 0, \dots, M-1 \\ 0 \leq \omega < 2\pi, \end{cases} \quad (4)$$

where $W_K^k = e^{-j\frac{2\pi k}{K}}$. In the following, the subscript K on W_K^k will be dropped for ease of notation. Equation (4) says that the aliasing images of the m th analysis filter overlaps with the corresponding synthesis filter only in their stopbands. This can only happen if the downsampling rate K relative to M satisfies

$$\frac{\pi}{M} + 2\epsilon \leq \frac{2\pi}{K}. \quad (5)$$

2.

$$\sum_{m=0}^{M-1} F_m(z)H_m(z) + F_m^*(z)H_m^*(z) \approx dz^{-k_0} \quad (6)$$

where d is a constant and k_0 is a delay imposed for causality. The inclusion of the second term above is equivalent to taking the real part in the time domain as shown at the output of Figure 1.

If $H_m(\omega)$ is an ideal zonal response, $\epsilon = 0$ and (5) can be satisfied for $K \leq 2M$. For practical filters where $\epsilon > 0$, there exists an integer $K < 2M$ such that (5) is satisfied. Thus, downsampling rates K up to *twice* the number of subbands is supported with the proposed single-sided analysis filters. Filters satisfying (4) and (6) can be readily designed. A filter design outline is given later in this paper.

3. DECOMPOSITION OF AN FIR LTI SYSTEM WITH SINGLE-SIDED FILTER BANKS

The quantities $U_m(z^{\frac{1}{K}})$ in Figure 1 are given as [1]

$$\mathbf{u}(z^{\frac{1}{K}}) = \frac{1}{K} \mathbf{H}^T(z^{\frac{1}{K}}) \mathbf{x}(z^{\frac{1}{K}}) \quad (7)$$

where $\mathbf{u}(z) = [U_0(z), \dots, U_{M-1}(z)]^T$, $\mathbf{H}(z)$ is the *modulation matrix* given by

$$\mathbf{H}(z) = \begin{bmatrix} H_0(z) & \dots & \dots & H_{M-1}(z) \\ H_0(zW) & \dots & \dots & H_{M-1}(zW) \\ \vdots & & & \vdots \\ H_0(zW^{K-1}) & \dots & \dots & H_{M-1}(zW^{K-1}) \end{bmatrix} \quad (8)$$

and the input vector $\mathbf{x}(z)$ is $[X(z), X(zW), \dots, X(zW^{K-1})]^T$. The output $\hat{S}(z)$ is therefore

$$\hat{S}(z) = \frac{1}{K} \mathbf{a}^T(z) \mathbf{x}(z) \quad (9)$$

where

$$\mathbf{a}(z) = \mathbf{H}(z) \mathbf{C}(z^K) \mathbf{f}(z), \quad (10)$$

$$\mathbf{C}(z^K) = \text{diag}[c_0(z^K), \dots, c_{M-1}(z^K)] \quad (11)$$

and

$$\mathbf{f}(z) = [F_0(z), \dots, F_{M-1}(z)]^T. \quad (12)$$

Let us now consider the k th element $a_k(z)$ of $\mathbf{a}(z)$ in (10). It is given as

$$a_k(z) = \frac{1}{K} \sum_{m=0}^{M-1} c_m(z^K) H_m(zW^k) F_m(z). \quad (13)$$

From (4), each set of frequency responses $H_m(zW^k)$ and $F_m(z)$ overlap only in their stopbands, for $k \neq 0$. Therefore,

$$a_k(z) \approx 0, \quad k = 1, \dots, K-1. \quad (14)$$

Note this result cannot hold for conventional double-sided (e.g., cosine-modulated) filterbanks [1], because aliasing components will always overlap (except for the impractical case where $\epsilon = 0$) regardless of the relationship between M and K . Therefore (4) is not satisfied in this case so (14) cannot hold. Equation (14) is a statement that aliasing error is (approximately) eliminated in the subbands, regardless of the values of the $c_m(z)$.

Since the overall filterbank structure is LTI [1], it suffices to specify only its impulse response to fully characterize the system. We can therefore set $X(z) = 1$, and from (9) (13) and (14) we then have

$$\hat{S}(z) = \sum_{m=0}^{M-1} H_m(z) c_m(z^K) F_m(z). \quad (15)$$

For $\hat{S}(z)$ to approximate some desired LTI system $S(z)$ whose impulse response is real, we can choose the $c_m(z^K)$ to satisfy

$$H_m(z) c_m(z^K) \approx H_m(z) S(z). \quad (16)$$

This is equivalent to choosing $c_m(z^K)|_{z=e^{j\omega}} = S(z)|_{z=e^{j\omega}}$, for $\omega \in \Omega_m^M$. Since $c_m(z^K)$ is periodic with period $2\pi/K$ but $H_m(z)$ is periodic with period 2π , equation (16) implies that it is sufficient to consider the frequency response of $c_m(z^K)$ over the $2\pi/K$ band that contains Ω_m^M . Since the bandwidth of Ω_m^M is always smaller than $2\pi/K$ for $K < 2M$ (from (5)) there is a range of frequencies over which the frequency response of $c(z^K)$ is arbitrary, regardless of the frequency response of $S(z)$.

To show that (16) indeed results in $\hat{S}(z)$ approximating $S(z)$, we substitute (16) into (15) to obtain

$$\hat{S}(z) \approx \frac{S(z)}{K} \sum_{m=0}^{M-1} H_m(z) F_m(z). \quad (17)$$

After taking the real part in the time domain, we see from (6) that this is equivalent to

$$\hat{S}(z) \approx \frac{S(z)}{K} dz^{-k_0} \quad (18)$$

so that $\hat{S}(z)$ is within a scale and a delay of $S(z)$ as required. We have therefore shown that the subbanded system of Figure 1 with an appropriately chosen diagonal system $\mathbf{C}(z)$ inserted between the analysis and synthesis filterbanks can approximate an arbitrary

LTI system. The $c_m(z)$ satisfying (15) are referred to as the *subband components* of $\hat{S}(z)$.

Even though this development has treated only the single-input single-output case, it can be extended to the multiple-input multiple output case in an obvious way. This can be done by incorporating a separate analysis filterbank for each required input, and a separate synthesis filterbank for each required output.

Due to limitations of space, the details are omitted, but it is straightforward to show [11] that $c_m(z)$ exist whose order is indeed $O(\frac{L}{K})$, where L is the order of $S(z)$.

Note also that, because aliasing distortion is suppressed rather than carefully cancelled as in conventional subbanding designs, the design of the filters $H_m(z)$ and $F_m(z)$ involve fewer constraints than is the case for more conventional subbanding systems. The only requirement is that (4) and (6) be satisfied. For example, we may choose the synthesis prototype to be the paraconjugate of the analysis filter prototype $H_p(z)$. This allows an analysis of the filterbank as an implementation of a tight frame expansion, which is desirable for the error analysis of subband adaptive filters [7]. With reference to Figure 2, the design procedure for the prototype filter $H_p(z)$ is

1. Minimize the stopband energy in the frequency regions $[\frac{2\pi}{K} - \epsilon, \pi]$ and $[\frac{\pi}{M} - \frac{2\pi}{K} + \epsilon, -\pi]$
2. Design $H_p(z)$ such that $H_p(z)\tilde{H}_p(z)$ is a Nyquist $2M$ filter

where $\tilde{H}_p(z)$ is the paraconjugate of $H_p(z)$.

4. EXAMPLE

We illustrate the proposed subband decomposition procedure with the AEC problem [4]–[5], [8]–[11]. This is well known to be a difficult adaptive filtering problem. An adaptive filter may be modelled as a 2-input single-output linear system. As such, two sets of analysis filters are required (for the desired and filter inputs respectively) and one synthesis filter bank. The AEC algorithm is implemented by replacing the $c_m(z)$ -block in Figure 1 with an adaptive filter in each subband, with the respective inputs and output connected to the appropriate filterbank.

We demonstrate two cases, each using an acoustic impulse response measured in a real room of 2000 samples in duration, at a sampling rate of 8 kHz. The input in each case is real speech, approximately 9 seconds in duration. The two test cases correspond to a) the fullband case where no subbanding is used, and b) the single-sided subband approach proposed here. In the latter case, $M = 10$ subbands are used, with the analysis and synthesis filters each containing 120 coefficients. The adaptive filter algorithm is the normalized LMS algorithm [2]. Further detail is given in [9].

Results showing echo cancellation vs. time for the fullband echo cancellation case (a) are shown in Figure 3. The bottom graphs show the echo return loss enhancement (ERLE) vs. time. In Figures 3 and Figure 4, the results have been obtained by averaging the power of the respective signals over a sliding window of 2000 samples in duration. The value of the learning rate μ for this case (a) is 1.0. It is seen from Figure 3 that the convergence rate is not as fast as desired- just under 6 seconds are required to achieve an echo cancellation level of 30dB. Further, the computational load is heavier than necessary. It is shown [9] that this configuration requires 32 MIPS¹ to implement.

¹MIPS=Millions of Instructions per Second. An instruction includes

The results for the proposed diagonal subband decomposition are shown in Figure 4. It is interesting to note that in this case, because of the single-sided filters, up to $2 \times$ undersampling can be exploited, as previously noted. In this case, we have used $K = 1.5M$. The significant improvement in convergence speed offered by this approach is apparent from the figure. A 30dB cancellation level is achieved in just over 3 seconds. However, the point to be emphasized is the reduction in MIP count afforded by the proposed method. In this case, the total MIP count is 8.3 [9], including filterbank overhead. This figure is achieved despite the penalty of complex arithmetic, which nominally requires $4 \times$ the MIP count for real arithmetic. The complex arithmetic penalty is abated by the use of the undersampled downsampling rate of $1.5M$. It is interesting to note that the MIP count for the proposed configuration is also lower than that for the method proposed in [4], which requires a count of 12 MIPS in total for similar experimental conditions [9]. This latter figure is higher than the proposed subband decomposition, despite the penalty of complex arithmetic, because, due to its *tridiagonal* structure, nearly three adaptive filters are required per subband, and a critical downsampling rate must be used.

5. CONCLUSIONS

In this paper we have shown that the use of single-sided filterbanks leads to a *diagonal* decomposition of a broad class of large problems into smaller, more tractable pieces. The field of application of filterbank theory is therefore significantly expanded. The effectiveness of the method has been demonstrated by the improved performance and the significantly reduced computational load for the AEC problem.

6. REFERENCES

- [1] P.P.Vaidyanathan, "Multirate systems and filter banks", Prentice Hall, New Jersey, 1993.
- [2] S. Haykin, "Adaptive Filter Theory", Prentice Hall, New Jersey, 3rd Ed., 1996.
- [3] S. Weiss, M. Hartneck and R.W. Stewart, "Design of near perfect reconstruction oversampled filter banks for subband adaptive filters", *IEEE Trans. Circ. Syst. II*, Aug. 1999, pp. 1081-1085.
- [4] A. Gilliore and M. Vetterli, "Adaptive filtering in subbands with critical sampling", *IEEE Trans. Sig. Proc.*, Aug. 1992, pp. 1862-1875.
- [5] Q. Jin, Z-Q. Luo, and K.M. Wong, "Optimum filter banks for signal decompositions and its application in Adaptive Echo Cancellation", *IEEE Trans. Sig. Proc.*, July 1996, pp. 1669-1679.
- [6] R.E. Crochiere and L.R. Rabiner, "Multirate Digital Signal Processing", Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1983.
- [7] S. Weiss, A. Stenger, R. Rabenstein and R. W. Stewart, "Steady state performance limitations of subband adaptive filters", to appear in *IEEE Trans. Sig. Proc.*

one real multiply plus one real add.

- [8] S.S. Pradhan and V.U. Reddy, "A new approach to sub-band adaptive filtering", *IEEE Trans. Sig. Proc.* Mar. 1999, pp.655-664.
- [9] M.T. Seibert, "Acoustic echo cancellation using single-sided subbanding" M.Eng Thesis, Dept. of Elect. and Comp. Eng., McMaster University, Hamilton, Ontario, Canada, October, 1999.
- [10] J. Reilly, M.Seibert, N.Ahmadvand, and G. Reesor, "Single-sided filters with application to Acoustic Echo Cancellation", Patent Application, submitted August 1998.
- [11] J. Reilly, M. Seibert, M. Wilbur, N. Ahmadvand, "The decomposition of large problems using single-sided subbanding", full paper version, in preparation.

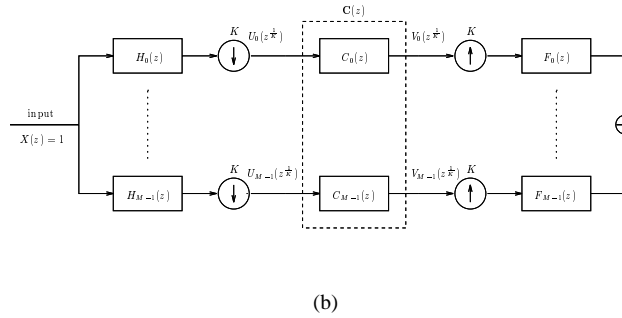
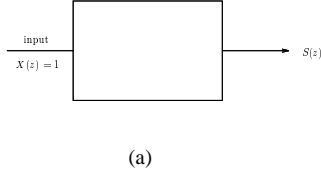


Figure 1: (a) An arbitrary LTI system we wish to approximate using the configuration of part (b). (b) A subband realization of a response $\hat{S}(z)$.

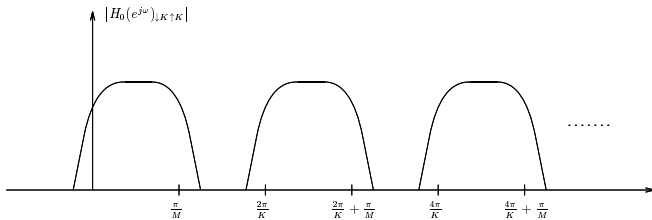


Figure 2: Amplitude spectrum of $H_0(e^{j\omega})$ after downsampling then upsampling

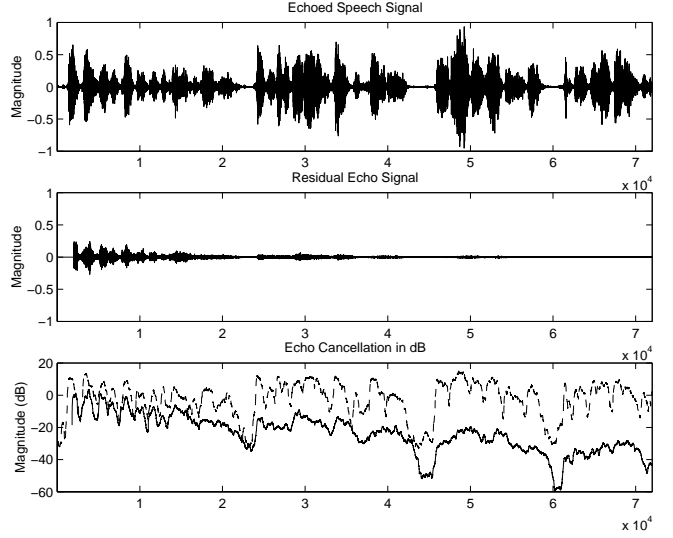


Figure 3: Echo canceller results for the fullband case. Top: echoed speech signal, Middle: residual echo after the canceller, Bottom: ERLE performance vs. time.

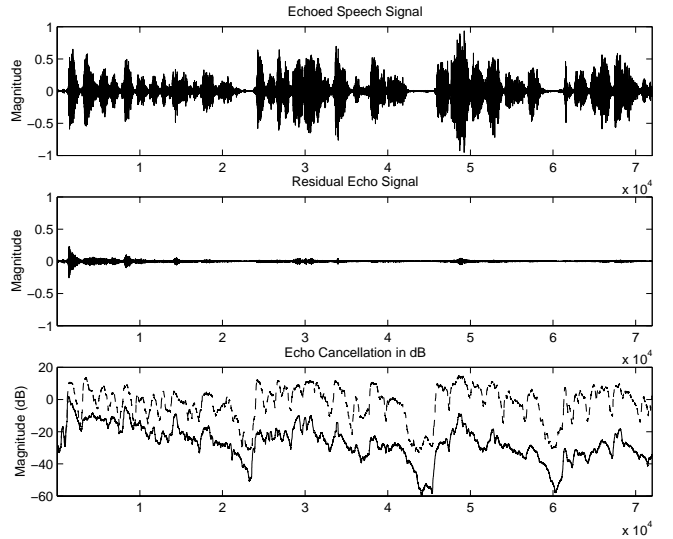


Figure 4: Echo canceller results for the single sided subbanding case.