APPLICATION OF PARTICLE FILTERS FOR TRACKING MOVING RECEIVERS IN WIRELESS COMMUNICATION SYSTEMS

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ABSTRACT

This paper presents a Bayesian approach for tracking the times-of-arrival (TOAs) of multiple *wideband* moving targets using a linear, passive sensor array, for use in smart antenna wireless applications. A novel form of posterior distribution describing the parameters of the received signal is developed. The nuisance parameters (source amplitudes and variances) of this distribution are integrated out to form a more computationally efficient estimator. The desired parameters are then tracked using *particle filtering* techniques. Simulation results demonstrate the effectiveness and robustness of this method.

1. INTRODUCTION

Over the past few years, the use of smart antennas has emerged as a valuable tool in mobile communications. The application of smart antennas can be enhanced if the directions of arrival of the desired receivers can be easily tracked. Many techniques have been developed for tracking the DOAs of *narrowband* signal sources observed with a sensor array. These include beamforming, signal-subspace techniques and maximum likelihood methods, as well as MCMC techniques, specifically *particle filters,* [1] [2] which are modern Bayesian methods based on numerically approximating the posterior distribution of interest. Of these mentioned techniques, the *wideband* tracking problem using arrays of sensors has not received much attention to date. Nevertheless, solutions to this problem are of interest for wireless communications (particularly in the CDMA scenario), military, naval, air traffic control operations, and the 911-problem.

In this paper, we propose a novel approach that leverages recent advances in sequential MCMC techniques, allied with the persistent increase in computer power, to solve the wideband tracking problem. Here, we formulate a novel model for the wideband received signal vector, which then leads to a posterior distribution in the target angles. Tracking the moving targets is then implemented in an efficient manner through the use of particle filters. In addition, the signal waveforms from multiple emitters can be separated and restored sequentially.

This paper is organized as follows. Section 2 presents the state-space model and the development of the posterior distribution of the unknown parameters. Section 3 describes the importance sampling procedure and introduces the optimal importance function. Section 4 presents simulation results and discussion. Section 5 presents the conclusions.

2. STATE SPACE MODEL

The sequential sampling approach we adopt admits a first order state-space hidden Markov model. The states $[\boldsymbol{\tau}(t); \boldsymbol{a}(n)]$ evolve according to:

$$\begin{aligned} \boldsymbol{\tau}(t) &= \boldsymbol{\tau}(t-1) + \sigma_v \boldsymbol{v}(t), && (1) \\ \boldsymbol{a}(n) &\sim \mathcal{N}\left(\mathbf{0}, \delta^2 \sigma_w^2 \boldsymbol{I}_K\right), && (2) \end{aligned}$$

where $\boldsymbol{\tau}(t) \in \mathcal{R}^{K\times 1}$ and $\boldsymbol{a}(n) \in \mathcal{R}^{K\times 1}$ are the times-of-arrival (TOAs) and the source amplitudes respectively of K (assumed known) plane waves impinging on an array of M sensors, $\boldsymbol{v}(t)$ is an iid Gaussian variable with zero mean and unit variance, σ_v^2 and σ_w^2 are the process and noise variances respectively, and δ^2 is a hyper-parameter, corresponding to an estimate of the SNR. According to [3], a snapshot at time t can be expressed as

$$\begin{aligned} \boldsymbol{y}(t) &= \sum_{k=0}^{K-1} \boldsymbol{s}_k(t - \tau_k(t)) + \sigma_w \boldsymbol{w}(t), && (3) \\ &\approx \sum_{k=0}^{K-1} \tilde{\boldsymbol{H}}(\tau_k(t)) \boldsymbol{s}_k(n) + \sigma_w \boldsymbol{w}(t), && (4) \end{aligned}$$

where[1] $\boldsymbol{w}(t)$ is an iid Gaussian variable with zero mean and unit variance which is uncorrelated with the signal, $\tilde{\boldsymbol{H}}(\tau_k(t)) \in$

[1]Note that for notational convenience, from this point onwards we replace the approximation with an equality.

$\mathcal{R}^{M\times L}$ is an Lth–order *interpolation matrix* for the sources [3], defined as

$$\tilde{\boldsymbol{H}}(\tau_k(t)) = \begin{cases} \boldsymbol{H}(\tau_k(t)), & \text{if } \tau_k(t) \geq 0 \\ \boldsymbol{E}_M \boldsymbol{H}(\tau_k(t)), & \text{if } \tau_k(t) < 0 \end{cases}, \quad (5)$$

where each row of $\boldsymbol{H}(\tau_k(t))$ represents L coefficients computed from an appropriately selected interpolation function [2], and $\boldsymbol{E}_M$ is an exchange matrix [4]. The quantities $\boldsymbol{s}_k(t)$, the kth signal, and $\boldsymbol{s}_k(n)$, the corresponding discrete–time version, are defined respectively as

$$\boldsymbol{s}_k(t-\tau_k) = \\ [s_k(t), s_k(t-\tau_k(t)), \ldots, s_k(t-(M-1)\tau_k(t))]^T, \quad (6)$$

$$\boldsymbol{s}_k(n) = [s_k(n), s_k(n-1), \ldots, s_k(n-(L-1))]^T. \quad (7)$$

The interpolation matrix $\tilde{\boldsymbol{H}}(\tau_k(t))$ in (4) interpolates the discrete–time sequences $\boldsymbol{s}_k(n)$ to give the desired sequences $\boldsymbol{s}_k(t-m\tau_k(t)), m = 0,\ldots,M-1, k = 0,\ldots,K-1$, which correspond to the signals from the kth source at the mth array element at time t.

We now re–order (4) into a more convenient form as follows. By defining $\tilde{\boldsymbol{H}}_l(\boldsymbol{\tau}(t)) \in \mathcal{R}^{M\times K}$ as

$$\tilde{\boldsymbol{H}}_l(\boldsymbol{\tau}(t)) = \left[\tilde{\boldsymbol{H}}_l(\tau_0(t)), \tilde{\boldsymbol{H}}_l(\tau_1(t)), \ldots, \tilde{\boldsymbol{H}}_l(\tau_{K-1}(t))\right], \quad (8)$$

which collects the lth columns of the interpolation matrices in (5) for K sources, we can then express (4) in the form

$$\boldsymbol{y}(t) = \sum_{l=0}^{L-1} \tilde{\boldsymbol{H}}_l(\boldsymbol{\tau}(t))\boldsymbol{a}(n-l) + \sigma_w \boldsymbol{w}(t), \quad (9)$$

where $\boldsymbol{a}(n) \triangleq [s_0(n), s_1(n), \ldots, s_{K-1}(n)]^T$. We define a vector $\boldsymbol{z}(n)$ (which is a function of only *past* source values) as follows

$$\boldsymbol{z}(t) \triangleq \boldsymbol{y}(t) - \sum_{l=1}^{L-1} \tilde{\boldsymbol{H}}_l(\boldsymbol{\tau}(t))\boldsymbol{a}(n-l), \quad (10)$$

and therefore according to (9) we have

$$\boldsymbol{z}(t) = \tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}(t))\boldsymbol{a}(n) + \sigma_w \boldsymbol{w}(t), \quad (11)$$

which represents the desired form of the wideband model. The unknown and time–varying TOAs, $\boldsymbol{\tau}(t)$, are to be sequentially estimated based on the observations $\boldsymbol{y}(t)$. It can be shown [3] that model $\boldsymbol{z}(t)$ can accommodate both the narrowband and wideband signals without change of structure and parameters. Also, because the data model is purely real, significant savings in hardware can be achieved, since quadrature mixing to IF frequencies is no longer required. Also computational requirements are reduced.

We define the vector of parameters $\boldsymbol{\theta}$ as

$$\boldsymbol{\theta}_{1:t} \triangleq \left(\{\boldsymbol{\tau}\}_{1:t}, \{\boldsymbol{a}\}_{1:t}, \sigma_v^2, \sigma_w^2\right), \quad (12)$$

where the notation $(.)_{1:t}$ indicates all the elements from time 1 to time t. Hence, the joint distribution of all the parameters is $\pi(\boldsymbol{\theta}_{1:t}) \triangleq p(\boldsymbol{\theta}_{1:t}|\boldsymbol{z}_{1:t})$, which can then be expanded using appropriately selected prior distributions of the parameters, according to Bayes' theorem as

$$\pi(\boldsymbol{\theta}_{1:t}) \propto p\left(\boldsymbol{z}_{1:t}|\boldsymbol{\tau}_{1:t}, \boldsymbol{a}_{1:t}, \sigma_v^2, \sigma_w^2\right) p\left(\boldsymbol{\tau}_{1:t}|\sigma_v^2\right) \\ \times p\left(\boldsymbol{a}_{1:t}|\boldsymbol{\tau}_{1:t}, \sigma_w^2\right) p(\sigma_w^2) p(\sigma_v^2), \quad (13)$$

where $p(z_{1:t}|\cdot)$ is the likelihood term, and the remaining distributions constitute the joint prior distribution for the parameters $\boldsymbol{\theta}$.

It is assumed that the observations, given the states, are iid and that the state conditional update likelihood is also iid. Therefore, we now assign distributions for each of the terms in (13) as

$$p(\boldsymbol{z}_{1:t}|\boldsymbol{\theta}_{1:t}) = \prod_{l=1}^{t} \mathcal{N}\left(\tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}_l)\boldsymbol{a}(n), \sigma_w^2 \boldsymbol{I}_M\right), \quad (14)$$

$$p\left(\boldsymbol{\tau}_{1:t}|\sigma_v^2\right) = \prod_{l=1}^{t} \mathcal{N}\left(\boldsymbol{\tau}_{l-1}, \sigma_v^2 \boldsymbol{I}_K\right), \quad (15)$$

$$p\left(\boldsymbol{a}_{1:t}|\boldsymbol{\tau}_{1:t}, \sigma_w^2\right) = \prod_{l=1}^{t} \mathcal{N}\left(\boldsymbol{0}, \delta^2\sigma_w^2 \left[\tilde{\boldsymbol{H}}_0^T(\boldsymbol{\tau}_l)\tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}_l)\right]^{-1}\right), \quad (16)$$

where $\mathcal{N}(\boldsymbol{\mu}|\boldsymbol{\Sigma})$ is the multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. The prior distribution for the source amplitude vector $\boldsymbol{a}$ is chosen as in [3] [5] [6]. The prior distribution on the variances σ_v^2 and σ_w^2 are both assumed to follow non–informative inverse Gamma distributions as follows

$$p(\sigma_w^2) \sim \mathcal{IG}\left(\frac{\nu_0}{2}, \frac{\gamma_0}{2}\right), \quad (17)$$

$$p(\sigma_v^2) \sim \mathcal{IG}\left(\frac{\nu_1}{2}, \frac{\gamma_1}{2}\right) \quad (18)$$

The above priors are noninformative when the hyperparameters ν and γ are set to zero. Combining these prior functions according to (13), and then simplifying the resulting

[2] For example, in the case of the uniform linear array, the interpolation function can be a windowed *sinc*$(\cdot)$ function.

functon [3] [6] yields

$$\pi\left(\boldsymbol{\theta}_{1:t}\right) \propto$$
$$\prod_{l=0}^{t} \frac{1}{\left(2\pi\sigma_w^2\right)^{M/2}} \exp\left\{\frac{-1}{2\sigma_w^2} \boldsymbol{z}_l^T \boldsymbol{P}_0^{\perp}\left(\boldsymbol{\tau_l}\right) \boldsymbol{z}_l\right\} \times$$
$$\prod_{l=0}^{t} \frac{1}{\left(2\pi\sigma_v^2\right)^{k/2}} \exp\left\{\frac{-1}{2\sigma_v^2}\left(\boldsymbol{\tau}_l-\boldsymbol{\tau}_{l-1}\right)^T\left(\boldsymbol{\tau}_l-\boldsymbol{\tau}_{l-1}\right)\right\} \times$$
$$\prod_{l=0}^{t} \frac{|\tilde{\boldsymbol{H}}_0^T(\boldsymbol{\tau}_l)\tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}_l)|}{\left(2\pi\delta^2\sigma_w^2\right)^{k/2}}$$
$$\exp\left\{\frac{-1}{2\delta^2\sigma_w^2}\left(\boldsymbol{a}_l-\boldsymbol{m}_a(l)\right)^T \boldsymbol{\Sigma}_0^{-1}\left(\boldsymbol{\tau_l}\right)\left(\boldsymbol{a}_l-\boldsymbol{m}_a(l)\right)\right\}$$
$$\times\, \sigma_w^{2^{-(\frac{\nu_0}{2}+1)}} \exp\left\{\frac{-\gamma_0}{2\sigma_w^2}\right\} \times \sigma_v^{2^{-(\frac{\nu_1}{2}+1)}} \exp\left\{\frac{-\gamma_1}{2\sigma_v^2}\right\}, \tag{19}$$

where

$$\boldsymbol{\Sigma}_0^{-1}\left(\boldsymbol{\tau}_l\right) = \left(1+\delta^{-2}\right)\tilde{\boldsymbol{H}}_0^T(\boldsymbol{\tau}_l)\tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}_l), \tag{20}$$

$$\boldsymbol{m}_a(l) = \boldsymbol{\Sigma}_0\left(\boldsymbol{\tau}_l\right)\tilde{\boldsymbol{H}}_0^T(\boldsymbol{\tau}_l)\boldsymbol{z}_l, \tag{21}$$
$$= \boldsymbol{\Sigma}_0\left(\boldsymbol{\tau}_l\right)\tilde{\boldsymbol{H}}_0^T\left(\boldsymbol{\tau}_l\right)\left(\boldsymbol{y}(l) - \sum_{j=1}^{L-1}\tilde{\boldsymbol{H}}_j(\boldsymbol{\tau}_l)\boldsymbol{a}(n-j)\right),$$

and

$$\boldsymbol{P}_0^{\perp}\left(\boldsymbol{\tau}_l\right) = \boldsymbol{I} - \frac{\tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}_l)\left[\tilde{\boldsymbol{H}}_0^T(\boldsymbol{\tau}_l)\tilde{\boldsymbol{H}}_0(\boldsymbol{\tau}_l)\right]^{-1}\tilde{\boldsymbol{H}}_0^T(\boldsymbol{\tau}_l)}{(1+\delta^{-2})}. \tag{22}$$

From (19) and (21), a *maximum a posteriori* esitmate of the amplitudes $\boldsymbol{a}(n)$, given all other parameters, is readily available as

$$\hat{\boldsymbol{a}}(n) \triangleq \boldsymbol{m}_a(n). \tag{23}$$

In this problem, the only parameters of interest are the $\boldsymbol{\tau}_{1:t}$, and the remaining ones can be considered as nuisance parameters and analytically integrated out. This integration on (19) yields a posterior distribution of the form

$$\boldsymbol{\pi}(\boldsymbol{\tau}_{1:t}) \propto$$
$$\prod_{l=1}^{t} \frac{1}{\sigma_w^{2M}(1+\delta^2)^{k_l}} \exp\left[\frac{-1}{\sigma_w^2}\boldsymbol{z}_l^T\boldsymbol{P}_s^{\perp}(\boldsymbol{\tau}_l)\boldsymbol{z}_l\right]$$
$$\times \prod_{l=1}^{t} \frac{1}{\sigma_v^{2^{(k_l/2)}}(2\pi)^{(k_l/2)}} \exp\left[\frac{-1}{2\sigma_v^2}(\boldsymbol{\tau}_l-\boldsymbol{\tau}_{l-1})^T(\boldsymbol{\tau}_l-\boldsymbol{\tau}_{l-1})\right]$$
$$\times\, \kappa(\sigma_v^2, \sigma_w^2) \tag{24}$$

where $\kappa(\sigma_v^2, \sigma_w^2)$ is a function only of σ_v^2 and σ_w^2. Eq. (24) is used to estimate the $\boldsymbol{\tau}$'s using the particle filtering (sequential importance sampling) approach discussed below. The σ_v^2 and σ_w^2 can be estimated according to a *maximum a posteriori* (MAP) procedure outlined in [6], and the amplitudes $\boldsymbol{a}$(n) can be obtained according to (23).

3. SEQUENTIAL IMPORTANCE SAMPLING

This section briefly describes the sequential sampling sampling (SIS) procedure, which is used to extract the TOA estimates for tracking. The reader is referred to [2] [6] [7] and the references therein for a more complete coverage of this topic.

Our objective is to generate a numerical approximation of the desired posterior distribution $\pi(\boldsymbol{\tau}_{1:t})$, in the form of a histogram, by drawing a large number N of samples, from an *importance function*, $q(\boldsymbol{\tau}_{1:t})$, whose support includes that of $\pi(\boldsymbol{\tau}_{1:t})$. It can be shown [2] [7] that the desired posterior distribution function $\pi(\boldsymbol{\tau}_{1:t})$ can be approximated using a set of importance weights $w^{(i)}(t) = \tilde{w}^{(i)}(t)/\sum_i^N \tilde{w}^{(i)}(t)$, $i = 1, ..., N$, where

$$\sum_{i=1}^{N} w^{(i)}(t) = 1, \quad \tilde{w}^{(i)}(t) = \frac{\pi(\boldsymbol{\tau}_t^{(i)})}{q(\boldsymbol{\tau}_t^{(i)})}. \tag{25}$$

Moreover, the posterior function $\pi(\boldsymbol{\tau}_{1:t})$ can be recursively updated using $w^{(i)}(t)$ for $i = 1, ..., N$ as

$$\tilde{w}^{(i)}(t) = \tilde{w}_{t-1}^{(i)} \times \frac{p\left(\boldsymbol{z}_t|\boldsymbol{\tau}_t^{(i)}\right)p\left(\boldsymbol{\tau_t^{(i)}}|\boldsymbol{\tau_{t-1}^{(i)}}\right)}{q\left(\boldsymbol{\tau_t^{(i)}}|\boldsymbol{\tau_{1:t-1}^{(i)}}, \boldsymbol{z_{1:t}}\right)}. \tag{26}$$

At each observation time t, samples (particles) are generated from an *optimal* importance function [2] [1], which satisfies a recurrence requirement in (26) and minimizes the variance of the weights generated by the recursion.

The optimal importance function can be chosen to be proportional to $p\left(\boldsymbol{z}_t|\boldsymbol{\tau}_t\right)p\left(\boldsymbol{\tau}_t|\boldsymbol{\tau}_{t-1}\right)$, the proportionality being independent of $\boldsymbol{\tau}_t$. We can approximate the optimal importance function locally by computing the gradient and Hessian at a point close to its mode [8]. Let the log of the optimal importance function be $L(\boldsymbol{\tau}_t) = L_z(\boldsymbol{\tau}_t) + L_\tau(\boldsymbol{\tau}_t)$, where $L_z(\boldsymbol{\tau}_t) = \log p\left(\boldsymbol{z}_t|\boldsymbol{\tau}_t\right)$, and $L_\tau(\boldsymbol{\tau}_t) = \log p\left(\boldsymbol{\tau}_t|\boldsymbol{\tau}_{t-1}\right)$. We use a second-order Taylor expansion about $\boldsymbol{\tau}_{t-1}$ to give

$$\begin{aligned} L(\boldsymbol{\tau}_t) \approx\ & L_z(\boldsymbol{\tau}_{t-1}) + L_\tau(\boldsymbol{\tau}_{t-1}) \\ & + \nabla^T L(\boldsymbol{\tau}_t)\left(\boldsymbol{\tau}_t - \boldsymbol{\tau}_{t-1}\right) \\ & + \frac{1}{2}\left(\boldsymbol{\tau}_t - \boldsymbol{\tau}_{t-1}\right)^T \nabla^2 L(\boldsymbol{\tau}_t)\left(\boldsymbol{\tau}_t - \boldsymbol{\tau}_{t-1}\right), \end{aligned} \tag{27}$$

where $\nabla L(\boldsymbol{\tau}_t) \in \mathcal{R}^{K\times 1}$ and $\nabla^2 L(\boldsymbol{\tau}_t) \in \mathcal{R}^{K\times K}$ are the gradient and the Hessian matrix of $L(\boldsymbol{\tau}_t)$, respectively. It can be shown that the optimal importance function, which is Gaussian, can be expressed as

$$q\left(\boldsymbol{\tau}_t^{(i)}|\boldsymbol{\tau}_{t-1}^{(i)}, \boldsymbol{z}_t\right) \sim \mathcal{N}\left(\boldsymbol{m}_t, \boldsymbol{\Sigma}_t\right), \tag{28}$$

where

$$\boldsymbol{\Sigma}_t = -\left(\nabla^2 L(\boldsymbol{\tau}_t)\right)^{-1}, \tag{29}$$

$$\boldsymbol{m}_t = \boldsymbol{\tau}_{t-1} + \boldsymbol{\Sigma}_t \nabla L(\boldsymbol{\tau}_t). \tag{30}$$

A well-known problem with the particle filter is that the algorithm involving (26) degenerates very quickly [2], where only a handful of particles have meaningful weights after a few iterations. To minimize the resulting variances of the estimates, resulting from lack of diversity in the importance weights of the particles, an MCMC resampling scheme based on the importance weights of the particles, that can be done very efficiently with $\mathcal{O}(N)$ operations, is employed [2] [6]. Unfortunately, the trajectories with high importance weights are statistically selected many times, limiting the true statistical diversity amongst the particles. An efficient way of limiting sample depletion is to apply an MCMC step [7] [9] on each particle at time t.

In this step, the Metropolis-Hasting [10] one-at-the-time MCMC scheme is employed for each particle, where a candidate sample of the unknown parameter, $\boldsymbol{\tau}^\star$, is drawn according to an appropriately selected proposal distribution, $d(\cdot)$. This candidate will be accepted as the new state with probability $\eta = \min\{1, r_{update}\}$, where r_{update} is

$$r_{update} = \frac{\pi(\boldsymbol{\tau}^\star)\, d\left(\boldsymbol{\tau}_t^{(i)}\right)}{\pi\left(\boldsymbol{\tau}_t^{(i)}\right) d(\boldsymbol{\tau}^\star)}. \tag{31}$$

As a result, true statistical diversity amongst the particles can be re-introduced.

4. SIMULATION RESULTS

The proposed algorithm is now applied to a wideband scenario to demonstrate the capability for tracking the $\boldsymbol{\tau}(t)$ and restoring signal waveforms, for $K = 2$ sources, where the other parameters are listed in Tables 4.1. The received array is composed of $M = 8$ elements, and the adjacent sensors of the array are spaced by $\lambda/2$ at the highest frequency of interest. We denote the normalized sampling frequency by $F_s = 1.0$ Hz. The TOA trajectories, $\boldsymbol{\tau}(t)$, are generated by (1), and the two sources amplitudes are Gaussian processes that are zero mean, each with variance $\delta^2\sigma_w^2$, and bandlimited to normalized frequency $[0.1, 0.4]$ Hz, i.e., the bandwidth is 0.3 Hz. The SNR is 13 dB, and the hyper-parameter $\delta^2 = 20$ is assumed known and constant.

A total of 10 independent trials, each consisting of 1,000 observations and using $N = 50$ particles, is used in the simulation. Figures 1-2 show the results for randomly selected trials. The proposed algorithm randomly initializes all unknown parameters. Figure 1 shows the comparison between the true and estimated tracks of $\boldsymbol{\tau}(t)$. Unlike ordinary MCMC, particle filters do not require a burn-in period

Parameter	Value
L	8
F_s (Hz)	1.0
σ_w^2	0.005
σ_v^2 (sec^2)	$(30000F_s)^{-1}$

Table 4.1. Parameters for the experiments.

[11]. According to Figure 1, it is clear that the TOAs are well tracked by their estimates throughout the entire tracking process. Moreover, Figure 2 shows that the signal amplitudes are well separated and restored by the algorithm. Note that to show the results clearly, only a portion of the restored amplitudes are displayed. Table 4.2 lists the mean-squared errors of the estimates of $\boldsymbol{\tau}(t)$ and $\boldsymbol{a}(n)$ over the 10 trials.

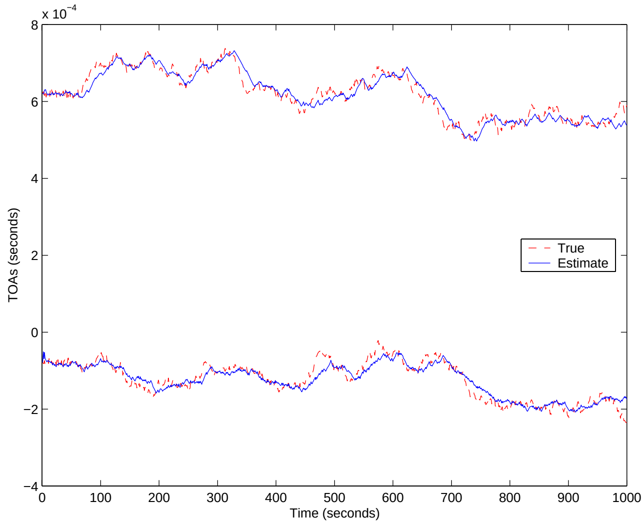

Fig. 1. Sequential estimates of the times-of-arrival, $\boldsymbol{\tau}(t)$.

Parameter	Mean-squared error (dB)
$\boldsymbol{\tau}(t)F_s$	-34.54, -40.27
$\boldsymbol{a}(n)$	-18.44, -18.46

Table 4.2. Comparison between the true and estimated parameters.

In addition, the proposed algorithm was applied to 50 different scenarios each of 500 observations, for different values of SNR, in order to estimate the variance of the estimate as a function of the SNR. The results are shown in Figure 3.

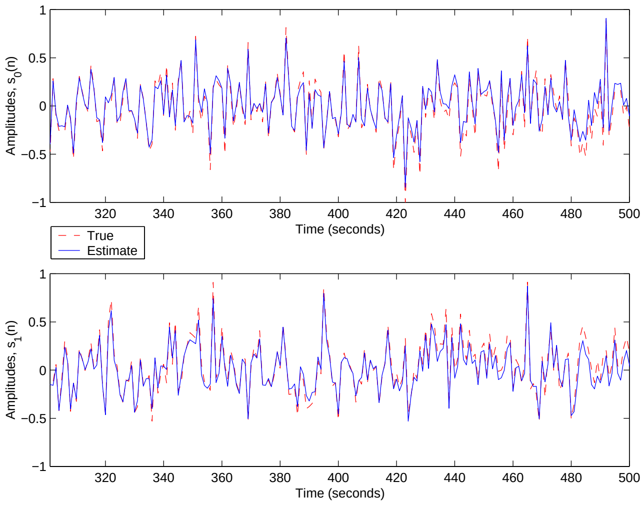

Fig. 2. Sequential estimates of the signal amplitudes, $\boldsymbol{a}(n)$.

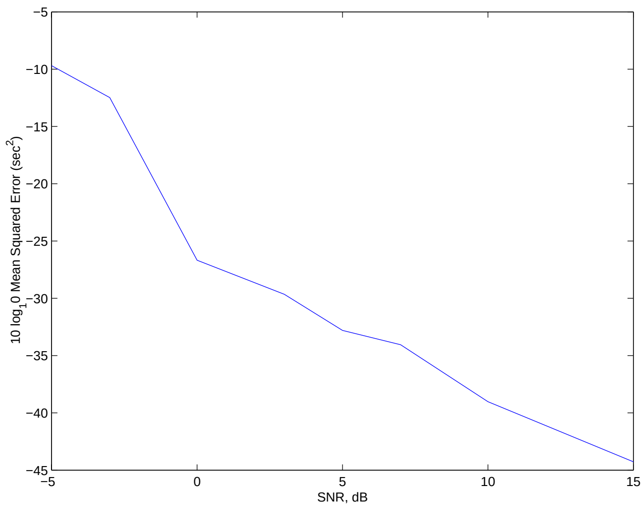

Fig. 3. Variances of the estimates vs. SNR.

5. CONCLUSIONS

A novel approach for wideband array signal processing is proposed. The proposed method would be useful for smart antenna applications where CDMA is used. A Bayesian approach is adopted, where a posterior density function which has the nuisance parameters integrated out is formulated. Using sequential MCMC techniques, the TOAs of wideband sources can be tracked, and the amplitudes are separated and restored. Simulation results support the effectiveness of the method, and demonstrate reliable tracking of the times of arrival of wideband sources in a white noise environment with a uniform linear array of sensors.

6. REFERENCES

[1] Arnaud Doucet, "On sequential simulation-based methods for Bayesian filtering.," Tech. Rep. TR.310, University of Cambridge, Department of Engineering, Signal Processing Group, England, 1998.

[2] Arnaud Doucet, Nando de Freitas, and Neil Gordon, Eds., *Sequential Monte Carlo in Practice*, Springer-Verlag, New York, 2001, To appear.

[3] William Ng, James. P. Reilly, Thia Kirubarajan, and Jean-René Larocque, "Wideband array signal processing using mcmc methods," *IEEE Transactions on Signal Processing*, 2002, Submitted for publication, also available at http://www.ece.mcmaster.ca/∼reilly.

[4] Gene H. Golub and Charles F. Van Loan, *MATRIX Computations, 2nd Edition*, The Johns Hopkins University Press, Baltimore, Maryland, 1993.

[5] C. Andrieu and A. Doucet, "Joint Bayesian model selection and estimation of noisy sinusoids via reversible jump MCMC," *IEEE Transactions on Signal Processing*, vol. 47, no. 10, pp. 2667–2676, Oct. 1999.

[6] Jean-René Larocque, James. P. Reilly, and William Ng, "Particle filter for the tracking of an unknown number of sources," *IEEE Transactions on Signal Processing*, vol. 50, no. 12, pp. 2926–2937, Dec. 2002.

[7] C. Andrieu, N. De Freitas, and A. Doucet, "Sequential MCMC for Bayesian model selection," in *Proceedings of the Internationl Workshop on Higher Order Statistics*, Ceasarea, Isreal, 1999.

[8] Matthew Orton and William Fitzgerald, "A Bayesian Apporach to Tracking Multiple Targets Using Sensor Arrays and Particle Filters," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 216–223, Feb. 2002.

[9] C. Andrieu, A. Doucet, W.J. Fitzgerald, and S.J. Godsill, "An introduction to the theory and applications of simulation based computational methods in Bayesian signal processing," in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, Seattle, WA, 1998.

[10] W.K. Hastings, "Monte Carlo sampling methods using Markov chains and their applications," *Biometrika*, vol. 57, no. 1, pp. 97–109, 1970.

[11] A. Doucet, N.J. Gordon, and V. Krishnamurthy, "Particle filters for state estimation of jump Markov linear systems," *IEEE Transactions on Signal Processing*, vol. 49, no. 3, pp. 613–624, Mar. 2001.