A Bayesian Approach to Blind Source Recovery

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***Abstract*— This paper presents a Bayesian approach for blind source recovery based on Rao-Blackwellised particle filtering techniques. The proposed state space model uses a time-varying autoregressive (TVAR) model for the sources, and a time-varying finite impulse response (FIR) model for the channel. The observed signals of the SISO, SIMO (Single Input, Multiple Output) or MIMO system are the convolution of the sources with the channels measured in additive noise. Sequential Monte Carlo (SMC) methods are used to implement a Bayesian approach to the nonlinear state estimation problem. The Rao-Blackwellisation technique is applied to directly recover the sources by marginalizing the AR and FIR coefficients from the joint posterior distribution. Simulation results and comparison with the PCRB are given to verify the performance of the proposed method. An alternate formulation of the standard particle filter is also introduced, referred to as block sequential importance sampling (BSIS).**

I. Introduction

In many applications in blind system identification, the source is often the desired signal. This is the case, e.g., in wireless communications systems and in recovering speech signals which have been recorded in reverberant enclosures. A common approach for source recovery is to first identify the channel, and then obtain an estimate of the source by applying the inverse of the channel estimate to the observed signal. This method is not feasible in cases for which the channel is ill-conditioned, rendering the channel inverse prone to large error. This is particularly true for acoustic reverberative channels when the long tails in the channel impulse response lead to ill-conditioning. This paper proposes a blind Bayesian approach that directly recovers the source signal, and therefore avoids the channel inversion problem.

A Bayesian filtering algorithm is developed for the nonlinear state space model using sequential Monte Carlo (SMC) methods, otherwise known as particle filtering. The use of SMC methods for nonlinear/non-Gaussian problems in signal processing was prompted by the introduction of the resampling step in a sequential framework [1]. Recent advances in computational power has led to applications to target tracking [1], speech processing [2] and wireless communications [3] problems. A particle filtering approach can result in significant computational complexity, however, they lend themselves well to a parallel implementation.

The nonlinear state space model is developed in Section 2. An introduction to SMC methods is presented in Section 3, and Rao-Blackwellisation is applied to the particle filter in Section 4. Simulation results are provided in Section V. The block sequential importance sampling (BSIS) formulation of the particle filter is developed in Section VI, followed by conclusions in Section VII.

II. State Space Model

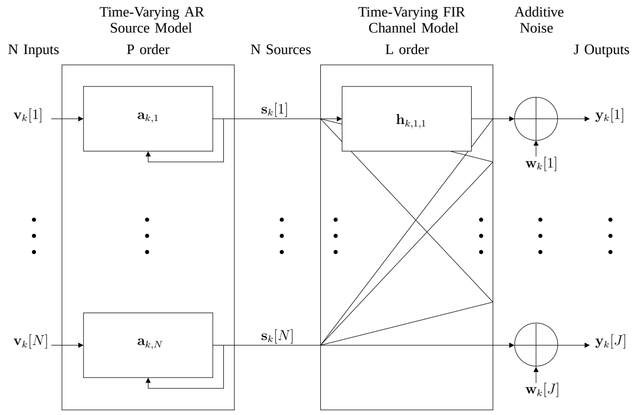

Fig. 1. Graphical representation of state space model

The state space model under consideration is shown graphically in Figure 1. The n_{th} source $\mathbf{s}_k[n]$ is assumed to evolve according to the following P-order time-varying autoregressive (TVAR) model:

$$\mathbf{s}_k[n] = \mathbf{a}_{k,n}^{\mathrm{T}} \mathbf{s}_{P,k-1,n} + \mathbf{v}_{k-1}[n]. \tag{1}$$

The source vector $\mathbf{s}_{P,k-1,n} \in \mathbb{R}^{P\times 1}$ is the concatenation of the most recent P samples at time $k-1$ for the n_{th} source, and the vector $\mathbf{a}_{k,n} \in \mathbb{R}^{P\times 1}$ contains the corresponding AR coefficients. The source noise $\mathbf{v}_{k-1}[n] \in \mathbb{R}$ is assumed to be white Gaussian distributed with mean zero and unknown variance $\sigma_{v,n}^2$. The source noise variances are assumed to be independent between sources. The following two matrix representations for the N sources are used:

$$\begin{aligned} \mathbf{s}_k &= \mathbf{A}_k \mathbf{s}_{P,k-1} + \mathbf{v}_{k-1} && (2) \\ &= \mathbf{S}_{k-1}\mathbf{a}_k + \mathbf{v}_{k-1}. && (3) \end{aligned}$$

The quantities $\mathbf{S}_{k-1}$ and $\mathbf{s}_{P,k-1}$ are formed from the source samples in the set of $\mathbf{s}_{P,k-1,n}$, $n = 1, 2, \ldots, N$, and $\mathbf{A}_k$, $\mathbf{a}_k$ are formed from the AR coefficients contained in the set of $\mathbf{a}_{k,n}$. Appropriate definitions are used in order to satisfy equation (1) for $n = 1, 2, \ldots, N$.

The time-varying AR coefficient vector $\mathbf{a}_k$ is itself assumed to evolve according to a first-order AR model as follows:

$$\mathbf{a}_k = a_a \mathbf{a}_{k-1} + \mathbf{v}_{a,k-1}, \tag{4}$$

with $0 < a_a < 1$ assumed known and the noise vector $\mathbf{v}_{a,k-1}$ Gaussian with mean zero and known covariance $\mathbf{\Sigma}_a$. In addition, the AR coefficients are constrained to be stable with all poles inside the unit circle.

The measurement equation for the j_{th} sensor is assumed to evolve according to the convolution of the sources with time-varying FIR channels in the presence of additive noise as follows:

$$\mathbf{y}_k[j] = \mathbf{h}_{k,j}^{\mathrm{T}} \mathbf{s}_{L,k} + \mathbf{w}_k[j]. \tag{5}$$

The source vector $\mathbf{s}_{L,k} \in \mathbb{R}^{NL \times 1}$ is the concatenation of the most recent L source vectors $\mathbf{s}_{k-\ell}, \ell = 0, 1, \ldots, L-1$ at time k, and the channel vector $\mathbf{h}_{k,j} \in \mathbb{R}^{NL \times 1}$ is formed from the N FIR filters $\mathbf{h}_{k,j,n}$ of length L from the n_{th} source to the j_{th} sensor. The measurement noise $\mathbf{w}_k[j] \in \mathbb{R}$ is assumed to be white Gaussian distributed with mean zero and unknown variance $\sigma_{w,j}^2$. The measurement noise variances are assumed to be independent between sensors. The matrix representations used for the measurements at the J sensors are:

$$\begin{aligned} \mathbf{y}_k &= \mathbf{H}_k \mathbf{s}_{L,k} + \mathbf{w}_k & (6) \\ &= \mathbf{T}_k \mathbf{h}_k + \mathbf{w}_k. & (7) \end{aligned}$$

The source matrix $\mathbf{T}_k$ is formed from $\mathbf{s}_{L,k}$, and $\mathbf{H}_k$,$\mathbf{h}_k$ are formed from the FIR coefficients contained in the set of $\mathbf{h}_{k,j}$ in order to satisfy (5) for $j = 1, 2, \ldots, J$.

The time-varying FIR coefficient vector $\mathbf{h}_k$ is assumed to evolve according to a first-order AR model as follows:

$$\mathbf{h}_k = a_h \mathbf{h}_{k-1} + \mathbf{v}_{h,k-1}, \tag{8}$$

with $0 < a_h < 1$ assumed known and the noise vector $\mathbf{v}_{h,k-1}$ Gaussian with mean zero and known covariance $\mathbf{\Sigma}_h$.

III. Sequential Monte Carlo Methods

A Bayesian approach to sequential state estimation is to recursively compute the posterior distribution of the states $\mathbf{x}_{1:k}$ given the measurements $\mathbf{y}_{1:k}$. When the state space model is linear-Gaussian, the Kalman filter provides the optimal Bayesian solution in closed-form. The given state space model is nonlinear since both the source and channel are unknown, so that SMC methods [4] are required. SMC methods numerically approximate the posterior distribution using a set of particles $\mathbf{x}_k^i$ and importance weights w_k^i for $i = 1, 2, \ldots, N_p$:

$$p(\mathbf{x}_{1:k}|\mathbf{y}_{1:k}) \approx \sum_{i=1}^{N_p} w_k^i \delta(\mathbf{x}_{1:k} - \mathbf{x}_{1:k}^i). \tag{9}$$

SMC methods are implemented using the sequential importance sampling (SIS) technique [5], which specifies a recursion for the importance weights:

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{y}_k|\mathbf{x}_k^i) p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i|\mathbf{x}_{1:k-1}^i, \mathbf{y}_{1:k})}. \tag{10}$$

The derivation assumes the state $\mathbf{x}_k$ is first-order Markov and the measurement $\mathbf{y}_k$ does not depend on past states $\mathbf{x}_{1:k-1}$. The importance weights are normalized such that $\sum_i w_k^i = 1$.

In practice, SIS algorithms suffer from the problem of importance weight degeneracy, in which after a few iterations of the recursion only one particle has a significant normalized importance weighting. The resampling step introduced in [1] reduces the weight degeneracy by duplicating particles with large weights and removing particles with small weights after the weight update in (10). The approximate effective sample size $\widehat{N}_{\text{eff}}$ [6] is used as a measure of degeneracy, with resampling occurring whenever it falls below a fixed threshold.

An undesired consequence of resampling is that particles with high importance weights can be selected numerous times. One method of reintroducing statistical diversity after the resampling procedure is the use of a Markov Chain Monte Carlo (MCMC) step [4].

IV. Rao-Blackwellised Particle Filtering

The Rao-Blackwellisation (RB) strategy [5] is applied to exploit the analytical structure in the proposed state space model. The RB technique marginalizes out conditionally linear-Gaussian state variables from the joint posterior distribution in order to reduce the state dimension for the particle filtering algorithm. This strategy can be shown to reduce the variance of the state estimates obtained using the particle filter [5]. This is due to the fact that the numerical particle filter is now only used to estimate the truly nonlinear states, while the remaining conditional linear-Gaussian states are estimated using the closed-form Kalman filter [7].

It can be seen from the proposed state space model that conditional on the sources $\mathbf{s}_{1:k}$ (which form $\mathbf{T}_k$) and the measurement noise covariance $\mathbf{\Sigma}_w$, equations (7)-(8) for the FIR coefficients $\mathbf{h}_k$ form a linear-Gaussian subsystem. Similarly, the pair of equations (3)-(4) for the AR coefficients $\mathbf{a}_k$ conditioned on the sources (which form $\mathbf{S}_{k-1}$) and the source noise covariance $\mathbf{\Sigma}_v$ also form a linear-Gaussian subsystem. The joint posterior distribution for the sources, FIR and AR coefficients is factorized using Bayes' rule to exploit this structure:

$$\begin{aligned} p(\mathbf{s}_{1:k}, \mathbf{a}_{1:k}, \mathbf{h}_{1:k}|\mathbf{y}_{1:k}) &= p(\mathbf{s}_{1:k}|\mathbf{y}_{1:k}) \\ p(\mathbf{a}_{1:k}|\mathbf{s}_{1:k}, \mathbf{y}_{1:k}) &p(\mathbf{h}_{1:k}|\mathbf{s}_{1:k}, \mathbf{y}_{1:k}) \end{aligned} \tag{11}$$

The dependence on the noise variances $\mathbf{\Sigma}_v$ and $\mathbf{\Sigma}_w$ are not shown explicitly since maximum a posteriori (MAP) estimates can be developed separately assuming non-informative inverse Gamma variance priors. The filtered distributions $p(\mathbf{a}_k|\mathbf{s}_{1:k}, \mathbf{y}_{1:k})$ and $p(\mathbf{h}_k|\mathbf{s}_{1:k}, \mathbf{y}_{1:k})$ are computed recursively in parallel for the decoupled conditionally linear-Gaussian problems using the standard Kalman filter:

$$\begin{aligned} p(\mathbf{a}_k|\mathbf{s}_{1:k}^i, \mathbf{y}_{1:k}) &= \mathcal{N}(\hat{\mathbf{a}}_{k|k}^i, \mathbf{\Phi}_{a,k|k}^i), & (12) \\ p(\mathbf{h}_k|\mathbf{s}_{1:k}^i, \mathbf{y}_{1:k}) &= \mathcal{N}(\hat{\mathbf{h}}_{k|k}^i, \mathbf{\Phi}_{h,k|k}^i). & (13) \end{aligned}$$

The quantities $\hat{\mathbf{a}}_{k|k}$,$\hat{\mathbf{h}}_{k|k}$ are the filtered means and $\mathbf{\Phi}_{a,k|k}$,$\mathbf{\Phi}_{h,k|k}$ are the filtered covariances from the Kalman recursions for the AR and FIR coefficients.

The marginalized posterior distribution $p(\mathbf{s}_{1:k}|\mathbf{y}_{1:k})$ is obtained using the Rao-Blackwellisation strategy for marginalizing out the conditionally linear-Gaussian AR and FIR coefficients. The resulting nonlinear estimation problem for the sources $\mathbf{s}_k$ is implemented using the particle filter. The development of the SIS method assumes the state transition model for $\mathbf{s}_k$ is first-order Markov and the measurement model does not explicitly depend on past states. The state equation (2) and the measurement equation (6) do not satisfy this requirement in general since they are dependent on P and L source samples, respectively. To satisfy the requirements of an SIS implementation, the new state variable $\mathbf{s}_{M,k} \in \mathbb{R}^{MN\times 1}$ is introduced for $M = max(P, L)$. The reformulated state transition and measurement equations are then:

$$\mathbf{s}_{M,k} = \tilde{\mathbf{A}}_k \mathbf{s}_{M,k-1} + \tilde{\mathbf{v}}_{k-1}, \quad (14)$$

$$\mathbf{y}_k = \tilde{\mathbf{H}}_k \mathbf{s}_{M,k} + \mathbf{w}_k, \quad (15)$$

where, using $\mathbf{0}_{a,b}$ to denote a matrix of zeros of dimension $a \times b$ if $a, b > 0$ and empty otherwise,

$$\tilde{\mathbf{A}}_k = \begin{bmatrix} [\mathbf{0}_{(M-1)N,N}, \mathbf{I}_{(M-1)N}] \\ [\mathbf{0}_{N,(L-P)N}, \mathbf{A}_k] \end{bmatrix}, \quad (16)$$

$$\tilde{\mathbf{v}}_{k-1} = [\mathbf{0}_{1,(M-1)N}, \mathbf{v}_{k-1}^{\mathrm{T}}]^{\mathrm{T}}, \quad (17)$$

$$\tilde{\mathbf{H}}_k = [\mathbf{0}_{J,(P-L)N}, \mathbf{H}_k]. \quad (18)$$

Using this formulation in terms of $\mathbf{s}_{M,k}$, we now develop the importance function and weight update used in the particle filter for estimation of the sources. An approximation to the optimal importance function is used to generate the particles. The optimal importance function is the function which minimizes the variance of the importance weights [5]:

$$q(\mathbf{s}_{M,k}|\mathbf{s}_{M,1:k-1}, \mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{s}_{M,k})p(\mathbf{s}_{M,k}|\mathbf{s}_{M,k-1}). \quad (19)$$

From the form of (14), only the quantity $\mathbf{s}_k$ of $\mathbf{s}_{M,k}$ is a random variable, while the remaining blocks are deterministic shifts of the blocks from the previous state $\mathbf{s}_{M,k-1}$. Thus, it is only required to consider generating particles for the current source vector $\mathbf{s}_k$ from an importance density of the form:

$$q(\mathbf{s}_k|\mathbf{s}_{M,1:k-1}, \mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{s}_{M,k})p(\mathbf{s}_k|\mathbf{s}_{M,k-1}), \quad (20)$$

where $p(\mathbf{y}_k|\mathbf{s}_{M,k})$ is the marginalized likelihood and $p(\mathbf{s}_k|\mathbf{s}_{M,k-1})$ is the marginalized prior. These distributions are determined by marginalizing over the FIR and AR coefficients, and are found to be [7]:

$$p(\mathbf{s}_k|\mathbf{s}_{M,k-1}) = \mathcal{N}(\mathbf{S}_{k-1}\hat{\mathbf{a}}_{k|k-1}, \mathbf{R}_k), \quad (21)$$

$$p(\mathbf{y}_k|\mathbf{s}_{M,k}) = \mathcal{N}(\mathbf{T}_k\hat{\mathbf{h}}_{k|k-1}, \mathbf{Q}_k), \quad (22)$$

where

$$\mathbf{R}_k = \mathbf{S}_{k-1}\mathbf{\Phi}_{a,k|k-1}\mathbf{S}_{k-1}^{\mathrm{T}} + \mathbf{\Sigma}_v, \quad (23)$$

$$\mathbf{Q}_k = \mathbf{T}_k\mathbf{\Phi}_{h,k|k-1}\mathbf{T}_k^{\mathrm{T}} + \mathbf{\Sigma}_w, \quad (24)$$

and $\hat{\mathbf{a}}_{k|k-1}$,$\hat{\mathbf{h}}_{k|k-1}$ are the predicted means and $\mathbf{\Phi}_{a,k|k-1}$,$\mathbf{\Phi}_{h,k|k-1}$ are the predicted covariances from the Kalman filter recursions. Even though the optimal importance function for $\mathbf{s}_k$ in (20) is the product of the two Gaussian distributions (21),(22), it is not Gaussian itself since the covariance term $\mathbf{Q}_k$ has a dependence on the variable of interest $\mathbf{s}_k$ (through $\mathbf{T}_k$). In order to derive a Gaussian importance function that has the necessary feature of being easy to sample from, the state-dependent covariance $\mathbf{Q}_k$ is approximated by $\hat{\mathbf{Q}}_k$ in which $\mathbf{s}_k$ is replaced with its predicted value from the transition prior:

$$\hat{\mathbf{s}}_{k|k-1} = \mathbf{S}_{k-1}\hat{\mathbf{a}}_{k|k-1} \quad (25)$$

To factorize the two distributions into an equivalent Gaussian distribution for $\mathbf{s}_k$, the variable $\mathbf{s}_k$ is isolated from the matrix $\mathbf{T}_k$ in the mean of (22) using the equivalent forms of the measurement equation in (6) and (7):

$$\begin{aligned}\mathbf{T}_k\hat{\mathbf{h}}_{k|k-1} &= \sum_{\ell=0}^{L-1} \hat{\mathbf{H}}_{k|k-1,\ell}\mathbf{s}_{k-\ell} \\ &= \hat{\mathbf{H}}_{k|k-1,0}\mathbf{s}_k + \hat{\mathbf{y}}_{k|k-1}\end{aligned} \quad (26)$$

where the predicted matrices of FIR coefficients at lag ℓ from the current time $\hat{\mathbf{H}}_{k|k-1,\ell}$ are formed from $\hat{\mathbf{h}}_{k|k-1}$, and the predicted measurement $\hat{\mathbf{y}}_{k|k-1}$ is defined as the summation excluding $\hat{\mathbf{H}}_{k|k-1,0}\mathbf{s}_k$. The resulting importance function is then Gaussian with mean $\boldsymbol{\mu_o}$ and covariance $\mathbf{\Sigma}_o$ given by:

$$\boldsymbol{\mu_o} = \hat{\mathbf{s}}_{k|k-1} + \mathbf{W}_k(\mathbf{y}_k - \hat{\mathbf{H}}_{k|k-1,0}\hat{\mathbf{s}}_{k|k-1} - \hat{\mathbf{y}}_{k|k-1}),$$

$$\mathbf{\Sigma}_o = \mathbf{R}_k - \mathbf{W}_k\hat{\mathbf{H}}_{k|k-1,0}\mathbf{R}_k,$$

which is in the form of a Kalman update on the predicted particles with gain given by:

$$\mathbf{W}_k = \mathbf{R}_k\hat{\mathbf{H}}_{k|k-1,0}^{\mathrm{T}}[\hat{\mathbf{H}}_{k|k-1,0}\mathbf{R}_k\hat{\mathbf{H}}_{k|k-1,0}^{\mathrm{T}} + \hat{\mathbf{Q}}_k]^{-1}. \quad (27)$$

The corresponding weight update from (10) is then:

$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{y}_k|\mathbf{s}_{M,k}^i)}{\hat{p}(\mathbf{y}_k|\mathbf{s}_{M,k}^i)}\hat{p}(\mathbf{y}_k|\mathbf{s}_{M,k-1}^i), \quad (28)$$

where

$$\hat{p}(\mathbf{y}_k|\mathbf{s}_{M,k}) = \mathcal{N}(\mathbf{T}_k\hat{\mathbf{h}}_{k|k-1}, \hat{\mathbf{Q}}_k) \quad (29)$$

The structure of the algorithm is shown in Figure 2.

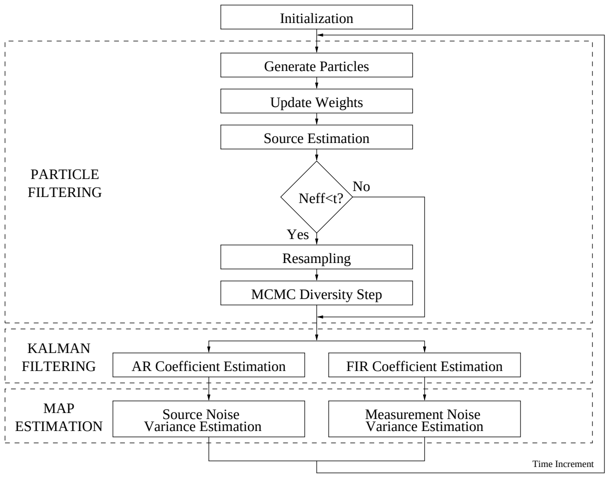

Fig. 2. Rao-Blackwellised Particle Filtering Algorithm Structure

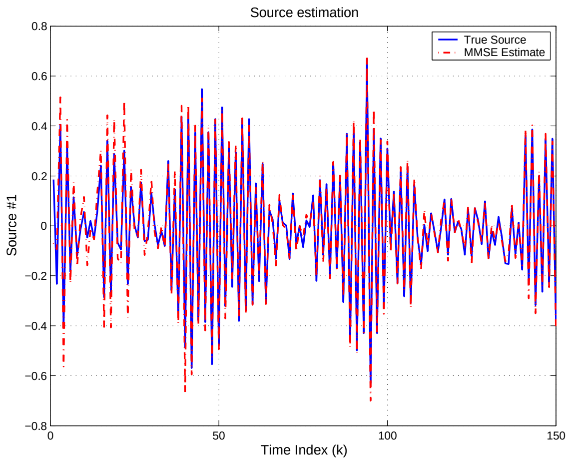

Fig. 3. Source estimation for SIMO system

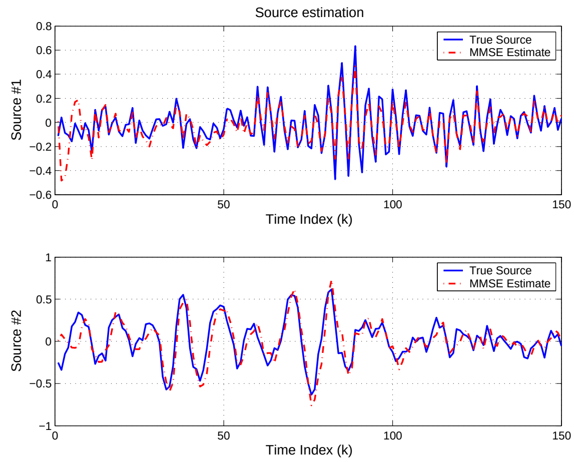

Fig. 4. Source estimation for MIMO system

V. Simulation Results

A SIMO system with $N = 1$ source and $J = 2$ sensors was run over time steps 1 to $K = 500$ for $N_t = 50$ Monte Carlo trials. The initial $P = 4$ order AR coefficient vector $\mathbf{a}_0$ was generated from a low-pass Butterworth filter with normalized cutoff frequency $w_n = 0.25$. The time-varying state $\mathbf{a}_k$ was then generated from the AR model in (4) using $a_a = 0.9999$ and $\mathbf{\Sigma}_a = 0.001\mathbf{I}_P$. The initial $L = 6$ order FIR channel vectors $\mathbf{h}_{0,j,n}$ were produced from independent draws from a zero-mean Gaussian distribution with exponentially decaying covariance matrix using $W = 0.15$:

$$\mathbf{\Sigma}_{h,0} = \mathrm{diag}\left([e^{-\frac{L-1}{WL}}, e^{-\frac{L-2}{WL}}, \ldots, e^{-\frac{0}{WL}}]\right). \quad (30)$$

The time-varying $\mathbf{h}_k$ was then generated from (8) using $a_h = 0.9999$ and $\mathbf{\Sigma}_h = (1-a_h^2)\mathbf{\Sigma}_{h,0}$. The noise variance parameters were $\sigma_v^2 = 0.01$, $\sigma_w^2 = 0.005$. The average signal-to-noise (SNR) ratio computed numerically over the Monte Carlo runs was 17.3 dB. The number of particles was $N_p = 50$.

The performance is measured using the mean square error (MSE) averaged over the time steps and Monte Carlo runs:

$$\mathrm{MSE} = 10\log_{10}\left(\frac{1}{N_t}\sum_{t=1}^{N_t}\left(\frac{1}{K}\sum_{k=1}^{K}\frac{\|\mathbf{s}_k^t - \hat{\mathbf{s}}_k^t\|_2^2}{N}\right)\right), \quad (31)$$

where $\mathbf{s}_k^t$ is the true source from the t^{th} Monte Carlo trial, $\hat{\mathbf{s}}_k^t$ is the minimum mean square error (MMSE) estimate, and N is the dimension of the state $\mathbf{s}_k$. Performance measures for the MMSE estimates of $\mathbf{h}_k$, $\mathbf{a}_k$, σ_v^2, and σ_w^2 also follow the form of (31). The MSE values are shown in Table I.

TABLE I

MSE SIMULATION RESULTS

Variable	$\mathbf{s}_k$	$\mathbf{h}_k$	$\mathbf{a}_k$	σ_v^2	σ_w^2
MSE	-23.08	-15.83	-12.09	-46.31	-72.40

Figure 3 compares the true source with the MMSE estimate from one trial. Figure 4 illustrates an example of a MIMO system with 2 sources and 4 sensors, using the same parameters from the SIMO case. The source MSE was -22.11 dB.

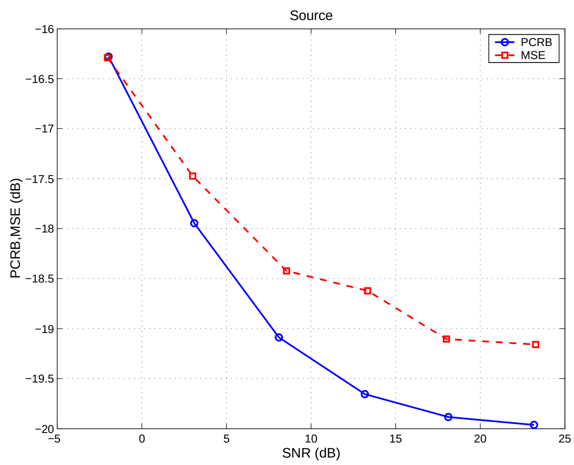

Fig. 5. Comparison of MSE with PCRB over SNR

The Posterior Cramér-Rao bound (PCRB) provides a lower bound on the covariance of an estimator of a random variable given the measurements. For the discrete-time nonlinear filtering problem, a recursion for the inverse of the Fisher information matrix which specifies the lower bound, denoted as $\mathbf{P}_k = \mathbf{J}_k^{-1}$, is presented in [8] (summarized in [4]) for the one-step-ahead prediction problem:

$$\mathbf{P}_{k+1} = \mathbf{F}_k(\mathbf{P}_k^{-1} + \mathbf{R}_k^{-1})^{-1}\mathbf{F}_k^{\mathrm{T}} + \mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k^{\mathrm{T}}. \quad (32)$$

The expectations required to compute the matrices in (32) are obtained by averaging 5000 *i.i.d.* statistical realizations of the system model. The benefit of the recursion in [8] is that it handles the case of a singular transition prior, as is the case for the proposed model.

Figure 5 compares the MSE of the SIMO one-step-ahead predicted source estimates with the PCRB over a range of SNR. To allow for closed-form evaluation of the SNR, a first-order AR model was used with $P = 1$ and $a_k = 0.9535$. The figure illustrates that the source MSE compares well with the PCRB, falling within 1 dB of the lower bound over the range of SNR.

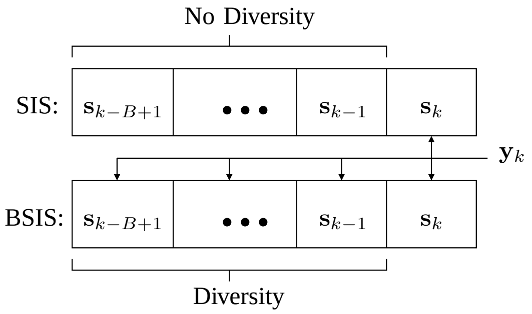

Fig. 6. Comparison of the SIS and BSIS particle filters

VI. Block Sequential Importance Sampling

The block sequential importance sampling (BSIS) formulation of the particle filter is designed to introduce additional measurement information and particle diversity into the particle generation process for past states. The motivation for this approach is the structure of (5), in which the measurements are formed from a convolution in terms of the current ($\mathbf{s}_k$) and past ($\mathbf{s}_{k-L+1:k-1}$) source samples. In this case, particle values for the past $L-1$ states directly influence the performance in estimating the current state.

The SIS particle filter assumes that the importance function is restricted to satisfy [5]:

$$q(\mathbf{s}_{1:k}|\mathbf{y}_{1:k}) = q(\mathbf{s}_k|\mathbf{s}_{1:k-1}, \mathbf{y}_{1:k})q(\mathbf{s}_{1:k-1}|\mathbf{y}_{1:k-1}). \tag{33}$$

This implicitly approximates the smoothed distribution $q(\mathbf{s}_{1:k-1}|\mathbf{y}_{1:k})$ with the filtered distribution $q(\mathbf{s}_{1:k-1}|\mathbf{y}_{1:k-1})$. This assumption is used to avoid the large computational cost of regenerating past particles for $\mathbf{s}_{1:k-1}$ based on the new measurement $\mathbf{y}_k$. As a result, the sequential algorithm proceeds by generating particles $\mathbf{s}_k^i$ from $q(\mathbf{s}_k|\mathbf{s}_{1:k-1}, \mathbf{y}_{1:k})$, and appending them to the particle history $\mathbf{s}_{1:k-1}^i$. It can be seen that the factorization property in (33) results in the form of the optimal importance function in (20), which specifies that particle values for $\mathbf{s}_{k-M+1:k-1}$ are reused from the previous time steps. The particles for the current state $\mathbf{s}_k$ are generated from the Gaussian approximation to the optimal importance function which incorporates the current measurement $\mathbf{y}_k$.

The BSIS approach proposes to generate particles at time k for the block of B most recent states $\mathbf{s}_{k-B+1:k}$, denoted as $\mathbf{s}_{B,k}$, from an importance function of the form:

$$q(\mathbf{s}_{1:k}|\mathbf{y}_{1:k}) = q(\mathbf{s}_{B,k}|\mathbf{s}_{1:k-B}, \mathbf{y}_{1:k})q(\mathbf{s}_{1:k-B}|\mathbf{y}_{1:k-B}). \tag{34}$$

The recursive importance weight update in terms of blocks of measurements $\mathbf{y}_{B,k}$ and states $\mathbf{s}_{B,k}$ can be shown to be:

$$w_k^i \propto w_{k-B}^i \frac{p(\mathbf{y}_{B,k}|\mathbf{s}_{B,k}^i)p(\mathbf{s}_{B,k}^i|\mathbf{s}_{k-B}^i)}{q(\mathbf{s}_{B,k}^i|\mathbf{s}_{1:k-B}^i, \mathbf{y}_{1:k})}. \tag{35}$$

The first benefit of this approach is that the current measurement is incorporated into the generation of new particle values for the past states $\mathbf{s}_{k-B+1:k-1}$. In the case of measurements which result from a convolution, the current measurement $\mathbf{y}_k$ can contain significant information about past states. This effectively performs smoothing on past states, however, with the additional goal of improving the numerical approximation to the filtered posterior distribution used to estimate the current state. An additional benefit is that particle diversity can be introduced on the past states using an MCMC step that generates candidates with an importance function of the form (34). In this case, new values for $\mathbf{s}_{k-B+1:k-1}$ can be generated to avoid the problem of sample impoverishment. Figure 6 compares the principles of the SIS and BSIS methods.

The potential improvement in estimation performance using a BSIS approach will come at the expense of increased computational complexity. The characteristics of the importance function and weight update indicate an increase in memory and computational requirements by a factor of $O(B)$, where B is a design parameter. Application to the proposed blind source recovery algorithm is under investigation. This framework may also be applicable to other state space models exhibiting similar features.

VII. Conclusions

The paper presents a Bayesian approach to directly recover sources which follow a TVAR model mixed by FIR channels and measured with additive noise. The blind estimation of the nonlinear model is implemented using sequential Monte Carlo methods. The performance of the particle filter is improved by exploiting the conditionally linear-Gaussian structure in the model using the Rao-Blackwellisation procedure, and through the development of a Gaussian approximation to the optimal importance function. Simulation results demonstrate the effectiveness of the method. The block sequential importance sampling framework is also introduced to exploit the convolution structure of the model.

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