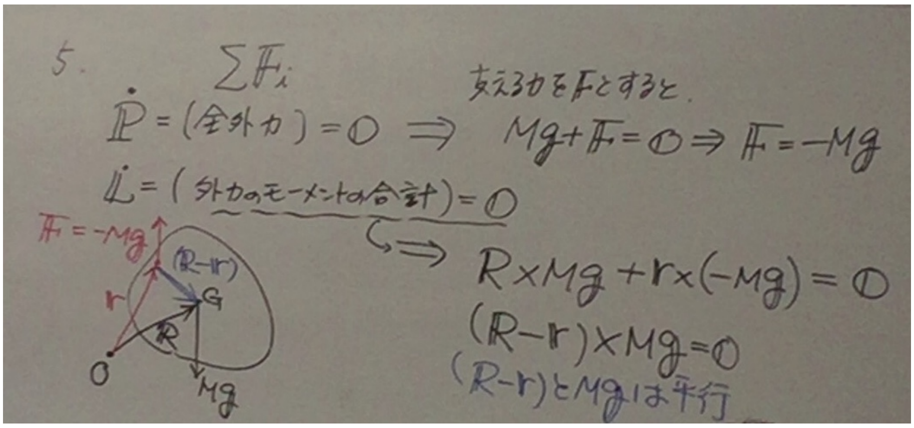
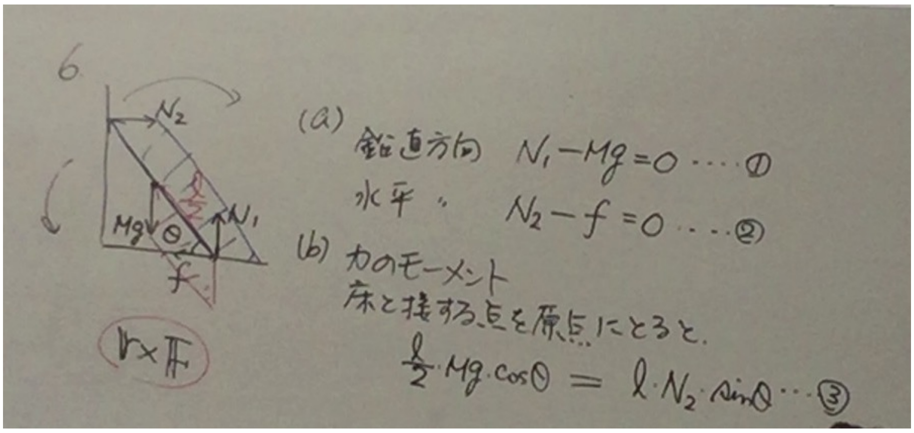
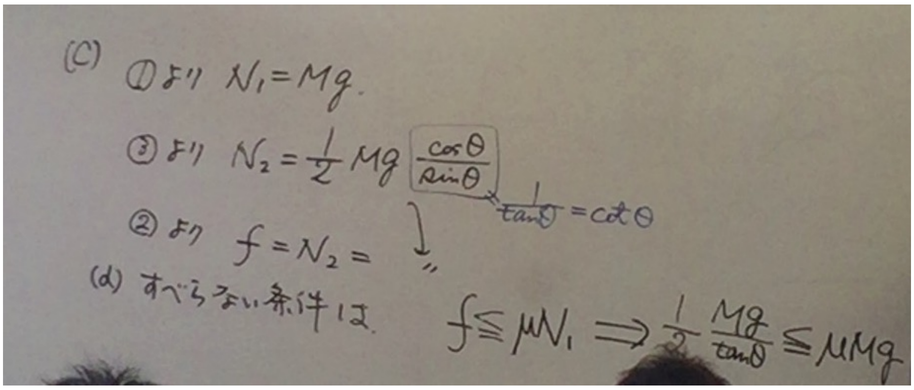
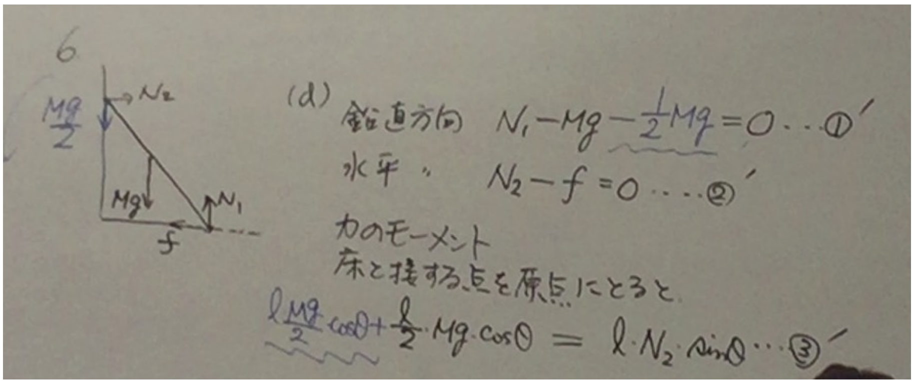
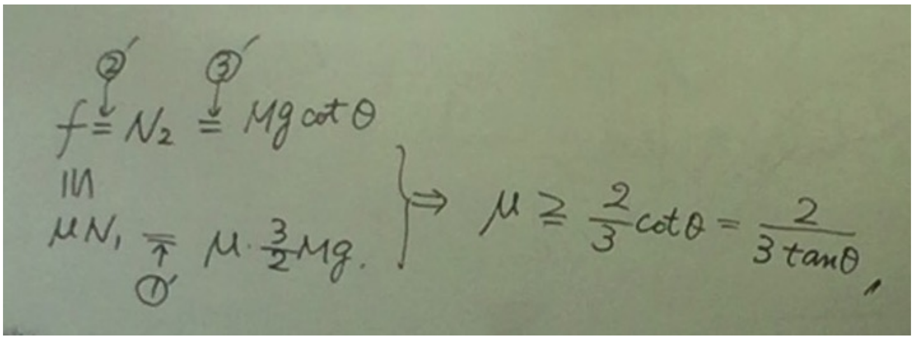
1 Prblm2


5.
$\sum \mathbb{F}_i$
$\dot{\mathbb{P}}$ = (全外力) = $\mathbb{O}$ ⇒
支える力を$\mathbb{F}$とすると
$M\mathbb{g} + \mathbb{F} = \mathbb{O} \Rightarrow \mathbb{F} = -M\mathbb{g}$
$\dot{\mathbb{L}}$ = (外力のモーメントの合計) = $\mathbb{O}$
⇒ $\mathbb{R} \times M\mathbb{g} + \mathbb{r} \times (-M\mathbb{g}) = \mathbb{O}$
$(\mathbb{R} - \mathbb{r}) \times M\mathbb{g} = \mathbb{O}$
$(\mathbb{R}-\mathbb{r})$と$M\mathbb{g}$は平行
$\mathbb{F} = -M\mathbb{g}$
$(\mathbb{R}-\mathbb{r})$
G
$\mathbb{r}$
$\mathbb{R}$
O
$M\mathbb{g}$

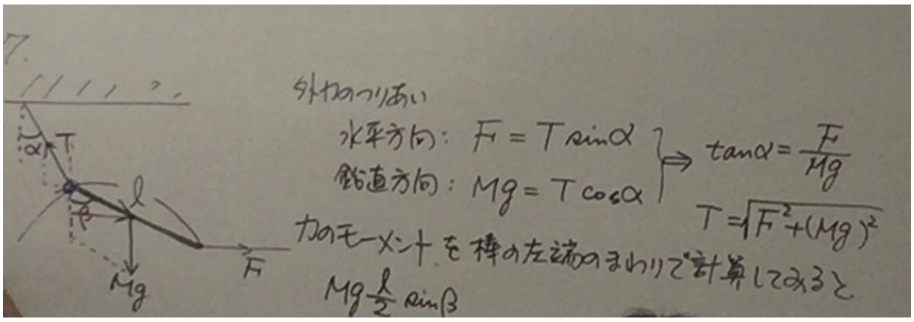

6.
N_2
$\frac{l}{2}$
N_1
Mg
θ
f
$\mathbb{r} \times \mathbb{F}$
(a)
鉛直方向 $N_1 - Mg = 0$ …… ①
水平 〃 $N_2 - f = 0$ …… ②
(b) 力のモーメント
床と接する点を原点にとると.
$\frac{l}{2} \cdot Mg \cdot \cos\theta = l \cdot N_2 \cdot \sin\theta$ … ③


(c)
①より $N_1 = Mg$.
③より $N_2 = \frac{1}{2} Mg \frac{\cos\theta}{\sin\theta}$
$\frac{1}{\tan\theta} = \cot\theta$
②より $f = N_2 =$ 〃
(d) すべらない条件は
$f \leqq \mu N_1 \Rightarrow \frac{1}{2} \frac{Mg}{\tan\theta} \leqq \mu Mg$


6.
$\frac{Mg}{2}$
N_2
Mg
N_1
f
(d)
鉛直方向 $N_1 - Mg - \frac{1}{2}Mg = 0$ … ①′
水平 〃 $N_2 - f = 0$ …… ②′
力のモーメント
床と接する点を原点にとると.
$l\frac{Mg}{2}\cos\theta + \frac{l}{2} \cdot Mg \cdot \cos\theta = l \cdot N_2 \cdot \sin\theta$ … ③′

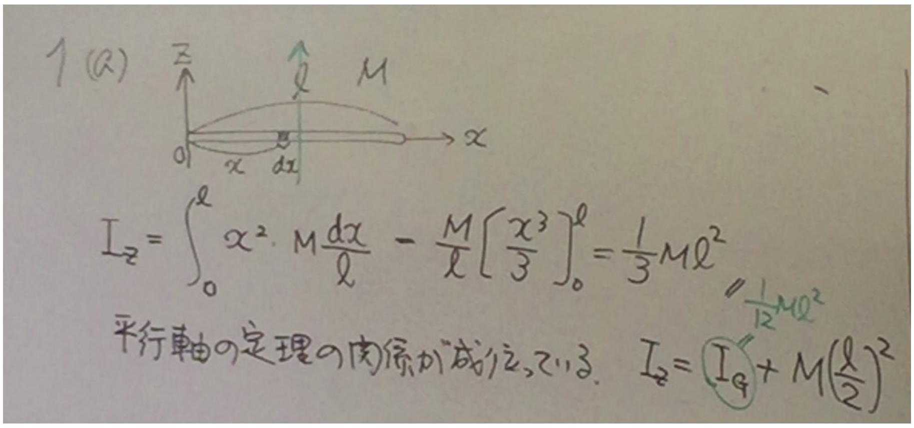


$f \leqq N_2 = Mg\cot\theta$
$\mu N_1 = \mu \cdot \frac{3}{2}Mg$
$\Rightarrow \mu \geqq \frac{2}{3}\cot\theta = \frac{2}{3\tan\theta}$

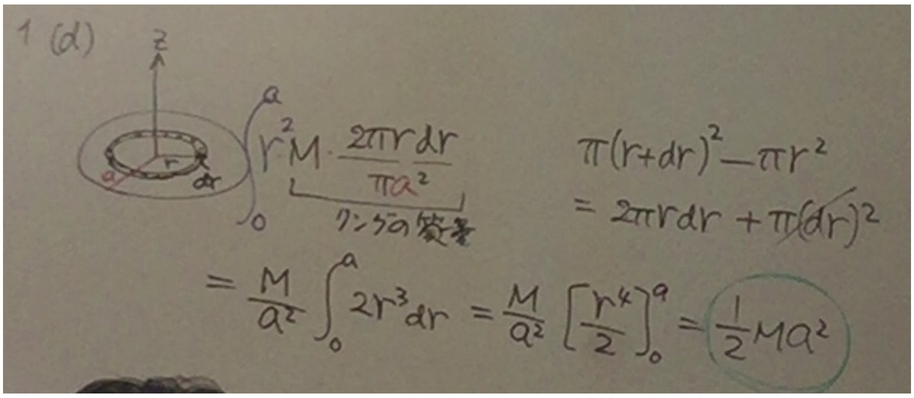


7.
T
α
β
l
Mg
F
外力のつりあい
水平方向： $F = T\sin\alpha$
鉛直方向： $Mg = T\cos\alpha$
$\Rightarrow \tan\alpha = \frac{F}{Mg}$
$T = \sqrt{F^2 + (Mg)^2}$
力のモーメントを棒の左端のまわりで計算してみると
$Mg\frac{l}{2}\sin\beta$

2 Prblm3

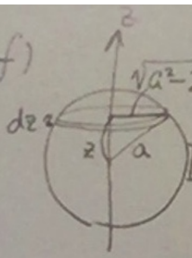


1 (c)
z
l
M
x
0
dx
$I_z = \int_0^l x^2 \cdot M\frac{dx}{l} = \frac{M}{l}\left[\frac{x^3}{3}\right]_0^l = \frac{1}{3}Ml^2$
平行軸の定理の関係が成り立っている。
$I_z = I_g + M\left(\frac{l}{2}\right)^2$
$\frac{1}{12}Ml^2$



1 (d)
z
r
a
dr
$\int_0^a r^2 \underbrace{M \cdot \frac{2\pi r\, dr}{\pi a^2}}_{\text{リングの質量}}$
$\pi(r+dr)^2 - \pi r^2 = 2\pi r\, dr + \pi(dr)^2$
$= \frac{M}{a^2}\int_0^a 2r^3 dr = \frac{M}{a^2}\left[\frac{r^4}{2}\right]_0^a = \frac{1}{2}Ma^2$

1 (f)



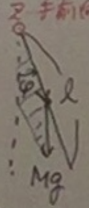
$\frac{1}{2}Ma^2$

$$I_z = \int_{-a}^{a} \frac{1}{2}(a^2-z^2)\cdot M\,\frac{\pi(a^2-z^2)\,dz}{\frac{4\pi}{3}a^3}$$

$$= \frac{3M}{8a^3}\int_{-a}^{a}(a^2-z^2)^2\,dz = \cdots\cdots = \frac{2}{5}Ma^2$$

3

(a) z 手前向き



外力のモーメントのz成分 N_z は

$$N_z = -\frac{l}{2}Mg\sin\varphi \qquad \boldsymbol{r}\times\boldsymbol{F}$$

(b) (8.5) $I_z\frac{d\omega}{dt} = N_z$ より $\boxed{\frac{1}{3}Ml^2\frac{d^2\varphi}{dt^2} = -\frac{l}{2}Mg\sin\varphi}$

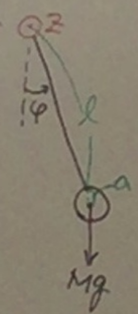
(c) $|\varphi| \ll 1$ のとき $\sin\varphi \cong \varphi$ なので

$$\frac{1}{3}Ml^2\frac{d^2\varphi}{dt^2} \cong -\frac{l}{2}Mg\,\varphi$$

$$\frac{d^2\varphi}{dt^2} = -\underbrace{\boxed{\frac{3g}{2l}}}_{\omega_0^2\text{ とおく}}\varphi$$

φの一般解は $\varphi = A\cos(\omega_0 t+\alpha) \Rightarrow$ 周期 $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{2l}{3g}}$

4.



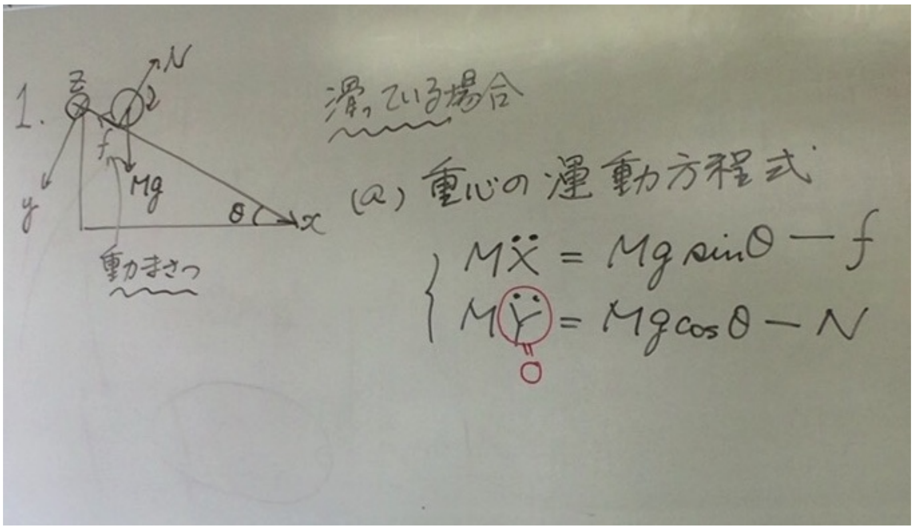
(a) 平行軸の定理を用いて,

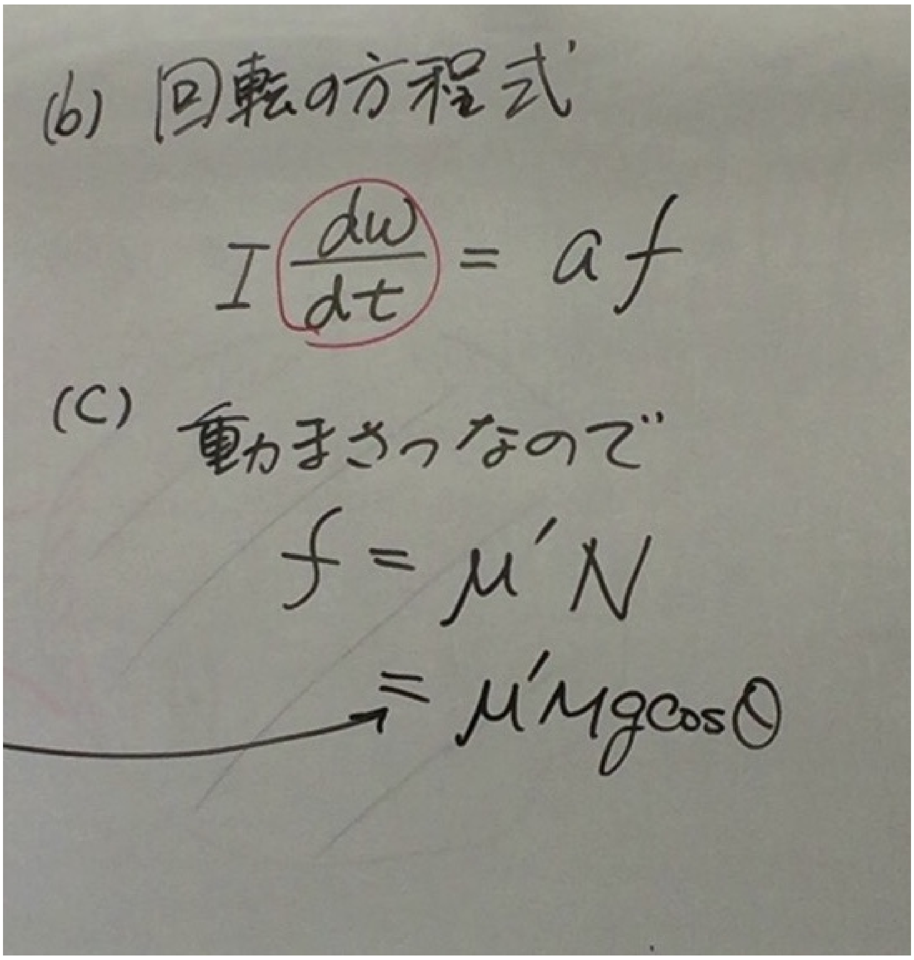
$$I_z = \frac{2}{5}Ma^2 + M\cdot(l+a)^2$$

(b) $N_z = -(l+a)Mg\sin\varphi$

(c) $I_z\frac{d\omega}{dt} = N_z \Rightarrow \cancel{I_z}\frac{d^2\varphi}{dt^2} \cong -\underbrace{\frac{(l+a)Mg}{I_z}}_{\omega_0^2\text{ とおく}}\varphi$

3 Prblm3


1.
N
f
Mg
y
θ
x
動まさつ
滑っている場合
(a) 重心の運動方程式
$M\ddot{x} = Mg\sin\theta - f$
$M\ddot{y} = Mg\cos\theta - N$
0


(b) 回転の方程式
$I\frac{d\omega}{dt} = af$
(C) 動まさつなので
$f = \mu' N$
$= \mu' Mg\cos\theta$

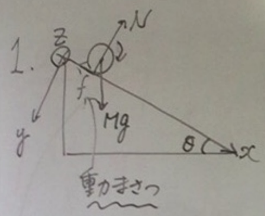
(d) $\ddot{X} \overset{①}{=} g\sin\theta - \dfrac{f}{M}$

$= g(\sin\theta - \mu'\cos\theta)$

$\dot{\omega} = \dfrac{a}{I}f = \dfrac{\mu' M g a\cos\theta}{I} > 0$

ωはだんだん大きくなる.

1.

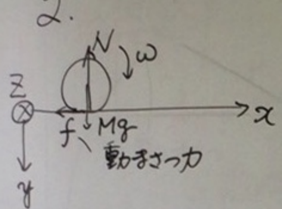


$\boxed{\tan\theta > \mu\left(\dfrac{Ma^2+I}{I}\right)}$

$\ddot{X} - a\dot{\omega} \underset{(d)}{=} g(\sin\theta - \mu'\cos\theta) - \mu' g\dfrac{Ma^2}{I}\cos\theta$

$= g\cos\theta\left\{\underbrace{\boxed{\tan\theta - \mu'\left(\dfrac{I+Ma^2}{I}\right)}}_{+}\right\}$

2.



(a) $M\underbrace{\ddot{Y}}_{0} = Mg - N$

$\therefore N = Mg$

(b) $M\ddot{X} = -f \left(= -\mu' N = -\mu' Mg\right)$

(c) $I\dot{\omega} = af$

(d) $f = \mu' N = \mu' M g$

(e) $\ddot{x} = -\dfrac{f}{M} = -\mu' g$

tについて積分すると

$$\dot{x} = -\mu' g t + \underbrace{C_1}_{=v_0}$$

$$= v_0 - \mu' g t$$

$$\dot{\omega} = \frac{af}{I} = \frac{\mu' M g a}{I} \Leftarrow \text{一定}$$

$$\omega = \frac{\mu' M g a}{I} t + \underbrace{C_2}_{=0}$$

(f) 接点で球が床に対して滑る速度

$$v_p = \dot{x} - a\omega$$

$$= (v_0 - \mu' g t) - \left(\frac{\mu' g M a^2}{\underbrace{I}_{=\frac{2}{5}Ma^2}}\right) t$$

$$v_p = v_0 - \frac{7}{2}\mu' g t$$

$v_p = 0$ となるのは $t = \frac{2v_0}{7\mu' g}$ のとき

(g) 転がり始めたあとのまさつ力 f' は？

$$\begin{cases} M\ddot{X} = -f' \\ I\dot{\omega} = af' \end{cases}$$

$$v_p = \dot{X} - a\omega = 0$$

滑らずに転がる

$$\Downarrow$$

$$\dot{v}_p = \ddot{X} - a\dot{\omega} = 0$$

$$= -\frac{f'}{M} - \frac{a^2 f'}{I}$$

$$= -\underbrace{\boxed{\left(\frac{1}{M} + \frac{a^2}{I}\right)}}_{+} f'$$

$$f' = 0$$

$$\begin{cases} M\ddot{X} = Mg - T \\ I\dot{\omega} = aT \\ a\omega = \dot{X} \rightarrow \dot{\omega} = \frac{\ddot{X}}{a} \end{cases}$$

$$M\ddot{X} = Mg - \frac{I}{a}\dot{\omega}$$

$$= Mg - \frac{I}{a^2}\ddot{X}$$

$$M\ddot{X}\underbrace{\left(1 + \frac{\overset{\frac{1}{2}Ma^2}{I}}{Ma^2}\right)}_{=\frac{3}{2}} = Mg$$

$$\ddot{X} = \frac{2}{3}g$$

$$T = M(g - \ddot{X})$$

$$= \frac{Mg}{3}$$