

$$m \begin{cases} \frac{d^2 x}{dt^2} = k(x_2 - x_1) & (1) \\ M \frac{d^2 x_2}{dt^2} = -k(x_2 - x_1) + k(x_3 - x_2) & (2) \\ m \frac{d^2 x_3}{dt^2} = -k(x_3 - x_2) & (3) \end{cases}$$

$x_1 = A_1 e^{i\omega t}, x_2 = A_2 e^{i\omega t}, x_3 = A_3 e^{i\omega t}$ とおくと

$$\begin{cases} -mA_1\omega^2 = k(A_2 - A_1) & (4) \end{cases}$$

$$\begin{cases} -MA_2\omega^2 = k(A_1 - 2A_2 + A_3) & (5) \end{cases}$$

$$\begin{cases} -mA_3\omega^2 = k(A_2 - A_3) & (6) \end{cases}$$

$x_1 = A_1 e^{i\omega t}, x_2 = A_2 e^{i\omega t}, x_3 = A_3 e^{i\omega t}$ とおくと

$$\begin{cases} -mA_1\omega^2 = k(A_2 - A_1) \\ -MA_2\omega^2 = k(A_1 - 2A_2 + A_3) \\ -mA_3\omega^2 = k(A_2 - A_3) \end{cases}$$

$$\omega^2 \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} \frac{k}{m} & -\frac{k}{m} & 0 \\ -\frac{k}{M} & \frac{2k}{M} & -\frac{k}{M} \\ 0 & -\frac{k}{m} & \frac{k}{m} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} a & -a & 0 \\ -b & 2b & -b \\ 0 & -a & a \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

固有値方程式は 固有値を λ とすると

$$\begin{vmatrix} a-\lambda & -a & 0 \\ -b & 2b-\lambda & -b \\ 0 & -a & a-\lambda \end{vmatrix} = 0$$

① $\lambda = 0, a, a+2b$

固有値 $\lambda = 0$ のとき

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ -A \end{pmatrix}$$

③

$$\begin{pmatrix} -\frac{a}{2b}A \\ A \\ -\frac{a}{2b}A \end{pmatrix} = \begin{pmatrix} -\frac{M}{2m}A \\ A \\ -\frac{M}{2m}A \end{pmatrix}$$

