1. フーリエ級数の解答 (問 3 まで)

1. $f(x) = \dfrac{\pi}{2} - \dfrac{4}{\pi}\sum_{n=\text{奇数}} \dfrac{\cos nx}{n^2} = \dfrac{\pi}{2} - \dfrac{4}{\pi}\left(\cos x + \dfrac{\cos 3x}{9} + \dfrac{\cos 5x}{25}\cdots\right)$

2. $f(x) = \dfrac{1}{2} + \dfrac{2}{\pi}\sum_{n=\text{奇数}} \dfrac{\sin nx}{n} = \dfrac{1}{2} + \dfrac{2}{\pi}\left(\sin x + \dfrac{\sin 3x}{3} + \dfrac{\sin 5x}{5}\cdots\right)$

3(1). $f(x) = 2\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}\sin nx}{n} = 2\left(\sin x - \dfrac{\sin 2x}{2} + \dfrac{\sin 3x}{3} - \dfrac{\sin 4x}{4}\cdots\right)$

3(2). $f(x) = \dfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \dfrac{(-1)^n \cos nx}{n^2} = \dfrac{\pi^2}{3} + 4\left(-\cos x + \dfrac{\cos 2x}{4} - \dfrac{\cos 3x}{9} + \dfrac{\cos 4x}{16}\cdots\right)$

3(3). $f(x) = \dfrac{2}{\pi} - \dfrac{4}{\pi}\sum_{n=1}^{\infty} \dfrac{\cos nx}{4n^2-1} = \dfrac{2}{\pi} - \dfrac{4}{\pi}\left(\dfrac{\cos x}{3} + \dfrac{\cos 2x}{15} + \dfrac{\cos 3x}{35}\cdots\right)$

3(4). $f(x) = \dfrac{2}{\pi} + \dfrac{4}{\pi}\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{4n^2-1}\cos nx = \dfrac{2}{\pi} + \dfrac{4}{\pi}\left(\dfrac{\cos x}{3} - \dfrac{\cos 2x}{15} + \dfrac{\cos 3x}{35} - \dfrac{\cos 4x}{63}\cdots\right)$

3(5). $$f(x) = \dfrac{e^{\pi}-e^{-\pi}}{2\pi} + \dfrac{e^{\pi}-e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\left\{\dfrac{(-1)^n}{n^2+1}\cos nx + \dfrac{(-1)^{n+1}n}{n^2+1}\sin nx\right\}$$
$$= \dfrac{e^{\pi}-e^{-\pi}}{\pi}\left\{\dfrac{1}{2} + \left(-\dfrac{\cos x}{2} + \dfrac{\cos 2x}{5} - \dfrac{\cos 3x}{10}\cdots\right) + \left(\dfrac{\sin x}{2} - \dfrac{2\sin 2x}{5} + \dfrac{3\sin 3x}{10}\cdots\right)\right\}$$

3(6). $$f(x) = \dfrac{\pi}{4} - \dfrac{2}{\pi}\sum_{n=\text{奇数}} \dfrac{\cos nx}{n^2} + \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}\sin nx}{n}$$
$$= \dfrac{\pi}{4} - \dfrac{2}{\pi}\left(\cos x + \dfrac{\cos 3x}{9} + \dfrac{\cos 5x}{25}\cdots\right) + \left(\sin x - \dfrac{\sin 2x}{2} + \dfrac{3\sin 3x}{3}\cdots\right)$$