CS 61A
Fall 2018

Trees & Sequences

Discussion 4: September 19, 2018 Solutions

1 List Comprehensions

A **list comprehension** is a compact way to create a list whose elements are the results of applying a fixed expression to elements in another sequence.

```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

It might be helpful to note that we can rewrite a list comprehension as an equivalent for statement. See the example to the right.

```
new_lst = []
for <name> in <iter exp>:
    if <filter exp>:
        new_lst += [<map_exp>]
return new_lst
```

Let's break down an example:

```
[x * x - 3 for x in [1, 2, 3, 4, 5] if x % 2 == 1]
```

In this list comprehension, we are creating a new list after performing a series of operations to our initial sequence `[1, 2, 3, 4, 5]`. We only keep the elements that satisfy the filter expression `x % 2 == 1` (1, 3, and 5). For each retained element, we apply the map expression `x*x - 3` before adding it to the new list that we are creating, resulting in the output `[-2, 6, 22]`.

Note: The `if` clause in a list comprehension is optional.

Questions

1.1 What would Python display?

```
>>> [i + 1 for i in [1, 2, 3, 4, 5] if i % 2 == 0]
```

```
[3, 5]
```

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```
>>> [i * i - i for i in [5, -1, 3, -1, 3] if i > 2]
```

```
[20, 6, 6]
```

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```
>>> [[y * 2 for y in [x, x + 1]] for x in [1, 2, 3, 4]]
```

```
[[2, 4], [4, 6], [6, 8], [8, 10]]
```

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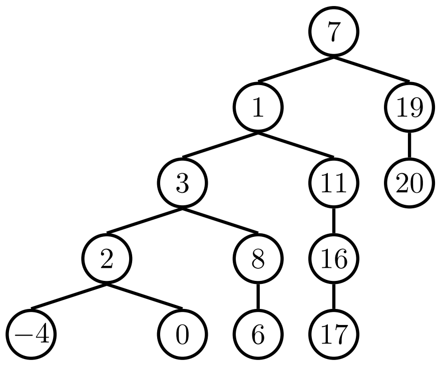
2 Trees



In computer science, **trees** are recursive data structures that are widely used in various settings. The diagram to the right is an example of a tree.

Notice that the tree branches downward. In computer science, the **root** of a tree starts at the top, and the **leaves** are at the bottom.

Some terminology regarding trees:

- **Parent node**: A node that has branches. Parent nodes can have multiple branches.
- **Child node**: A node that has a parent. A child node can only belong to one parent.
- **Root**: The top node of the tree. In our example, the node that contains 7 is the root.
- **Label**: The value at a node. In our example, all of the integers are values.
- **Leaf**: A node that has no branches. In our example, the nodes that contain -4, 0, 6, 17, and 20 are leaves.
- **Branch**: A subtree of the root. Note that trees have branches, which are trees themselves: this is why trees are *recursive* data structures.
- **Depth**: How far away a node is from the root. In other words, the number of edges between the root of the tree to the node. In the diagram, the node containing 19 has depth 1; the node containing 3 has depth 2. Since there are no edges between the root of the tree and itself, the depth of the root is 0.
- **Height**: The depth of the lowest leaf. In the diagram, the nodes containing -4, 0, 6, and 17 are all the "lowest leaves," and they have depth 4. Thus, the entire tree has height 4.

In computer science, there are many different types of trees. Some vary in the number of branches each node has; others vary in the structure of the tree.

Implementation

A tree has both a value for the root node and a sequence of branches, which are also trees. In our implementation, we represent the branches as a list of trees. Since a tree is an abstract data type, our choice to use lists is just an implementation detail.

- The arguments to the constructor `tree` are the value for the root node and a list of branches.
- The selectors for these are `label` and `branches`.

Note that `branches` returns a list of trees and not a tree directly. It's important to distinguish between working with a tree and working with a **list of** trees.

We have also provided a convenience function, `is_leaf`.

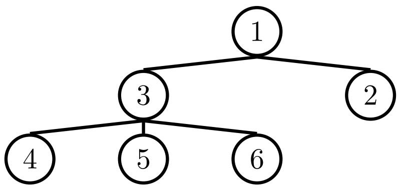
```
# Constructor
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches

# Selectors
def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

# For convenience
def is_leaf(tree):
    return not branches(tree)
```

Let's try to create the tree below.



```
# Example tree construction
t = tree(1,
      [tree(3,
          [tree(4),
           tree(5),
           tree(6)]),
      tree(2)])
```

Questions

2.1 Write a function that returns the largest number in a tree.

```
def tree_max(t):
    """Return the maximum label in a tree.

    >>> t = tree(4, [tree(2, [tree(1)]), tree(10)])
    >>> tree_max(t)
    10
    """


    return max([label(t)] + [tree_max(branch) for branch in branches(t)])
```

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2.2 Write a function that returns the height of a tree. Recall that the height of a tree is the length of the longest path from the root to a leaf.

```
def height(t):
    """Return the height of a tree.

    >>> t = tree(3, [tree(5, [tree(1)]), tree(2)])
    >>> height(t)
    2
    """


    if is_leaf(t):
        return 0
    return 1 + max([height(branch) for branch in branches(t)])
```

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2.3 Write a function that takes in a tree and squares every value. It should return a new tree. You can assume that every item is a number.

```
def square_tree(t):
    """Return a tree with the square of every element in t"""


    sq_branches = [square_tree(branch) for branch in branches(t)]
    return tree(label(t)**2, sq_branches)
```

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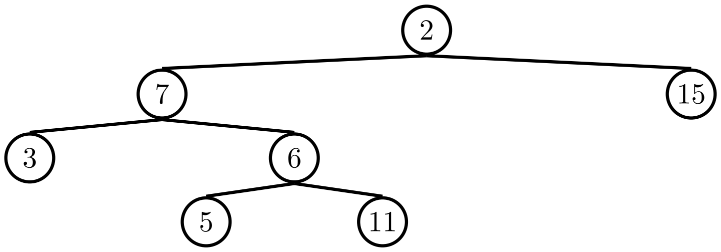
2.4 Write a function that takes in a tree and a value `x` and returns a list containing the nodes along the path required to get from the root of the tree to a node containing `x`.

If `x` is not present in the tree, return `None`. Assume that the entries of the tree are unique.

For the following tree, `find_path(t, 5)` should return `[2, 7, 6, 5]`



```
def find_path(tree, x):
    """
    >>> t = tree(2, [tree(7, [tree(3), tree(6, [tree(5), tree(11)])] ), tree(15)])
    >>> find_path(t, 5)
    [2, 7, 6, 5]
    >>> find_path(t, 10)  # returns None
    """

    if _____________________________:

        return _____________________________

    _____________________________:

        path = _____________________________

        if _____________________________:

            return _____________________________
```

```
def find_path(tree, x):
    if label(tree) == x:
        return [label(tree)]
    for b in branches(tree):
        path = find_path(b, x)
        if path:
            return [label(tree)] + path
```

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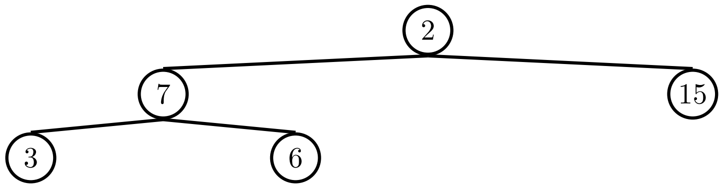
2.5 Write a function that takes in a tree and a depth `k` and returns a new tree that contains only the first `k` levels of the original tree.

For example, if `t` is the tree shown in the previous question, then `prune(t, 2)` should return the following tree.



```
def prune(t, k):


    if k == 0:
        return tree(label(t), [])
    else:
        return tree(label(t), [prune(branch, k - 1) for branch in branches(t)])
```

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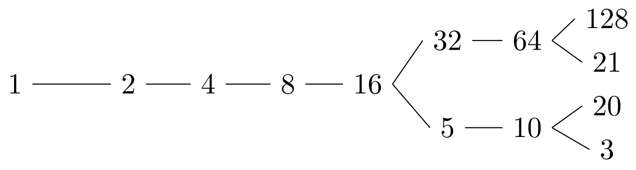
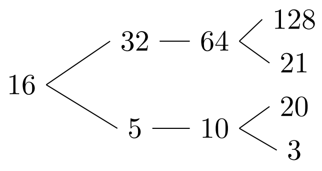
2.6 We can represent the hailstone sequence as a tree in the figure below, showing the route different numbers take to reach 1. Remember that a hailstone sequence starts with a number n, continuing to $n/2$ if n is even or $3n + 1$ if n is odd, ending with 1. Write a function `hailstone_tree(n, h)` which generates a tree of height `h`, containing hailstone numbers that will reach `n`.

Hint: A node of a hailstone tree will always have at least one, and at most two branches (which are also hailstone trees). Under what conditions do you add the second branch?



`hailstone_tree(1, 7)`



`hailstone_tree(16, 3)`

```
def hailstone_tree(n, h):
    """Generates a tree of hailstone numbers that will
       reach N, with height H.
```

```
>>> hailstone_tree(1, 0)
[1]
>>> hailstone_tree(1, 4)
[1, [2, [4, [8, [16]]]]]
>>> hailstone_tree(8, 3)
[8, [16, [32, [64]], [5, [10]]]]
"""


if h == 0:
    return tree(n)
branches = [hailstone_tree(n * 2, h - 1)]
if (n - 1) % 3 == 0 and ((n - 1) // 3) % 2 == 1 and (n - 1) // 3 > 1:
    branches += [hailstone_tree((n - 1) // 3, h - 1)]
return tree(n, branches)
```