UNIVERSITY PARTNER

UNIVERSITY OF WOLVERHAMPTON

HERALD COLLEGE KATHMANDU

<u>4MM013 - Computational Mathematics</u>

Mathematics Assignment-2

Full Marks: 20

University ID : 2332244

Submitted by : Naomi Thing

Submitted on : 2023/04/29

1. Using Cramer's rule obtain the solutions to the following set of equations:

$$2\,x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

(4)

Mathematics Assignment 2

Naomi Thing - 2332244

1) Solve

$2x_1 + x_2 - x_3 = 0$ —— (i)

$x_1 + x_3 = 4$ —— (ii)

$x_1 + x_2 + x_3 = 0$ —— (iii)

writing the given eq^{ns} into matrix form,

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

Now,

According to cramer's rule,

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= 2(0-1) - 1(1-1) - 1(1-0)$$

$$= 2.(-1) - 1.0 - 1.1$$

$$= -2 - 0 - 1$$

$$= -3$$

Then,

$$\Delta x_1 = \begin{vmatrix} 0 & 1 & -1 \\ 4 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 0(0-1) - 1(4-0) - 1(4-0)$$

$$= 0 - 4 - 4$$

$$= -8$$

Naomi Thing - 2332244

$$\Delta x_2 = \begin{vmatrix} 2 & 0 & -1 \\ 1 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix}$$

$$= 2(4-0) - 0(1-1) - 1(0-4)$$

$$= 8+4$$

$$= 12$$

$$\Delta x_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 4 \\ 1 & 1 & 0 \end{vmatrix} = 2\begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= 2(0-4) - 1(0-4) + 0(1-0)$$

$$= -8+4$$

$$= -4$$

Now,

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-8}{-3} = \frac{8}{3}$$

$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{12}{-3} = -4$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{-4}{-3} = \frac{4}{3}$$

2.

a) Solve the following using Gauss elimination:

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

(4)

b) Find the inverse of the matrix from (a) using elimination.

Naomi Thing - 2332244

2.) Soln,

Given,

$$x_1 + x_2 + x_3 = 2 \quad \text{--- (i)}$$
$$2x_1 + 3x_2 + 4x_3 = 3 \quad \text{--- (ii)}$$
$$x_1 - 2x_2 - x_3 = 1 \quad \text{--- (iii)}$$

Writing the eqns in augmented forms,

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & -2 & -1 & 1 \end{array}\right]$$

Then,

Echelon form,

$$R_2 = R_2 - 2R_1$$
$$R_3 = R_3 - R_1$$

we get,

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array}\right]$$

Again,

$$R_3 = R_3 + 3R_2$$

we get,

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$$

Now,

doing backward substitution,

Naomi Thing - 2332244

$4x_3 = -4$

or, $x_3 = \frac{-\cancel{4}\,1}{\cancel{4}}$

$\therefore x_3 = -1$

Again,

$x_2 + 2x_3 = -1$

or, $x_2 + 2.(-1) = -1$

or, $x_2 = -1 + 2$

$\therefore x_2 = 1$

Now,

$x_1 + x_2 + x_3 = 2$

or, $x_1 + 1 + (-1) = 2$

or, $x_1 + \cancel{1} - \cancel{1} = 2$

$\therefore x_1 = 2$

$\therefore x_1 = 2,\ x_2 = 1,\ x_3 = -1$ //.

Naomi Thing - 2332244

b) Soln,

Given,

$x_1 + x_2 + x_3 = 2 \longrightarrow$ (I)

$2x_1 + 3x_2 + 4x_3 = 3 \longrightarrow$ (II)

$x_1 - 2x_2 - x_3 = 1 \longrightarrow$ (III)

Now, writing the eqns in ~~augmented~~ AI form,

$$[AI] = \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

Naomi Thing - 2332244

Then, to make echelon form,

$R_2 = R_2 - 2R_1$

$R_3 = R_3 - R_1$

we get,

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right]$$

Then, $R_3 = R_3 + 3R_2$, we get,

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

Again,

$R_1 = 4R_1 - R_3$

$R_2 = 2R_2 - R_3$

$$\left[\begin{array}{ccc:ccc} 4 & 4 & 0 & 11 & -3 & -1 \\ 0 & 2 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

Again,

$R_1 = R_1 - 2R_2$

$$\left[\begin{array}{ccc:ccc} 4 & 0 & 0 & 5 & -1 & 1 \\ 0 & 2 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right]$$

Now,

dividing R_1 by 4, R_2 by 2 and R_3 by 4,

we get,

Naomi Thing - 2332244

$$IA^{-1} = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 5/4 & -1/4 & 1/4 \\ 0 & 1 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & -7/4 & 3/4 & 1/4 \end{array}\right]$$

We know,

$$AI = IA^{-1}$$

So, $$A^{-1} = \begin{bmatrix} 5/4 & -1/4 & 1/4 \\ 3/2 & -1/2 & -1/2 \\ -7/4 & 3/4 & 1/4 \end{bmatrix}$$

3. Determine whether the following sequence converges or diverges.

$$t_n = (-1)^{n+1}\,\frac{n+1}{n^2+3}$$

(4)

Naomi Thing - 2332244

3) Given,

$$t_n = (-1)^{n+1}\,\frac{n+1}{n^2+3}$$

To check whether the term converges or diverges,

$$\lim_{n\to\infty} t_n = \lim_{n\to\infty} (-1)^{n+1}\,\frac{n+1}{n^2+3}$$

$$= \lim_{n\to\infty} (-1)^{n+1}\,\frac{(n+1)/n}{(n^2+3)/n}$$

$$= \lim_{n\to\infty} (-1)^{n+1}\,\frac{1+1/n}{n+3/n}$$

$$= (-1)^{\infty+1}\,\frac{1+1/\infty}{\infty+3/\infty}$$

$$= (-1)^{\infty+1}\,\frac{1+0}{\infty+0}$$

$$= (-1)^{\infty+1}\,\frac{1}{\infty}$$

Naomi Thing - 2332244

$$= (-1)^{\infty+1} \cdot 0$$

$= 0$; which is positive real number

∴ So, this given term (t_n) converges, i.e; convergent.

4. Find the Maclaurin series expansion of **Sinx,** also calculate the radius of convergence. (4).

Naomi Thing - 2332244

4) Here,

we know,

the maclaurin series expansion of the sinx is given by,

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) + \ldots$$

Now,

to find maclaurin series expansion of sinx,

$f(x) = \sin x \Rightarrow f(0) = \sin 0 = 0$

$f'(x) = \cos x \Rightarrow f'(0) = \cos 0 = 1$

$f''(x) = -\sin x \Rightarrow f''(0) = -\sin 0 = 0$

$f'''(x) = -\cos x \Rightarrow f'''(0) = -\cos 0 = -1$

$f''''(x) = \sin x \Rightarrow f''''(0) = \sin 0 = 0$

So,

$$\sin x = 0 + x.1 + \frac{x^2}{2!} \cdot 0 + \frac{x^3}{3!} \cdot (-1) + \frac{x^4}{4!} \cdot 0 + \ldots \infty$$

$$= 0 + x + 0 - \frac{x^3}{3!} + 0 + \ldots \infty$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \infty$$

Naomi Thing - 2332244

Then,

to find radius of convergence of sinx,

In summation form,

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Using ratio test,

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} \cdot x^{2(n+1)+1}}{(2(n+1)+1)!}}{\frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^{n+1} \cdot x^{2n+1+2} \cdot (2n+1)!}{(-1)^n \cdot x^{2n+1} \cdot (2n+2+1)!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(-1)^n} \cdot (-1)^1 \cdot \cancel{x^{2n+1}} \cdot x^2 \cdot \cancel{(2n+1)!}}{\cancel{(-1)^n} \cdot \cancel{x^{2n+1}} \cdot (2n+3)(2n+2)\cancel{(2n+1)!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2}{(2n+3)(2n+2)} \right|$$

$$= \frac{x^2}{(2\cdot\infty+3)(2\cdot\infty+2)}$$

$$= 0$$

∴ The radius of convergence of sinx is ∞.

The End