README for ssd_solidangle.cpp

N. Ozawa

September 24, 2019

This is a script to 1. Calculate the α detection efficiency of the beam profile monitor and 2. Find the optimum geometry parameters z_0 and x_0 that maximizes the detection efficiency. Here, "detection efficiency" is defined as the percentage of particles that hit the SSD which are emitted from the surface of the MCP/mesh. In the general literature, "detection efficiency" would be defined as the fraction of α particles that generate a detectable signal at the Si detector out of the number of Fr ions that reached the surface of the MCP/mesh. In our particular case, this efficiency ε_{tot} could be factorized in the following way:

$$\varepsilon_{tot} \quad = \quad \varepsilon_{mesh}\varepsilon_{decay}\varepsilon_{direction}\varepsilon_{geometry}\varepsilon_{detection}$$

where ε_{mesh} is the probability that the incoming Fr ion is captured on the mesh surface, ε_{decay} is the average ratio of Fr ions that decay and emit an α particle, $\varepsilon_{direction} = \frac{1}{2}$ is the ratio of α particles that are emitted in the beamline upstream direction rather than the downstream direction which would be undetectable with the SSD, $\varepsilon_{geometry}$ is the ratio of α particles that enter the SSD holder/box out of the ones that were emitted in the upstream direction, and $\varepsilon_{detection}$ is the ratio of α particles that generate a signal which depends on the detector characteristics. In the script ssd_solidangle.cpp, only $\varepsilon_{geometry}$ is calculated. This is nearly equivalent to the solid angle Ω of the hole opened on the lid of the SSD holder/box observed from the center of the MCP. Since the hole is off-axis with respect to the MCP center, it is approximated with $\Omega = 2\pi(1 - \cos\theta)$, $\cos\theta \approx \frac{\sqrt{z_0{}^2+x_0{}^2}}{\sqrt{\left(\frac{R_{SSD}}{\sqrt{2}}\right)^2+z_0{}^2+x_0{}^2}}$. The factor $\sqrt{2}$ for the R_{SSD} is used to approximate the off-axis solid angle with a 45 degree tilt. The meaning of each parameter is to be referenced below.
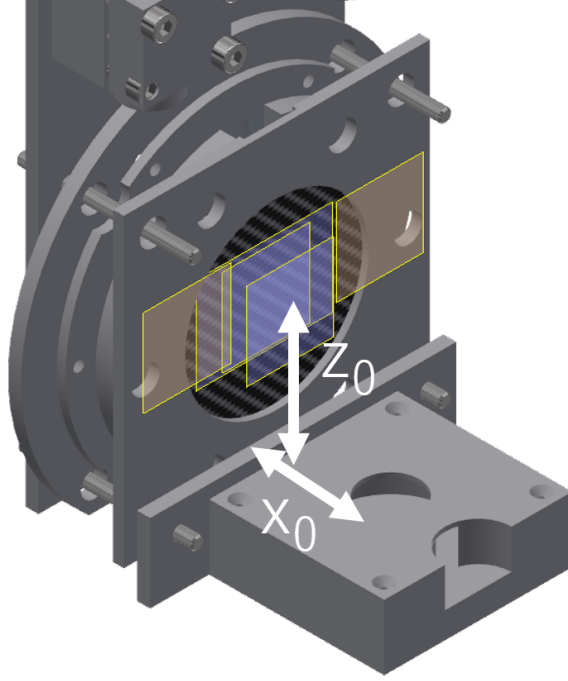
Figure 1: The planned design for the beam profile monitor.

Figure 1 depicts the design of the beam profile monitor. The plate in the center (height M_H and width M_W) is the topmost plate holding the MCP. A metal mesh covers this plate which catches the incoming Fr beam. The α particles emitted from the Fr are assumed to emerge from the surface of the plate $\left\{ (x_M,\ y_M,\ z_M) \mid x_M = 0,\ -\frac{M_W}{2} \le y_M \le \frac{M_W}{2},\ z_0 - \frac{M_H}{2} \le z_M \le z_0 + \frac{M_H}{2} \right\}$ following the quasi-Gaussian distribution $(X_c, Y_c, \sigma_x, \sigma_y)$ on the MCP coordinates given by the beam simulations. Here, z_0 is the distance between the center of the mesh and the lower surface of the lid of the box holding the SSD. The α particle is assumed to be emitted in random directions $\vec{a_M} = (a_x,\ a_y,\ a_z)$, $a_x \ge 0$ from the surface, following a uniform half-sphere direction distribution.

The SSD is placed inside the box in the lower right corner in figure 1. A hole of diameter $2R_{\text{SSD}}$ in the lid of thickness z_l is the entrance for α rays to be detected by the SSD. In the current coordinates, the upper surface of the lid is defined as $\left\{ (x_{lu},\ y_{lu},\ z_{lu}) \mid z_{lu} = z_l,\ (x_l u - x_0)^2 + y_l u^2 > R_{\text{SSD}}^2 \right\}$ and the lower surface as $\left\{ (x_{ll},\ y_{ll},\ z_{ll}) \mid z_{ll} = 0,\ (x_{ll} - x_0)^2 + y_{ll}^2 > R_{\text{SSD}}^2 \right\}$. Here x_0 is the distance between the mesh and the center of the hole in the lid.

In this setup, the geometrical parameters z_0 and x_0 can be adjusted to yield the maximum detection efficiency. In order to analyze this, a method of calcu-

lating the detection efficiency is developed in the following way. Note that this method only considers the α particles emitted from the surface of the mesh in the direction of the incoming beam, and enters the SSD box. It does not consider the efficiency of the Fr ions being captured at the mesh, and the detection efficiency of the detector for the α particles that entered the box.

For each particle:

1. Set point $\vec{P} = (0,\ y_M,\ z_M)$ following a Gaussian random distribution $(X_c,\ Y_c,\ \sigma_X,\ \sigma_Y)$ as $y_M \sim N(X_c,\ \sigma_X)$ and $z_M \sim N(z_0 + Y_c,\ \sigma_Y)$, constrained within $-\frac{M_W}{2} \leq y_M \leq \frac{M_W}{2}$ and $z_0 - \frac{M_H}{2} \leq z_M \leq z_0 + \frac{M_H}{2}$ as starting point of the α particle.

2. Set direction of α particle as $\vec{a_M} = (a_x,\ a_y,\ a_z)$ with a_x as a non-negative random number, and a_y and a_z as random numbers. The random numbers are generated following a normal distribution.

3. Define $t_u = \frac{z_l - z_M}{a_z}$ so that $\vec{P} + t_u \vec{a_M}$ is on the surface $z = z_l$ i.e. the upper surface of the lid of the SSD box. Similarly, define $t_l = -\frac{z_M}{a_z}$ so that $\vec{P} + t_l \vec{a_M}$ is on the surface $z = 0$ i. e. the lower surface of the lid.

4. Define $x_{lu} = t_u a_x$, $y_{lu} = y_M + t_u a_y$, $x_{ll} = t_l a_x$, and $y_{ll} = y_M + t_l a_y$. The α particle passes the upper surface of the lid at point $(x_{lu},\ y_{lu},\ z_l)$ and the lower surface of the lid at point $(x_{ll},\ y_{ll},\ 0)$.

5. If $(x_{lu} - x_0)^2 + y_{lu}^2 > R_{\text{SSD}}^2$, the α particle hits the upper lid and does not reach the SSD. Similarly, if $(x_{ll} - x_0)^2 + y_{ll}^2 > R_{\text{SSD}}^2$, the α particle hits the lid and does not reach the SSD.

Repeat for all the N_{Fr} particles defined and count the number $N_{Detected}$ of them that reached the SSD. The detection efficiency ε_i for this set i of N_{Fr} particles is defined as $\varepsilon_i = \frac{N_{Detected}}{N_{Fr}}$. This value is expected to be sample-dependent, since the particle positions and directions are randomly selected. In order to obtain a more reliable value, the average of $N_{Average}$ samples is calculated as the final detection efficiency: $\varepsilon = \sum_{i=1}^{N_{Average}} \varepsilon_i$.

In the code, the value of ε is calculated for various combinations of z_0 and x_0. For the TOF-BPM at CYRIC, $z_0 = 29$ mm and $x_0 = 33$ mm.

Footnote: Generating Random Trajectories

In the code, random trajectories are defined by giving a random "starting point"

$$\vec{P_0} = \begin{pmatrix} 0 \\ y_M \\ z_M \end{pmatrix}$$

and a random "direction"

$$\vec{d} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

so that using some parameter t, $\vec{P}_0 + t\vec{d}$ creates a trajectory of the emitted α particle. Based on the Fr ion simulations, y_M and z_M are assumed to follow a normal distribution. $\vec{d}$ is then given randomly, so that the direction is uniformly distributed on a half-sphere surface in the beamline upstream direction. This could be done by selecting the parameters a_x, a_y, and a_z randomly, following a normal distribution[1]. This could be understood in the following way.

Suppose a_x, a_y, and a_z are all generated following a normal distribution

$$N_k(0,\ R) = \frac{1}{\sqrt{2\pi}R} \exp\left(-\frac{a_k{}^2}{2R^2}\right) \quad (k = x, y, z)$$

with some positive number R. Since all three parameters are individually determined, the set of numbers (a_x, a_y, a_z) are generated following a 3D normal distribution

$$N_x(0,\ R)N_y(0,\ R)N_z(0,\ R) = \frac{1}{(2\pi)^{\frac{3}{2}}R^3} \exp\left(-\frac{a_x{}^2 + a_y{}^2 + a_z{}^2}{2R^2}\right).$$

When we take a sample sphere of $a_x{}^2 + a_y{}^2 + a_z{}^2 = r^2$ where r is some constant, the distribution itself becomes a constant, which corresponds to a uniform distribution over the 3D spherical surface of radius r.

This is not the case when $\vec{d}$ is generated using uniformly distributed random numbers. Suppose a_x, a_y, and a_z are generated following a uniform distribution
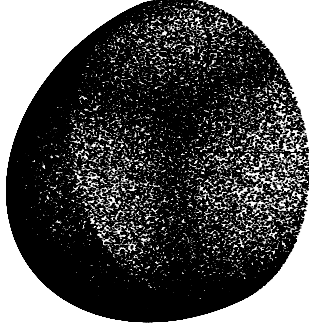
$$U_k(0,\ R) = \begin{cases} \frac{1}{2R} & (-R \le a_k \le R) \\ 0 & \text{(otherwise)} \end{cases} \quad (k = x, y, z).$$

Following the same method as above, the set of numbers (a_x, a_y, a_z) follow the distribution

$$U_x(0,\ R)U_y(0,\ R)U_z(0,\ R) = \begin{cases} \frac{1}{8R^3} & ((-R \le a_x \le R) \cap (-R \le a_y \le R) \cap (-R \le a_z \le R)) \\ 0 & \text{(otherwise)} \end{cases} .$$

This corresponds to a uniform distribution over the volume of a cube, not the surface of a sphere. This is demonstrated using the code random_tester.cpp producing the follwing scatter plot. When $\vec{d}$ is generated using a uniform distribution, a cube-shaped shadow appears, corresponding to a non-uniformly distributed points on the sphere surface.

Uniformly Distributed Direction　　　　　　　Normally Distributed Direction
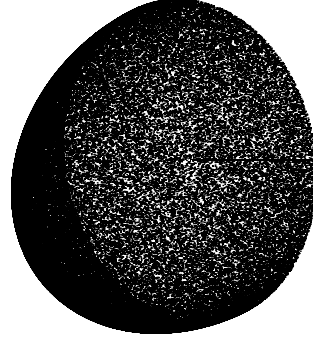
Case with no mesh

In the case where a mesh is not applied, the Fr ions will be attatched to the surface of the MCP front panel. Since the open-area ratio (OAR) of our MCP is 63%, approximatly 37% of the Fr that reached the MCP will be attatched to the surface. This corresponds to the replacement

$$\varepsilon_{mesh} \to \varepsilon_{MCP} = 0.37$$

for the detection efficiency. Furthermore, the calculation method for $\varepsilon_{geometry}$ must be altered so that the α particles originate at the surface of the MCP. As a simple approximation, this could be achieved by just extending the length X_0 in the code ssd_solidangle,cpp. Applying this method, the efficiency of

$$\varepsilon_{geometry} = 0.0077$$

has been obtained.
Therefore,

$$\varepsilon_{\alpha} = \varepsilon_{MCP}\varepsilon_{direction}\varepsilon_{geometry} = 0.0014245$$

is the ratio of the α particles emitted on the MCP-IN surface that reaches the SSD surface.

References

[1] M. E. Muller, *Comm. Assoc. Comp. Mach.* **2**, 19, (1959).