The evaluation of neutron radiation caused by the reaction $^{18}\mathrm{O} + {}^{197}\mathrm{Au} \rightarrow {}^{210}\mathrm{Fr} + 5n$

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The reaction ^{197}Au (n,γ) ^{198}Au can be used as a method of detecting neutron radiation. The daughter nucleus ^{198}Au emits a γ ray of energy around 411 keV at a half-life of τ_h =2.7 days. This process could be formulated as

$$\frac{dN_{ex}(t)}{dt} = \begin{cases} A\left(N_0 - N_{ex}(t)\right) - \frac{1}{\tau_l}N_{ex}(t) & (0 < t < t_0) \\ -\frac{1}{\tau_l}N_{ex}(t) & (t_0 < t < t_1) \end{cases},$$

where N_0 is the original number of ^{197}Au atoms inside the film, $N_{ex}(t)$ is the number of the ^{198}Au in its excited state which will emit the γ ray, A is the occurence rate of the reaction ^{197}Au (n,γ) ^{198}Au, and $\tau_l = \frac{\tau_h}{\ln 2}$ is the lifetime of ^{198}Au. It is assumed that the neutron irradiation starts at $t = 0$, the irradiation is stopped at $t = t_0$, and the γ rays are measured at $t = t_1$.

The reaction rate A can be determined by the intensity of the neutron beam $F_n(E_n)$, the solid angle of the gold film that is used for detection ε, the cross section $\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)$ of the neutron capture, the density $N_{^{197}\mathrm{Au}}$ of the ^{197}Au inside the gold film, and the thickness T of the film. The cross section data could be obtained from eg. the EXFOR database.

First, the reaction rate at the surface $(D = 0)$ of the film can be given by

$$\tilde{A}(D = 0)dD = F_n(E_n)\varepsilon\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}dD.$$

Since the flux of the neutron beam is damped as it penetrates the film,

$$\begin{aligned} A &= \int_0^T \tilde{A}(D)dD \\ &= \int_0^T F_n(E_n)\varepsilon e^{-\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}D}\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}dD \\ &= F_n(E_n)\varepsilon\left(1 - e^{-\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}T}\right). \end{aligned}$$

Now, the equation will be solved for the neutron irradiation period ($0 < t < t_0$). By reorganizing the terms,

$$\frac{dN_{ex}(t)}{dt} + \left(A + \frac{1}{\tau_l}\right) N_{ex}(t) = AN_0.$$

This equation can be solved by using the particular solution $u(t) = u_0 e^{-\left(A+\frac{1}{\tau_l}\right)t}$ of $N_{ex}(t)$ which is the solution when the RHS is equated to 0. By redefining $u_0 = u_0(t)$ and reevaluating the equation using $N_{ex}(t) = u_0(t)e^{-\left(A+\frac{1}{\tau_l}\right)t}$,

$$\begin{aligned}
\frac{du_0(t)}{dt} e^{-\left(A+\frac{1}{\tau_l}\right)t} &- \left(A + \frac{1}{\tau_l}\right) u_0(t) e^{-\left(A+\frac{1}{\tau_l}\right)t} + \left(A + \frac{1}{\tau_l}\right) u_0(t) e^{-\left(A+\frac{1}{\tau_l}\right)t} = AN_0 \\
\frac{du_0(t)}{dt} &= AN_0 e^{\left(A+\frac{1}{\tau_l}\right)t} \\
u_0(t) &= \frac{AN_0}{A + \frac{1}{\tau_l}} e^{\left(A+\frac{1}{\tau_l}\right)t} + u_0(0) \\
N_{ex}(t) &= \left\{ \frac{AN_0}{A + \frac{1}{\tau_l}} e^{\left(A+\frac{1}{\tau_l}\right)t} + u_0(0) \right\} e^{-\left(A+\frac{1}{\tau_l}\right)t}.
\end{aligned}$$

Since the neutron irradiation starts at $t = 0$,

$$\begin{aligned}
N_{ex}(t=0) &= \frac{AN_0}{A + \frac{1}{\tau_l}} + u_0(0) = 0 \\
u_0(0) &= -\frac{AN_0}{A + \frac{1}{\tau_l}}
\end{aligned}$$

therefore,

$$\begin{aligned}
N_{ex}(t) &= \frac{AN_0}{A + \frac{1}{\tau_l}} \left(e^{\left(A+\frac{1}{\tau_l}\right)t} - 1 \right) e^{-\left(A+\frac{1}{\tau_l}\right)t} \\
&= \frac{AN_0}{A + \frac{1}{\tau_l}} \left(1 - e^{-\left(A+\frac{1}{\tau_l}\right)t} \right)
\end{aligned}$$

is the solution during the neutron irradiation. Using the value at $t = t_0$, the time evolution of the system can be formulated as

$$N_{ex}(t) = \begin{cases} \frac{AN_0}{A+\frac{1}{\tau_l}} \left(1 - e^{-\left(A+\frac{1}{\tau_l}\right)t} \right) & (0 < t < t_0) \\ \frac{AN_0}{A+\frac{1}{\tau_l}} \left(1 - e^{-\left(A+\frac{1}{\tau_l}\right)t_0} \right) e^{-\frac{t-t_0}{\tau_l}} & (t_0 < t < t_1) \end{cases}.$$

The data that was obtained at CYRIC was the strength of the γ ray emmision from the excited ^{198}Au nuclei at time $t = t_1$, which is equivalent to

$$-\left.\frac{dN_{ex}(t)}{dt}\right|_{t=t_1} = \frac{AN_0}{A\tau_l + 1} \left(1 - e^{-\left(A+\frac{1}{\tau_l}\right)t_0} \right) e^{-\frac{t_1-t_0}{\tau_l}}$$

$$
\begin{aligned}
&= \frac{N_0}{\tau_l}\frac{1}{1+\frac{1}{A\tau_l}}\left(1-e^{-\left(A+\frac{1}{\tau_l}\right)t_0}\right)e^{-\frac{t_1-t_0}{\tau_l}} \\
&= \frac{N_0\alpha}{\tau_l}\left(1-e^{-\frac{t_0}{(1-\alpha)\tau_l}}\right)e^{-\frac{t_1-t_0}{\tau_l}}.
\end{aligned}
$$

Here, a new parameter $\alpha = \frac{1}{1+\frac{1}{A\tau_l}}$ has been defined. Assuming $\alpha \ll 1$, it can be simplified to

$$
-\left.\frac{dN_{ex}(t)}{dt}\right|_{t=t_1} \approx \frac{N_0}{\tau_l}\left(1-e^{-\frac{t_0}{\tau_l}}\right)e^{-\frac{t_1-t_0}{\tau_l}}\alpha.
$$

Thus the intensity of the neutron beam at the foil can be calculated by

$$
\begin{aligned}
N_0F_n(E_n)\varepsilon &= \frac{AN_0}{1-\exp\left[-\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}T\right]} \\
&= \frac{\alpha}{(1-\alpha)\tau_l}\frac{N_0}{1-\exp\left[-\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}T\right]} \\
&\approx \frac{\alpha}{\tau_l}\frac{N_0}{1-\exp\left[-\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}T\right]} \\
&= \frac{1}{\left(1-e^{-\sigma_{^{197}\mathrm{Au}(n,\gamma)^{198}\mathrm{Au}}(E_n)N_{^{197}\mathrm{Au}}T}\right)\left(1-e^{-\frac{t_0}{\tau_l}}\right)e^{-\frac{t_1-t_0}{\tau_l}}}\left(-\left.\frac{dN_{ex}(t)}{dt}\right|_{t=t_1}\right)
\end{aligned}
$$

using the obtained value, based on the assumption that the neutron flux is constant during the irradiation. The number of neutrons that is captured at the gold film can be estimated by AN_0.

The data at shorter distances or higher neutron energies are expected to contain more error. The former is because of the inaccurate fit of the radiation data at short distances, and the latter is because of inaccurate fit of cross sections at higher neutron energies. Otherwise, this method should work for lower-energy neutrons (below sub MeV) and long distances (more than a few meters).

By using the obtained value of α, the number of excited ^{198}Au nuclei contained in the gold foil can be simulated as

$$
N_{ex}(t) = \begin{cases} N_0\alpha\left(1-e^{-\frac{t}{(1-\alpha)\tau_l}}\right) & (0<t<t_0) \\ N_0\alpha\left(1-e^{-\frac{t_0}{(1-\alpha)\tau_l}}\right)e^{-\frac{t-t_0}{\tau_l}} & (t_0<t) \end{cases}.
$$