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① prove that $2h + 1 \leq n$ when n = number of nodes
h = height of tree

Base case : a proper tree with 0 height has 1 node
and a tree with 1 height has to add at least 2 nodes
to a tree with 0 height, or (1-1) height.

$$h = 0 \; ; \; n_0 = 1$$

$$h = 1 \; ; \; n_1 = (n_0) + 2$$

proof : if a tree with i height

$$h = i \; ; \; n_i = (n_{i-1}) + 2$$

$$= (1 + 2(i-1)) + 2$$

$$= 1 + 2i - 2 + 2$$

$$= 1 + 2i \quad \blacksquare$$

② $h + 1 \leq n_E$ when n_E = number of external nodes
h = height of tree

Base case : a proper tree with 0 height has 1 external node
and a tree with 1 height, has to add at least
2 external nodes to 1 node in $n-1$th level
of the tree, and has 2 external nodes,

$$h = 0 \; ; \; n_E = 1$$

$$h = 1 \; ; \; n_E = (n_{E_0}) + 2 - 1 = 1 + 2 - 1 = 2$$

a tree with 2 height has 3 external nodes since it has to add ~~[illegible]~~ at least 2 external node to 1 node and that one node has become internal node.

~~[illegible]~~ $h = 2$; $n_E = 2 + \underset{\uparrow \text{new internal node}}{2} - 1$ ← ~~new~~ node become internal node

proof: if a tree with i height

$h = i$; $n_{E_i} = \cancel{[illegible]} (n_{E_{i-1}}) + 2 - 1$

$= i + 2 - 1 = i + 1$ ■

③ proof that : $h \leq n_I$ when h = height of tree

n_i = number of internal nodes

Base case : a proper tree with 0 height has 0 internal node since it has only 1 node. And a tree with 1 height has 1 internal nodes, sinc one node in 0-th level has become internal node.

$h = 0$; $n_{i_0} = 0$

$h = 1$; $n_{i_1} = (n_{i_0}) + 1 = 1$

proof : if a tree with j height

$h = j$; ~~[illegible]~~ $n_{i_j} = (n_{i_{j-1}}) + 1 = j - 1 + 1 = j$ ■

④ proof that : $h \leq \frac{n-1}{2}$ when h = height of the tree

n = numbe of nodes

assume that $2h + 1 \leq n$ by ①

and $h \leq \frac{n-1}{2}$ ■