Data Structures

Lecture 16.1: Trees (cont.)

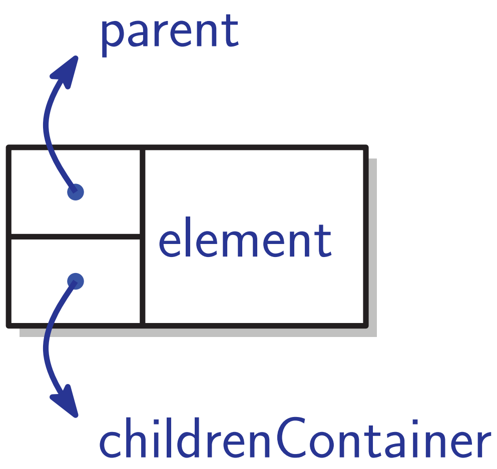
Nopadon Juneam
Department of Computer Science
Kasetsart university

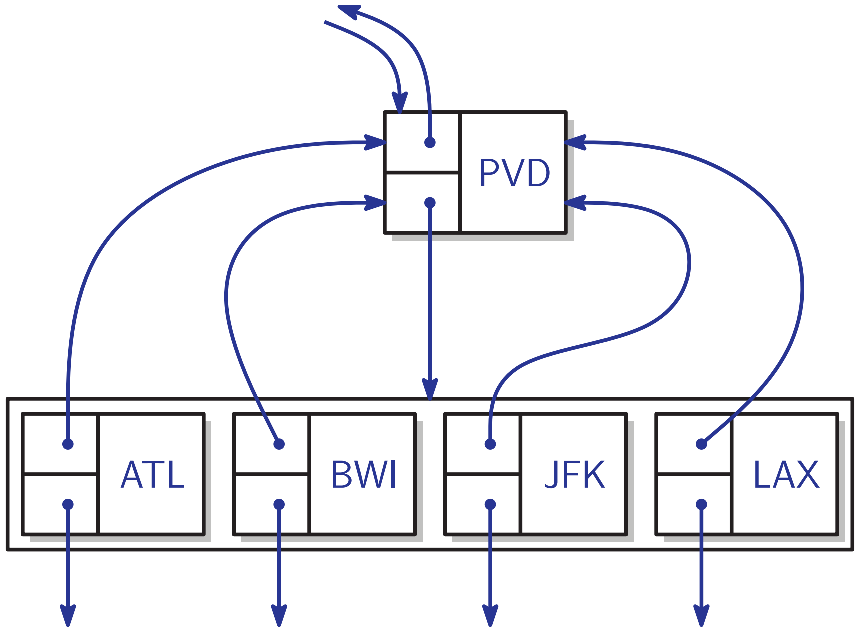
Outlines

- Ordered trees & linked structures
- Basic operations on ordered trees

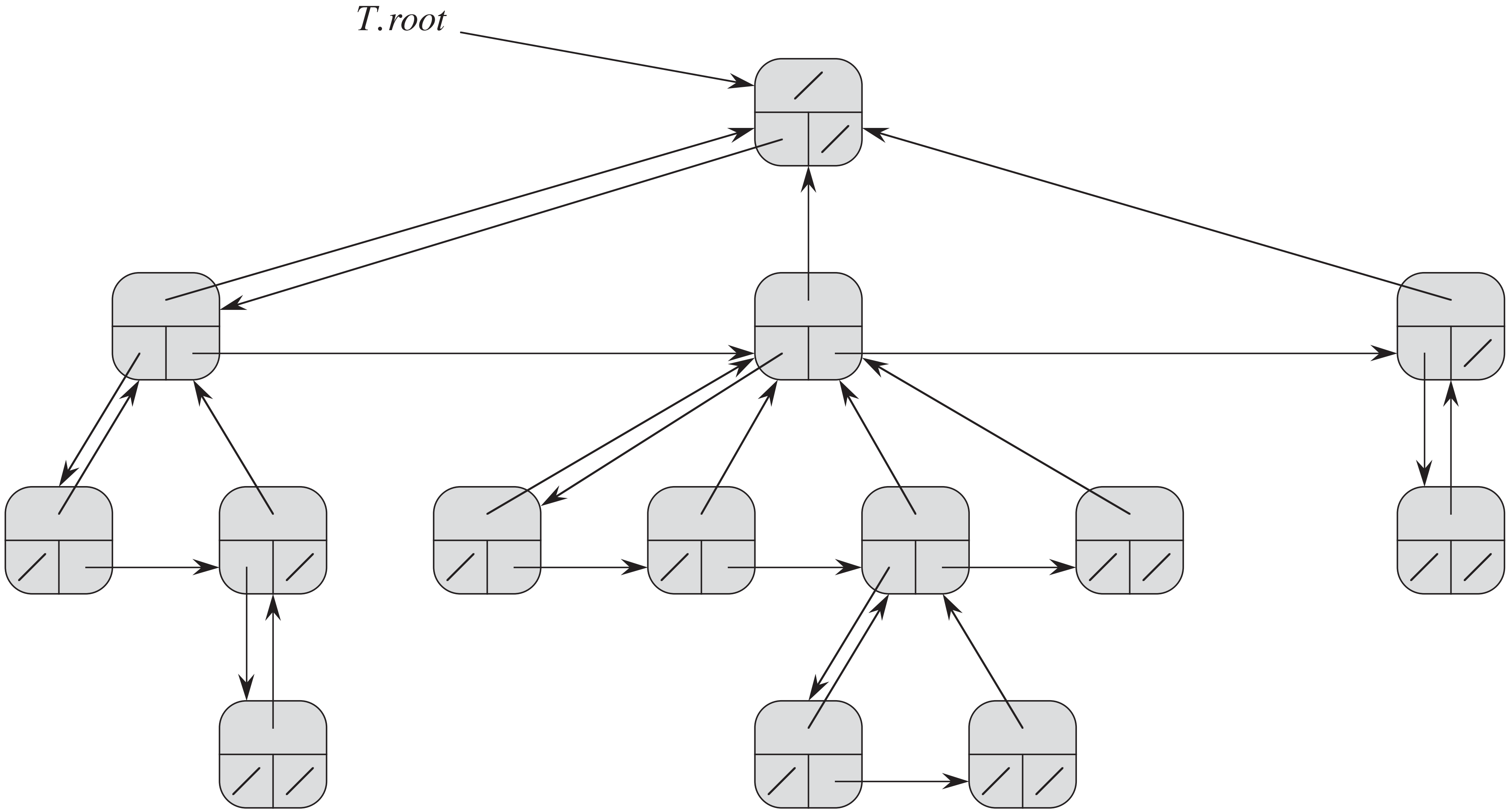
Linked Structure for General Trees

- A natural way to realize a tree *T* is to use a ***linked structure***, where we represent each node of *T* by a n object *p* with the following fields:
    - A reference to the node’s element.
    - A link to the node’s parent.
    - Some kind of collection (for example, a list or array) to store links to the node’s children.

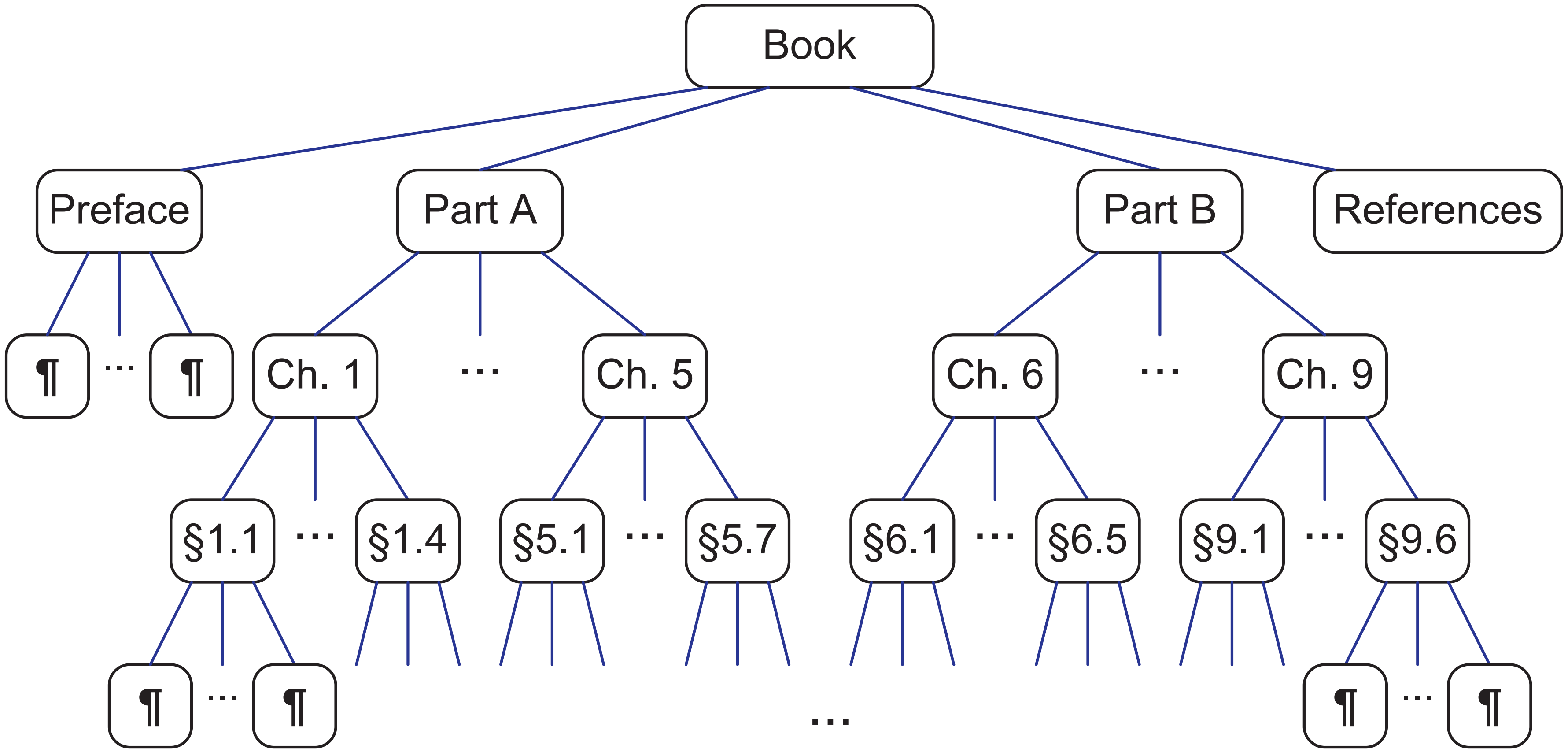




Linked Structure for Rooted Trees



Ordered Trees



- An ***ordered tree*** is a rooted tree in which the children of each node are ordered. That is, if a node has *k* children, then there is a first child, a second child, . . . , and a *k*-th child.

Basic Operations on Ordered Trees (1)

- Basic operations commonly performed a ordered tree:
  - `createRoot(r, T):` create the root *r* of the tree *T*;
  - `createNode(u, p, T):` create a node u whose parent is *p* in the tree;
  - get`Parent(u, T):` return the parent of *u* in *T*;
  - `getChild(u, k, T):` return the *k*-th child of *u* in *T*;

Basic Operations on Ordered Trees (2)

- `isRoot(u, T):` check whether a given node *u* is the root of *T*;
- `isExternal(u, T):` check whether a given node *u* is an external node (leaf) of *T*;
- `depth(u, T):` return the depth of node *u* in *T*;
- `height(T):` return the height of *T*;

Operation: createRoot

- `createRoot(r, T)`: create the root *r* of the tree *T*;

```
struct node* createRoot(int key)
{
  // Allocate memory for new node
  struct node* node = (struct node*)
                  malloc(sizeof(struct node));
  // Assign key to this node
  node->key = key;
  // Initialize parent
  node->parent = NULL;
  // Initialize left child, and right sibling as NULL

  node->leftChild = NULL;
  node->rightSibling = NULL;
  return(node);
}
```

```
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  …
  free(node1);
  return 0;
}
```

Operation: createNode

- `createNode(u, p, T)`: create a node u whose parent is *p* in the tree *T*;

```c
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```c
struct node* createNode(int key, struct node* parent)
{
  // Allocate memory for new node
  struct node* node = (struct node*)malloc(sizeof(struct node));
  // Assign key to this node
  node->key = key;
  // Initialize parent
  node->parent = parent;
  // Initialize left child, and right sibling as NULL
  node->leftChild = NULL;
  node->rightSibling = NULL;
  // Set this node as a child to its parent
  if(node->parent != NULL) {
        if(node->parent->leftChild != NULL) {
            struct node* child = node->parent->leftChild;
            while(child->rightSibling != NULL) {
                child = child->rightSibling;
            }
            child->rightSibling = node;
        }
        else {
            node->parent->leftChild = node;
        }
    }
 return node;
}
```

```c
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  struct node* node2 =createNode(2, node1);
  struct node* node3 =createNode(3, node1);
  struct node* node4 =createNode(4, node2);
  /* 4 becomes left child of 2
           1
        /       \
       2         3
     /
    4
  */
  …
  return 0;
}
```

Operations: getParent

- `getParent(u, T):` return the parent of *u* in *T*;

```
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```
struct node* getParent(struct
node* node)
{
    return node->parent;
}
```

```
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  struct node* node2 =createNode(2, node1);
  struct node* node3 =createNode(3, node1);
  struct node* node4 =createNode(4, node2);
  /* 4 becomes left child of 2
           1
       /       \
      2          3
    /
   4
  */
 printf("%d\n", getParent(node2)->key); //
output "1"
  …
 return 0;
}
```

Operations: getChild

- `getChild(u, k, T):` return the *k*-th child of *u* in *T*;

```
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```
struct node* getChild(struct node* node,
int k)
{
    struct node* child = node->leftChild;
    for(int i=1; i<k; i++) {
        child = child->rightSibling;
    }
    return child;
}
```

```
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  struct node* node2 =createNode(2, node1);
  struct node* node3 =createNode(3, node1);
  struct node* node4 =createNode(4, node2);
  /* 4 becomes left child of 2
           1
       /       \
      2          3
    /
   4
  */
 printf("%d\n", getChild(node1,2)->key); // output “3”
 …
 return 0;
}
```

Operation: isRoot

- `isRoot(u, T)`: check whether a given node *u* is the root of *T*;

```
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```
void isRoot(struct node* node)
{
    if(node->parent == NULL)
        printf("Yes\n");
    else
        printf("No\n");
}
```

```
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  struct node* node2 =createNode(2, node1);
  struct node* node3 =createNode(3, node1);
  struct node* node4 =createNode(4, node2);
  /* 4 becomes left child of 2
           1
       /       \
      2          3
    /
   4
  */
 isRoot(node1); // output "Yes"
 isRoot(node2); // output “No”
 …
 return 0;
}
```

Operation: isExternal (isInternal)

- `isExternal(u, T)`: check whether a given node *u* is an external node (leaf) of *T*;

```
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```
void isExternal(struct node*
node)
{
    if(node->leftChild == NULL)
        printf("Yes\n");
    else
        printf("No\n");
}
```

```
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  struct node* node2 =createNode(2, node1);
  struct node* node3 =createNode(3, node1);
  struct node* node4 =createNode(4, node2);
  /* 4 becomes left child of 2
           1
       /       \
      2          3
    /
   4
  */
 isExternal(node2); // output "No"
 isExternal(node3); // output “Yes"
 …
 return 0;
}
```

Operation: depth

- `depth(u, T):` return the depth of node *u* in *T*;

```
struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};
```

```
int depth(struct node* node)
{
    int depth = 0;
    while(node->parent != NULL) {
        node = node->parent;
        depth++;
    }
    return depth;
}
```

```
int main()
{
  /*create root*/
  struct node* node1 = createRoot(1);
  struct node* node2 =createNode(2, node1);
  struct node* node3 =createNode(3, node1);
  struct node* node4 =createNode(4, node2);
  /* 4 becomes left child of 2
           1
       /       \
      2          3
    /
   4
  */
 printf("%d\n", depth(node4)); // output "2"
 …
 return 0;
}
```

Complexity of Operations on Ordered Trees

Operations	Complexity
`createRoot`	$O(1)$
`createNode`	$O(k)$
`getParent`	$O(1)$
`getChild`	$O(k)$
`isRoot`	$O(1)$
`isExternal`	$O(1)$
`depth`	$O(n)$, where n is the number of nodes of a tree
`height`	$O(n)$