Data Structures

Lecture 15: Trees

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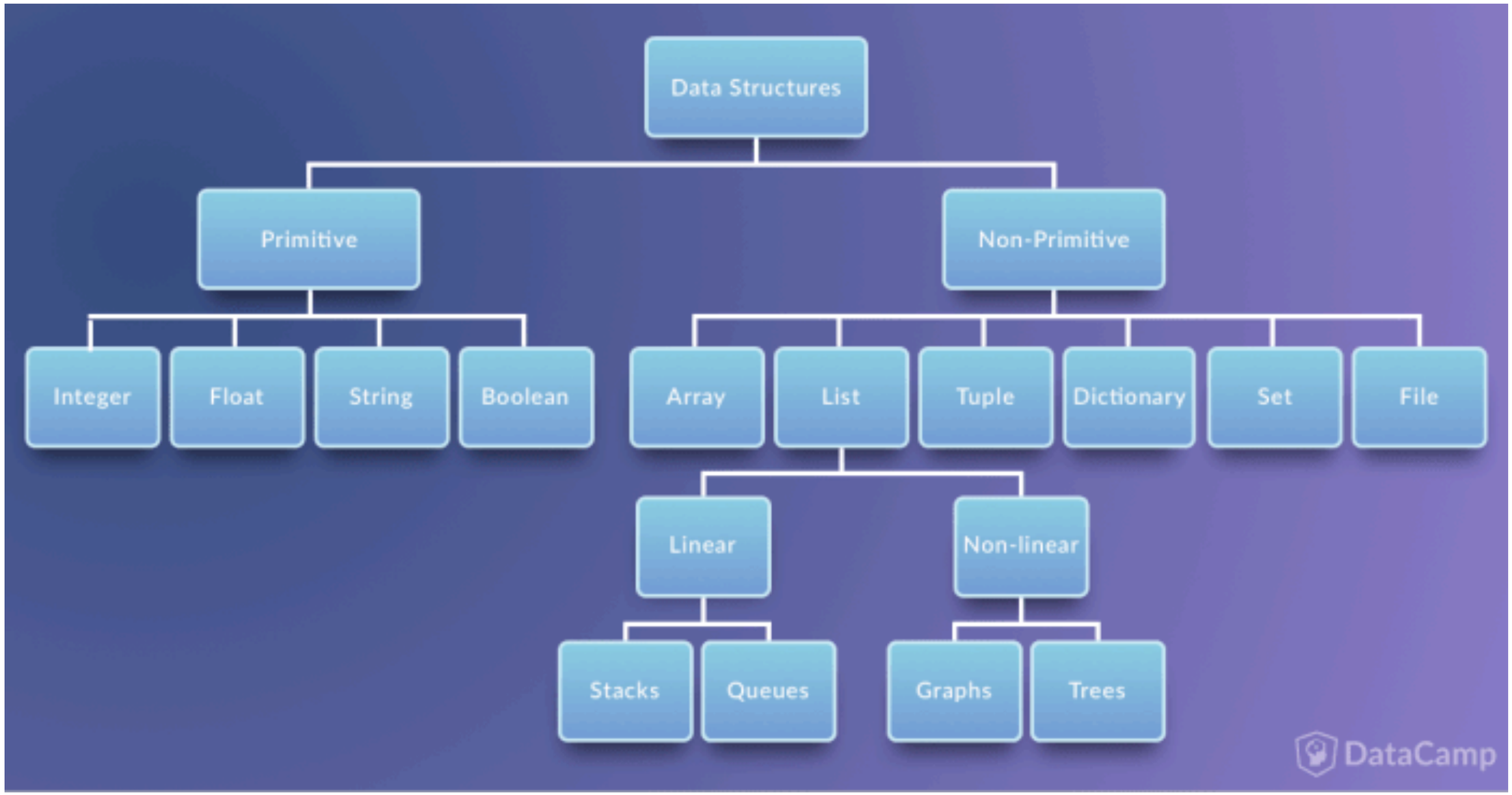
Outlines

- Trees: basic terminology and notations
  - Free Trees
  - Rooted Trees
    - Ordered Trees
- Data structures for representing trees
  - Linked structures
- Basic operation on trees
  - Create a rooted tree

Classification of Data Structures

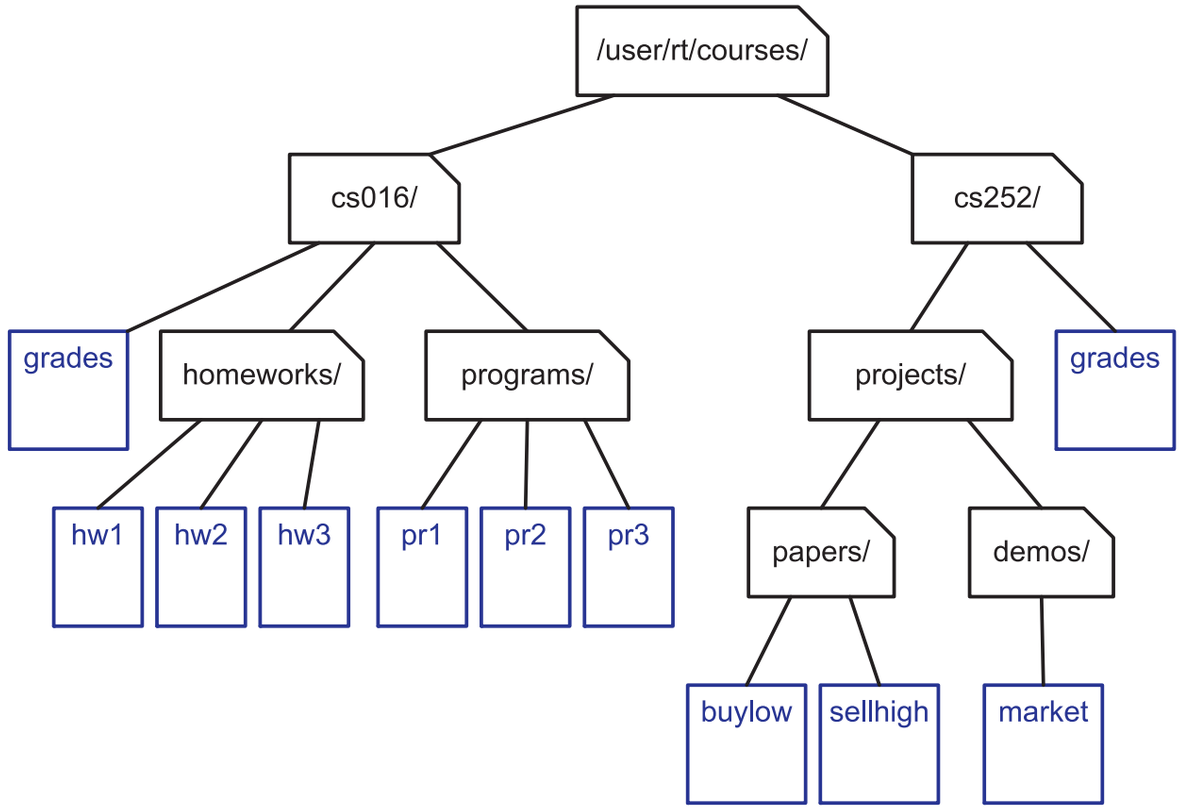


Trees: Informal Introduction (1)

- Trees are *non-linear*, but ***hierarchical*** data structure

- Trees are a breakthrough in data organization; they allow implementing a host of algorithms which are much faster than when using linear data structures

- Trees also provide a natural organization for file systems, GUI, databases, websites, etc.

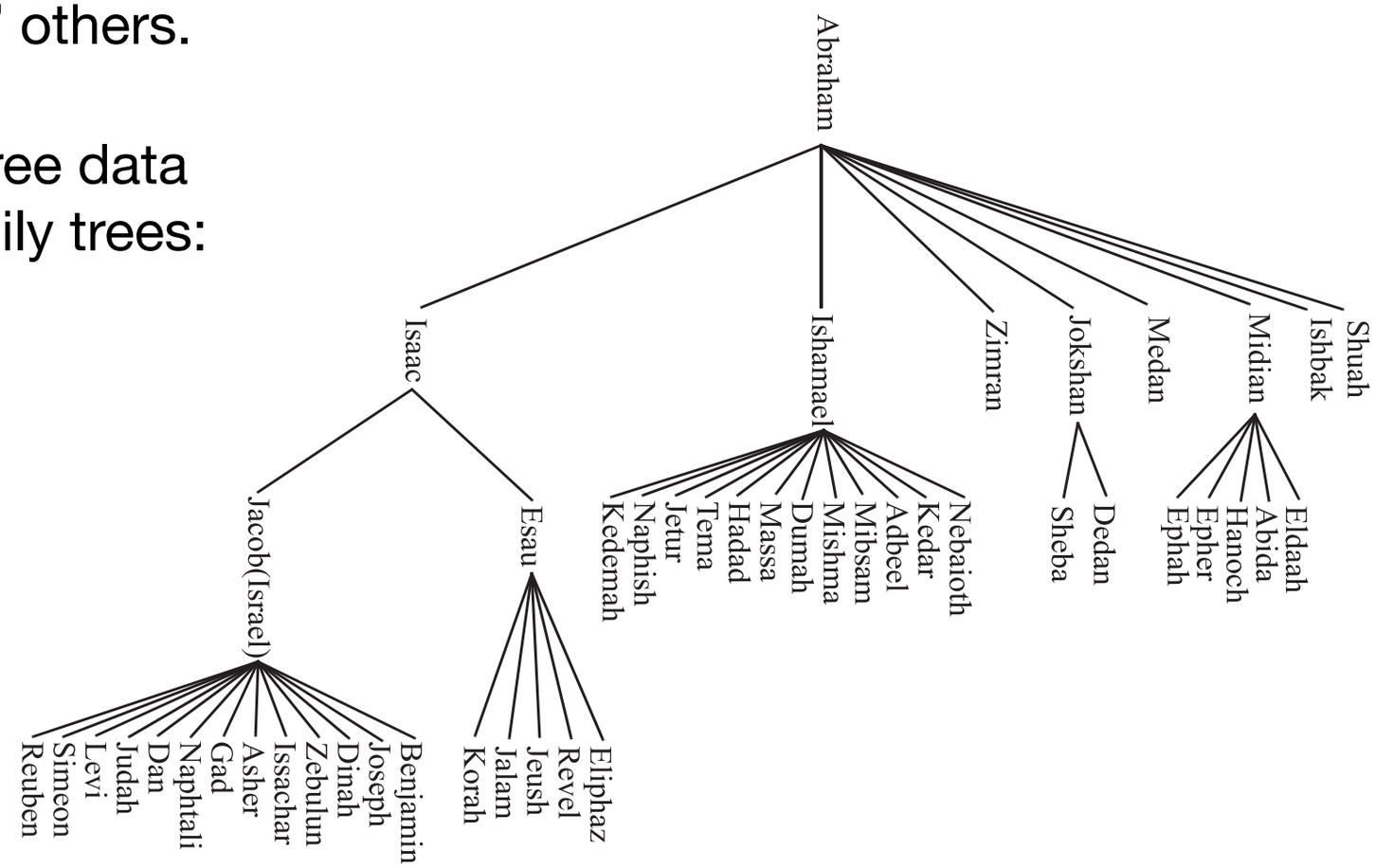


Trees: Informal Introduction (2)

- The relationships in a tree are hierarchical, with some objects being "*above*" and some "*below*" others.

- The main terminology for tree data structures comes from family trees:

  - **Parent**
  - **Child**
  - **Ancestor**
  - **Descendant**
  - **Siblings**



Trees (Free Trees)

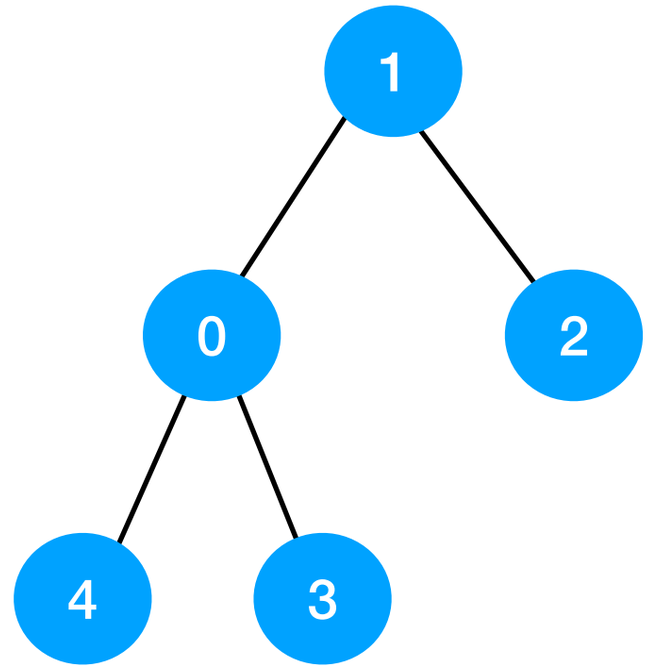
- A ***free tree*** (or ***tree***) is a connected undirected graph that has no cycle

- In the example, a free tree is given by

  - The set of vertices
    $V = \{0, 1, 2, 3, 4\}$

  - The set of edges
    $E = \{\{1,0\}, \{1,2\}, \{0,3\}, \{0,4\}\}$

- ***Remark*: Since a free tree is a graph, we can use data structures for graph to represent it.

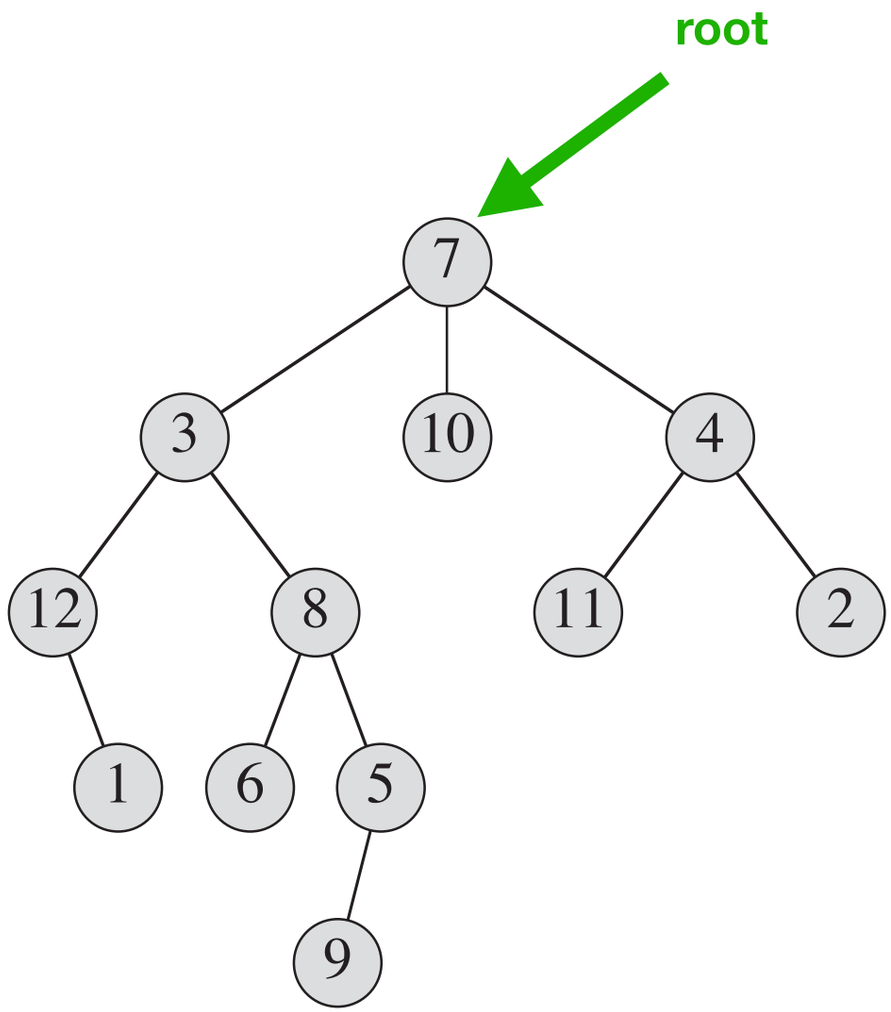


Rooted Trees

- A ***rooted tree*** is a free tree in which one of the vertices is distinguished from the others.

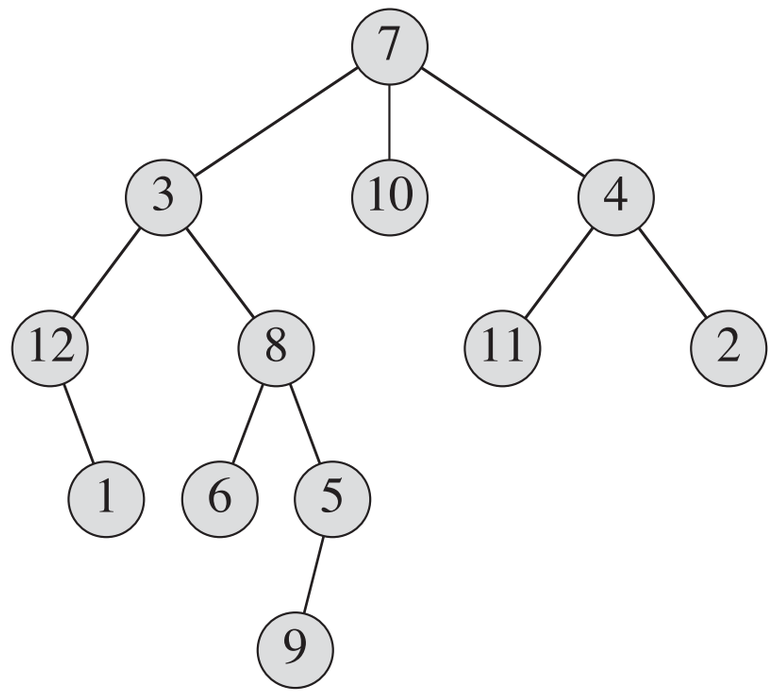
- We call the distinguished vertex the ***root*** of the tree (the top element of the tree)

- We often refer to a vertex of a rooted tree as a ***node*** of the tree.



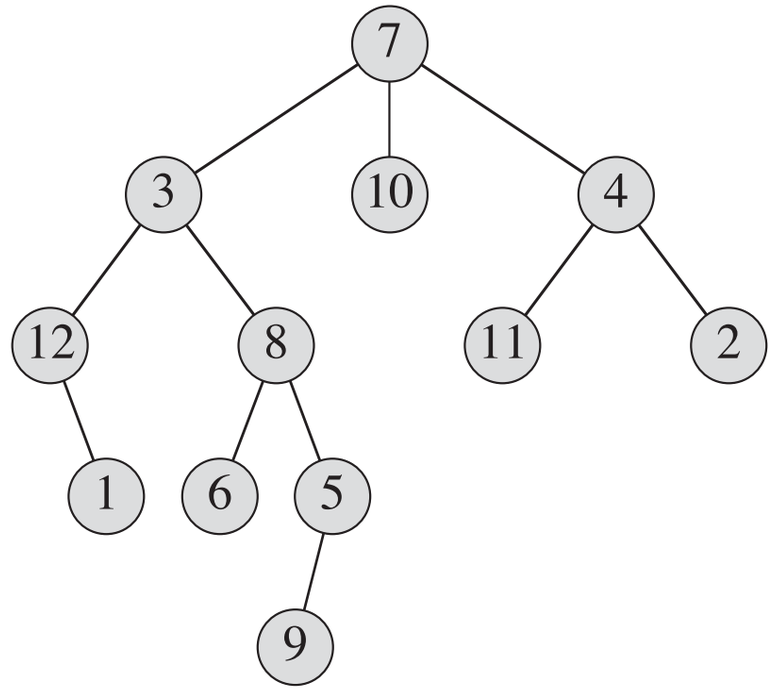
Rooted Tree Terminology (1)

- Consider a node *x* in a rooted tree *T* with root *r*:
  - We call *any* node *y* on the unique simple path from *r* to *x* an ***ancestor*** of *x*
  - If *y* is an ancestor of *x*, then *x* is a ***descendant*** of *y* (every node is both an ancestor and a descendant of itself)
  - The ***subtree rooted at x*** is the tree induced by descendants of *x*, rooted at *x*
    - For example, the subtree rooted at node 8 in the figure contains nodes 8, 6, 5, and 9



Rooted Tree Terminology (2)

- If the last edge on the simple path from the root *r* of a tree *T* to a node *x* is (*y*, *x*), then *y* is the ***parent*** of *x*, and x is a ***child*** of *y*
  - The root is the only node in *T* with no parent
- If two nodes have the same parent, they are ***siblings***
- A node with no children is a ***leaf*** or ***external node***
- A non-leaf node is an ***internal node***



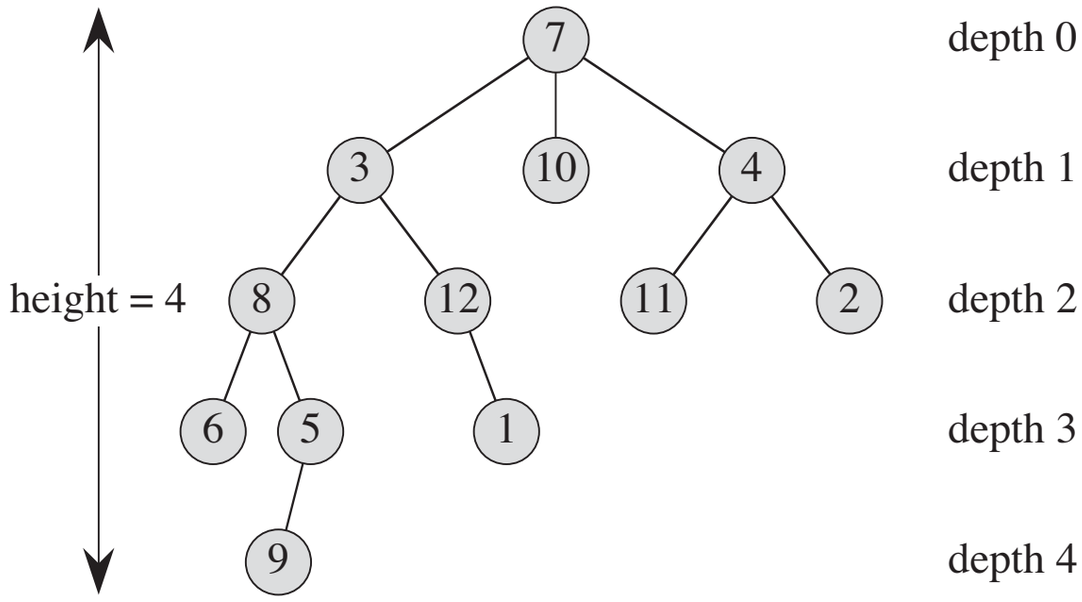
Rooted Tree Terminology (3)

- The number of children of a node *x* in a rooted tree *T* equals the ***degree*** of *x*

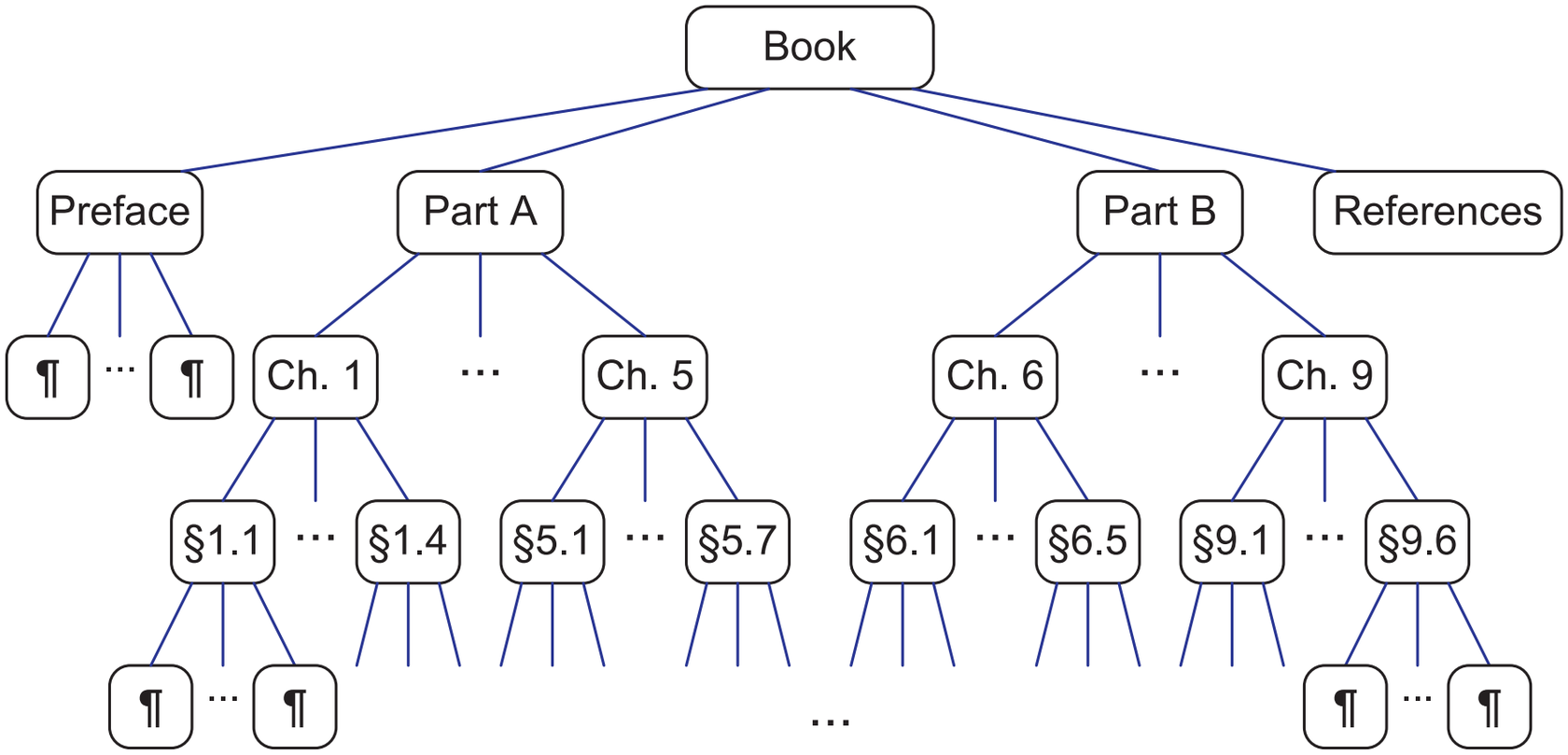
- The length of the simple path from the root *r* to a node *x* is the ***depth*** of *x* in *T*; A ***level*** of a tree consists of all nodes at the same depth

- The ***height*** of a node in a tree is the number of edges on the longest simple downward path from the node to a leaf, and the height of a tree is the height of its root

  - The height of a tree is also equal to the largest depth of any node in the tree



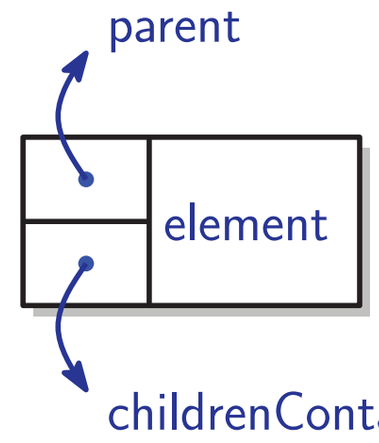
Ordered Trees



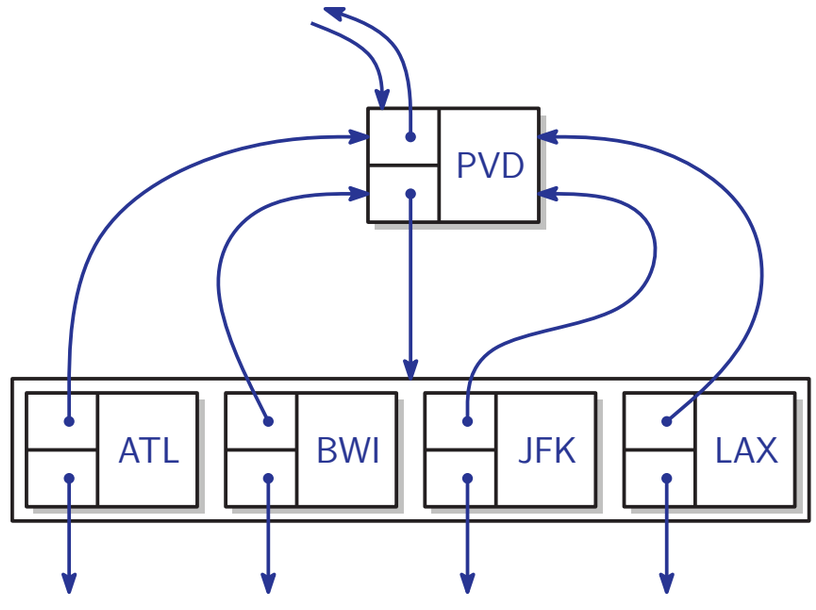
- An ***ordered tree*** is a rooted tree in which the children of each node are ordered. That is, if a node has k children, then there is a first child, a second child, . . . , and a k-th child

Linked Structure for General Trees

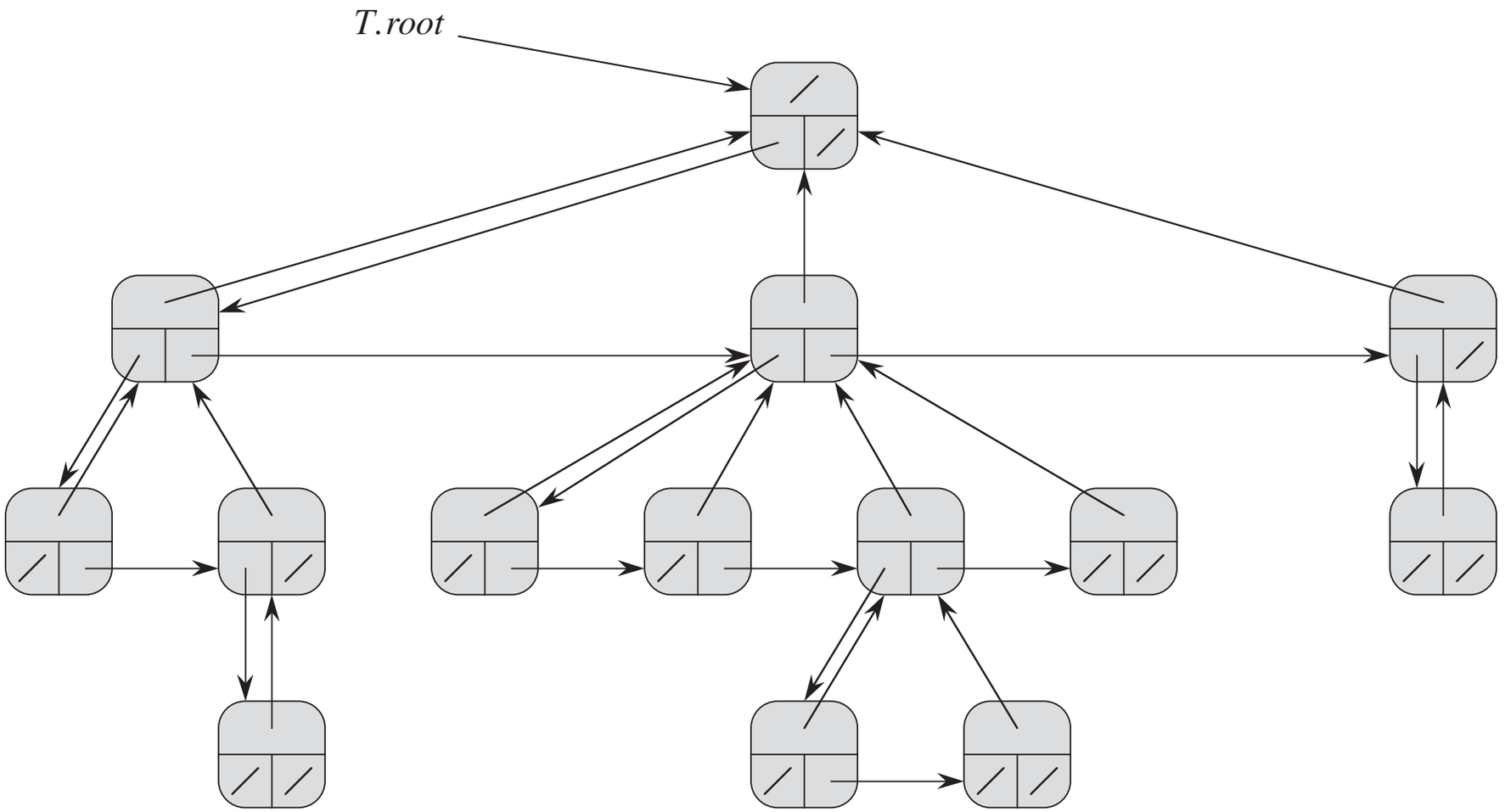
- A natural way to realize a tree T is to use a ***linked structure***, where we represent each node of T by a n object p with the following fields:



  - A reference to the node's element
  - A link to the node's parent
  - Some kind of collection (for example, a list or array) to store links to the node's children



Linked Structure for Rooted Trees



Basic Operation on Trees: Create a Rooted Tree (1)

```c
#include<stdlib.h>

struct node
{
    int key;
    struct node* parent;
    struct node* leftChild;
    struct node* rightSibling;
};

struct node* createNode(int key, struct node* parent) {
  // Allocate memory for new node
  struct node* node = (struct node*)malloc(sizeof(struct node));
  // Assign key to this node
  node->key = key;
  // Initialize parent
  node->parent = parent;
  // Initialize left child, and right sibling as NULL
  node->leftChild = NULL;
  node->rightSibling = NULL;
  // Set this node as a child to its parent
  if(node->parent != NULL) {
        if(node->parent->leftChild != NULL) {
            struct node* child = node->parent->leftChild;
            while(child->rightSibling != NULL) {
                child = child->rightSibling;
            }
            child->rightSibling = node;
        }
        else {
            node->parent->leftChild = node;
        }
    }
 return node;
}
```

Basic Operation on Trees: Create a Rooted Tree (2)

```
int main()
{
  /*create root*/
  struct node *root = createNode(1, NULL);
  /* following is the tree after the above statement

        1
  */

  createNode(2, root);
  createNode(3, root);
  /* 2 and 3 become children of 1
           1
         /   \
        2     3
  */

  createNode(4, root->leftChild);
  /* 4 becomes left child of 2
           1
         /   \
        2     3
       /
      4
  */
  …
  return 0;
}
```