Math 151 Final Exam Review Problems
Coverage: Cumulative
Math Lab may help!

1. *(2.2)* Find each limit, if it exists. Don't use a table.

   a) $\displaystyle\lim_{x\to 3^+} \frac{2}{\ln(x-3)}$ b) $\displaystyle\lim_{x\to -3^-} \frac{x}{\sqrt{x^2-9}}$

   c) $\displaystyle\lim_{x\to 5} \frac{x+1}{x-5}$ d) $\displaystyle\lim_{x\to 5} \frac{x+1}{(x-5)^2}$

   Note: Most of these problems involve limits that equal to a number over 0 (after direct substitution).

2. *(2.3, 2.6)* Find each limit, if it exists. Don't use a table.

   a) $\displaystyle\lim_{x\to -2} \frac{x-2}{x^2+4}$ b) $\displaystyle\lim_{x\to 0^+} \frac{e^{2x}-1}{e^x-1}$

   c) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$ d) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{5x^6-2}}{x-x^3}$

   e) $\displaystyle\lim_{x\to\infty} [\ln(2+x)-\ln(1+3x)]$

   Note: Most of these problems involve limits that equal to 0/0 or ∞/∞. So l'Hospital's Rule can also be used.

3. *(4.4)* Find each limit using l'Hospital's Rule.

   a) $\displaystyle\lim_{x\to -\infty} xe^{-2x}$ b) $\displaystyle\lim_{x\to 0^+} \sqrt{x}\ln x$

4. *(2.4)* Find the limit, then use the $\epsilon-\delta$ definition to prove the limit.

   $$\lim_{x\to -3}\left(-\frac{2}{3}x+7\right)$$

5. *(2.5)* Determine the intervals/points on which the function **is not** continuous. Explain why. Also determine the intervals on which the function **is** continuous.

   a) $f(x) = \dfrac{x-1}{x\sqrt{x+1}}$

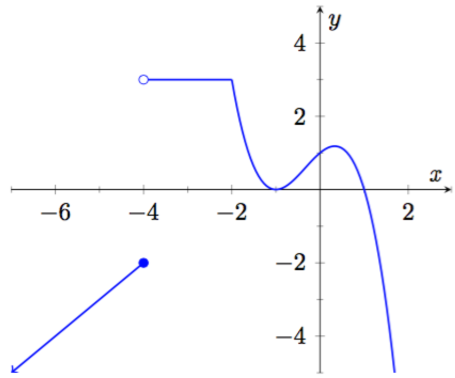
   b) $f(x) = \begin{cases} \dfrac{3x^2-x-2}{x^2-1}, & x\le 1 \\ 0, & x>1 \end{cases}$

6. *(2.5)* Use the Intermediate Value Theorem to prove or disprove that the function $\dfrac{5}{x} = \cos\left(\dfrac{\pi x}{4}\right)$ has a root in the interval $[6,8]$.

7. *(2.7,2.8)* Find the derivative using the limit definition. Do not use any differentiation rules.

   $$f(x) = \frac{1}{x+2}$$

8. *(2.8)* The graph of f is given. Sketch the graph of its derivative f'.



9. *(3.1-3.6)* Differentiate.

   a) $r(x) = \cos^2(x/2)+\cos(x^2/2)$

   b) $f(x) = \dfrac{x\sqrt{4x^2+7}}{x+4}$

   c) $h(x) = 5^x + x^5 + \log_5 x + \log_5 \pi$

   d) $g(t) = \dfrac{1+\sin t}{1-\tan t}$

   e) $y = x\arccos(2x) - 2\sqrt{1-x^2}$

10. *(3.1)* Find an equation of the tangent line to the curve $y = x^4+1$ that is perpendicular to the line $x-32y=15$.

11. *(3.1-3.5)* Find an equation of the tangent line to the graph of $f(x)=\arctan(x^2)$ at $x=-1$.

12. *(3.5)* Use implicit differentiation to find dy/dx:

    $$4xy+\ln(x^2y)=7$$

13. *(3.6)* Use logarithmic differentiation to find dy/dx:

    $$y=(\ln x)^{\ln x}$$

14. *(3.9)* A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 5 cm deep?

15. *(3.10)* The radius of a sphere is measured as 9 cm, with a possible error of ± 0.025 cm. Use differentials to approximate the possible propagated error in computing the surface area of the sphere.

16. *(3.10)* Approximate $\sqrt[3]{1.03}$

    (a) Using a linear approximation.

    (b) Using a differential.

17. *(4.1)* Find the absolute extrema of the function on the given closed interval.

    $$f(x)=\sqrt{x}(x-3), \quad [0,4]$$

18. *(4.2)* Determine whether the Mean Value Theorem can be applied for f on the closed interval $[a,b]$. If the Mean Value Theorem can be applied, find all values of c in the open interval (a,b) guaranteed by the theorem.

$$f(x) = x\ln x, \quad [1,2]$$

19. *(4.3)* Find all open intervals on which the function is increasing or decreasing, then determine all points of relative extrema.

    (a) $f(x) = 2^x(2-x)$

    (b) $g(x) = \cos^2 2x, \quad 0 < x < 2\pi$

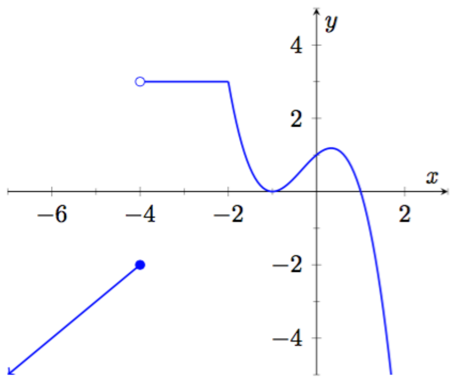
20. *(4.3)* Find all open intervals on which the function has upward concavity or downward concavity, then find the points of inflection (if any).

$$f(x) = x + \cos x, \quad [0, 2\pi]$$

21. *(4.3)* Use the Second Derivative Test to find all relative extrema when applicable.

$$f(x) = 2x + \frac{18}{x}$$

22. *(4.3)* The graph of f' is given.



    (a) Find all critical numbers. Explain.

    (b) On what intervals is f increasing? Decreasing?

    (c) At what values of x does f have a local maximum? Local minimum?

    (d) On what intervals is f concave upward? Concave downward?

    (e) State the x-coordinate(s) of the point(s) of inflection.

23. *(4.5)* Analyze and sketch the graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes.

    a) $f(x) = 4x^3 - x^4$ b) $f(x) = \dfrac{10 - x - 3x^2}{x^2 - 4}$

24. *(Various sections)* Sketch the graph of a function that satisfies the given conditions:

$f(0) = 0,\ f'(-2) = f'(1) = f'(9) = 0,$
$\lim_{x\to\infty} f(x) = 0,\ \lim_{x\to 6} f(x) = -\infty,$
$f'(x) < 0$ on $(-\infty,-2)$, $(1,6)$, and $(9,\infty)$,
$f'(x) > 0$ on $(-2,1)$ and $(6,9)$,
$f''(x) > 0$ on $(-\infty,0)$ and $(12,\infty)$,
$f''(x) < 0$ on $(0,6)$ and $(6,12)$

25. *(4.7)* A rectangular piece of paper with perimeter 100 cm is to be rolled to form a cylindrical tube. Find the dimensions of the paper that will produce a tube with maximum volume.

26. *(4.9)* Find f that satisfies the given conditions.

    a) $f''(x) = 1 - 6x + 48x^2, \quad f(0) = 1, \quad f'(0) = 2$

    b) $f''(x) = x^{-2}, \quad x > 0, \quad f(1) = 0, \quad f(2) = 0$

27. *(App E)* Find the sum: $\displaystyle\sum_{i=5}^{200} i(i+1)(i+2)$

28. *(5.1)* Use the right endpoints and $n = 5$ to approximate the integral.

$$\int_1^3 e^{x^2}\,dx$$

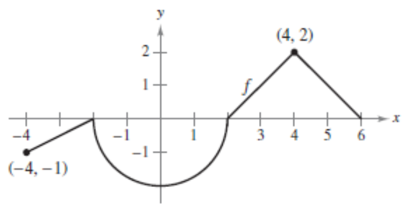
(Also review the left endpoints.)

29. *(5.2)* Sketch the region bounded by the graph of $y = 5 - x^2$ and the x-axis over the interval $[-2, 1]$.

    (a) Use the limit process to find the area of the region.

    (b) Verify your answer in part (a) using FTC2.

30. *(5.2)* The graph of f is below. Use it to determine $\displaystyle\int_{-4}^{4} f(x)dx$.



31. *(5.3)* Find the derivative of the function using FTC1.

    a) $f(x) = \displaystyle\int_{x^2}^{1} \frac{1}{t^3+1}\,dt$

    b) $g(x) = \displaystyle\int_{2^x}^{\pi x} \cos(t^2+1)\,dt$

32. *(5.3,5.5)* Use FTC2 to evaluate the definite integral.

    (a) $\displaystyle\int_0^1 6x^3\sqrt{x^4+3}\,dx$

    (b) $\displaystyle\int_{e^2}^{e^4} \frac{1}{x\ln\sqrt[3]{x}}\,dx$

33. *(4.9,5.4,5.5)* Evaluate.

a) $\displaystyle\int \frac{x^4+8}{x^3}\,dx$ b) $\displaystyle\int \sin^3 x \cos x\,dx$

c) $\displaystyle\int x^2\sqrt{x+1}\,dx$ d) $\displaystyle\int \frac{\sin x}{1+\cos x}\,dx$

e) $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$ f) $\displaystyle\int \frac{x-1}{3x^2+2x-5}\,dx$

g) $\displaystyle\int \frac{1-2x}{\sqrt{1-x^2}}\,dx$ h) $\displaystyle\int \frac{1}{x(\ln x)^2}\,dx$

(Look over other exercises in these sections as well.)

ANSWERS

1. (a) 0
   (b) $-\infty$
   (c) DNE
   (d) ∞

2. (a) $-\frac{1}{2}$
   (b) 2
   (c) $\frac{1}{2\sqrt{5}}$
   (d) $\sqrt{5}$
   (e) $-\ln 3$

3. (a) $\frac{1}{2}$
   (b) 0

4. $L = 9$, $\delta = 3\epsilon/2$

5. (a) Discont. on $(-\infty, -1]$ and $x = 0$. Not in domain. Continuous on $(-1, 0) \cup (0, \infty)$.
   (b) Discont. at $x = -1$ (not in domain) and at $x = 1$ (definition not satisfied). Continuous on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

6. Consider $f(x) = -\dfrac{5}{x} + \cos\left(\dfrac{\pi x}{4}\right)$. Domain of f is $\mathbb{R} - \{0\}$, so f is continuous on $[6, 8]$. Since $f(4) = -5/6$ and $f(4) = 3/8$ and $f(4) \le 0 \le f(6)$, by IVT, $\exists\, c$ between 6 and 8 such that $f(c) = 0$. That is $-\dfrac{5}{c} + \cos\left(\dfrac{\pi c}{4}\right) = 0$ for some $6 \le c \le 8$. In other words, c is a root of the equation $\dfrac{5}{x} = \cos\left(\dfrac{\pi x}{4}\right)$.

7. $f'(x) = \lim\limits_{h \to 0} \dfrac{\dfrac{1}{x+h+2} - \dfrac{1}{x+2}}{h} = -\dfrac{1}{(x+2)^2}$

8. Left to you to explore.

9. (a) $r'(x) = -\cos(x/2)\sin(x/2) - x\sin(x^2/2)$
   $= -\dfrac{\sin x}{2} - x\sin(x^2/2)$
   (b) $f'(x) = \left(\dfrac{1}{x} + \dfrac{4x}{4x^2+7} - \dfrac{1}{x+4}\right)\dfrac{x\sqrt{4x^2+7}}{x+4}$
   (c) $h'(x) = \ln 5 \cdot 5^x + 5x^4 + \dfrac{\log_5 x}{\ln 5}$
   (d) $g'(t) = \dfrac{\cos t - \sin t + \sec^2 t + \sec^2 t \sin t}{(1 - \tan t)^2}$
   (e) $y' = \arccos(2x) - \dfrac{2x}{\sqrt{1-4x^2}} + \dfrac{2x}{\sqrt{1-x^2}}$

10. $y - 17 = -32(x + 2)$

11. $y - \dfrac{\pi}{4} = -(x + 1)$

12. $\dfrac{dy}{dx} = -\dfrac{4xy^2 + 2y}{x(4xy + 1)}$

13. $y' = (\ln x)^{\ln x}\left(\dfrac{1 + \ln(\ln x)}{x}\right)$

14. $\dfrac{8}{9\pi}$ cm/s

15. $dV = 8\pi(9)(\pm 0.025) = \pm 1.8$ cm^2

16. $\dfrac{101}{100}$

17. Absolute min $= -2$ (at $x = 1$);
    Absolute max $= 2$ (at $x = 4$)

18. **1.** Domain of f: $(0, \infty)$, so f is continuous on $[1, 2]$.
    2. $f'(x) = 1 + \ln x$ with domain $(0, \infty)$. So f' is continuous on (1,2). So f is differentiable on $(1, 2)$.
    3. MVT is applicable and $c = \dfrac{4}{e}$

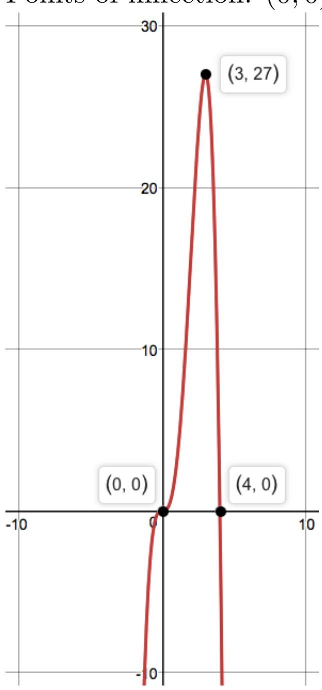
19. (a) Inc on $\left(-\infty, 2 - \dfrac{1}{\ln 2}\right)$
    Dec on $\left(2 - \dfrac{1}{\ln 2}, \infty\right)$
    Point of relative max: $\left(2 - \dfrac{1}{\ln 2}, \dfrac{2^{2-1/\ln 2}}{\ln 2}\right)$
    (b) Inc on $(\pi/4, \pi/2) \cup (3\pi/4, \pi) \cup (5\pi/4, 3\pi/2) \cup (7\pi/4, 2\pi)$
    Dec on $(0, \pi/4) \cup (\pi/2, 3\pi/4) \cup (\pi, 5\pi/4) \cup (3\pi/2, 7\pi/4)$
    Rel max: $(\pi/2, 1)$, $(\pi, 1)$, $(3\pi/2, 1)$
    Rel min: $(\pi/4, 0)$, $(3\pi/4, 0)$, $(5\pi/4, 0)$, $(7\pi/4, 0)$

20. Upward concavity on $(\pi/2, 3\pi/2)$
    Downward concavity on $(0, \pi/2) \cup (3\pi/2, 2\pi)$
    Points of inflection: $(\pi/2, \pi/2)$ and $(3\pi/2, 3\pi/2)$

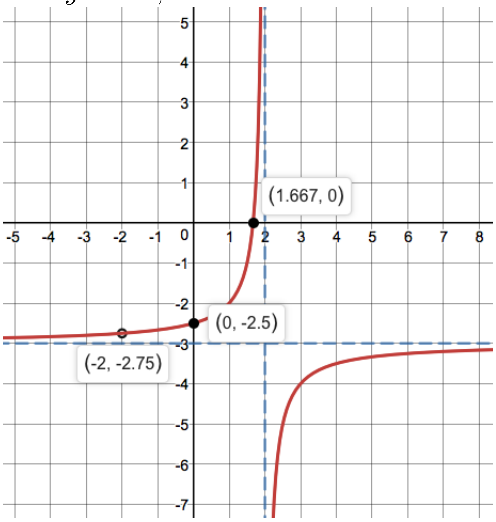
21. Point of relative max $= (-3, -12)$ $(f''(-3) < 0)$
    Point of relative min $= (3, 12)$ $(f''(3) > 0)$

22. (a) $x = -4, -1, 1$
    (b) Inc on $(-4, 1)$; Dec on $(-\infty, -4) \cup (1, \infty)$
    (c) Local max at $x = 1$; No local min
    (d) CU on $(-\infty, -4) \cup (-1, 0.4)$; CD on $(-2, -1) \cup (0.4, \infty)$
    (e) POI at $x = -1, 0.4$

23. (a) x-int: $(0, 0)$, $(4, 0)$; y-int: $(0, 0)$
    Point of rel. max: $(3, 27)$;
    Point of rel. min: None
    Points of inflection: $(0, 0)$ and $(2, 16)$



(b) x-int: $(5/3, 0)$; y-int: $(0, -5/2)$
Points of relative extrema: None
Points of inflection: None
HA: $y = -3$; VA: $x = 2$



24. Answer may vary

25. $100/3$ cm by $50/3$ cm

26. (a) $f(x) = \dfrac{x^2}{2} - x^3 + 4x^4 + 2x + 1$

    (b) $f(x) = -\ln x + \ln 2 \cdot x - \ln 2$

27. 412110090

28. $\dfrac{2}{5}\left\{e^{(7/5)^2} + e^{(9/5)^2} + e^{(11/5)^2} + e^{(13/5)^2} + e^{9}\right\}$

29. (a) Area $= \displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(1 + \frac{12i}{n} + \frac{9i^2}{n^2}\right)\frac{3}{n}\right] = 12$

    (b) 12

30. $1 - 2\pi$

31. (a) $f'(x) = -\dfrac{2x}{x^6+1}$

    (b) $g'(x) = \pi\cos(\pi^2 x^2 + 1) - \cos(4^x + 1) \cdot 2^x \cdot \ln 2$

32. (a) $8 - 3\sqrt{3}$

    (b) $3\ln 2$

33. (a) $\dfrac{x^2}{2} - \dfrac{4}{x^2} + C$

    (b) $\dfrac{\sin^4 x}{4} + C$

    (c) $\dfrac{2}{7}(x+1)^{7/2} - \dfrac{4}{5}(x+1)^{5/2} + \dfrac{2}{3}(x+1)^{3/2} + C$

    (d) $-\ln|1 + \cos x| + C$

    (e) $-e^{1/x} + C$

    (f) $\dfrac{1}{3}\ln|3x + 5| + C$

    (g) $\arcsin x + 2\sqrt{1 - x^2}$

    (h) $-\dfrac{1}{\ln x} + C$