Problem Set 7
Math 146 Spring 2023
Due: Friday, May 5, 11:59 PM

Note: All are Matlab problems that require code submissions. All answers should be submitted in a write-up.

1. (50 points)

   (a) Use Matlab to find the singular value decomposition of the matrix A given below.

   $$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

   (b) Write code to make a set of points giving a unit circle S (Hint: Use a common parametrization of a circle, given in terms of the angle θ, and then pick a list of θ values that are equally spaced. Plot S.

   (c) Apply the matrix A to S to get the resulting ellipse, and plot it.

   (d) Does your image match what you saw in the SVD? Explain.

2. (75 points) Take the almost singular linear system $A\mathbf{x} = \mathbf{b}$ given below.

   $$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & 8 & 12 & 2 \\ 1 & 2 & 3 + 10^{-15} & 4 \\ 2 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 66 \\ 30 \\ 7 \end{bmatrix}$$

   (a) Use Matlab's backslash to estimate a solution to this system. What do you expect the size of the relative error to be? Based on this relative error, what part of your solution can you trust?

   (b) Use Matlab to find the SVD of the matrix A. At what singular value do you think you should truncate? Explain. What will be the resulting rank r of your matrix A_r? What is the condition number of the resulting A_r (Hint: You do not need to form A_r to answer this.)

   (c) Write Matlab code to find the solution to the now over-determined problem resulting from the rank-r approximation of A. Use `format long` to see more digits of your solution.

   (d) Does your solution satisfy $A\mathbf{x} = \mathbf{b}$? Explain. If it does not satisfy the solution, what is the residual? Why would we choose to "solve" this system this way?

3. (75 points)

   (a) Load the two pictures from Matlab's repository of images, titled "mandrill.mat" and "durer.mat". Use `colormap(gray)` and `imagesc` for all processes in this problem. What are the sizes and ranks of the associated matrices (X)? Write a Matlab script to computing the truncated SVD. For both pictures, examine a range of rank values at which to truncate, r, starting with 2 and going up by powers of 2 to 256. Provide the output. (You may want to use `subplot` to help with the presentation.)

(b) Comment on the performance of the truncated SVD for each image. Explain the difference in the effectiveness of the technique for the two images for small r.

(c) State how much storage is required as a function of r and how much storage is required for the original image.