Problem Set 3
Math 146 Spring 2023
Due Tuesday, February 21, 11:59 PM

Note: Number 2 is a "pen and paper" problem for which you should show work, where appropriate. Numbers 1 & 3 are Matlab problems that require code submissions. All answers should be submitted in a write-up.

1. (30 points)
   (a) Write a Matlab function for solve $A\mathbf{x} = \mathbf{b}$ by using Gaussian Elimination without pivoting, and backward/forward substitution. Your function should take as inputs: square matrix A and vector $\mathbf{b}$. Its output should be the solution $\mathbf{x}$. Hint: Within this function, you can call other functions saved in your folder. Use the functions you made for Problem Seet 2, problems 5-7.
   (b) Validate your code by forming a 100×100 matrix for A made up of random numbers between 0 and 50 and a similarly formed vector for b. Solve for $\mathbf{x}$ using both your function and the Matlab backslash and find the relative difference between the solutions, relative to the max norm. Comment on your results, using the comments from Problem Set 2, Number 5 to help guide your analysis.

2. (35 points) Solve the system

$$\begin{bmatrix} 0.00005 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \tag{1}$$

   step-by-step, with and without partial pivoting, assuming that, at ever step, you only store 4 digits. Compare the results.

3. (a) (40 points) Write a Matlab function to use Gaussian Elimination with partial pivoting to perform LU decomposition for a matrix A. Your function should take as input A, an $n \times n$ matrix. Its output should be the lower triangular and upper triangular matrices, L and U, and the permutation matrix, P. Validate your code. Explain how you validated it and give any relevant output.
   (b) (25 points) Write a Matlab function to solve $A\mathbf{x} = \mathbf{b}$ by using Gaussian Elimination with partial pivoting, and backward/forward substitution. Your function should take as inputs: square matrix A and vector $\mathbf{b}$. Its output should be the solution $\mathbf{x}$. Validate your code. Explain how you validated it and give any relevant output.
   (c) (30 points) Use your code from part (b) to solve the linear system $A\mathbf{x} = \mathbf{b}$ for `A= [20:2:36; 100:2:116; -20*ones(1,9); linspace(0.001, 0.005,9);21:2:37; 23:2:39; 24:2:40 ; linspace(-0.6, 0,9); linspace(-17,20,9) ]` and `b=[1380; 4980; -900; 0.165; 1425; 1515; 1560; -9; 345]`. The exact solution to this is $x_{exact} = [1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9]^{\mathsf{T}}$. All with respect to the 2-norm, find the relative error

for your solution, the relative error for the Matlab solution found by using backslash, and the relative difference between your solution and the Matlab solution. Comment on your results.

(d) (40 points) Perform a computational experiment in which in a single trial, you generate a random square matrix of a given size. Generate a random vector $\mathbf{z}$ and compute $\mathbf{b} = A\mathbf{z}$. Then use your code in part (b) to solve the linear system $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$. Compute the relative error, $||\mathbf{x} - \mathbf{z}||/||\mathbf{z}||$, with respect to the 2 norm. Also compute the condition number of the matrix. Do this for matrices of various sizes from $n = 5$ to 500. Make a scatter plot of the relative error vs. the condition number on a log-log plot. (Matlab command `loglog`). Look at the scaling between the condition number and the relative error. Does the scaling depend on the matrix size? Do the results match your expectation? Discuss.