

# Automatic Harmonic Analysis of Classical String Quartets From Symbolic Score

Néstor Nápoles López

Supervisors: Xavier Serra / Rafael Caro







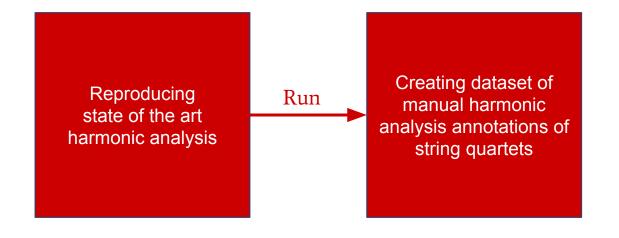
Reproducing state of the art harmonic analysis



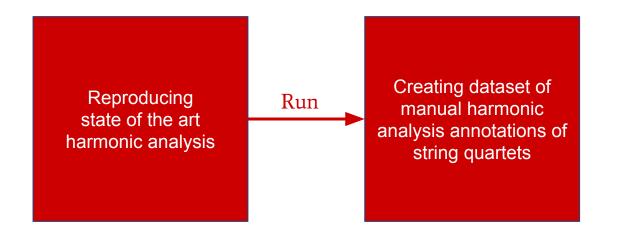
Reproducing state of the art harmonic analysis

Creating dataset of manual harmonic analysis annotations of string quartets



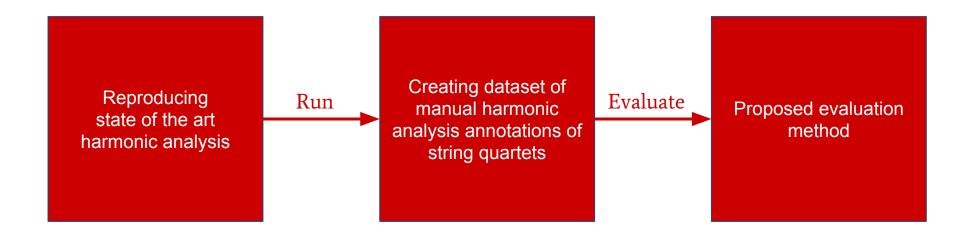






Proposed evaluation method







# Structure of the presentation

- 1. Automatic Harmonic Analysis
- 2. The dataset: Six string quartets Op.20, Joseph Haydn
- 3. Running the analysis
- 4. Evaluation
- 5. Results
- 6. Contributions



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#### **Analysis**

Interpreting the structures in music,

resolving into relatively simpler elements,

investigating the relevant functions of those elements.

Ian D. Bent and Anthony Pople. "Analysis." Grove Music Online. Oxford Music Online. Oxford University Press. Web. 28 Jun. 2017. <a href="http://www.oxfordmusiconline.com/subscriber/article/grove/music/41862">http://www.oxfordmusiconline.com/subscriber/article/grove/music/41862</a>>



#### Harmony

The simultaneous sounding of notes.

"Harmony." The Oxford Dictionary of Music, 2nd ed. rev. Ed. Michael Kennedy. Oxford Music Online. Oxford University Press. Web. 28 Jun. 2017. <a href="http://www.oxfordmusiconline.com/subscriber/article/opr/t237/e4724">http://www.oxfordmusiconline.com/subscriber/article/opr/t237/e4724</a>>.



#### Harmonic analysis

Interpreting the **harmonic** structures in music,

resolving into relatively simpler elements, i.e., labels,

investigating the relevant functions of those elements.

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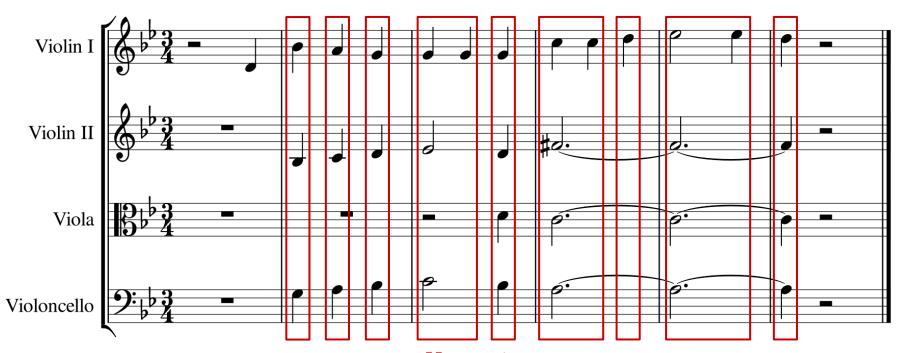












Harmonic structures



#### Harmonic analysis

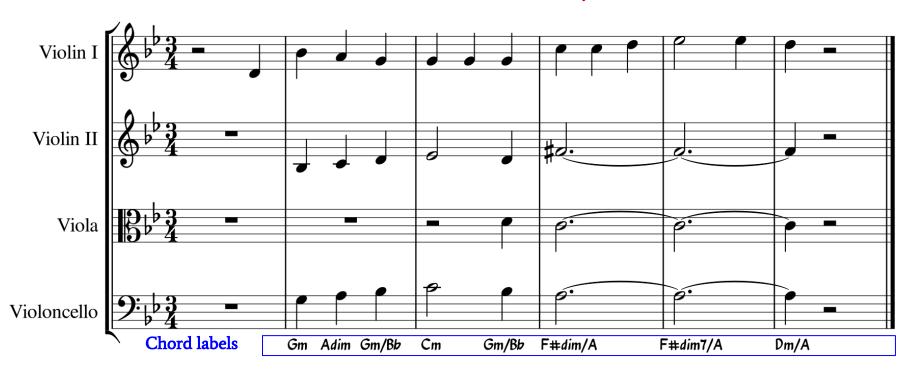
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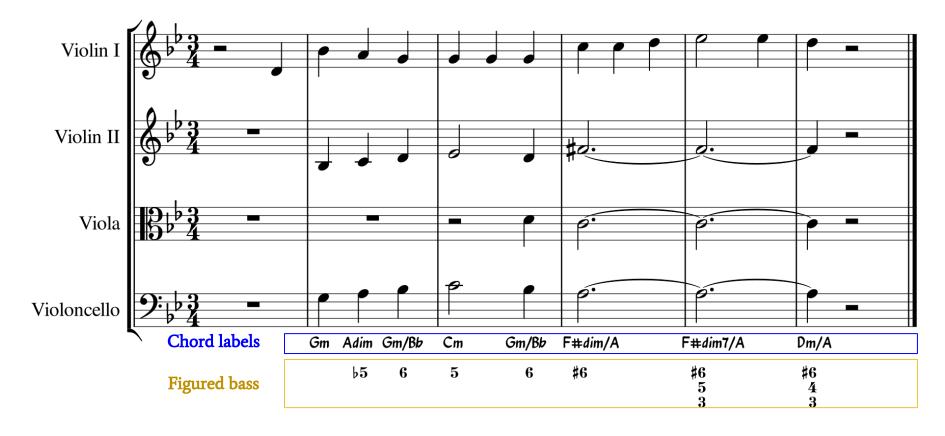
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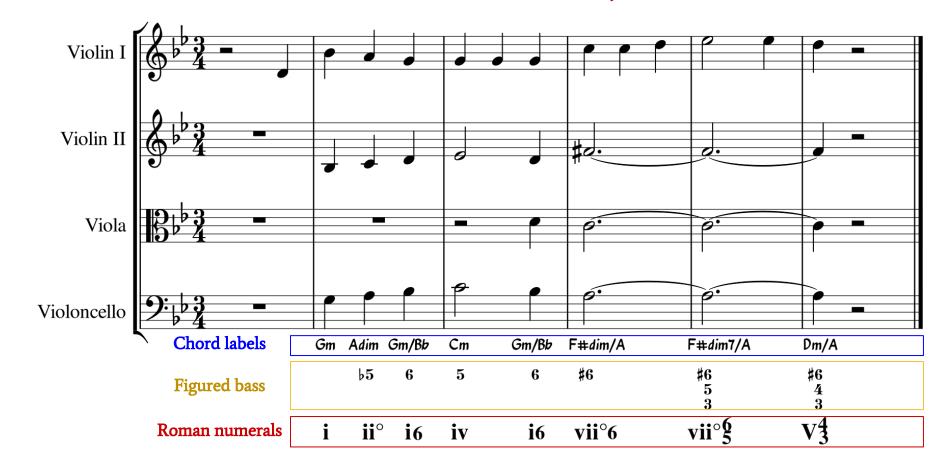














#### Harmonic analysis

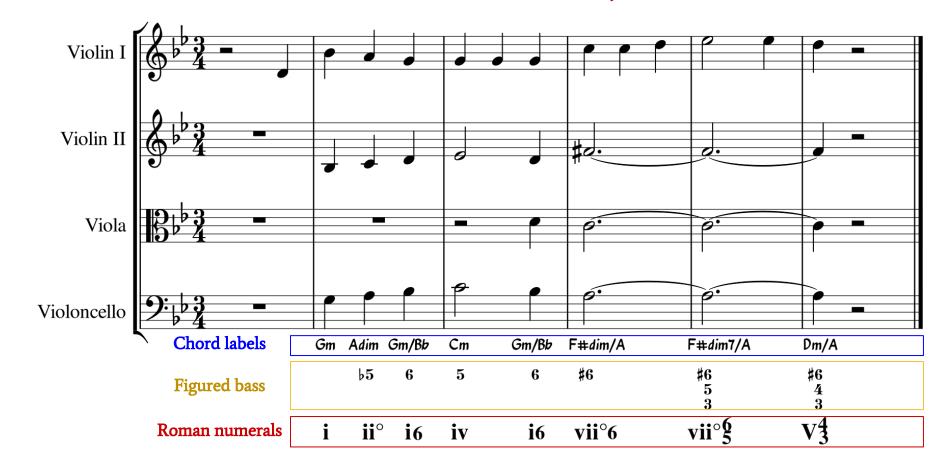
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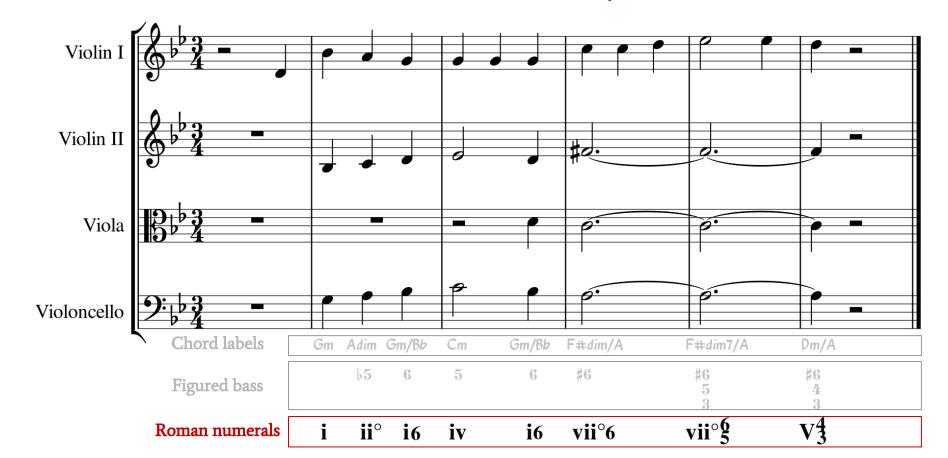
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#### Harmonic analysis

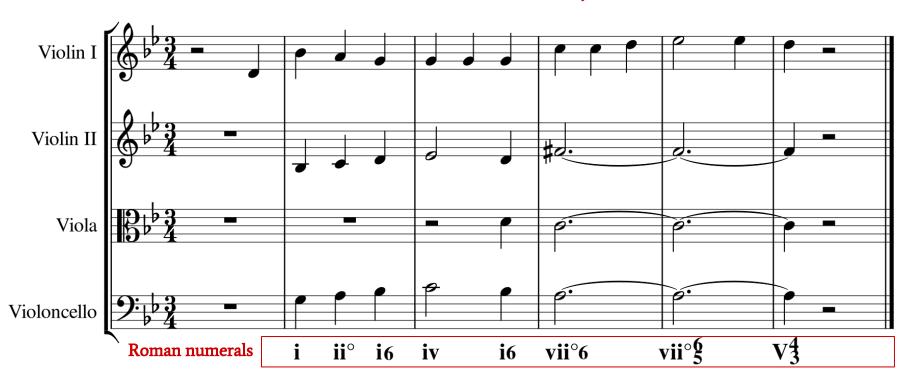
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#### Harmonic analysis

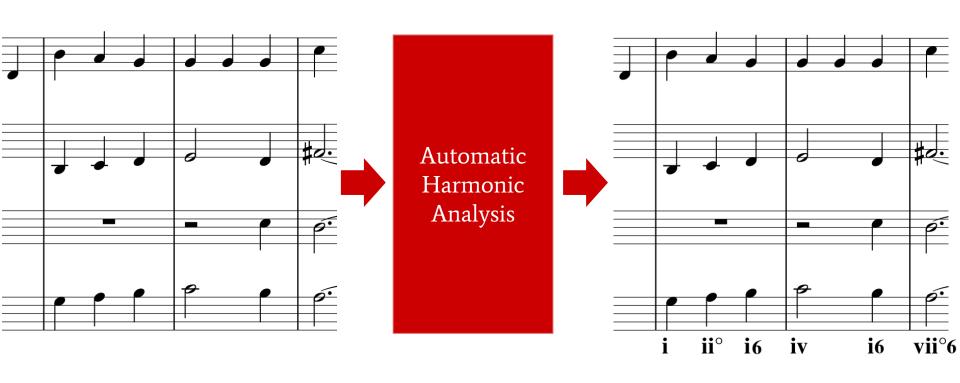
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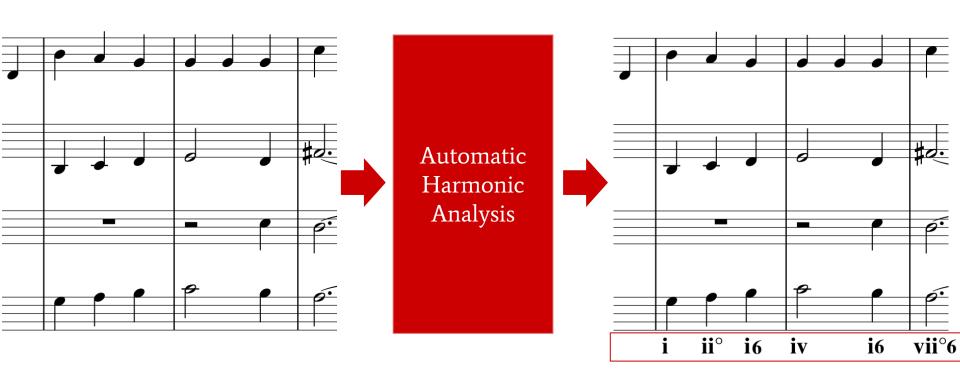
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Six string quartets from Joseph Haydn

Op.20 No.1

Op.20 No.2

Op.20 No.3

Op.20 No.4

Op.20 No.5



Six string quartets from Joseph Haydn

Op.20 No.1

Op.20 No.2

Op.20 No.3

Op.20 No.4

Op.20 No.5



Six string quartets from Joseph Haydn

Op.20 No.1

Op.20 No.2

Op.20 No.3

Op.20 No.4

Op.20 No.5



# Six string quartets from Joseph Haydn

Op.20 No.1

Op.20 No.2

Op.20 No.3

Op.20 No.4

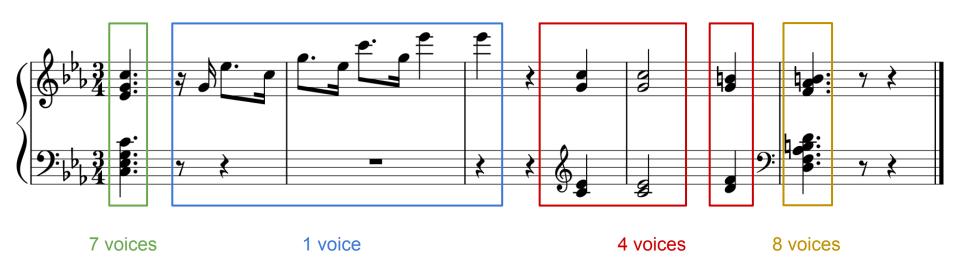
Op.20 No.5



#### The String Quartet

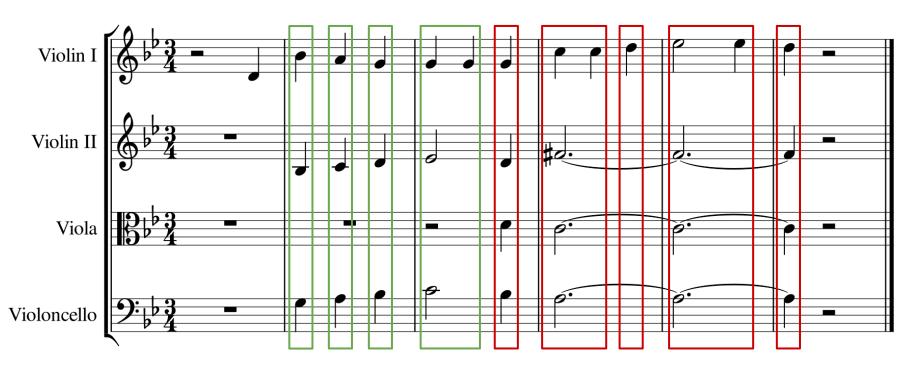
- Require a broad knowledge of harmony
- One of the most prominent genres developed during the Classical period
- Four voices (almost) all the time
- Explicit stream segregation
  - Each voice is separated explicitly in the symbolic score
  - O Its own midi channel, spine, node, etc.





Excerpt from Ludwig van Beethoven's Sonata Op.10 No.1 - I. Molto Allegro e con brio, mm. 1-5





3 voices

4 voices



#### Joseph Haydn

- Colloquially named "The father of the string quartet"
- Major figure of the classical period of western art music
  - Exemplifies many of the characteristic features of the style
- Mentor of Wolfgang Amadeus Mozart and Ludwig van Beethoven
  - The two other major figures of the classical period



#### Six string quartets Op.20

- They provided innovations to the compositional technique of string quartets
- Good distribution of musical forms and structure
  - O Sonata movements, fugues, theme and variations, minuets, etc.
- Mastery in the use of musical elements
  - O Syncopation, imitation, counterpoint
- Representative style, less experimental than later works, e.g., Op.33



#### KernScores

http://kern.ccarh.org/

Virtual library of musical scores in Humdrum \*\*kern syntax

Allocates musical scores for 19 out of 24 pieces of the String quartets Op.20



# Humdrum \*\*kern syntax

		•
**kern	**kern	**kern
*k[b-e-]	*k[b-e-]	*k[b-e-]
		*g:
		*clefG2
*M3/4		*M3/4
2r	2r	2r
$4\mathrm{r}$	$4\mathrm{r}$	4d
=1	=1	=1
2.r	4B-	4b-
	4c	4a
141	4d	4g
=2	=2	=2
2r	2e-	4g
		4g
4d	4d	4g
=3	=3	=3
[2.c]	[2.f#	4cc
		4cc
141		4dd
=4	=4	=4
$2.c\_$	$2.\mathrm{f\#}$ _	2ee-
		4ee-
=5	=5	=5
4c]	$4\mathrm{f}\#]$	4dd
2r	2r	4r
*		4d
	*k[b-e-] *g: *clefC3 *M3/4 2r 4r =1 2.r =2 2r . 4d =3 [2.c =4 2.c =5 4c] 2r	*k[b-e-] *k[b-e-] *g: *g: *clefC3 *clefG2 *M3/4 *M3/4 2r 2r 4r 4r =1 =1 2.r 4B 4c . 4d =2 =2 2r 2e 4d 4d =3 =3 [2.c [2.f# =4 =4 2.c_ 2.f# =5 =5 4c] 4f#  2r 2r



# **Automatic Harmonic Analysis**





# Each column represents a voice

## Dataset

## Humdrum \*\*kern syntax

**kern	**1	**1	10000
Kern	**kern	**kern	**kern
*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]
*g:	*g:	*g:	*g:
*clefF4	*clefC3	*clefG2	*clefG2
*M3/4	*M3/4	*M3/4	*M3/4
2r	2r	2r	2r
$4\mathrm{r}$	$4\mathrm{r}$	$4\mathrm{r}$	4d
=1	=1	=1	=1
4G	2.r	4B-	4b-
4A		4c	4a
4B-		4d	4g
=2	=2	=2	=2
2c	2r	2e-	4g
		•	4g
4B-	4d	4d	4g
=3	=3	=3	=3
[2.A	[2.c]	[2.f#	4cc
			4cc
r.	140		4dd
=4	=4	=4	=4
$2.A_{\perp}$	$2.c_{-}$	$2.\mathrm{f\#}$	2ee-
			4ee-
=5	=5	=5	=5
4A]	4c]	$4\mathrm{f}\#]$	4dd
2r	2r	2r	4r
*	*	٠	4d



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Transcribing 5 remaining scores



## The Humdrum \*\*harm syntax

http://www.music-cog.ohio-state.edu/Humdrum/representations/harm.rep.html

Very detailed syntax for roman numerals

Allows to annotate roman numerals in a machine-friendly

and (sort of) human-friendly format



# Humdrum \*\*kern, \*\*harm syntax

**harm	**commentary	**kern	**kern	**kern	**kern
*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]
*g:	*g:	*g:	*g:	*g:	*g:
*	*	*clefF4	*clefC3	*clefG2	*clefG2
*M3/4	*M3/4	*M3/4	*M3/4	*M3/4	*M3/4
8.58		2r	2r	2r	2r
		$4\mathrm{r}$	$4\mathrm{r}$	$4\mathrm{r}$	4d
=1	=1	=1	=1	=1	=1
i		4G	2.r	4B-	4b-
iio	Not really diminished, missing the fifth	4A		4c	4a
ib	N.	4B-	1.60	4d	4g
=2	=2	=2	=2	=2	=2
iv		2c	2r	2e-	4g
					4g
ib	u u	4B-	4d	4d	4g
=3	=3	=3	=3	=3	=3
viiob		[2.A]	[2.c]	[2.f#	4cc
					4cc
V7c	u_	2	141		4dd
=4	=4	=4	=4	=4	=4
viioD7b		$2.A_{\perp}$	$2.c\_$	$2.\mathrm{f\#}$ _	2ee-
	8,				4ee-
=5	=5	=5	=5	=5	=5
V7c		4A]	4c]	$4\mathrm{f}\#]$	4dd
•		2r	2r	2r	$4\mathrm{r}$
•			•	•	4d



## Humdrum \*\*kern, \*\*harm syntax

Manual
annotation of
harmony

	•		•		
**harm	**commentary	**kern	**kern	**kern	**kern
*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]
*g:	*g:	*g:	*g:	*g:	*g:
*	*	*clefF4	*clefC3	*clefG2	*clefG2
*M3/4	*M3/4	*M3/4	*M3/4	*M3/4	*M3/4
	. ^	2r	2r	2r	2r
		$4\mathrm{r}$	$4\mathrm{r}$	$4\mathrm{r}$	4d
$\stackrel{\cdot}{=}1$	=1	=1	=1	=1	=1
i		4G	2.r	4B-	4b-
iio	Not really diminished, missing the fifth	4A		4c	4a
ib		4B-	121	4d	4g
=2	=2	=2	=2	=2	=2
iv	,	2c	2r	2e-	4g
					4g
ib	·	4B-	4d	4d	4g
=3	=3	=3	=3	=3	=3
viiob		[2.A]	[2.c	[2.f#	4cc
	÷				4cc
V7c	¥ _	2	141	141	4dd
=4	=4	=4	=4	=4	=4
viioD7b		$2.A_{\perp}$	$2.c\_$	$2.\mathrm{f\#}$ _	2ee-
	i				4ee-
=5	=5	=5	=5	=5	=5
V7c		4A]	4c]	$4\mathrm{f}\#]$	4dd
	,	2r	2r	2r	$4\mathrm{r}$
•	,	•			4d



## Summary of the dataset

Six string quartets

24 pieces of music

146 pages of music notation (Altmann Edition)

4961 roman numerals annotations

Commentaries (ambiguous sections, structural analysis, etc.)

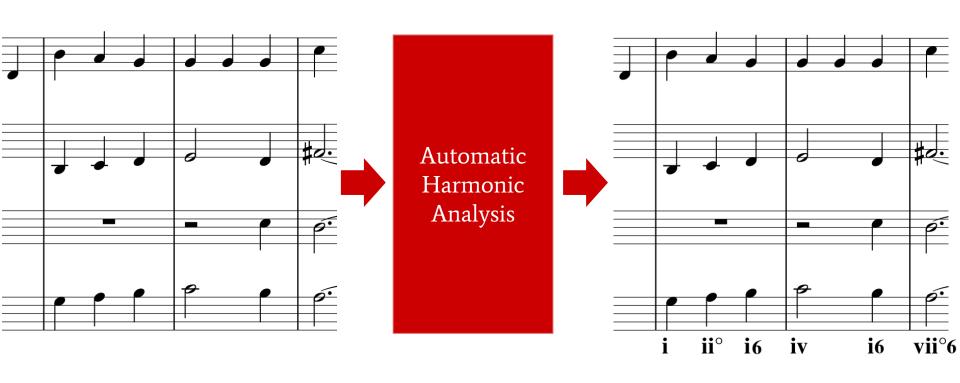
https://github.com/napulen/haydn\_op20\_harm



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**kern	**kern	**kern	**kern
*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]
*g:	*g:	*g:	*g:
*clefF4	*clefC3	*clefG2	*clefG2
*M3/4	*M3/4	*M3/4	*M3/4
2r	2r	2r	2r
$4\mathrm{r}$	$4\mathrm{r}$	4r	4d
=1	=1	=1	=1
4G	2.r	4B-	4b-
4A		4c	4a
4B-		4d	4g
=2	=2	=2	=2
2c	2r	2e-	4g
			4g
4B-	4d	4d	4g
=3	=3	=3	=3
[2.A]	[2.c]	$[2.\mathrm{f}\#$	4cc
			4cc
7			4dd
=4	=4	=4	=4
$2.A_{\perp}$	$2.c_{-}$	$2.f\#$ _	2ee-
			4ee-
=5	=5	=5	=5
4A]	4c]	$4\mathrm{f}\#]$	4dd
2r	2r	2r	4r
			4d



# Automatic Harmonic Analysis



	1			
**harm	**kern	**kern	**kern	**kern
*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-]	*k[b-e-
*g:	*g:	*g:	*g:	*g:
*	*clefF4	*clefC3	*clefG2	*clefG2
*M3/4	*M3/4	*M3/4	*M3/4	*M3/4
	2r	2r	2r	2r
÷	$4\mathrm{r}$	$4\mathrm{r}$	$4\mathrm{r}$	4d
=1	=1	=1	=1	=1
i	4G	2.r	4B-	4b-
iio	4A		4c	4a
ib	4B-		4d	4g
=2	=2	=2	=2	=2
iv	2c	2r	2e-	4g
				4g
ib	4B-	4d	4d	4g
=3	=3	=3	=3	=3
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				4cc
V7c				4dd
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viioD7b	$2.A_{\perp}$	$2.c_{-}$	$2.\mathrm{f\#}_{\_}$	2ee-
				4ee-
=5	=5	=5	=5	=5
V7c	4A]	4c]	4f#]	4dd
	2r	2r	2r	$4\mathrm{r}$
				4d
,	J			



Approach	Type of Model	Year	Implementation	Available
Terry Winograd	Grammar-based	1968	LISP	No
John Maxwell	Rule-based	1992*	LISP	No
David Temperley	Rule-based	1997**	С	Yes
Christopher Raphael	Probabilistic	2003	С	Partially
Plácido Illescas	Rule-based	2008	Java	Partially

<sup>\*</sup> Maxwell presented this work at his PhD dissertation in 1984. However, most researchers cite the publication from 1992 in the book "Understanding music with AI".

<sup>\*\*</sup>A lot of work has been done after Temperley in 1997, traceable to as recent as 2017. This reference is just the starting point for this approach.

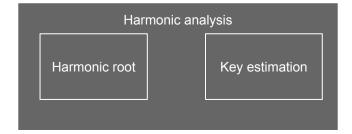


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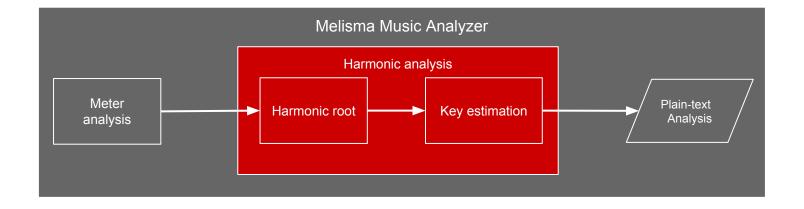
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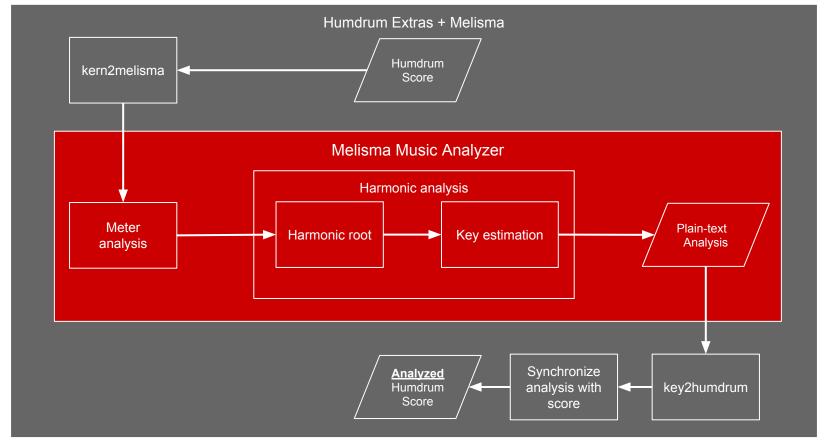
David Temperley





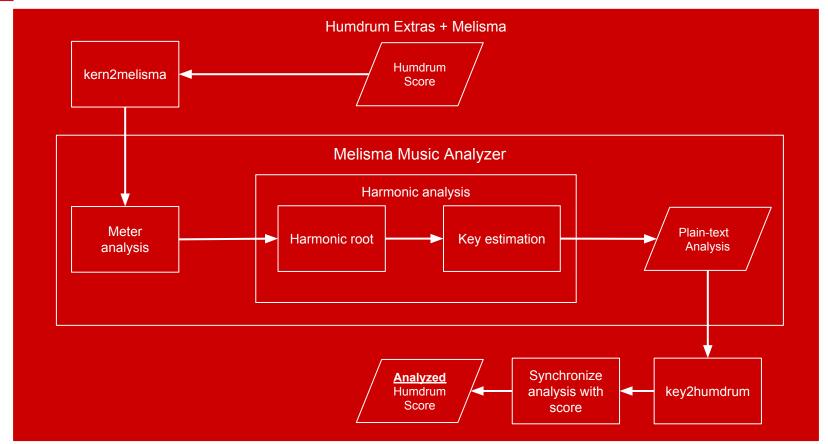
## David Temperley + Daniel Sleator





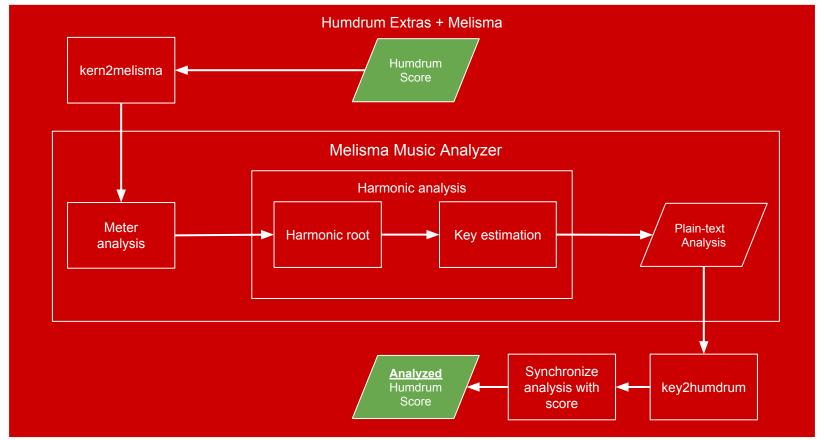
David Temperley + Daniel Sleator + Craig Sapp





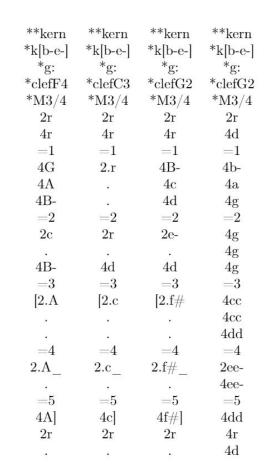
David Temperley + Daniel Sleator + Craig Sapp

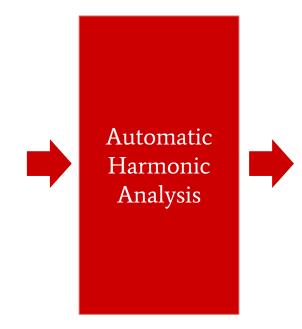


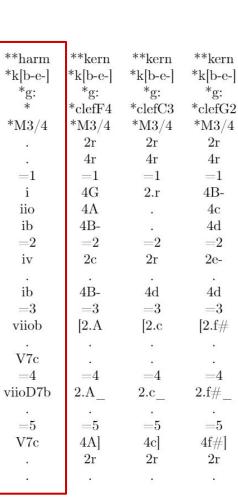


David Temperley + Daniel Sleator + Craig Sapp









\*\*kern

\*k[b-e-]

\*g:

\*clefG2

\*M3/4

2r

4d

4b-

4a

4g

4g

4g

4g

4cc

4cc

4dd

=4

2ee-

4ee-

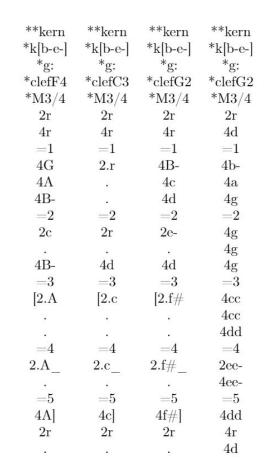
=5

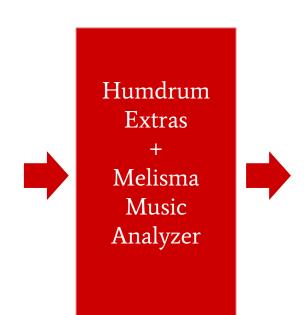
4dd

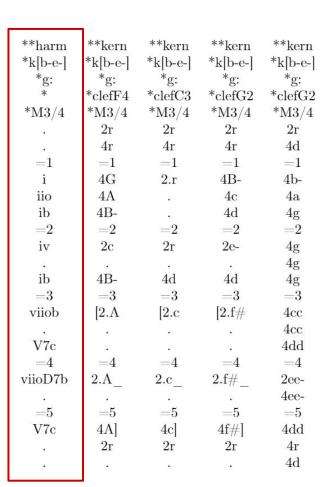
4r

4d

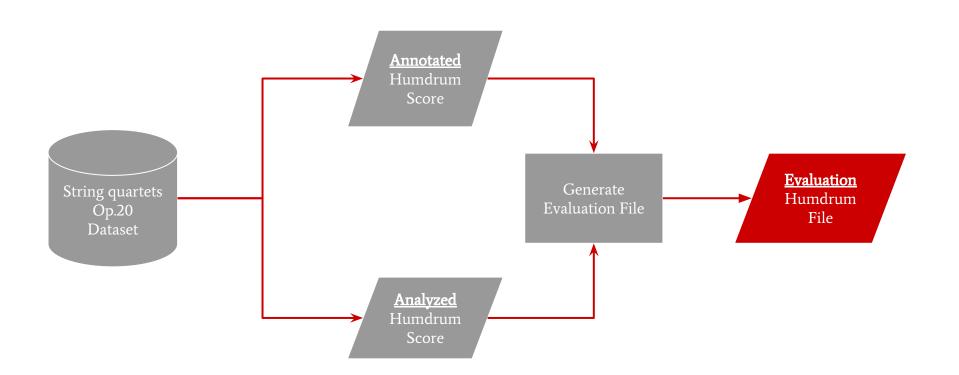














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#### Evaluation file

It is a valid Humdrum file. However, it is no longer a musical score

Contains two pairs of \*\*harm \*\*root spines

upf. Music
Technology
Group

# Evaluation

**harm	**root	**harm	**root
	•		
(*)		×	
	•		
•			
=1	=1	=1	=1
=1 i	g		
	20		· f
iio	a	iv	f
•			
ib	g	0	-
		*	Si .
=2	=2	=2	=2
iv	c	ib	C
	2	<b>U</b>	4
	*	V7c	g
	•		
ib	g		
	2	0	12



**harm	**ro
(4)	•
(*)	
3.50	•
•	•
20-22	¥3
=1	=1
i	g
iio	a
•	
ib	g
=2	=2
iv	c
ib	g

Manual annotations

**harm	**root
*	
*	
v	
*	
=1	=1
	· f
iv	f
0	
0	
*	
=2	=2
ib	c
e e	0
V7c	g
*	
0	
	ů.

Automatic analysis



## Summary of evaluation

Normalization of time units,

resolution of harmonic roots,

string comparison of harmonic roots per time unit

output the percentage of matches over the time units



#### Normalization of time units

Shortest note of the musical score



#### Resolution of harmonic roots

Cmajor: I == Gmajor: IV

Cmajor: V/V == Gmajor: V

Cmajor: V7/vi == Dminor: V7/V



#### Resolution of harmonic roots

Developed a parser for the \*\*harm syntax <a href="https://github.com/napulen/harmparser">https://github.com/napulen/harmparser</a>

Input a \*\*harm expression and output its characteristics

Using this to resolve the harmonic roots of different labels, even if they are relative to different keys



# Structure of the presentation

- 1. Automatic Harmonic Analysis
- 2. The dataset: Six string quartets Op.20, Joseph Haydn
- 3. Running the analysis
- 4. Evaluation
- 5. Results
- 6. Contributions



# Results

	110001100		
	Matching time units	Total time units	Score
Op.20 No.1			
I	2261	3489	64.80%
II	221	805	27.45%
III	1319	1730	76.24%
IV	332	1289	25.76%
Op.20 No.2			
I	1254	5137	24.41%
II	1119	2017	55.48%
III	713	1033	69.02%
IV	309	1945	15.89%
Op.20 No.3			
I	2426	4322	56.13%
II	121	535	22.62%
III	3386	4069	83.21%
IV	535	1681	31.83%
Op.20 No.4			
I	2464	3663	67.27%
II	1992	3922	50.79%
III	65	223	29.15%
IV	1651	6289	26.25%
Op.20 No.5			
I	4950	7729	64.04%
II	793	1201	66.03%
III	2617	3061	85.49%
IV	798	1473	54.18%
Op.20 No.6			
I	1808	3985	45.37%
Π	6217	7585	81.96%
III	405	505	80.20%
IV	1361	3041	44.76%



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## Contributions

- 1. Reproducing the harmonic analysis done in the backend of KernScores
- 2. Finding, and in some cases, fixing issues
- 3. The Op.20 dataset of manual annotations
  - **a.** With help of Rafael Caro for the first movement of each quartet
- **4.** A basic evaluation process
- 5. A parser for the \*\*harm syntax

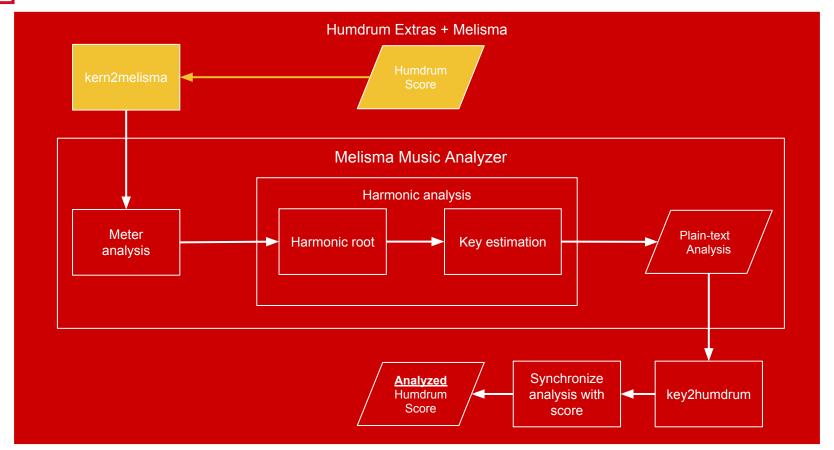


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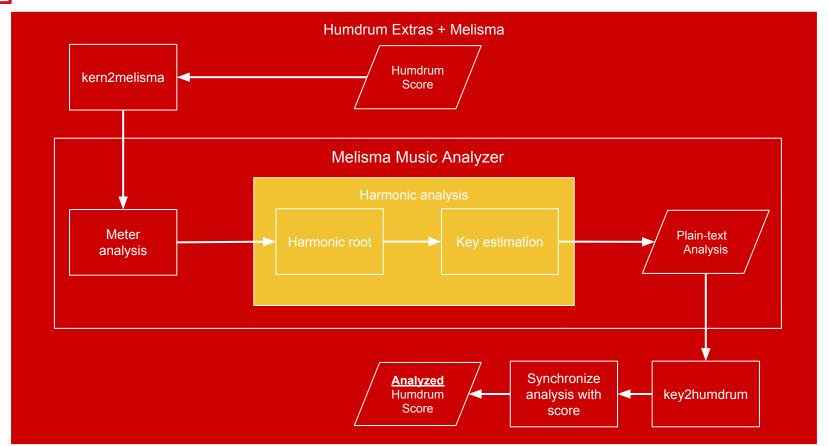


## Running the analysis



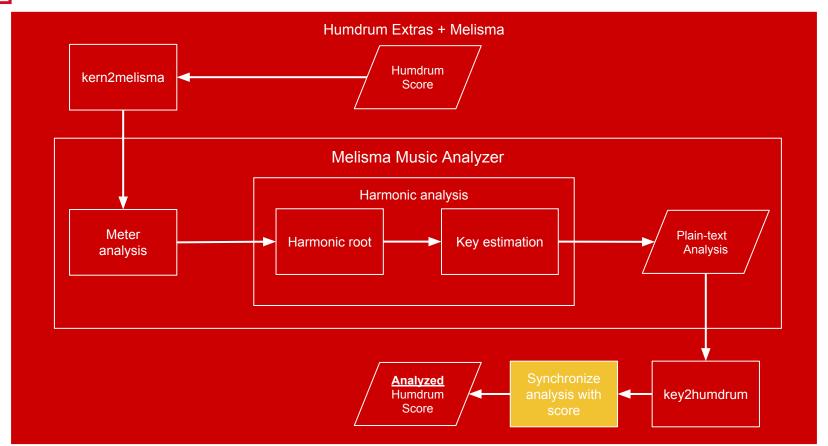


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# Thank you



# Backup Slides



Isn't there any more recent research than that?



### Isn't there any more recent research than that?

Sort of, but not entirely.

They are considering a pre-processed music score, especially ignoring the difficult problem of non-harmonic tones

They output a different kind of label notation

### \*\*harm parser

```
^ (
    (?P<accidental>
                              # Named group _accidental
    [\#-]\{0,2\})
    (?P<root>
                             # Named group _root_
     i|ii|iii|iv|v|vi|vii|
                            # Minor mode degrees
     I|II|III|IV|V|VI|VII|
                            # Major mode degrees
                             # Special chords
     N|Gn|Lt|Fr|Tr)
    (?P<attribute>
                             # Named group _attribute_
    [o+]?)
    ((?P<intervals>
                             # Named group intervals
    d+\lceil mMPAD\rceil d+\rceil
                             # Detect minor, Major, Augmented or Diminished intervals
                             # Double-augmented intervals
    AA\d+|
                             # Double-diminished intervals
    DD\d+)
    *)
                             # Not a limit on how many intervals can be added
    (?P<inversion>
                             # Named group _inversions_
    [b-d]?)
                             # Only third inversions possible so far
    (\[
                             # Open brackets
    (?P<alternative>
                             # Named group _alternative_
                             # Match at least one time for any expression inside brackets
    ([^\\[^\]])+)
                             # Close brackets
    \1
    )?
                             # If no alternative expression, then no brackets should appear at all
                             # Slash implies a secondary function
    (?P<secondary>
                             # Named group _secondary_
                             # Get all the expression after the slash symbol
    ([\s\S])+)
    )?
                             # If no secondary function, then the slash symbol should not appear
)$
```