Name: _____ Date: _____

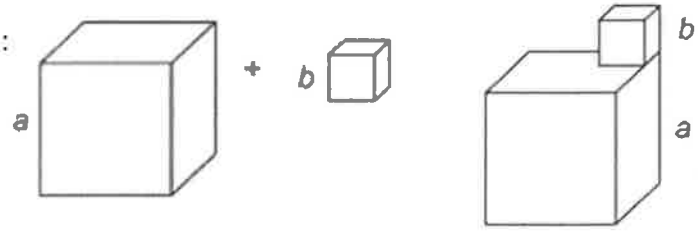
Algebra II Lesson 1-4

FACTORING PERFECT CUBES

Do Now: Fill in the parentheses to create an equivalent expression.

1. $x^3+125=(\ )^3+(\ )^3$ 2. $x^3-64=(\ )^3-(\ )^3$ 3. $8x^3+343=(\ )^3+(\ )^3$

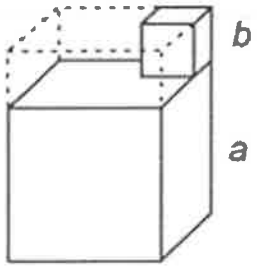
Step 1: Pictorially, the sum of cubes looks like this:



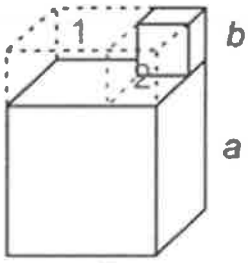
Step 2: The formula for volume is *length* × *width* × *height*. Find the volume of the sum of these two cubes.

Volume = _____.

Step 3: Now we will write that volume in a different way.



$$V = a \times a \times (a+b)$$
$$= a^2(a+b)$$



$$V = a^2(a+b) - \left[\underbrace{ab(a-b)}_{\text{Prism 1}} + \underbrace{b^2(a-b)}_{\text{Prism 2}}\right]$$

Step 4: Now let's simplify this!

$V = a^3 + a^2b - a^2b + ab^2 - ab^2 + b^3$

$V = a^3 + a^2b - a^2b - ab^2 + ab^2 + b^3$

$V = a^2(a+b) - ab(a+b) + b^2(a+b)$

$V = (a+b)(a^2 - ab + b^2)$

"Same, opposite, Always positive"

A Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

same sign — opposite sign — always +

A Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

same sign — opposite sign — always +

Example #1: $x^3 + 125 = (x+5)(x^2 - 5x + (5)^2)$

S O AP

$(x+5)(x^2 - 5x + 25)$

$a = x \quad b = 5$

x	f(x)
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

Example #2: $2x^4 - 128x$

$2x(x^3 - 64) = 2x(x-4)(x^2 + 4x + (4)^2)$

S O AP

$a = x \quad b = 4$

$= 2x(x-4)(x^2 + 4x + 16)$

Example #3: $8x^3 + 343 = (2x+7)((2x)^2 - (2x)(7) + (7)^2)$

S O AP

$a = 2x \quad b = 7$

$(2x+7)(4x^2 - 14x + 49)$

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Algebra II Lesson 1-4

FACTORING PERFECT CUBES PRACTICE

***Directions*:** Factor each of the following using either the sum or difference of cubes formula.

1. $8x^3+27$ $= (2x+3)((2x)^2-(2x)(3)+(3)^2)$ (S, O, AP)

$a=2x$ $b=3$ $= (2x+3)(4x^2-6x+9)$

Perfect Cubes in the Calc..

$Y_1 = X^3$

x	f(x)
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

2. x^5-125x^2 $= x^2(x-5)(x^2+5x+(5)^2)$ (S, O, AP)

$x^2(x^3-125) = x^2(x-5)(x^2+5x+25)$

$a=x$

$b=5$

3. $64x^3+343$ $= (4x+7)((4x)^2-(4x)(7)+(7)^2)$ (S, O, AP)

$a=4x$ $b=7$ $(4x+7)(16x^2-28x+49)$

4. $5x^6-135x^3$

$5x^3(x^3-27) = 5x(x-3)(x^2+3x+(3)^2)$ (S, O, AP)

$a=x$ $b=3$ $= 5x(x-3)(x^2+3x+9)$

Answers

1. $(2x+3)(4x^2-6x+9)$
2. $x^2(x-5)(x^2+5x+25)$
3. $(4x+7)(16x^2-28x+49)$
4. $5x^3(x-3)(x^2+3x+9)$