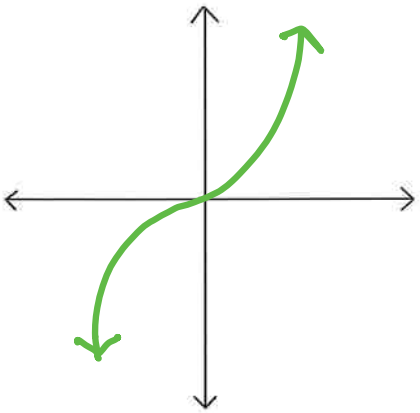
Lesson 2-3

Polynomial Functions

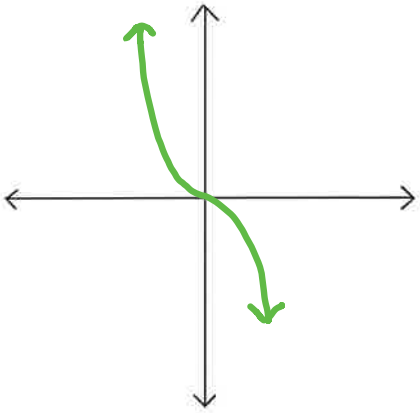
Do Now

Graph the following functions in your calculator and **draw a sketch** of each of them on the given axes below.

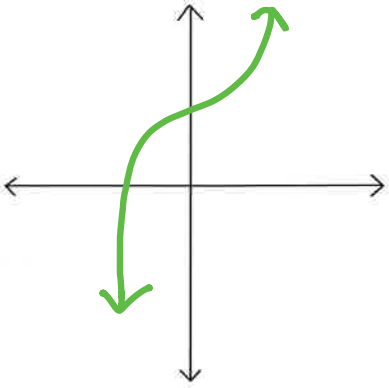
a) $f(x)=x^3$



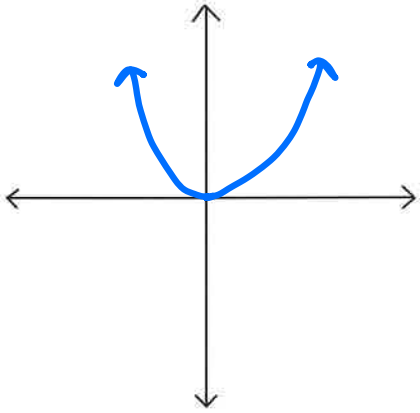
b) $f(x)=-x^3-x$



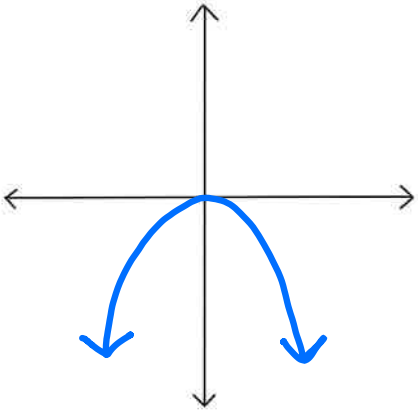
c) $f(x)=x^5+5x^3+2$



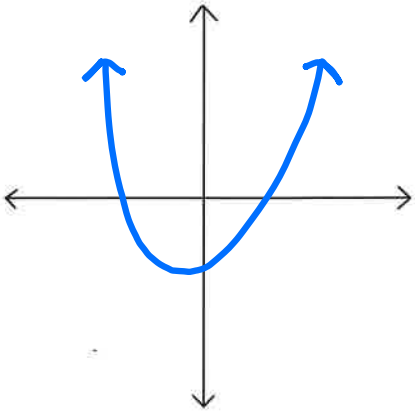
d) $f(x)=x^2$



e) $f(x)=-x^8-x^2$



f) $f(x)=x^4+2x^2-3$



Think about it...

Based on your graphs above, what do you notice?

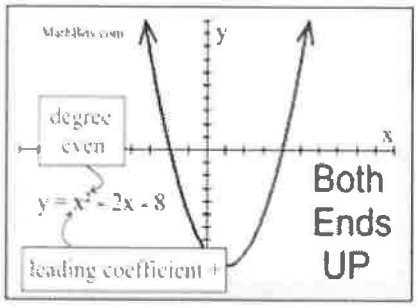
The top graphs have one end pointing up and the other end pointing down.

The bottom graphs have both ends pointing in the same direction

Even Degree Polynomials

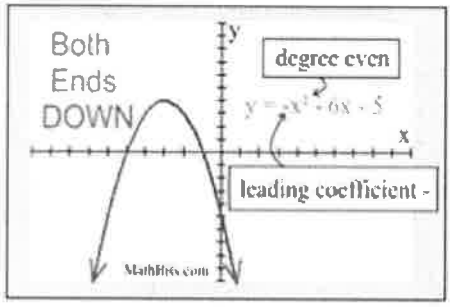
"ends" behave similar to a quadratic

When the leading coefficient is **positive**, **both "ends"** are **up**.



↑ ↑

When the leading coefficient is **negative**, both "ends" are **down**.

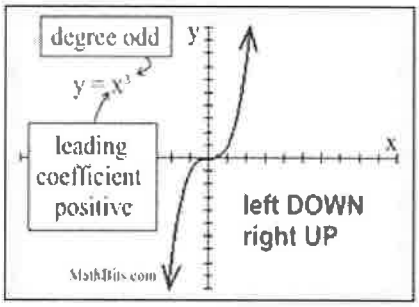


↓ ↓

Odd Degree Polynomials

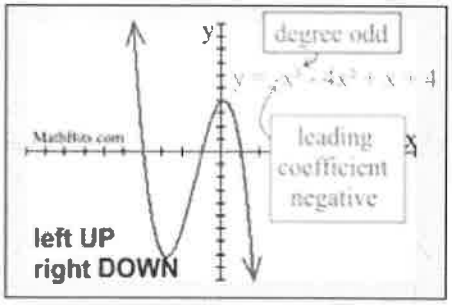
"ends" behave similar to a cubic (or linear)

When the leading coefficient is **positive**, the **left end** is **down** and the **right end** is **up**.



↓ ↑

When the leading coefficient is **negative**, the **left end** is **up** and the **right end** is **down**.



↑ ↓

The **end behavior** of polynomials refers to the appearance of a graph as it is followed indefinitely in either horizontal direction.

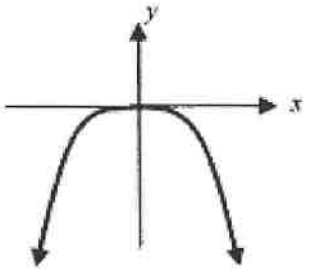
No Calculator: Which of the following functions is shown in the graph below? Explain your choice.

(1) $y = -4x^7$

(2) $y = -3x^{10}$

(3) $y = 6x^2$

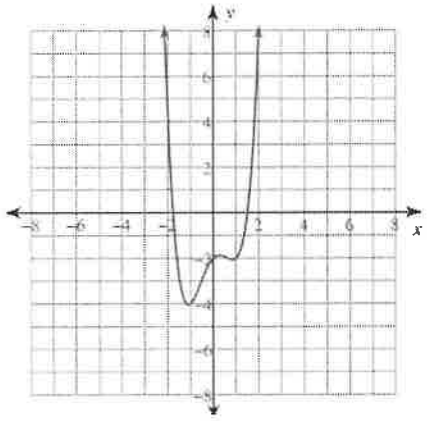
(4) $y = 5x^9$



↓ ↓ means even degree + negative leading coefficient!

***Exercise* #1:** Given each graph, determine whether the degree of the function is even or odd, state the sign of the leading coefficient and state the end behavior.

a.



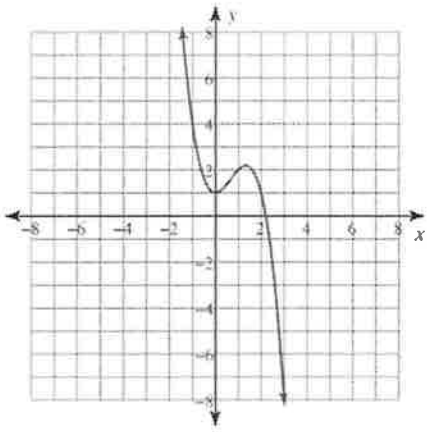
Degree: even

Sign of Leading Coefficient: positive

$x \to \infty, \quad f(x) \to$ ∞

$x \to -\infty, \quad f(x) \to$ ∞

b.



Degree: odd

Sign of Leading Coefficient: negative

$x \to \infty, \quad f(x) \to$ $-\infty$

$x \to -\infty, \quad f(x) \to$ ∞

***Exercise* #2:** Given each equation, determine the degree and the end behavior.

a. $f(x) = 4x^3 - x^5 - 2x - 2$

$-x^5 + 4x^3 - 2x - 2$

Degree: odd, negative

$x \to \infty, \quad f(x) \to$ $-\infty$

$x \to -\infty, \quad f(x) \to$ ∞

↑ ↓

b. $f(x) = -x^2 - 8x - 15$

Degree: even, negative

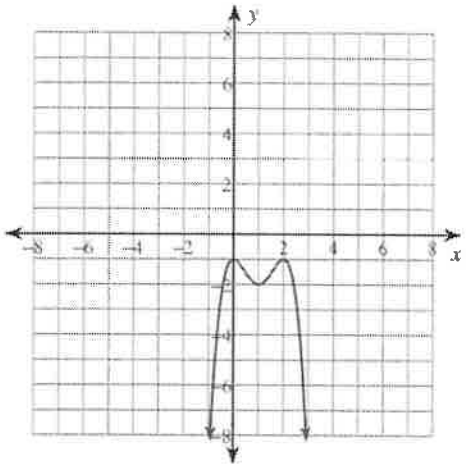
$x \to \infty, \quad f(x) \to$ $-\infty$

$x \to -\infty, \quad f(x) \to$ $-\infty$

↓ ↓

***Exercise* #3:** Given each graph, determine whether the degree of the function is even or odd, state the sign of the leading coefficient and state the end behavior.

a



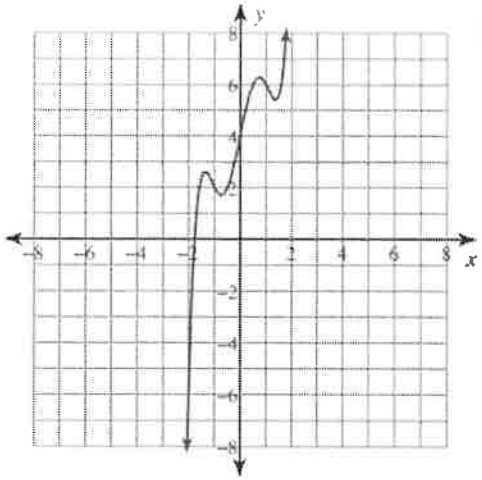
Degree: even

Sign of Leading Coefficient: negative

$x \to \infty, \quad f(x) \to$ $-\infty$

$x \to -\infty, \quad f(x) \to$ $-\infty$

b.



Degree: odd

Sign of Leading Coefficient: positive

$x \to \infty, \quad f(x) \to$ ∞

$x \to -\infty, \quad f(x) \to$ $-\infty$

***Exercise* #4:** Given each equation, determine the degree and the end behavior.

c. $f(x) = 10x^2 + x^3 + 32x + 34$ ↙

Degree: odd (positive),

$x \to \infty, \quad f(x) \to$ ∞

$x \to -\infty, \quad f(x) \to$ $-\infty$

↓ ↑

d. $f(x) = x^4 - 4x^2 - x + 3$

Degree: even positive,

$x \to \infty, \quad f(x) \to$ ∞

$x \to -\infty, \quad f(x) \to$ ∞

↑ ↑

SUMMARY: The End Behavior of a Polynomial Function with Leading Term ax^n :

End Behavior	n is Even (not zero)	n is Odd
a is positive	Both Ends UP ↑↑	Left Down, Right Up ↓↑
a is negative	Both Ends Down ↓↓	Left Up, Right Down ↑↓

Matching Activity!

Polynomial Function	Graph	End Behavior
1. $y = x^3 - 4x^2 + 4$	J	↓ ↑
2. $y = 2x^2 + 3x - 8$	I	↑ ↑
3. $y = -x^4 + 2x^3 + 2x^2 - x + 2$	A	↓ ↓
4. $y = -x^3 + 2x^2 + 3x + 1$	E	↑ ↓
5. $y = -x^5 + 5x^3 + x^2 - x - 2$	H	↑ ↓
6. $y = 2x^4 - 6x^2 - 4$	G	↑ ↑
7. $y = x^4 + 2x^3 - 3x^2 - x + 2$	D	↑ ↑
8. $y = (x+2)(x-1)(x-2)$	C	↓ ↑
9. $y = -x^3(x-2)$	F	↓ ↓
10. $y = x^3 - 1$	B	↓ ↑