Operations with Complex Numbers

Do Now: Express $4xi + 5yi^8 + 6xi^3 + 2yi^4$ in simplest $a + bi$ form.

$4xi + 5y(1) + 6x(-i) + 2y(1)$

$\underline{\underline{4xi}} + \underline{5y} - \underline{\underline{6xi}} + \underline{2y}$

$7y - 2xi$

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1. The expression $x(3i^2)^3 + 2xi^{12}$ is equivalent to

(1) 2x + 27xi $x(27i^6) + 2x(1)$ ③ −25x

(2) −7x $-27x + 2x$ (4) −29x

$-25x$

2. The expression $6xi^3(-4xi + 5)$ is equivalent to

1) $2x - 5i$
②) $-24x^2 - 30xi$ $-24x^2i^4 + 30xi^3$
3) $-24x^2 + 30x - i$
4) $26x - 24x^2i - 5i$ $-24x(1) + 30x(-i)$

$-24x - 30xi$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

3. Given i is the imaginary unit, $(2-yi)^2$ in simplest form is

1) $y^2 - 4yi + 4$
2) $-y^2 - 4yi + 4$ (circled)
3) $-y^2 + 4$
4) $y^2 + 4$

$(2-yi)(2-yi)$

$4 - 2yi - 2yi + y^2 i^2$

$4 - 4yi + y^2(-1)$

$4 - 4yi - y^2$

$-y^2 - 4yi + 4$

4. Write $(5 + 2yi)(4 - 3i) - (5 - 2yi)(4 - 3i)$ in $a + bi$ form, where y is a real number.

$20 - 15i + 8yi - 6yi^2 - (20 - 15i - 8yi + 6yi^2)$

$20 - 15i + 8yi - 6y(-1) - (20 - 15i - 8yi + 6y(-1))$

$20 - 15i + 8yi + 6y - 20 + 15i + 8yi + 6y$

$12y + 16yi$

5. Write $\frac{2}{3}i^2(\sqrt{-16} + 3) - 3i^3$ in simplest $a + bi$ form.

$\frac{2}{3}(-1)(4i + 3) - 3(-i)$

$-\frac{2}{3}(4i + 3) + 3i$

$-\frac{8}{3}i - 2 + 3i$

$-2 + \frac{1}{3}i$

$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

1. Express $6xi - 7yi^{12} - 2xi^6 + 7yi^4$ in simplest $a + bi$ form.

$6xi - 7y(1) - 2x(-1) + 7y(1)$

$6xi - \cancel{7y} + 2x + \cancel{7y}$

$2x + 6xi$

2. Simplify into simplest $a + bi$ form: $x(2i^4)^3 + 5x(i^9)$

$8xi^{12} + 5xi^9$

$8x(1) + 5x(i)$

$8x + 5xi$

3. Multiply $x + yi$ by its conjugate, and express the product in simplest form.

$(x+yi)(x-yi)$

$x^2 + xyi - xyi - y^2i^2$

$x^2 - y^2(-1)$

$x^2 + y^2$

4. If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$2xi(i - 4i^2)$

$2xi^2 - 8xi^3$

$2x(-1) - 8x(-i)$

$-2x + 8xi$

i
-1
$-i$
1

5. Simplify $xi(i - 7i)^2$, where i is the imaginary unit.

$$Xi \, (i - 7i)(i - 7i)$$
$$Xi \, (i^2 - 7i^2 - 7i^2 + 49i^2)$$
$$Xi \, (i^2 - 14i^2 + 49i^2)$$
$$Xi \, (36i^2)$$
$$36xi^3$$
$$- 36Xi$$

i
i
$-i$

6. The expression $(x + i)^2 - (x - i)^2$ is equivalent to

1) 0
2) −2
3) $-2 + 4xi$
4) $4xi$ ⟵ circled

$$(X+i)(X+i) - (X-i)(X-i)$$
$$X^2 + Xi + Xi + i^2 - (X^2 - Xi - Xi + i^2)$$
$$X^2 + 2Xi - 1 - (X^2 - 2Xi - 1)$$
$$\cancel{X^2} + 2Xi \cancel{-1} - \cancel{X^2} + 2Xi \cancel{+1}$$

$$4Xi$$

7. Write $(3 + 2yi)(6 - 3i) - (3 - 2yi)(6 - 3i)$ in $a + bi$ form, where y is a real number.

$$18 - 9i + 12yi - 6yi^2 - (18 - 9i - 12yi + 6yi^2)$$
$$18 - 9i + 12yi + 6y - (18 - 9i - 12yi - 6y)$$
$$\cancel{18} - \cancel{9i} + 12yi + 6y - \cancel{18} + \cancel{9i} + 12yi + 6y$$

$$24yi + 12y$$
$$12y + 24yi \quad \leftarrow a+bi \text{ form!}$$

Challenge:

⟶ Doesn't have an i

Given $(a + bi)(c + di)$ is a purely real number and the product of b and c is −5, find the product of a and d.

$$ac + adi + bci + bdi^2$$
$$ac + adi + bci - bd$$
$$ac - bd + \underline{adi + bci}$$
$$(ad + bc)i$$
$$\underbrace{}_{= 0}$$

$$ad + bc = 0$$
$$ad + (-5) = 0$$
$$\boxed{ad = 5}$$