

Polynomial Long Division

When we divide two polynomials, we get another polynomial, $q(x)$ and a remainder, $r(x)$.

Recall the **quotient-remainder form**:

$$\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$$

When dividing two first degree polynomials both the quotient and remainder are constants.

Do Now: Consider the expression $\frac{x+8}{x+3}$.

- a. Perform long division and write the result in quotient-remainder form.

$$\begin{array}{r} x+3 \overline{) x+8} \\ \underline{-(x+3)} \\ 5 \end{array} \quad \frac{x}{x} = 1$$
$$1 + \frac{5}{x+3}$$

- b. Re-write the expression in quotient-remainder form by simplification.

$$\frac{x+8}{x+3}$$
$$\frac{x+3+5}{x+3}$$
$$\frac{x+3}{x+3} + \frac{5}{x+3}$$
$$1 + \frac{5}{x+3}$$

*you can only do this if you are dividing polynomials of the same degree

Exercise #1: Given $\frac{4x+13}{x+2}$, state the quotient and remainder.

$$\begin{array}{r} 4 \\ x+2 \overline{) 4x+13} \\ \underline{-(4x+8)} \\ 5 \end{array}$$

$$4 + \frac{5}{x+2}$$

$$\begin{array}{r} 4x+13 \\ x+2 \\ \hline 4x+8+5 \\ \hline x+2 \end{array}$$

$$\frac{4x+8}{x+2} + \frac{5}{x+2}$$

$$\frac{4(\cancel{x+2})}{\cancel{x+2}} + \frac{5}{x+2}$$

$$4 + \frac{5}{x+2}$$

Exercise #2: Given $f(x) = 3x^4 + 2x + x^3 - 5x^2 + 3$ and $g(x) = x^2 + 2x + 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

* Needs to be in standard form!

$$\begin{array}{r} 3x^2 - 5x - 1 \\ x^2 + 2x + 2 \overline{) 3x^4 + x^3 - 5x^2 + 2x + 3} \\ \underline{-(3x^4 + 6x^3 + 6x^2)} \downarrow \\ -5x^3 - 11x^2 + 2x \\ \underline{-(-5x^3 - 10x^2 - 10x)} \downarrow \\ -1x^2 + 12x + 3 \\ \underline{-(-1x^2 - 2x - 2)} \\ 14x + 5 \end{array}$$

$$3x^2 - 5x - 1 + \frac{14x + 5}{x^2 + 2x + 2}$$

Need to fill in missing terms

Exercise #3: Given $f(x) = 4x^4 + 5x - 4$ and $g(x) = x^2 + 3x - 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

$$\begin{array}{r} 4x^2 - 12x + 44 \\ x^2 + 3x - 2 \overline{) 4x^4 + 0x^3 + 0x^2 + 5x - 4} \\ \underline{-(4x^4 + 12x^3 - 8x^2)} \downarrow \\ -12x^3 + 8x^2 + 5x \\ \underline{-(-12x^3 - 36x^2 + 24x)} \downarrow \\ 44x^2 - 19x - 4 \\ \underline{-(44x^2 + 132x - 88)} \\ -151x + 84 \end{array}$$

$$4x^2 - 12x + 44 - \frac{151x + 84}{x^2 + 3x - 2}$$

Reminder...

Check for standard form
& for missing terms!

Name: _____
Algebra II

Date: _____
Lesson 1-9

POLYNOMIAL LONG DIVISION PRACTICE

1. Given $\frac{3x-5}{x-4}$, state the quotient and remainder.

$$\begin{array}{r} 3 \\ x-4 \overline{) 3x-5} \\ \underline{-(3x-12)} \\ 7 \end{array}$$

$$\frac{3x}{x} = 3$$

$$\boxed{3 + \frac{7}{x-4}}$$

← Put into standard form!

2. Given $f(x) = x^4 + 3x^3 + x - 4x^2 - 7$ and $g(x) = x^2 + 3x - 1$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

$$\begin{array}{r} x^2 - 3 \\ x^2+3x-1 \overline{) x^4+3x^3-4x^2+x-7} \\ \underline{-(x^4+3x^3-x^2)} \downarrow \downarrow \\ -3x^2+x-7 \\ \underline{-(-3x^2-9x+3)} \\ 10x-10 \end{array}$$

$$\frac{-3x^2}{x^2} = -3$$

$$\boxed{x^2 - 3 + \frac{10x-10}{x^2+3x-1}}$$

← Fill in missing terms

3. Given $f(x) = 2x^4 + 3x^3 + 5x - 1$ and $g(x) = x^2 + 3x + 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

$$\begin{array}{r}
 2x^2 - 3x + 5 \\
 x^2 + 3x + 2 \overline{) 2x^4 + 3x^3 + 0x^2 + 5x - 1} \\
 \underline{-(2x^4 + 6x^3 + 4x^2)} \downarrow \\
 -3x^3 - 4x^2 + 5x \\
 \underline{-(-3x^3 - 9x^2 - 6x)} \downarrow \\
 5x^2 + 11x - 1 \\
 \underline{-(5x^2 + 15x + 10)} \\
 -4x - 11
 \end{array}$$

$$2x^2 - 3x + 5 + \frac{-4x - 11}{x^2 + 3x + 2}$$

ANSWERS

1. $3 + \frac{7}{x-4}$

2. $x^2 - 3 + \frac{10x-10}{x^2+3x-1}$

3. $2x^2 - 3x + 5 + \frac{-4x-11}{x^2+3x+2}$