# Even and Odd Functions (algebraically)

**Do Now:**

1.) Given each equation, determine the degree and the end behavior.

a. $f(x) = -x^2 - 4x^3 + 15x + 16$

$f(x) = -4x^3 - x^2 + 15x + 16$

Degree: 3 (odd)

Sign of leading coefficient: negative

$x \to \infty, \quad f(x) \to$ $-\infty$

$x \to -\infty, \quad f(x) \to$ $\infty$

$\uparrow \quad \downarrow$

b. $f(x) = x^4 - 3x^2 - 5x + 10$

Degree: 4 (positive)

Sign of leading coefficient: positive

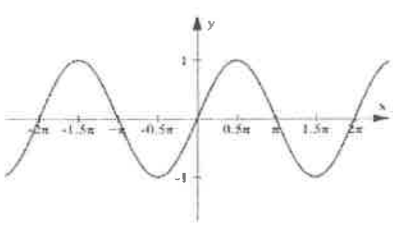
$x \to \infty, \quad f(x) \to$ $\infty$

$x \to -\infty, \quad f(x) \to$ $\infty$

$\uparrow \uparrow$

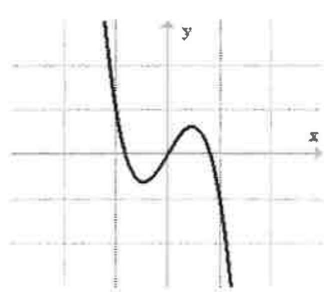
Determine if the following are even functions, odd functions, or neither.

2.)



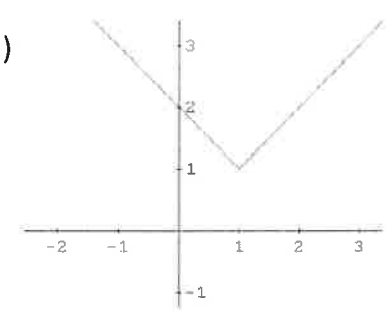

Odd

3.)




odd

4.)




neither

## Even Functions

→ Exact

- symmetric with respect to the y-axis
- $f(-x) = f(x)$ which means that an input of x gives the same output as an input of $-x$

To evaluate algebraically whether if a function is even, plug in $-x$ for $x$.
If $f(x) = f(-x)$, then the function is even.

**Example 1**: Is the function $f(x) = x^2 + 7$ even?

Evaluate: $f(\ x) = x^2 + 7$

$f(-x) = (-x)^2 + 7$

$= x^2 + 7$

They are exactly the same, so it is an even function

**Example 2**: Is the function $f(x) = (x-3)^2 + 7$ even?

No, it is not even

Evaluate: $f(\ x) = (x-3)^2 + 7 = (x-3)(x-3) + 7$
$x^2 - 3x - 3x + 9 + 7 = x^2 - 6x + 16$

$f(-x) = (-x-3)^2 + 7 = (-x-3)(-x-3) + 7$
$x^2 + 3x + 3x + 9 + 7 = x^2 + 6x + 16$

**Example 3**: Is the function $f(x) = \frac{5}{x^2}$ even?

Evaluate: $f(\ x) = \frac{5}{x^2}$

$f(-x) = \frac{5}{(-x)^2} = \frac{5}{x^2}$

They are exactly the same, so it is an even function

## ODD FUNCTIONS

↳ Opposite

- symmetric with respect to the origin
- $f(-x) = -f(x)$ which means that an input of $-x$ is equal to $-f(x)$

To evaluate algebraically whether if a function is odd, plug in $-x$ for $x$.
If $f(-x) = -f(x)$, then the function is odd.

Example 4: Is the function $f(x) = x^5 + 3x^3 - 7x$ odd?

Evaluate: $f(\ x) = x^5 + 3x^3 - 7x$

$f(-x) = (-x)^5 + 3(-x)^3 - 7(-x)$

$-x^5 - 3x^3 + 7x$

$-(x^5 + 3x^3 - 7x)$

They are opposite, so it is an odd function

Example 5: Is the function $f(x) = x^3 + 1$ odd?

Evaluate: $f(\ x) = x^3 + 1$

$f(-x) = (-x)^3 + 1$

$-x^3 + 1$

Not opposite, so it is not odd

Example 6: Is the function $f(x) = \frac{7}{x}$ odd?

Evaluate: $f(\ x) = \frac{7}{x}$

$f(-x) = \frac{7}{-x} = -\frac{7}{x}$

They are opposite, so it is an odd function

SUMMARY:

Always plug in $(-x)$ for $x$

Even → Exact
Odd → Opposite

# Even and Odd Functions matching workspace

$f(x) = -3x^4 + 6x^2 - 7$

$f(-x) = -3(-x)^4 + 6(-x)^2 - 7$

$f(-x) = -3x^4 + 6x^2 - 7$

$f(x) = f(-x)$ even

$f(x) = 2x^5 - 5x^3 + 2x$

$f(-x) = 2(-x)^5 - 5(-x)^3 + 2(-x)$

$f(-x) = -2x^5 + 5x^3 - x$

$= -(2x^5 - 5x^3 + x)$

$f(-x) = -f(x)$ odd

$f(x) = -7x^6 + 4x^8 - 2$

$f(-x) = -7(-x)^6 + 4(-x)^8 - 2$

$f(-x) = -7x^6 + 4x^8 - 2$

$f(x) = f(-x)$ even

$f(x) = 12x^4 + x^6 + 3x^3$

$f(-x) = 12(-x)^4 + (-x)^6 + 3(-x)^3$

$f(-x) = 12x^4 + x^6 - 3x^3$

$= -(-12x^4 - x^6 + 3x^3)$

neither

$f(x) = x^4 + 3x^6 - 2x^9$

$f(-x) = (-x)^4 + 3(-x)^6 - 2(-x)^9$

$f(-x) = x^4 + 3x^6 + 2x^9$

$= -(-x^4 - 3x^6 - 2x^9)$

neither

$f(x) = -3x^5 + 6x^3 - 4x^9$

$f(-x) = -3(-x)^5 + 6(-x)^3 - 4(-x)^9$

$f(-x) = +3x^5 - 6x^3 + 4x^9$

$= -(-3x^5 + 6x^3 - 4x^9)$

$f(-x) = -f(x)$ odd

Even and Odd Functions Extension

**EVEN AND ODD FUNCTIONS**

A function is known as **even** if $f(-x)=f(x)$ for every value of $x$ in the domain of $f(x)$.

A function is known as **odd** if $f(-x)=-f(x)$ every value of $x$ in the domain of $f(x)$.

In #1 – 6, algebraically determine if each of the following functions are even, odd or neither.

1. $f(x)=2x$

$2(-x)$

$=-2x$

odd

2. $f(x)=x^2+x+3$

$(-x)^2+(-x)+3$

$x^2-x+3$

Neither

3. $f(x)=x^2+7$

$(-x)^2+7$

$x^2+7$

even

4. $f(x)=2x^8-4x^2+15$

$2(-x)^8-4(-x)^2+15$

$2x^8-4x^2+15$

even

5. $f(x)=x^5+3x^3-x$

$(-x)^5+3(-x)^3-(-x)$

$-x^5-3x^3+x$

odd

6. $f(x)=x^6+3x^2-8$

$(-x)^6+3(-x)^2-8$

$x^6+3x^2-8$

even

Given the following functions, find the requested information:

7.) If $f(x)$ is odd and $f(6)=1$, then find $f(-6)$.

opposite

$-1$

8.) If $f(x)$ is even and $f(2)=-3$, then find $f(-2)$.

exact

$-3$

9.) If $f(x)$ is odd and $f(-5)=8$, then find $f(5)$.

opposite

$-8$

10.) If $f(x)$ is even and $f(-10)=-1$, then find $f(10)$.

exact

$-1$

11. Sketch the function $f(x) = -x^2(x-5)(x+3)^2$

Degree: 5 (odd) ↑ ↓

negative

$X=0$ $M=2$ B

$X=5$ $M=1$ C

$X=-3$ $M=2$ B