Solving Square Root Applications

Warm up:

Solve for x: $(\sqrt{x-4})^2 = (\sqrt{2x-13})^2$

$$x - 4 = 2x - 13$$
$$-x \quad +13 \quad -x \quad +13$$
$$9 = x$$

check:

$$\sqrt{9-4} = \sqrt{2(9)-13}$$
$$\sqrt{5} = \sqrt{5} \checkmark$$

Equations involving square roots arise in many applied contexts. As always, the key to solving these equations lies in the applications of inverse operations. The key inverse relationship in these equations is that between taking a square root and squaring.

***Exercise* #1**: In an amusement park ride, a rider suspended by cables swings back and forth from a tower. The maximum speed *v* (in meters per second) of the rider can be approximated by $v = \sqrt{2gh}$, where *h* is the height (in meters) at the top of each swing and *g* is the acceleration due to gravity ($g \approx 9.8$ m/sec^2). Determine the height, to the *nearest hundredth*, at the top of the swing of a rider whose maximum speed is 15 meters per second.

$$V = \sqrt{2gh}$$
$$15 = \sqrt{2(9.8)h}$$
$$(15)^2 = (\sqrt{19.6h})^2$$
$$\frac{225}{19.6} = \frac{19.6h}{19.6}$$
$$h = 11.47959184\ldots$$
$$\boxed{h = 11.48}$$

Exercise #2: The speed s (in miles per hour) of a car can be given by $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the stopping distance (in feet). The table shows the coefficient of friction for different surfaces. How much further was the stopping distance of a car traveling 45 miles per hour on wet asphalt compared to dry asphalt? Justify your answer.

Surface	Coefficient of friction, f
dry asphalt	0.75
wet asphalt	0.30
snow	0.30
ice	0.15

dry asphalt:

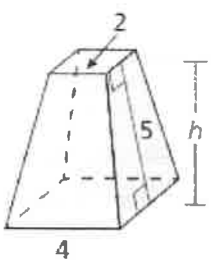
$$45 = \sqrt{30(.75)\,d}$$
$$(45)^2 = \left(\sqrt{22.5d}\right)^2$$
$$\frac{2025}{22.5} = \frac{22.5d}{22.5}$$
$$90 = d$$

wet asphalt:

$$45 = \sqrt{30(.30)\,d}$$
$$(45)^2 = \left(\sqrt{9d}\right)^2$$
$$\frac{2025}{9} = \frac{9d}{9}$$
$$225 = d$$

$$225 - 90 = \boxed{135}$$

Exercise #3: You are trying to determine the height of a truncated pyramid, which cannot be measured directly. The height h and slant height l of the truncated pyramid are related by the formula $l = \sqrt{h^2 + \frac{1}{4}(b_2 - b_1)^2}$. In the given formula, b_1 and b_2 are the side lengths of the upper and lower bases of the pyramid, respectively. When $l = 5$, $b_1 = 2$, and $b_2 = 4$, what is the height of the pyramid, in simplest radical form?



$$5 = \sqrt{h^2 + \tfrac{1}{4}(4-2)^2}$$
$$5 = \sqrt{h^2 + \tfrac{1}{4}(2)^2}$$
$$(5)^2 = \left(\sqrt{h^2 + 1}\right)^2$$
$$25 = h^2 + 1$$
$$-1 \qquad -1$$
$$\sqrt{24} = \sqrt{h^2}$$
$$\pm\sqrt{24} = h$$
$$\sqrt{4 \cdot 6}$$
$$\pm 2\sqrt{6} = h$$

reject negative

$$\boxed{h = 2\sqrt{6}}$$

Solving Square Root Applications Practice

1. In a hurricane, the mean sustained wind velocity v (in meters per second) can be modeled by $v(p)=6.3\sqrt{1013-p}$, where p is the air pressure (in millibars) at the center of the hurricane. Find the air pressure, to the *nearest ten-thousandth*, at the center of the hurricane when the mean sustained wind velocity is 54.5 meters per second.

$$54.5 = 6.3\sqrt{1013-p}$$
$$(8.650793...)^2 = (\sqrt{1013-p})^2$$
$$74.836230... = 1013 - p$$
$$-938.163769... = -p$$
$$938.163769... = p$$
$$\boxed{p = 938.1638}$$

2. The Beaufort wind scale was devised to measure wind speed. The Beaufort numbers B, which range from 0 to 12, can be modeled by $B=1.69\sqrt{s+4.25}-3.55$, where s is the wind speed (in miles per hour). What is the wind speed for $B=0$ to the *nearest thousandth*?

$$0 = 1.69\sqrt{s+4.25} - 3.55$$
$$3.55 = 1.69\sqrt{s+4.25}$$
$$(2.1005917...)^2 = (\sqrt{s+4.25})^2$$
$$4.4124855... = s + 4.25$$
$$-4.25 \qquad -4.25$$
$$.1624855... = s$$
$$\boxed{s = .162}$$

Beaufort number	Force of wind
0	calm
3	gentle breeze
6	strong breeze
9	strong gale
12	hurricane

→ x – values

Write the domain of wind speeds represented by the Beaufort number. Round all values to the *nearest thousandth.*

$$12 = 1.69\sqrt{s+4.25} - 3.55$$
$$15.55 = 1.69\sqrt{s+4.25}$$
$$(9.20118343...)^2 = (\sqrt{s+4.25})^2$$
$$84.66177655 = s + 4.25$$
$$-4.25 \qquad -4.25$$
$$80.411776... = s$$

Domain:

$[.162, 80.412]$

OR

$.162 \le x \le 80.412$

3. **Challenge Question:** A burning candle has a radius of r inches and was initially h_0 inches tall. After t minutes, the height of the candle has been reduced to h inches. These quantities are related by the formula $r = \sqrt{\dfrac{kt}{\pi(h_0 - h)}}$ where k is a constant. Rewrite the formula, solving for h in terms of t, k, r, and h_0.

$$(r)^2 = \left(\sqrt{\frac{Kt}{\pi(h_0 - h)}}\right)^2$$

$$\frac{r^2}{1} = \frac{Kt}{\pi(h_0 - h)}$$

$$Kt = r^2\pi(h_0 - h)$$

$$\frac{Kt}{r^2\pi} = h_0 - h$$

$$-h_0 \qquad -h_0$$

$$\frac{\frac{Kt}{r^2\pi} - h_0}{-1} = \frac{-h}{-1}$$

$$\boxed{-\frac{Kt}{r^2\pi} + h_0 = h}$$

Suppose the radius of a candle is 0.875 inch, its initial height is 6.5 inches, and $k = 0.04$. Use your formula to determine the height of the candle, to the *nearest tenth of an inch*, after burning 45 minutes.

$r = .0875$

$h_0 = 6.5$

$K = .04$

$t = 45$

$$\frac{-.04(45)}{(.875)^2\pi} + 6.5 = h$$

$$5.751646961\ldots = h$$

$$\boxed{h = 5.8}$$