Lesson 3-5

Solving Radical Equations

A radical equation is an equation in which the variable is hiding inside a radical symbol (in the radicand)

Which of the following is a radical equation?

$\sqrt{x} + 3 = 10$

$x + \sqrt{3} = 10$

Steps to Solve a Radical Equation Algebraically

1. Algebraically **isolate** the radical in the equation.
2. **Square** both sides of the resulting equation
3. Now **solve** for *x* using an appropriate method for the type of equation that you have
4. **Check** your answers. You may have extraneous roots! (To be explained later this period.)

Example 1: $(\sqrt{x-1})^2 = (2)^2$

$x - 1 = 4$

$+1 \quad +1$

$x = 5$

Check

$\sqrt{5-1} = 2$

$\sqrt{4} = 2$

$2 = 2$ ✓

Example 2: $\sqrt{2x-1}+5=2$

$-5 \quad -5$

$(\sqrt{2x-1})^2=(-3)^2$

$2x-1=9$

$+1 \quad +1$

$2x=10$

$x=5$

Reject

No Solution

Check

$\sqrt{2(5)-1}+5=2$

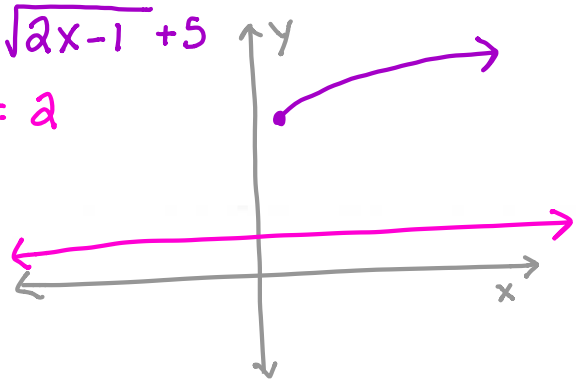
$\sqrt{9}+5=2$

$3+5=2$

$8 \neq 2$

$Y_1=\sqrt{2x-1}+5$

$Y_2=2$



Example 3: $-3=\sqrt{30-2x}-x$

$+x \quad +x$

$(x-3)^2=(\sqrt{30-2x})^2$

$(x-3)(x-3)=30-2x$

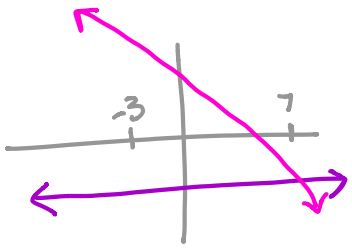
$x^2-6x+9=30-2x$

$x^2-4x-21=0$

$(x+3)(x-7)=0$

$x=-3$ | $x=7$

Reject



Check

$-3=\sqrt{30-2(-3)}-(-3)$

$-3=\sqrt{36}+3$

$-3=6+3$

$-3 \neq 9$

$-3=\sqrt{30-2(7)}-7$

$-3=\sqrt{16}-7$

$-3=4-7$

$-3=-3$ ✓

Example 4: $\left(2\sqrt{x+8}\right)^2 = \left(3\sqrt{x-2}\right)^2$

$4(x+8) = 9(x-2)$

$4x + 32 = 9x - 18$

$-5x = -50$

$x = 10$

Check

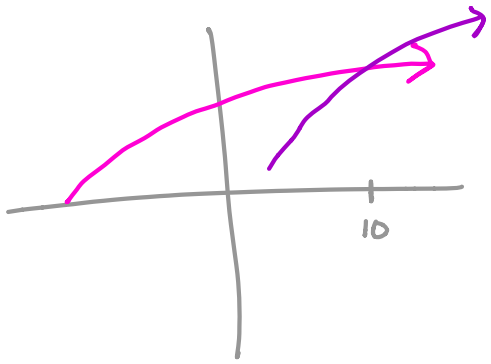
$2\sqrt{10+8} = 3\sqrt{10-2}$

$2\sqrt{18} = 3\sqrt{8}$

$2\sqrt{9\cdot 2} = 3\sqrt{4\cdot 2}$

$2(3)\sqrt{2} = 3(2)\sqrt{2}$

$6\sqrt{2} = 6\sqrt{2}$ ✓



Solving Radical Equations Partner Practice

Directions: Solve each equation. Check your answers for extraneous solutions!

1) $\left(\sqrt{4x}\right)^2=(6)^2$

$\frac{4x}{4}=\frac{36}{4}$

$\boxed{x=9}$

check:

$\sqrt{4(9)}=6$

$\sqrt{36}=6$

$6=6$ ✓

2) $6-\sqrt{x}=11$

$-\sqrt{x}=5$

$\left(\sqrt{x}\right)^2=(-5)^2$

$x=25$

Reject

$\boxed{\text{No Solution}}$

check:

$6-\sqrt{25}=11$

$6-5=11$

$1\neq 1$

3) $5+\sqrt{4x-3}=10$

$\left(\sqrt{4x-3}\right)^2=(5)^2$

$4x-3=25$

$4x=28$

$\boxed{x=7}$

check:

$5+\sqrt{4(7)-3}=10$

$5+\sqrt{28-3}=10$

$5+\sqrt{25}=10$

$5+5=10$

$10=10$ ✓

4) $\sqrt{3x-2}+5=3$

$\left(\sqrt{3x-2}\right)^2=(-2)^2$

$3x-2=4$

$3x=6$

$x=2$

Reject

$\boxed{\text{No Solution}}$

check:

$\sqrt{3(2)-2}+5=3$

$\sqrt{6-2}+5=3$

$\sqrt{4}+5=3$

$2+5=3$

$7\neq 3$

5) $\left(\sqrt{15+x^2}\right)^2=(5-x)^2$

$15+x^2=(5-x)(5-x)$

$15+x^2=25-5x-5x+x^2$

$15+x^2=x^2-10x+25$

$-25\quad -25$

$-10=-10x$

$1=x$

Check:

$\sqrt{15+(1)^2}=5-1$

$\sqrt{15+1}=4$

$\sqrt{16}=4$

$4=4$ ✓

6) $\sqrt{5x-3}-\sqrt{x+17}=0$

$\left(\sqrt{5x-3}\right)^2=\left(-\sqrt{x+17}\right)^2$

$5x-3=x+17$

$-x\ +3\quad -x\ +3$

$4x=20$

$x=5$

Check:

$\sqrt{5(5)-3}-\sqrt{5+17}=0$

$\sqrt{25-3}-\sqrt{22}=0$

$\sqrt{22}-\sqrt{22}=0$

$0=0$ ✓

7) $(x-1)^2=\left(\sqrt{5x-9}\right)^2$

$(x-1)(x-1)=5x-9$

$x^2-2x+1=5x-9$

$x^2-7x+10=0$

$(x-2)(x-5)=0$

$x=2$ | $x=5$

Check:

$2-1=\sqrt{5(2)-9}$

$1=\sqrt{10-9}$

$1=\sqrt{1}$

$1=1$ ✓

$5-1=\sqrt{5(5)-9}$

$4=\sqrt{25-9}$

$4=\sqrt{16}$

$4=4$ ✓

8) $\left(\sqrt{x^2+8}\right)^2=\left(2\sqrt{2x-1}\right)^2$

$x^2+8=4(2x-1)$

$x^2+8=8x-4$

$x^2-8x+12=0$

$(x-2)(x-6)=0$

$x=2$ | $x=6$

Check:

$\sqrt{(2)^2+8}=2\sqrt{2(2)-1}$

$\sqrt{4+8}=2\sqrt{4-1}$

$\sqrt{12}=2\sqrt{3}$

$\sqrt{4\cdot 3}$

$2\sqrt{3}=2\sqrt{3}$ ✓

9) $(x)^2=(\sqrt{6x+7})^2$

$x^2 = 6x+7$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x=7$ | $x\neq -1$ Reject

Check:

$7=\sqrt{6(7)+7}$

$7=\sqrt{42+7}$

$7=\sqrt{49}$

$7=7$ ✓

$-1=\sqrt{6(-1)+7}$

$-1=\sqrt{-6+7}$

$-1=\sqrt{1}$

$-1\neq 1$

10) $(\sqrt[3]{2x})^3=(-2)^3$

$\frac{2x}{2}=\frac{-8}{2}$

$x=-4$

Check:

$\sqrt[3]{2(-4)}=-2$

$\sqrt[3]{-8}=-2$

$-2=-2$ ✓

11) $(\sqrt[3]{2x+1})^3=(3)^3$

$2x+1=27$

$2x=26$

$x=13$

Check:

$\sqrt[3]{2(13)+1}=3$

$\sqrt[3]{26+1}=3$

$\sqrt[3]{27}=3$

$3=3$ ✓

12) $(\sqrt{x}-4)^2=(\sqrt{9x})^2$

$(\sqrt{x}-4)(\sqrt{x}-4)=9x$

$x-4\sqrt{x}-4\sqrt{x}+16=9x$

$x-8\sqrt{x}+16=9x$

$\frac{-8\sqrt{x}}{-8}+\frac{16}{-8}=\frac{8x}{-8}$

$\sqrt{x}-2=-x$

$+2 \quad +2$

$(\sqrt{x})^2=(-x+2)^2$

$x=(-x+2)(-x+2)$

$x=x^2-4x+4$

$0=x^2-5x+4$

$0=(x-1)(x-4)$

$x\neq 1$ Reject | $x\neq 4$ Reject

No Solution

Check:

$\sqrt{4}-4=\sqrt{9(4)}$

$2-4=\sqrt{36}$

$-2\neq 6$

$\sqrt{1}-4=\sqrt{9(1)}$

$1-4=\sqrt{9}$

$-3\neq 3$