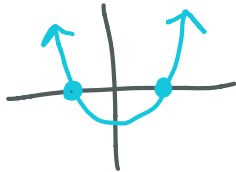Solving Equations with Complex Solutions

Warm Up: Find the roots of each equation below. Leave your answers in simplest radical form in terms
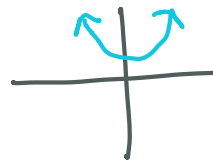of *i* when appropriate. Check your answers.

1. $x^2 - 9 = 0$

$\underline{\quad +9 \quad +9 \quad}$

$X^2 = 9$

$X = \pm 3$

2. $x^2 + 9 = 0$

$\underline{\quad -9 \quad -9 \quad}$

$X^2 = -9$

$X = \pm 3i$

3. $2x^2 + 47 = 11$

$\underline{\quad -47 \quad -47 \quad}$

$2X^2 = -36$

$\sqrt{X^2} = \sqrt{-18}$

$X = \pm\sqrt{-18}$

$X = i\sqrt{18}$

$X = i\sqrt{9 \cdot 2}$

$X = 3i\sqrt{2}$

4. $4x^2 + 20 = 0$

$4X^2 = -20$

$\sqrt{X^2} = \sqrt{-5}$

$X = \pm\sqrt{-5}$

$X = \pm i\sqrt{5}$

- -
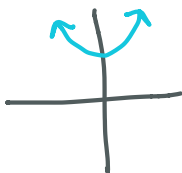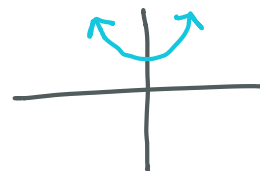
**Exercise #1:** Which graph represents a quadratic function with imaginary solutions?

(1)

(3)

(2)

(4)
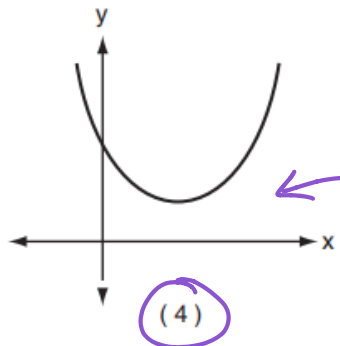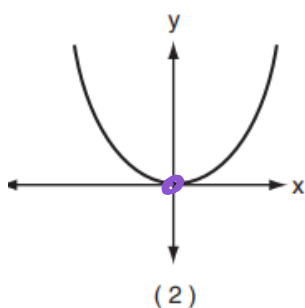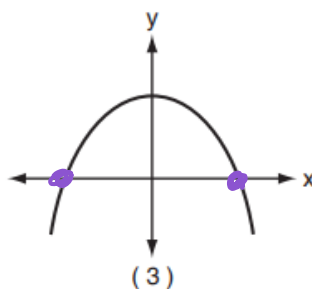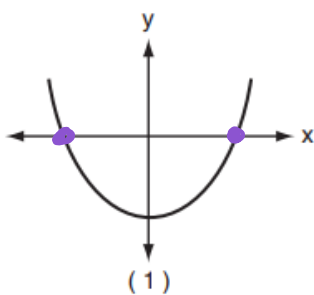
← No x- intercepts!

Exercise #2: Solve algebraically for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

$$\begin{array}{r} +16x \quad +16x \\ \hline x^4 + 4x^3 + 4x^2 + 16x = 0 \end{array}$$

$$x(x^3 + 4x^2 + 4x + 16) = 0$$

$$x(x^2(x+4) \{ + 4(x+4)) = 0$$

$$x(x+4)(x^2+4) = 0$$

$\boxed{X=0}$	$\begin{array}{l} x+4=0 \\ \boxed{x=-4} \end{array}$	$\begin{array}{l} x^2+4=0 \\ x^2 = -4 \\ x = \pm\sqrt{-4} \\ \boxed{x = \pm 2i} \end{array}$

Exercise #3: A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

4 real roots →

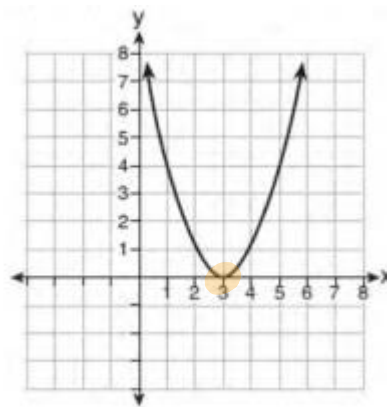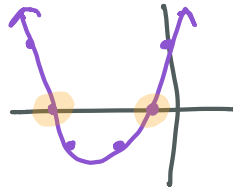(1) ✗

(3)

(2)

4 real roots ←

(4) ✗
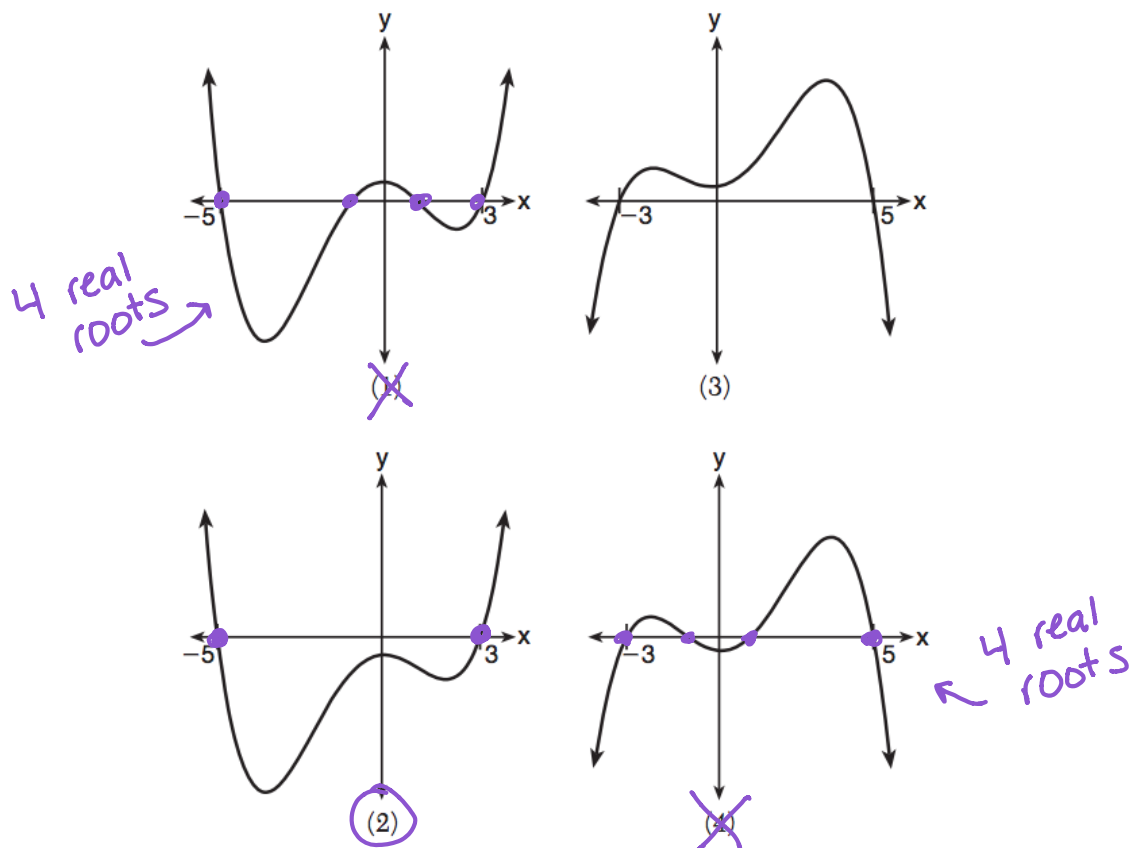
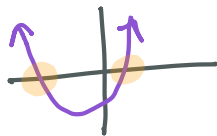***Exercise #4:*** Which representation of a quadratic has imaginary roots?

(1)

x	y
−2.5	2
−2.0	0
−1.5	−1
−1.0	−1
−0.5	0
0.0	2

(3)

(2) $2(x + 3)^2 = 64$

(4) $2x^2 + 32 = 0$

- -
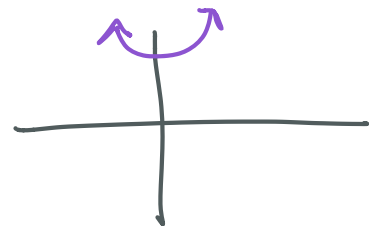
Solving Equations with Complex Solutions Practice

1. What is the solution set of the equation $3x^5 - 48x = 0$?

$3x(x^4 - 16) = 0$
$3x(x^2 + 4)(x^2 - 4) = 0$ (3) $\{0, \pm 2, \pm 2i\}$
$3x(x^2 + 4)(x + 2)(x - 2) = 0$
$3x = 0 \quad | \quad x^2 + 4 = 0 \quad | \quad x = -2 \quad | \quad x = 2$
$x = 0 \quad | \quad x^2 = -4$
$\qquad\qquad x = \sqrt{-4}$
$\qquad\qquad x = \pm 2i$

(1) $\{0, \pm 2\}$

(2) $\{0, \pm 2, 3\}$

(4) $\{\pm 2, \pm 2i\}$

2. Which equation has $1 - i$ as a solution?

(1) $x^2 + 2x - 2 = 0$ (3) $x^2 - 2x - 2 = 0$

(2) $x^2 + 2x + 2 = 0$ (4) $x^2 - 2x + 2 = 0$

3. Solve: $x^4 - 3x^2 = 4$

$$X^4 - 3X^2 - 4 = 0$$

$$X^4 + 1X^2 - 4X^2 - 4 = 0$$

$$X^2(X^2+1) \mid -4(X^2+1) = 0$$

$$(X^2+1)(X^2-4) = 0$$

$$X^2+1 = 0 \mid X^2-4 = 0$$

$$X^2 = -1 \qquad X^2 = 4$$

$$X = \pm\sqrt{-1} \qquad \boxed{X = \pm 2}$$

$$\boxed{X = \pm 1i}$$

4. Solve: $x^4 + 4x^2 = 32$.

$$X^4 + 4X^2 - 32 = 0$$

$$X^4 - 4X^2 + 8X^2 - 32 = 0$$

$$X^2(X^2-4) \mid 8(X^2-4) = 0$$

$$(X^2-4)(X^2+8) = 0$$

$$(X+2)(X-2)(X^2+8) = 0$$

$$\boxed{X = -2} \mid \boxed{X = 2} \mid X^2+8 = 0$$

$$X^2 = -8$$

$$X = \pm\sqrt{-8}$$

$$X = \pm i\sqrt{4 \cdot 2}$$

$$\boxed{X = \pm 2i\sqrt{2}}$$

5. Which equation represents a quadratic function with imaginary solutions?

(1) $x^2 + 3x - 5 = 0$ (3) $(x-3)^2 - 7 = 0$ *look at each graph

(2) $x^2 - 4x = 5(x+2)$ (4) $x^2 + 2x + 2 = 0$

A   M

6. Solve the polynomial equation $0 = x^4 + 13x^2 + 36$?

$$x^4 + 4x^2 + 9x^2 + 36 = 0$$
$$x^2(x^2+4) + 9(x^2+4) = 0$$
$$(x^2+4)(x^2+9) = 0$$

$x^2 + 4 = 0$ | $x^2 + 9 = 0$
$x^2 = -4$ $x^2 = -9$
$x = \pm\sqrt{-4}$ $x = \pm\sqrt{-9}$

$\boxed{x = \pm 2i}$ $\boxed{x = \pm 3i}$

Degree = 4

⊛ Make sure you have 4 solutions!

7. Find the zeros of the following function: $y = x^3 + 2x^2 + 16x + 32$?

$$x^2(x+2) | + 16(x+2)$$

$$(x+2)(x^2+16) = 0$$

$\boxed{x = -2}$ $x^2 + 16 = 0$
$x^2 = -16$
$x = \pm\sqrt{-16}$
$\boxed{x = \pm 4i}$