Name: hey

Algebra II

Date: ____________

Lesson 1-14

Polynomial Identities

A closer look at Questions #2 and #3 from HW #1-13……

#2 from HW

work with one side until it looks like the other!

- Consider the polynomial identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Show that this is an identity.

$(a+b)(a+b)(a+b)$

$(a+b)(a^2+ab+ba+b^2)$

$(a+b)(a^2+2ab+b^2)$

$a^3+2a^2b+ab^2+ba^2+2ab^2+b^3$

$a^3+3a^2b+3ab^2+b^3 \quad = a^3+3a^2b+3ab^2+b^3$ ✓

#3 from HW

- Algebraically prove that $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$ is an identity by manipulating the right side of the equation.

$x^6-x^4y^2+x^2y^4+y^2x^4-x^2y^4+y^6$

✓ $x^6+y^6 = x^6+y^6$

Because polynomials consist of basic operations on variables, they can be manipulated using the associative, commutative, and distributive properties (as you have done many times). These operations can result in what are known as **polynomial identities**. An identity is defined more broadly below:

IDENTITIES

An **identity** is an **equation** that is **true** for all values of the replacement variable or variables.

Exercise #1: Algebraically prove that $x^2 - y^2 = (x-y)(x+y)$.

$$x^2 - y^2 = x^2 + \underline{xy} - \underline{xy} - y^2$$

$$x^2 - y^2 = x^2 - y^2 \checkmark$$

⊛ Note:

you can not just plug in a value and show that it is true....

Exercise #2: Verify the following identity for all values of *a*, *b*, and *c*:

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$$

$$(a+b+c)(a+b+c) = a^2+b^2+c^2+2(ab+bc+ac)$$

	a	b	c
a	a^2	ab	ac
b	ab	b^2	bc
c	ac	bc	c^2

$$a^2+b^2+c^2+2ab+2bc+2ac = a^2+b^2+c^2+2(ab+bc+ac)$$

$$a^2+b^2+c^2+2(ab+bc+ac) = a^2+b^2+c^2+2(ab+bc+ac) \checkmark$$

***Exercise* #3:** Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$1 + \frac{1}{x^3+8} = 1 + \frac{1}{x^3+8} \checkmark$$

$$\begin{array}{r} 1 \\ x^3+0x^2+0x+8 \overline{\big)\, x^3+0x^2+0x+9} \\ -(x^3+0x^2+0x+8) \\ \hline 1 \end{array}$$

$$1 + \frac{1}{x^3+8}$$

POLYNOMIAL IDENTITIES PRACTICE

1) Algebraically prove the following identity: $x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$$x^3 - y^3 = x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{x^2y} - \cancel{xy^2} - y^3$$

$$x^3 - y^3 = x^3 - y^3 \checkmark$$

2) Algebraically prove the following identity: $(x^2-y^2)^2 + (2xy)^2 = (x^2+y^2)^2$

* Expand Both Sides

$$(x^2-y^2)^2 + (2xy)^2 = (x^2+y^2)^2$$

$$(x^2-y^2)(x^2-y^2) + 4x^2y^2 = (x^2+y^2)(x^2+y^2)$$

$$x^4 - x^2y^2 - x^2y^2 + y^4 + 4x^2y^2 = x^4 + x^2y^2 + x^2y^2 + y^4$$

$$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 \checkmark$$

Challenge Question

Algebraically determine the values of *h* and *k* to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x-4)(2x^2 + hx + 3) + k$$

$$\cancel{2x^3} - 10x^2 + 11x - 7 = \cancel{2x^3} + hx^2 + \cancel{3x} - \cancel{8x^2} - 4hx - \cancel{12} + k$$

$$\cancel{-2x^3} + 8x^2 - 3x + 12 \quad \cancel{-2x^3} \quad \cancel{-3x} \quad \cancel{+8x^2} \quad \cancel{+12}$$

$$-2x^2 + 8x + 5 = hx^2 - 4hx + k$$

$h = -2$

$k = 5$