Imaginary Numbers

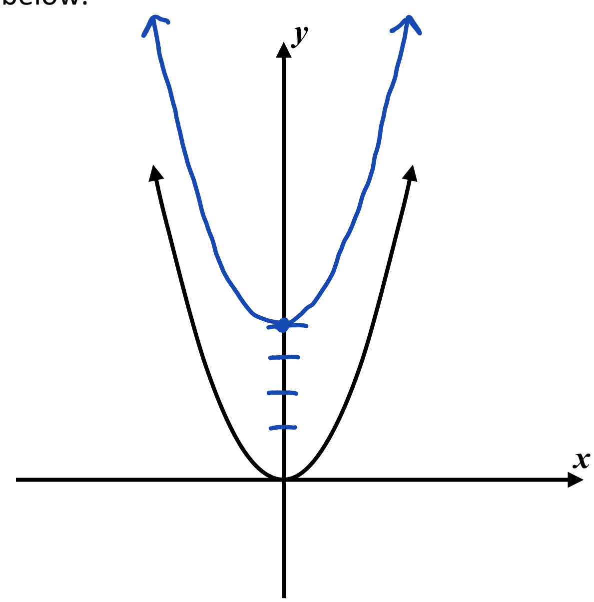
Warm up:

On the axes below, a sketch of $y = x^2$ is shown. Now, consider the parabola whose equation is given in function notation as $y = x^2 + 4$.

(a) How is the graph of $y = x^2$ shifted to produce the graph of $f(x)$?

Shifted up 4 units

(b) Create a quick sketch of $f(x)$ on the axes below.



(c) What can be said about the x-intercepts of the function $y = f(x)$?

There are no x-intercepts

(d) Algebraically, show that these intercepts do not exist, in the Real Number System, by solving the quadratic $x^2 + 4 = 0$.

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \pm\sqrt{-4}$$

you can't take the square root of a negative #

Since we cannot solve this equation using Real Numbers, we introduce a new number, called i, the basis of **imaginary numbers**. Its definition allows us to now have a result when finding the square root of a negative real number. Its definition is given below.

The Definition of the Imaginary Number I

$$i = \sqrt{-1}$$

A pure imaginary number can be written in bi form where b is a real number and i is $\sqrt{-1}$

Examples: $2i,\ -5i,\ 3i\sqrt{3},\ \frac{3}{2}i$

Example 1: Simplify each in terms of i.

(a) $\sqrt{-9}$

$3i$

(b) $2\sqrt{-49}$

$2 \cdot 7i$

$14i$

(c) $3\sqrt{-162}$

$3i\sqrt{162}$

$3i\sqrt{\underline{\underline{81}} \cdot 2}$

$3i \cdot 9\sqrt{2}$

$27i\sqrt{2}$

Complex Numbers

A complex number is any number that can be written in the standard form $a + bi$, where a and b are real numbers and i is the imaginary unit.

Complex Number: Standard $a + bi$ form	a	bi
$7 + 2i$	7	$2i$
$1 - 5i$	1	$-5i$
$8i$	0	$8i$
$\frac{-2+3i}{5} = \frac{-2}{5} + \frac{3i}{5}$	$\frac{-2}{5}$	$\frac{3i}{5}$

Example 2: Express in simplest $a + bi$ form

(a) $4 + \sqrt{-36}$

$4 + 6i$

real

imaginary

(b) $8 - \sqrt{-24}$

$8 - i\sqrt{24}$

$8 - i\sqrt{\underline{\underline{4}} \cdot 6}$

$8 - 2i\sqrt{6}$

Not like terms
(we cant subtract these)

Example 3: Simplify each of the following powers of i.

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

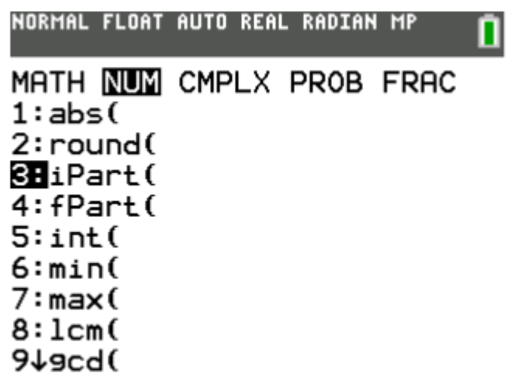
$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

Powers of i on your calculator:

Math > NUM 3:iPart(



Example 4: Determine the value of n in simplest form: $i^{13} + i^{18} + i^{31} + n = 0$

$(\cancel{i}) + (-1) + (-\cancel{i}) + n = 0$

$-1 + n = 0$

$\boxed{n = 1}$

Imaginary Numbers Practice

1. Simplify each of the following powers of *i* into either $-1, 1, i,$ or $-i$.

(a) i^{25} (b) i^{55} (c) i^{16} (d) i^{34}

i $-i$ 1 -1

2. The expression $2i^2 + 3i^3$ is equivalent to

(1) $-2 - 3i$ $2(-1)+3(-i)$ (3) $-2 + 3i$

(2) $2 - 3i$ $-2-3i$ (4) $2 + 3i$

3. Express in simplest form:

(a) $i^{100} + i^{101} + i^{102}$

$\cancel{(1)}+(i)+\cancel{(-1)}$

i

(b) $i^8 + i^9 + i^{10} + i^{11}$

$\cancel{(1)}+\cancel{(i)}+\cancel{(-1)}+\cancel{(-i)}$

0

(b) (c) $i^{16} + i^6 - 2i^5 + i^{13}$

$\cancel{1}\cancel{-1}-2(i)+i$

$-2i+i$

$-1i$

(d) $\frac{i^3}{i^{16}} = \frac{-i}{1} = -i$

4. In simplest form $\sqrt{-300}$ is equivalent to

(1) $3i\sqrt{10}$ $i\sqrt{300}$ (3) $10i\sqrt{3}$

$i\sqrt{100 \cdot 3}$

(2) $5i\sqrt{12}$ $10i\sqrt{3}$ (4) $12i\sqrt{5}$