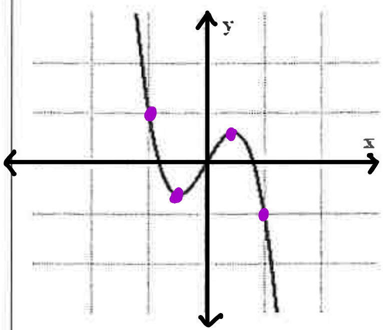
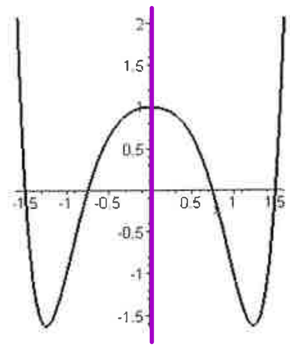
# Even and Odd Functions Graphically

**Do Now #1:** Each of the following functions are labeled **even**, **odd**, or **neither**. Look at the graphs and draw conclusions about what makes a function **even** and what makes a function **odd**. Write your thoughts below.

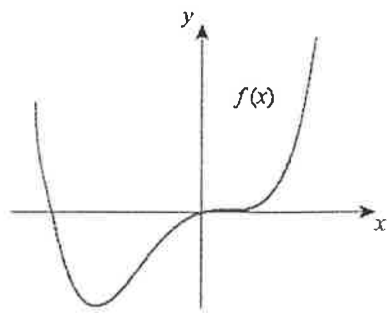
**Odd:**




**Even:**




**Neither:**




A function is **EVEN** if: It is symmetric over the y- axis

A function is **ODD** if: It is symmetric over the origin

**Do Now #2:** Each of the following functions are labeled **even**, **odd**, or **neither**. Look at the tables and draw conclusions about what makes a function **even** and what makes a function **odd**. Write your thoughts below.

**Odd:** opposite sign

| x | y |
|---|---|
| -3 | -54 |
| -2 | -16 |
| -1 | -2 |
| 0 | 0 |
| 1 | 2 |
| 2 | 16 |

**Even:** exactly the same

| x | y |
|---|---|
| -3 | 686 |
| -2 | 46 |
| -1 | -2 |
| 0 | 2 |
| 1 | -2 |
| 2 | 46 |

**Neither:**

| x | y |
|---|---|
| -3 | -8720 |
| -2 | -503 |
| -1 | -2 |
| 0 | 1 |
| 1 | 4 |
| 2 | 505 |

A function is **EVEN** if: $f(x) = -f(-x)$

A function is **ODD** if: $f(x) = f(-x)$

# Even Functions

An even function is symmetric with respect to the $y$-axis.
When a function is even, $f(-x)$ will be equal to $f(x)$.

$$f(x) = f(-x)$$

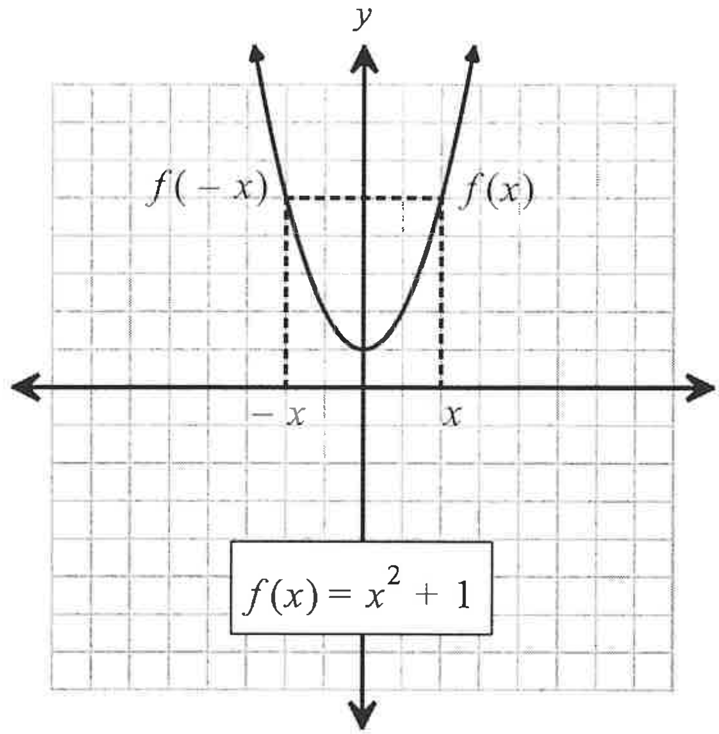
| $x$ | $f(x)$ |
|---|---|
| -3 | 10 |
| -2 | 5 |
| -1 | 2 |
| 0 | 1 |
| 1 | 2 |
| 2 | 5 |
| 3 | 10 |

This is the curve $f(x) = x^2 + 1$

Evaluate $f(4)$: 17

Evaluate $f(-4)$: 17

Notice that $f(4) = f(-4)$. This will be true for all values of $x$.

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Determines end behavior!

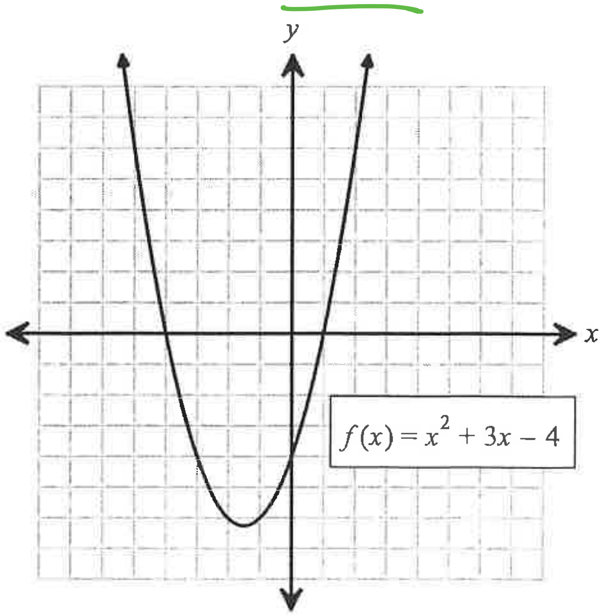
An **even degree polynomial** does **not** always make an **even function**.

For example, $f(x) = x^2 + 3x - 4$ is an even degree polynomial, but ***not*** an even function.

Notice how $f(x) = x^2 + 3x - 4$ does not have symmetry with respect to the $y$-axis.

(has a line of symmetry, but not w/ the y-axis)

| $x$ | $f(x)$ |
|---|---|
| -3 | -4 |
| -2 | -6 |
| -1 | -6 |
| 0 | -4 |
| 1 | 0 |
| 2 | 6 |
| 3 | 14 |



# Odd Functions

An odd function is symmetric with respect to the origin. When a function is odd, $f(-x)$ will be equal to $-f(x)$.

$$f(-x) = -f(x)$$

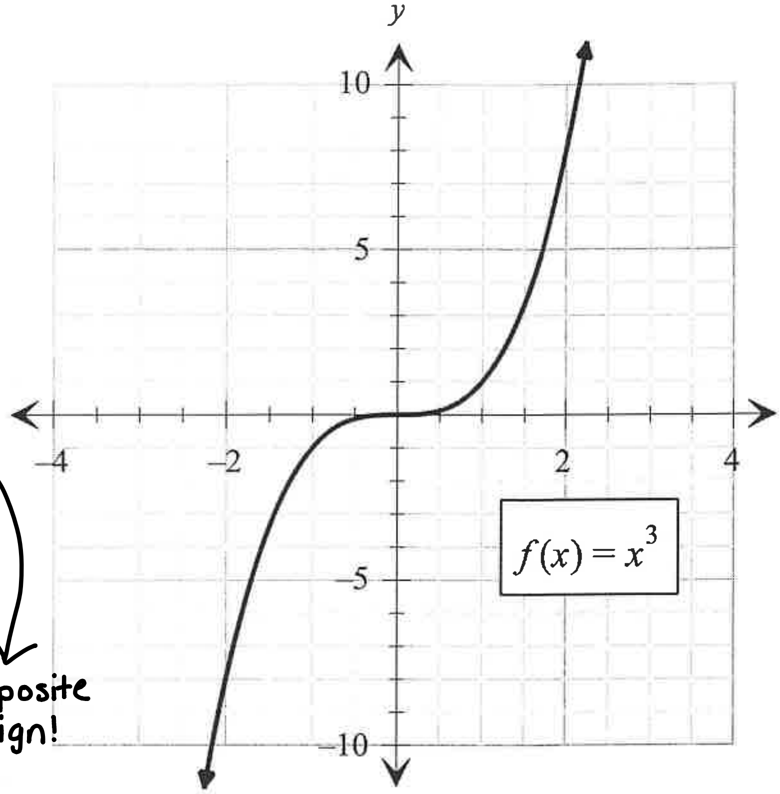
This is the curve $f(x) = x^3$

| $x$ | $f(x)$ |
|---|---|
| -3 | -27 |
| -2 | -8 |
| -1 | -1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |

Evaluate $f(4)$: 64

Evaluate $f(-4)$: −64

opposite Sign!




Notice that $f(-4) = -f(4)$. This will be true for all values of $x$.

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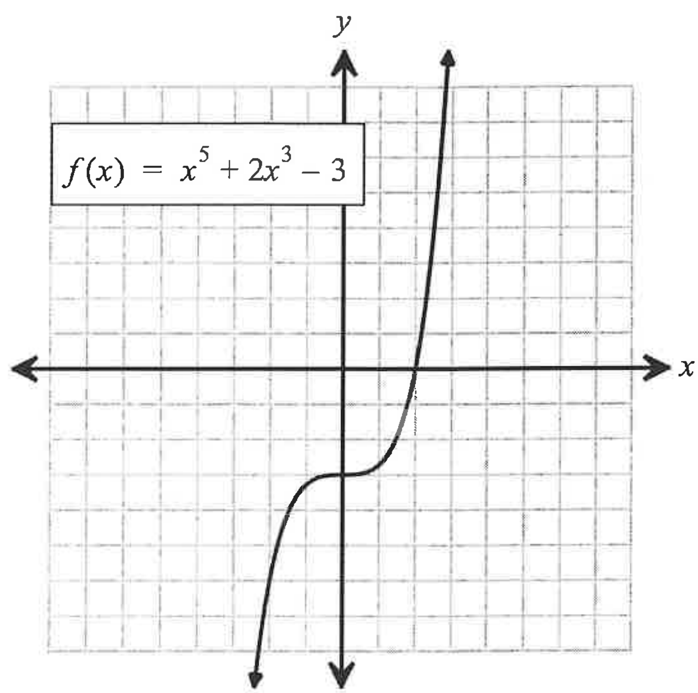
→ Tells us the end behavior!

Just like with even functions, an **odd degree polynomial** does **not** always make an **odd function**.

For example, $f(x) = x^5 + 2x^3 - 3$ is an odd degree polynomial, but ***not*** an odd function.

Notice how $f(x) = x^5 + 2x^3 - 3$ does not have symmetry with respect to the origin.

| $x$ | $f(x)$ |
|---|---|
| -3 | -300 |
| -2 | -51 |
| -1 | -6 |
| 0 | -3 |
| 1 | 0 |
| 2 | 45 |
| 3 | 294 |



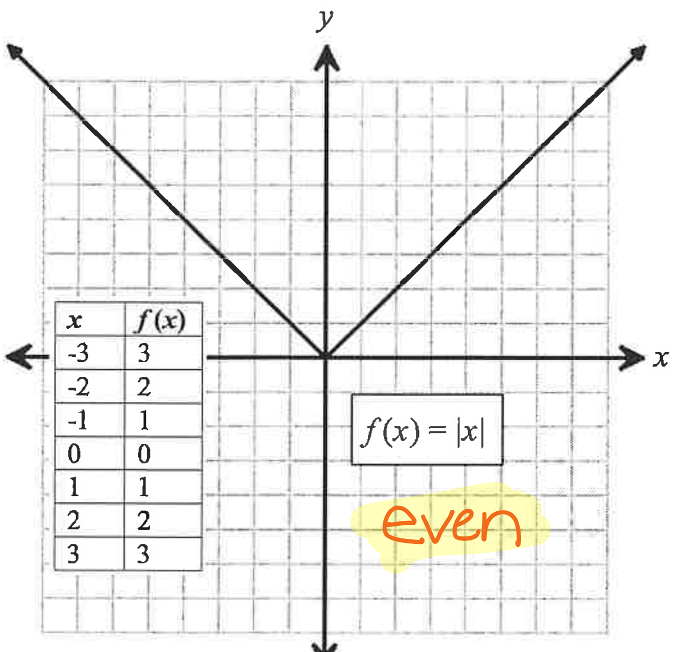
## Even and Odd Functions

A function is known as **even** if the function's graph has symmetry with respect to the *y*-axis.

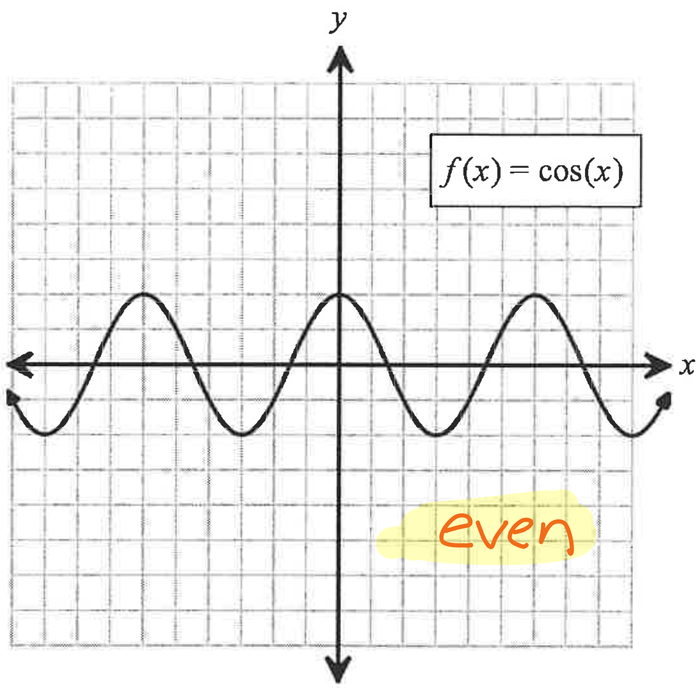
A function is known as **even** if $f(-x) = f(x)$ for every value of $x$ in the domain of $f(x)$.

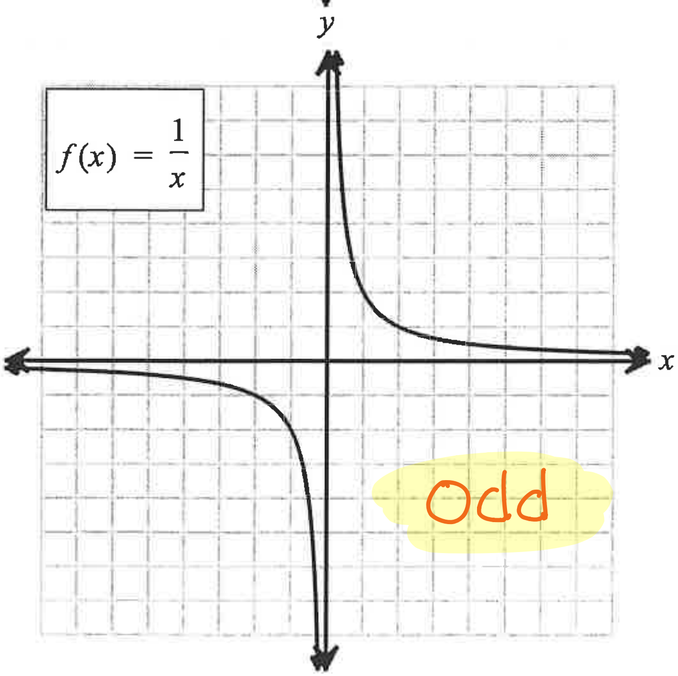
A function is known as **odd** if the function's graph has symmetry with respect to the origin. "Flip test"

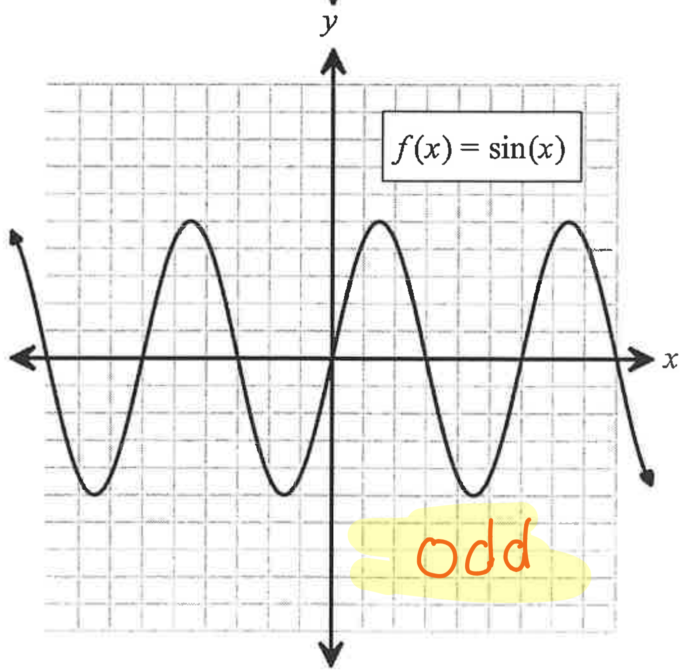
A function is known as **odd** if $f(-x) = -f(x)$ every value of $x$ in the domain of $f(x)$.




| $x$ | $f(x)$ |
|---|---|
| -3 | 3 |
| -2 | 2 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |



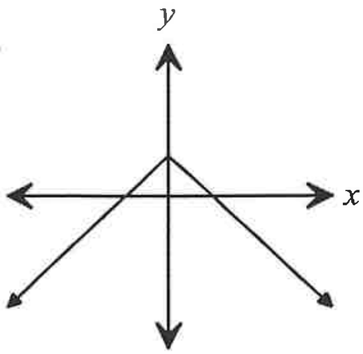



# Even and Odd Functions Graphically Practice

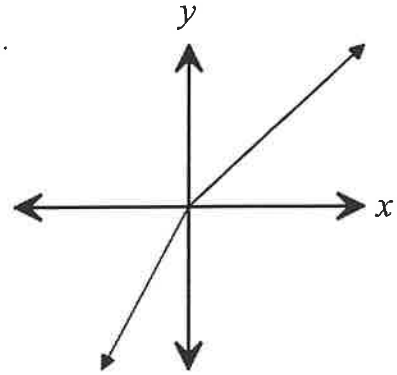
In # 1 – 9, identify each of the following functions as even, odd or neither.

1.



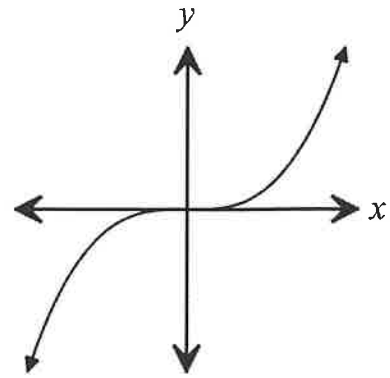

even

2.



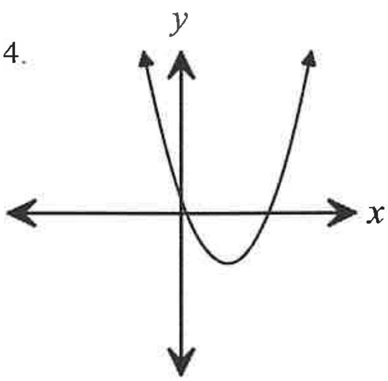

Neither

3.



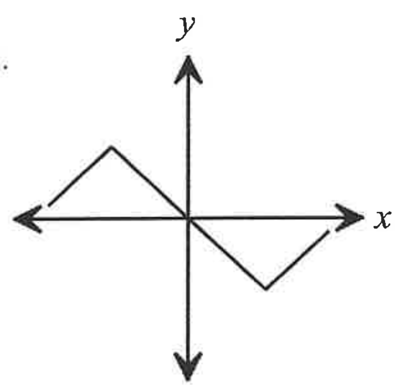

odd

4.



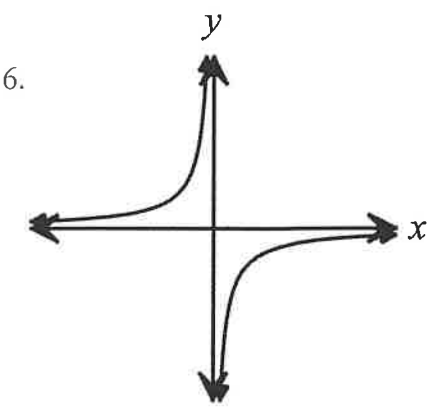

Neither

5.



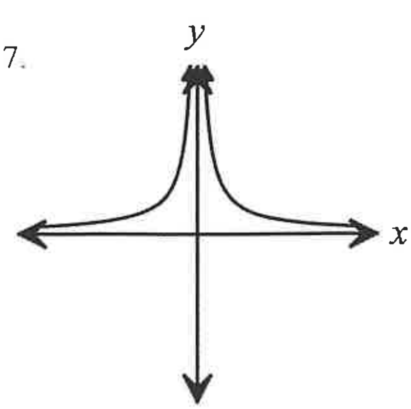

odd

6.



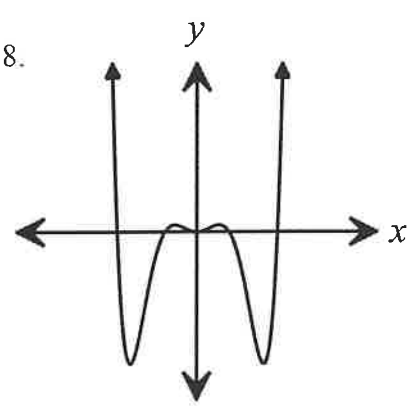

odd

7.



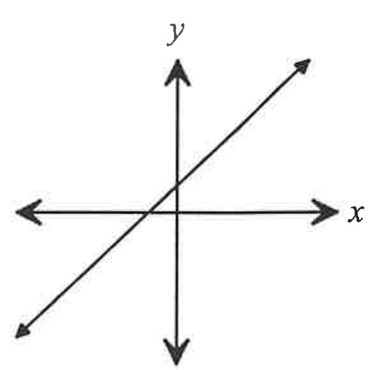

even

8.




even

9.




Neither

Use your table feature on your calculator to determine if the following functions are even, odd, or neither.

10. $f(x) = x^5 - 3x^3 + 2x$

odd

11. $f(x) = x^6 - 5x^2 + 2$

even

12. $h(x) = 4x^7 - x^3 + 1$

neither

13. $g(x) = \frac{3}{x^2}$

even

14. $f(x) = |x - 1| + 2$

neither

15. $k(x) = \frac{1}{x^3}$

odd

16. If $f(x)$ is odd and $f(-3) = 5$, then find $f(3)$.

$f(3) = -5$ ← opposite sign

17. If $f(x)$ is even and $f(8) = 1$, then find $f(-8)$.

$f(-8) = 1$ ← same sign

18. Given the partially filled out table below f(x), fill out the rest of it based on the function type.

(a) Even → exact

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 8 | 5 | -4 | 2 | -4 | 5 | 8 |

(b) Odd → opposite

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 8 | -5 | -4 | 2 | 4 | 5 | -8 |