Lesson 2-4

Graphing Polynomial Functions

Multiplicity of Roots (or Zeros)

The real roots (zeros) of a polynomial correspond with the *x*-intercepts of the polynomial graph.

The number of times a factor appears in a polynomial is referred to as its **multiplicity**.

When the multiplicity is an **even number**, the graph will just **touch** ("bounce") the *x*-axis.

When the multiplicity is an **odd number**, the graph will **cross** the *x*-axis.

***Exercise* #2:** Find each zero and state its multiplicity: $P(x) = x^2(x-3)^2(x+1)(x+4)^3$

X=0 M=2 | X=3 M=2 | X=-1 M=1 | X=-4 M=3

→ When the function is not in standard form we add the exponents

What is the degree of the polynomial? 8

positive even degree ↑ ↑

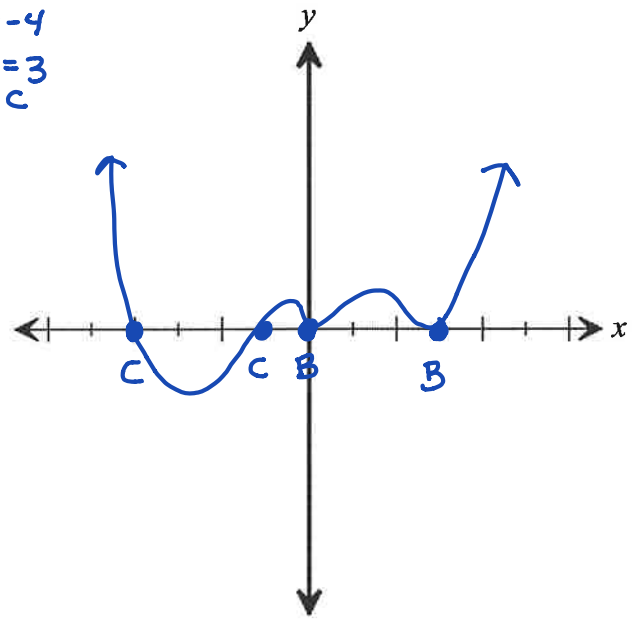
Determine and state the end behavior. $x \to \infty, \quad f(x) \to \infty$

$x \to -\infty, \quad f(x) \to \infty$

Sketch the graph of $y=P(x)$ without the use of your calculator.

X=0 M=2 B | X=3 M=2 B | X=-1 M=1 C | X=-4 M=3 C

↑ ↑



***Exercise* #3:** Sketch the graph of the function $f(x) = (x + 1)(x - 2)(x + 4)(x - 3)^2$.

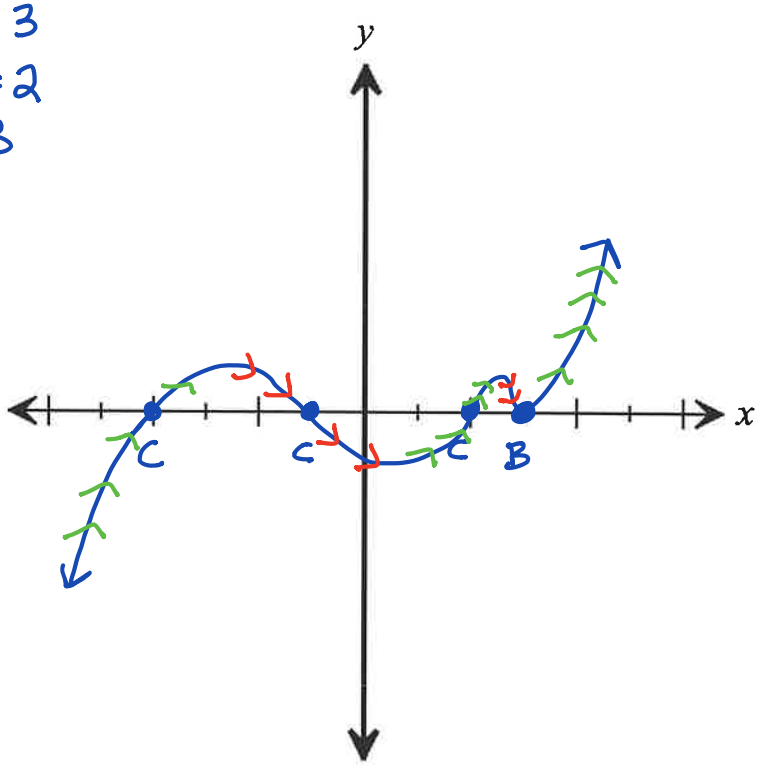
X = -1 M = 1 C

X = 2 M = 1 C

X = -4 M = 1 C

X = 3 M = 2 B

Degree: 5 (odd)
positive
↓ ↑

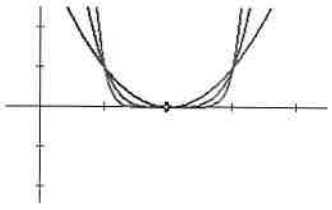


Identify how many intervals over which the function is increasing and how many intervals over which the function is decreasing.

Increasing 3 Decreasing 2

A polynomial to the nth degree can have **at most** $n - 1$ turning points!

Graph Behavior Near Roots



Surrounding EVEN multiplicities: As even multiplicities increase, the graph will become increasingly "flatter" near the root value.

Surrounding ODD multiplicities: As odd multiplicities increase, the graph will become increasingly "flatter" near the root value.

Graphing Polynomial Functions Extension....

1. Answer the following questions based on the polynomial function $P(x) = x(x + 2)^2(x - 3)^3$

   a. State the roots of the polynomial. Indicate whether the graph crosses the x-axis at each root or just touches the x-axis.

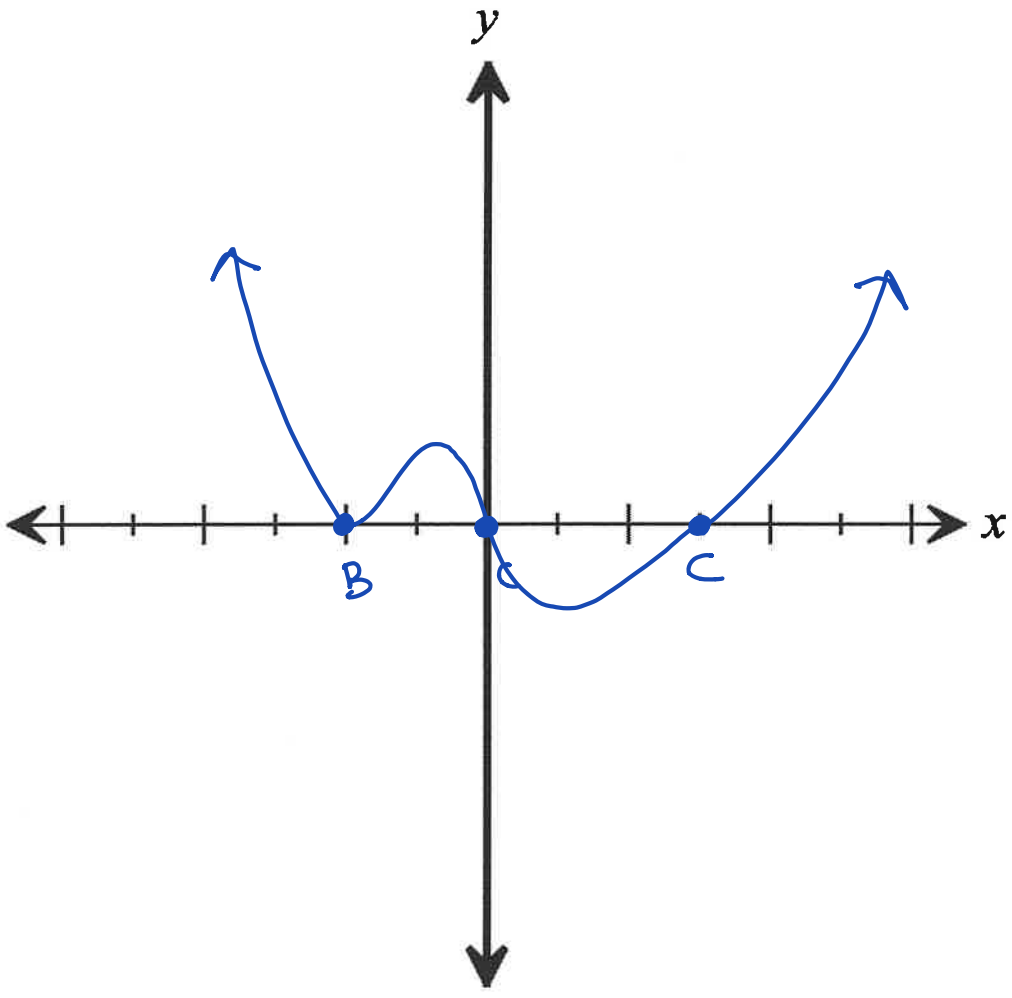
$X = 0$ $X = -2$ $X = 3$

$M = 1$ $M = 2$ $M = 3$

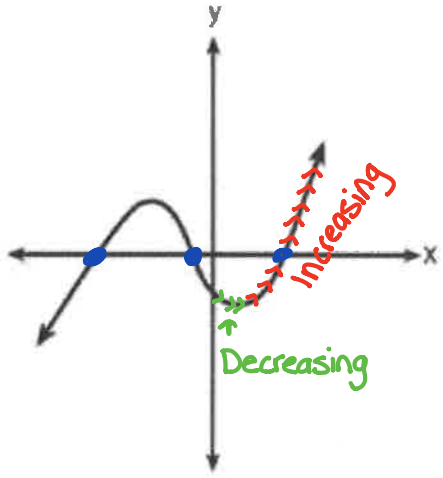
C B C

Degree: 6 (even) positive ↑↑

   b. Draw a sketch of the graph.



2. Which of the following characteristics does **not** pertain to the graph shown at the right?



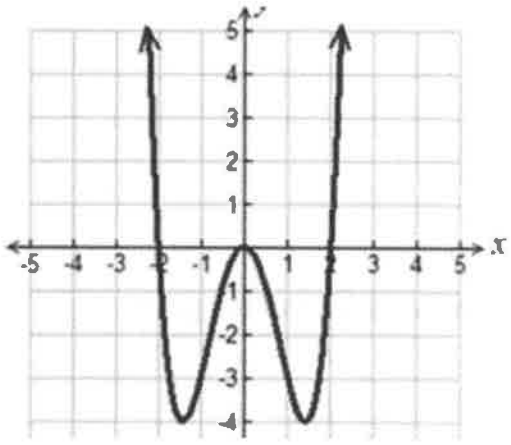
(1) as $x \to -\infty$, $f(x) \to -\infty$ ✓

(2) as $x \to \infty$, $f(x) \to \infty$ ✓

(3) the function has three real zeros ✓

(4) the function is increasing across the positive *x*-axis

3. Which of the following characteristics does **not** pertain to the graph shown at the right? [Assume all roots are real.]



(1) Repeated (double) root at $x = 0$ ✓

(2) multiplicity of $x = 0$ is 3 → can't be 3 because the graph bounces at x=0

(3) as $x \to -\infty$, $f(x) \to \infty$ ✓

(4) degree 4 ✓ (4 roots)

4. Answer the following based on the polynomial function sketched below:

a) Is the degree of the polynomial function even or odd? even

b) Describe the end behavior for this function.

$x \to \infty$, $f(x) \to$ ∞

$x \to -\infty$, $f(x) \to$ ∞

c) Is the leading coefficient of this function positive or negative? Explain your answer.

Negative because both ends are pointing down!