Name: ______________________________ Date: __________

Algebra II Lesson 1-10

Synthetic Division & The Remainder Theorem

Warm up: Divide $(2x^3+4x^2+5x-1)$ by $(x-3)$ and check your answer!

$$\begin{array}{r}
2x^2+10x+35 \\
x-3\overline{)2x^3+4x^2+5x-1} \\
-(2x^3-6x^2) \downarrow \\
\hline
10x^2+5x \\
-(10x^2-30x)\downarrow \\
\hline
35x-1 \\
-(35x-105) \\
\hline
104
\end{array}$$

$$\boxed{2x^2+10x+35+\frac{104}{x-3}}$$

Check:

$(x-3)(2x^2+10x+35)+104$

$2x^3+10x^2+35x-6x^2-30x-105+104$

$2x^3+4x^2+5x-1$ ✓

Synthetic Division is a "short-hand" version of long division for polynomials.

Requirements:

1. The divisor must be a polynomial of degree one (linear). The exponent (on *x*) must be 1 (nothing else).

2. This method is most efficient when the coefficient of the divisor variable, *x*, is a one.

To use Synthetic Division:

$$\frac{2x^3-5x^2-x+3}{x+3}$$

exponent must be 1

coefficient should be 1

MathBits

Steps to Success:

1. Copy the coefficients from the terms in descending order. **Use zeros for missing terms.**
2. Find the root associated with the divisor.
3. Bring down the first coefficient.
4. Multiply the root value times the first coefficient and add it to the second coefficient.
5. Multiply the root value times this sum and add to the next coefficient.
6. Continue until the last coefficient is used.
7. Solution: The final solution uses the values in the bottom row as coefficients for the answer. Since you are dividing by a polynomial of degree 1, the degree of the solution will be 1 less than the degree of the dividend.

Note: The last value in the bottom row is the remainder and is written as a fraction. If the last value is 0, there is no remainder, and the divisor is a factor of the dividend.

Example 1: Divide $(2x^3+4x^2+5x-1)$ by $(x-3)$ using synthetic division.

$x-3=0$
$x=3$

$$\begin{array}{r|rrrr} 3 & 2 & 4 & 5 & -1 \\ & \downarrow & 6 & 30 & 105 \\ \hline & 2 & 10 & 35 & 104 \\ & x^2 & x & c & R \end{array}$$

$$\boxed{2x^2+10x+35+\frac{104}{x-3}}$$

Example 2:

← Fill in missing terms

(a) Divide $(2x^4+4x^2-1)$ by $(x-1)$ using synthetic division.

$x=1$

$$\begin{array}{r|rrrrr} 1 & 2 & 0 & 4 & 0 & -1 \\ & \downarrow & 2 & 2 & 6 & 6 \\ \hline & 2 & 2 & 6 & 6 & 5 \\ & x^3 & x^2 & x & c & R \end{array}$$

$$\boxed{2x^3+2x^2+6x+6+\frac{5}{x-1}}$$

(b) If $f(x)=2x^4+4x^2-1$, evaluate $f(1)$.

$f(1)=5$

(c) What do you notice about this value and the remainder from part a?

When we evaluated $f(1)$ we got the Remainder!

THE POLYNOMIAL REMAINDER THEOREM:

When the polynomial $f(x)$ is divided by the binomial $(x-a)$,
the remainder will always be $f(a)$.

Let's take a quick look back at Example 1… If $f(x)=2x^3+4x^2+5x-1$, evaluate $f(3)$

$$\begin{array}{r}
2x^2+10x+35 \\
x-3\overline{)\,2x^3+4x^2+5x-1} \\
-(2x^3-6x^2) \\
\hline
10x^2+5x \\
-(10x^2-30x) \\
\hline
35x-1 \\
-(35x-105) \\
\hline
104
\end{array}$$

$f(3)=104$

Remainder!

Its a factor if it goes in even (Remainder = 0)

***Example* 3:** Determine if $(x+5)$ is a factor of $(3x^3+17x^2+6x-20)$.

Using Synthetic Division:

$$\begin{array}{r|rrrr}
-5 & 3 & 17 & 6 & -20 \\
 & \downarrow & -15 & -10 & -20 \\
\hline
 & 3 & 2 & -4 & 0
\end{array}$$

Remainder = 0

Using the Remainder Theorem:

$3(-5)^3+17(-5)^2+6(-5)-20$

$=0$

Remainder = 0

x+5 is a factor!

***Example* 4:** What is the remainder of $\dfrac{x^2-11x+22}{x-9}$?

(1) -3

(3) -9

(2) 5

(4) 4

$(9)^2-11(9)+22$

$=4$

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Algebra II Lesson 1-9

Synthetic Division & The Remainder Theorem Practice

1. Determine if $(x-3)$ is a factor of $(9x^3-9x+3)$.

$9(3)^3-9(3)+3 = 219$

remainder

$$\begin{array}{r|rrrr} 3 & 9 & 0 & -9 & 3 \\ & \downarrow & 27 & 81 & 216 \\ \hline & 9 & 27 & 72 & \boxed{219} \end{array}$$

No, x-3 is not a factor of $9x^3-9x+3$ since there is a remainder of 219 when it is divided by x-3.

2. What is the remainder of $\dfrac{x^2-8x+18}{x-2}$?

$(2)^2-8(2)+18 = 6$

$$\begin{array}{r|rrr} 2 & 1 & -8 & 18 \\ & \downarrow & 2 & -12 \\ \hline & 1 & -6 & \boxed{6} \end{array}$$

$\boxed{6}$

3. What is the remainder of $\dfrac{3x^2+7x-20}{x+4}$?

$3(-4)^2+7(-4)-20 = 0$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ & \downarrow & -12 & 20 \\ \hline & 3 & -5 & \boxed{0} \end{array}$$

$\boxed{\text{There is no remainder.}}$

Therefore x+4 is a factor of $3x^2+7x-20$.

4. Solve the following problems using synthetic division.

a. $(x^2-11x+28)\div(x-4)$

$$\begin{array}{r|rrr} 4 & 1 & -11 & 28 \\ & \downarrow & 4 & -28 \\ \hline & 1 & -7 & \boxed{0} \end{array}$$

$\boxed{x-7}$

b. $(m^2-2m-39)\div(m+5)$

$$\begin{array}{r|rrr} -5 & 1 & -2 & -39 \\ & \downarrow & -5 & 35 \\ \hline & 1 & -7 & \boxed{-4} \end{array}$$

$\boxed{m-7-\dfrac{4}{m+5}}$

c. $(2x^4-x^3+2x^2-3x+7)\div\left(x-\dfrac{1}{2}\right)$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -1 & 2 & -3 & 7 \\ & \downarrow & 1 & 0 & 1 & -1 \\ \hline & 2 & 0 & 2 & -2 & \boxed{6} \end{array}$$

$\boxed{2x^3+2x-2+\dfrac{6}{x-\frac{1}{2}}}$

d. $\dfrac{x^4-16}{x-2}$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -16 \\ & \downarrow & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & \boxed{0} \end{array}$$

$\boxed{x^3+2x^2+4x+8}$

5. Determine if $\left(x+\frac{3}{4}\right)$ is a factor of $(8x^3-6x^2-5x+3)$.

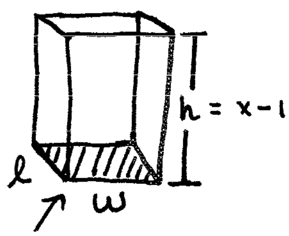
$$8\left(-\tfrac{3}{4}\right)^3-6\left(-\tfrac{3}{4}\right)^2-5\left(-\tfrac{3}{4}\right)+3=0$$

Therefore $x+\frac{3}{4}$ is a factor of $8x^3-6x^2-5x+3$ since $x=-\frac{3}{4}$ is a zero.

6. The volume of a rectangular prism is $(2x^3+2x^2-5x+1)$ cubic feet. The height is $(x-1)$ feet. What is the area of the base?

$$\text{Volume} = l\cdot w\cdot h = \underbrace{l\cdot w}\cdot(x-1) = 2x^3+2x^2-5x+1$$

We need to find the other factors of the volume!



area of base $= l\cdot w$

$$\begin{array}{r|rrrr} 1 & 2 & 2 & -5 & 1 \\ & \downarrow & 2 & 4 & -1 \\ \hline & 2 & 4 & -1 & \boxed{0} \end{array}$$

$$2x^2+4x-1 = l\cdot w$$

$$\boxed{\text{Area of the base} = l\cdot w = 2x^2+4x-1}$$

ANSWERS

1. No
2. 6
3. No remainder
4. (a) $x-7$

   (b) $m-7+\frac{-4}{m+5}$

   (c) $2x^3+2x-2+\frac{6}{x-\frac{1}{2}}$

   (d) x^3+2x^2+4x+8
5. Yes
6. $2x^2+4x-1$