Date: _____ Lesson 1-14

Polynomial Identities

A closer look at Questions #2 and #3 from HW #1-13.....



work with one side until it looks (

Consider the polynomial identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Show that this is an identity.

$$(a+b)(a+b)(a+b)$$

 $(a+b)(a^2+ab+ba+b^2)$
 $(a+b)(a^2+2ab+b^2)$
 $a^3+2a^2b+ab^2+ba^2+2ab^2+b^3$
 $a^3+3a^2b+3ab^2+b^3 = q^3+3a^2b+3ab^2+b^3$

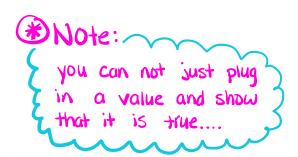


• Algebraically prove that $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$ is an identity by manipulating the right side of the equation.

Because polynomials consist of basic operations on variables, they can be manipulated using the associative, commutative, and distributive properties (as you have done many times). These operations can result in what are known as polynomial identities. An identity is defined more broadly below:

IDENTITIES

An identity is an equation that is true for all values of the replacement variable or variables.



Exercise #2: Verify the following identity for all values of a, b, and c:

$$(a+b+c)^{2} = a^{2} + b^{2} + c^{2} + 2(ab+bc+ac)$$

$$(a+b+c)(a+b+c) = a^{2} + b^{2} + c^{2} + a(ab+bc+ac)$$

$$a \quad b \quad c$$

$$a^{2} \quad ab \quad ac$$

$$b \quad ab \quad b^{2} \quad bc$$

$$c \quad ac \quad bc \quad c^{2}$$

$$a^{2} + b^{2} + c^{2} + aab+abc+aac = a^{2} + b^{2} + c^{2} + a(ab+bc+ac)$$

$$a^{2} + b^{2} + c^{2} + a(ab+bc+ac) = a^{2} + b^{2} + c^{2} + a(ab+bc+ac)$$

Exercise #3: Algebraically prove that
$$\frac{x^3+9}{x^3+8}=1+\frac{1}{x^3+8}$$
, where $x\neq -2$.

$$1 + \frac{1}{\chi^{3} + 8} = 1 + \frac{1}{\chi^{3} + 8}$$

$$\frac{\chi^{3} + 0\chi^{2} + 0\chi + 8)\chi^{3} + 0\chi^{2} + 0\chi + 9}{-(\chi^{3} + 0\chi^{2} + 0\chi + 8)}$$

$$1 + \frac{1}{\chi^{3+8}}$$

POLYNOMIAL IDENTITIES PRACTICE

1) Algebraically prove the following identity:
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$\chi^{3} - \gamma^{3} = \chi^{3} + \chi^{2} + \chi^{2} - \chi^{2} - \chi^{2} - \gamma^{3}$$

$$\chi^{3} - \gamma^{3} = \chi^{3} - \gamma^{3} \checkmark$$

2) Algebraically prove the following identity:
$$(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$$



$$(x^{2}-y^{2})^{2}+(2xy)^{2}=(x^{2}+y^{2})^{2}$$

$$(x^{2}-y^{2})(x^{2}-y^{2})+4x^{2}y^{2}=(x^{2}+y^{2})(x^{2}+y^{2})$$

$$x^{4}-x^{2}y^{2}-x^{2}y^{2}+y^{4}+4x^{2}y^{2}=x^{4}+x^{2}y^{2}+x^{2}y^{2}+y^{4}$$

$$x^{4}+2x^{2}y^{2}+y^{4}=x^{4}+2x^{2}y^{2}+y^{4}$$



Algebraically determine the values of h and k to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

$$2x^{3}-10x^{2}+11x-7=2x^{3}+hx^{2}+3x-8x^{2}-4hx-14+K$$

 $-2x^{3}+8x^{2}-3x+12-2x^{3}$
 $-3x+8x^{2}$

$$-2x^{2}+8x+5=hx^{2}-4hx+K$$