

# Solving Rational Equations

**Example #1:** Solve for all values of  $x$ :  $\frac{5x^2+8x-4}{3x^2+11x+6} - \frac{1}{x+3} = \frac{x}{3x+2}$

$$\frac{(x+2)(5x-2)}{(3x+2)(x+3)} - \frac{1}{x+3} = \frac{x}{3x+2}$$

$$5x^2+8x-4 - (3x+2) = x(x+3)$$

$$5x^2+8x-4-3x-2 = x^2+3x$$

$$\begin{array}{r} 5x^2+8x-4 \\ -x^2-3x \hline 4x^2+2x-6 \end{array} = x^2+3x$$

$$4x^2+2x-6 = 0$$

$$2(2x^2+x-3) = 0$$

$$2(2x^2+3x-2x-3) = 0$$

$$2(x(2x+3)-1(2x+3)) = 0$$

$$2((2x+3)(x-1)) = 0$$

$$2 \neq 0 \quad \boxed{x = -\frac{3}{2}} \quad \boxed{x = 1}$$

**Example #2:** Solve algebraically for  $x$ :  $\frac{1}{x+3} - \frac{2}{3-x} = \frac{4}{x^2-9}$

$$\frac{1}{x+3} + \frac{2}{x-3} = \frac{4}{(x+3)(x-3)}$$

$$x-3 + 2(x+3) = 4$$

$$x-3+2x+6 = 4$$

$$3x+3 = 4$$

$$3x = 1$$

$$\boxed{x = \frac{1}{3}}$$

M:18  
A  
 $3x^2+11x+6$   
 $3x^2+2x+9x+6$   
 $x(3x+2)+3(3x+2)$   
 $(3x+2)(x+3)$

M:20  
A  
 $5x^2+8x-4$   
 $5x^2+10x-2x-4$   
 $5x(x+2)-2(x+2)$   
 $(x+2)(5x-2)$

**Example #3:** The focal length,  $F$ , of a camera's lens is related to the distance of the object from the lens,  $J$ , and the distance to the image area in the camera,  $W$ , by the formula below.

$$\frac{1}{J} + \frac{1}{W} = \frac{1}{F}$$

LCD:  $JWF$

$$\cancel{J}WF + \cancel{J}WF = \cancel{J}WF$$

When this equation is solved for  $J$  in terms of  $F$  and  $W$ ,  $J$  equals

(1)  $\frac{FW}{F-W}$

(3)  $F-W$

(2)  $\frac{FW}{W-F}$

(4)  $\frac{1}{F} - \frac{1}{W}$

$$\frac{WF}{-JF} + \frac{JF}{-JF} = \frac{JW}{-JF}$$

$$WF = JW - JF$$

$$\frac{WF}{W-F} = \frac{J(W-F)}{W-F}$$

$$\frac{WF}{W-F} = J$$

### Solving Rational Equations Practice

LCD =  $3x(x+1)$

1) Solve algebraically:  $\frac{2}{3x} + \frac{4}{x} = \frac{7}{x+1}$

$$\cancel{3x(x+1)} \frac{2}{\cancel{3x}} + \cancel{3x(x+1)} \frac{4}{\cancel{x}} = \cancel{3x(x+1)} \frac{7}{\cancel{x+1}}$$

$$2(x+1) + 12(x+1) = 21x$$

$$2x+2+12x+12=21x$$

$$14x+14=21x$$

$$\frac{14}{7} = \frac{7x}{7}$$

$$2 = x$$

2) Solve algebraically:  $\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$

LCD:  $x(x+2)$

$$\cancel{x(x+2)} \frac{3}{\cancel{x}} + \cancel{x(x+2)} \frac{x}{\cancel{x+2}} = -\cancel{x(x+2)} \frac{2}{\cancel{x+2}}$$

$$3(x+2) + x^2 = -2x$$

$$3x+6+x^2=-2x$$

$$x^2+5x+6=0$$

$$(x+3)(x+2)=0$$

$$x = -3 \quad | \quad x = -2$$

3) Solve algebraically:  $\frac{x}{x-4} - \frac{1}{x+3} = \frac{28}{x^2-x-12}$   
 $(x+3)(x-4)$

$$\frac{\cancel{x}}{\cancel{x-4}} - \frac{\cancel{1}}{\cancel{x+3}} = \frac{\cancel{28}}{(\cancel{x+3})(\cancel{x-4})}$$

LCD:  $(x+3)(x-4)$

$$x(x+3) - (x-4) = 28$$

$$x^2 + 3x - x + 4 = 28$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

$$x = -6 \quad | \quad x = 4$$

4) Solve for F in terms of P and Q:  $\frac{2}{F} - \frac{1}{P} = \frac{1}{Q}$

$$\frac{\cancel{2}PQ}{\cancel{F}} - \frac{\cancel{1}PQ}{\cancel{P}} = \frac{\cancel{1}PQ}{\cancel{Q}}$$

$$2PQ - FQ = FP$$

$$2PQ = FP + FQ$$

$$\frac{2PQ}{P+Q} = \frac{F(P+Q)}{P+Q}$$

$$\frac{2PQ}{P+Q} = F$$

### Answers

1.  $x = 2$

2.  $x = -3$

3.  $x = -6$

4.  $F = \frac{2PQ}{P+Q}$