

SOLVING RATIONAL EQUATIONS

Warm Up:

1. Solve for x using the method of cross multiplication.

$$\frac{4x+5}{2} = \frac{x-1}{5}$$

$$\begin{aligned} 2(x-1) &= 5(4x+5) \\ 2x-2 &= 20x+25 \\ -2x-25 &= 20x+25 \\ -27 &= 18x \\ \frac{-27}{18} &= \frac{18x}{18} \\ x &= -1.5 \end{aligned}$$

2. Add without a calculator:

$$LCD = 30$$

$$\left(\frac{6}{6}\right)\frac{2}{5} + \frac{1}{6}\left(\frac{5}{5}\right)$$

$$\frac{12}{30} + \frac{5}{30} = \frac{17}{30}$$

Exercise #1: Consider the equation $\frac{1}{2} - \frac{9}{4x} = \frac{3}{4x}$.

What is the Least Common Denominator?

$$4x$$

Multiply both sides of this equation by the LCD to "clear" the equation of the denominators. Now, solve the resulting linear equation. Be sure to check your answer.

$$2(4x) \frac{1}{2} - \frac{(4x)9}{4x} = \frac{(4x)3}{4x}$$

$$\begin{aligned} 2x - 9 &= 3 \\ +9 &+9 \\ \hline 2x &= 12 \\ \frac{2x}{2} &= \frac{12}{2} \\ x &= 6 \end{aligned}$$

Steps to Success

- 1) Find the LCD of all denominators
- 2) Multiply each term by the LCD
- 3) Simplify (no fractions should remain)
- 4) Solve
- 5) Check for **extraneous roots!**

Why must we check the solutions of a rational equation?

To make sure the denominator does not equal zero (undefined)

Exercise #2: Solve for x : $\frac{1}{2} + \frac{3}{x} - \frac{1}{x^2} = \frac{1}{4x} + \frac{1}{2x^2}$

$$\overset{2}{(4x^2)} \frac{1}{2} + \overset{(4x^2)}{\frac{3}{x}} - \overset{(4x^2)}{\frac{1}{x^2}} = \overset{(4x^2)}{\frac{1}{4x}} + \overset{(4x^2)}{\frac{1}{2x^2}}$$

$$2x^2 + 12x - 4 = x + 2$$

$$\begin{array}{r} 2x^2 + 12x - 4 \\ -x \quad -2 \\ \hline \end{array}$$

$$2x^2 + 11x - 6 = 0$$

$$2x^2 + 12x - 1x - 6 = 0$$

$$2x(x+6) - 1(x+6) = 0$$

$$(x+6)(2x-1) = 0$$

$$\boxed{x = -6} \quad \boxed{x = \frac{1}{2}}$$

Exercise #3: Solve for x : $\frac{x+1}{x+5} + \frac{18}{x^2+8x+15} = \frac{9}{x+3}$

$$LCD = (x+5)(x+3)$$

$$\overset{(x+5)(x+3)}{\cancel{x+5}} \frac{x+1}{\cancel{x+5}} + \overset{(x+5)(x+3)}{\cancel{(x+5)(x+3)}} \frac{18}{\cancel{(x+5)(x+3)}} = \overset{(x+5)(x+3)}{\cancel{(x+5)(x+3)}} \frac{9}{\cancel{x+3}}$$

$$(x+3)(x+1) + 18 = 9(x+5)$$

$$x^2 + x + 3x + 3 + 18 = 9x + 45$$

$$x^2 + 4x + 21 = 9x + 45$$

$$\begin{array}{r} x^2 + 4x + 21 \\ -9x \quad -45 \\ \hline \end{array}$$

$$x^2 - 5x - 24 = 0$$

$$(x-8)(x+3) = 0$$

$$\boxed{x = 8} \quad \boxed{x = -3}$$

← why can't $x = -3$?
Reject

SOLVING RATIONAL EQUATIONS PRACTICE

1

1. Solve for x: $\frac{3}{4} + \frac{1}{2x} = \frac{1}{2x} + \frac{1}{3x^2}$

$LCD = 12x^2$

$$\begin{aligned} \frac{3}{4} + \frac{1}{2x} &= \frac{1}{2x} + \frac{1}{3x^2} \\ 9x^2 + 6x &= 6x + 4 \\ -6x &-6x \\ \hline 9x^2 &= 4 \\ -4 &-4 \\ \hline 9x^2 - 4 &= 0 \\ (3x+2)(3x-2) &= 0 \\ x = -\frac{2}{3} & \quad x = \frac{2}{3} \end{aligned}$$

2

2. Solve for x: $\frac{x}{x-8} + \frac{6}{x-2} = \frac{x^2}{x^2-10x+16}$
 $(x-8)(x-2)$

$LCD: (x-8)(x-2)$

$$\begin{aligned} \frac{x}{x-8} + \frac{6}{x-2} &= \frac{x^2}{(x-8)(x-2)} \\ x(x-2) + 6(x-8) &= x^2 \\ x^2 - 2x + 6x - 48 &= x^2 \\ \cancel{x^2} + 4x - 48 &= \cancel{x^2} \\ \hline 4x - 48 &= 0 \\ 4x &= 48 \\ \frac{4x}{4} &= \frac{48}{4} \\ x &= 12 \end{aligned}$$

3

3. Solve for c: $\frac{3c}{c^2-4} + \frac{1}{c-2} = \frac{2}{c+2}$
 $(c+2)(c-2)$

$LCD: (c+2)(c-2)$

$$\begin{aligned} \frac{3c}{(c+2)(c-2)} + \frac{1}{c-2} &= \frac{2}{c+2} \\ 3c + c+2 &= 2(c-2) \\ 4c + 2 &= 2c - 4 \\ -2c &-2c \\ \hline 2c &= -6 \\ c &= -3 \end{aligned}$$

4. Solve for a : $2 - \frac{5}{2a} = \frac{2a}{a+1}$

$LCD = 2a(a+1)$

$$\frac{2}{1} - \frac{5}{2a} = \frac{2a}{a+1}$$

$$4a(a+1) - 5(a+1) = 4a^2$$

$$4a^2 + 4a - 5a - 5 = 4a^2$$

$$\begin{array}{r} 4a^2 - a - 5 = 4a^2 \\ -4a^2 \quad -4a^2 \hline \end{array}$$

$$-a - 5 = 0$$

$$-a = 5$$

$$a = -5$$

5. Solve for x : $\frac{4}{x^2+4x-12} + \frac{x-1}{x+6} = \frac{1}{x-2}$

$LCD: (x+6)(x-2)$

$$\frac{4}{(x+6)(x-2)} + \frac{x-1}{x+6} = \frac{1}{x-2}$$

$$4 + (x-2)(x-1) = (x+6)$$

$$4 + x^2 - x - 2x + 2 = x + 6$$

$$\begin{array}{r} x^2 - 3x + 6 = x + 6 \\ -x \quad -6 \quad -x \quad -6 \hline \end{array}$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x=0 \quad | \quad x=4$$

6. Solve for x : $\frac{x-3}{x-7} - \frac{1}{x} = \frac{28}{x^2-7x}$

$LCD: x(x-7)$

$$\frac{x-3}{x-7} - \frac{1}{x} = \frac{28}{x(x-7)}$$

$$x(x-3) - (x-7) = 28$$

$$x^2 - 3x - x + 7 = 28$$

$$\begin{array}{r} x^2 - 4x + 7 = 28 \\ -28 \quad -28 \hline \end{array}$$

$$x^2 - 4x - 21 = 0$$

$$(x-7)(x+3) = 0$$

$$x=7 \quad | \quad x=-3$$

Reject