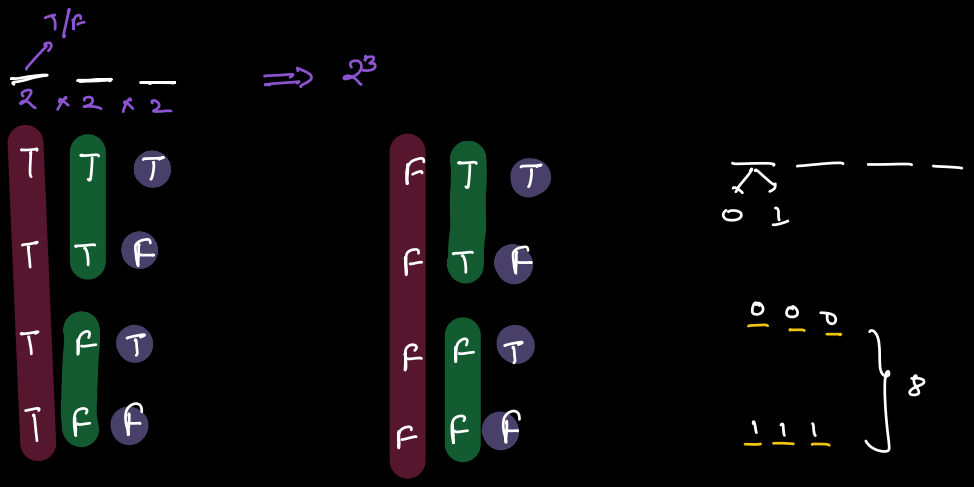
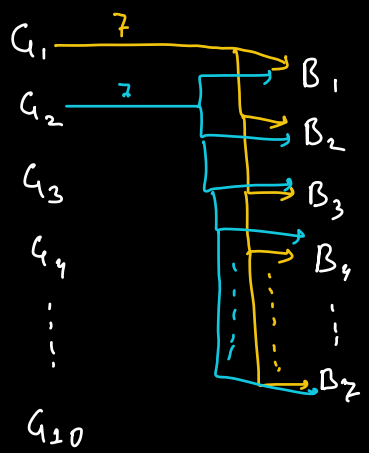
Quiz 1 Given 3 T/F questions.
In how many ways can we answer them.



Quiz 2 There are 10 girls & 7 boys in a hall.
Count ways to form one G-B pairs.



(G-B)

10 × 7 ⇒ 70 pairs

Quiz 3

Count no of paths from Hyd to Delhi

To go from Hyd to Delhi

- Pick one path from Hyd to Mum ⇒ 3

(And)

- Pick one path from Mum to Delhi = 4

$3 \times 4 \Rightarrow 12$

Quiz 4

Count no of paths from Hyd to Delhi

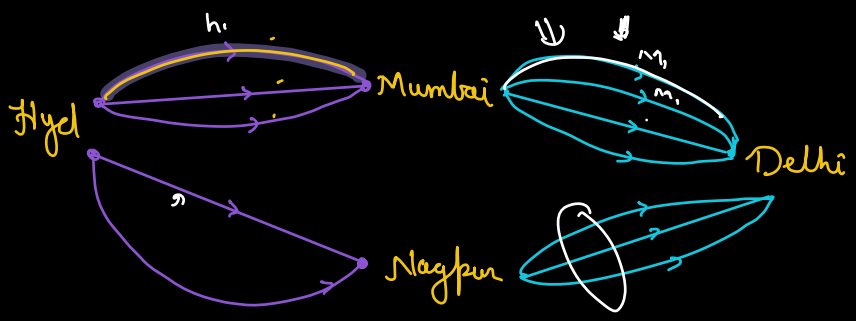
To go from Hyd to Delhi

- Pick one path from Hyd to Nag ⇒ 2

(And)

- Pick one path from Nag to Delhi = 3

$2 \times 3 = 6$



Hyd to Delhi

Hyd —3→ Mumbai
AND
Mumbai —4→ Delhi

OR

Hyd → Nag
AND
Nag → Delhi

Hyd → Mum → Delhi

⇓

12

Hyd → Nag → Delhi

⇓

6

⇒ 18

AND	⇒	×
OR	⇒	+

Quiz 5

You can gift one of the following combos:

- [1 Pen AND 1 Book]

OR

- [1 flower AND 1 Chocolate]

OR

- [1 Ring]

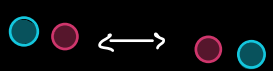
Pen : 3
Book : 5
Flower : 7
Chocolate : 3
Ring : 3

$(3 \times 5) + (7 \times 3) + (3)$

$\Rightarrow 39$

Permutation

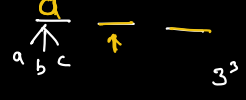
(Arrangement of objects)



$(i, j) \neq (j, i)$

Quiz 6

Count no. of ways to arrange 3 characters a, b & c



a b c
a c b

b a c
b c a

c a b
c b a

$3 \times 2 \times 1 = 6$

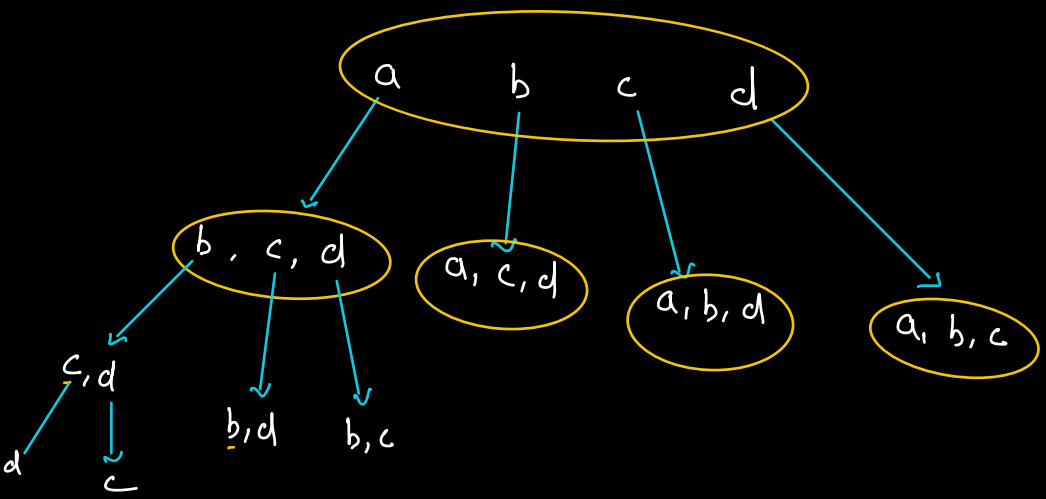
Quiz 7 — a, b, c, & d

1st Pos ⇒ 4

2nd Pos ⇒ 4×3

3rd Pos ⇒ 4×3×2

4th Pos ⇒ 4×3×2×1



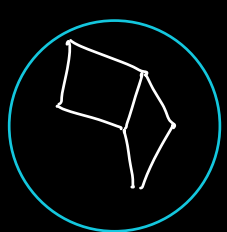
No. of ways to arrange **n** **distinct** objects in **n** places.

$N \times (N-1) \times (N-2) \times \text{----} \times 2 \times 1 \Longrightarrow N!$ (factorial)

Quiz 8 — No of ways to arrange 0 objects ⇒ 1

3 ₹ 100 Notes

$0! = 1$



Quiz 9 Given 5 distinct characters.
Count no of ways of arranging them in 2 places.

a, b, c, d, e

$\underline{\quad}\ \underline{\quad}$

a : {b, c, d, e}
b : {a, c, d, e}
c : {a, b, d, e}
d : {a, b, c, e}
e : {a, b, c d}

$\underline{5} \times \underline{4} \Rightarrow 20$

$\underline{5} \times \underline{4} \times \underline{3} \Rightarrow 60$

N distinct objects & 3 positions $\longrightarrow N \times (N-1) \times (N-2)$

N distinct objects & 4 positions $\longrightarrow N \times (N-1) \times (N-2) \times (N-3)$

⋮

N distinct objects & r positions $\longrightarrow N \times (N-1) \times (N-2) \times \cdots \times (N-(r-1))$

$$N \times (N-1) \times (N-2) \times \cdots \times (N-r+1) = \frac{N \times (N-1) \times (N-2) \times \cdots \times (N-r+1) \times (N-r) \times (N-r-1) \times \cdots \times 1}{(N-r) \times (N-r-1) \times \cdots \times 1}$$

N distinct objects & r positions $= \dfrac{N!}{(N-r)!}$ $\longleftarrow {}^{N}P_{r}$

$$ {}^{N}P_{r} = \frac{N!}{(N-r)!} $$

Combinations

(Selection of objects)

$(i, j) == (j, i)$ ✓

(order does not matter)

Quiz 10 Count no of ways of selecting 3 plays out of 4

P_1 P_2 P_3 P_4

P_1 P_2 P_3

P_1 P_2 P_4

P_1 P_3 P_4

P_2 P_3 P_4

No of ways of arranging 4 plays in 3 slots.

P_1 P_2 P_3	P_1 P_2 P_4	P_1 P_3 P_4	P_2 P_3 P_4
P_1 P_3 P_2	P_1 P_4 P_2	P_1 P_4 P_3	P_2 P_4 P_3
P_2 P_1 P_3	P_2 P_1 P_4	P_3 P_4 P_1	P_3 P_2 P_4
P_2 P_3 P_1	P_2 P_4 P_1	P_3 P_1 P_4	P_3 P_4 P_2
P_3 P_1 P_2	P_4 P_1 P_2	P_4 P_1 P_3	P_4 P_3 P_2
P_3 P_2 P_1	P_4 P_2 P_1	P_4 P_3 P_1	P_4 P_2 P_3
P_1 P_2 P_3	P_1 P_2 P_4	P_1 P_3 P_4	P_2 P_3 P_4

Total arrangements ⟶ 24

No of selections ⟶ $24/3!$

Ways to arrange N objects in r places.

$$^{N}P_{r} = \frac{N!}{(N-r)!}$$

Ways to arrange r items in r places $= r!$

No of ways to select r items from N items $= \frac{N!}{(N-r)!} \times \frac{1}{r!}$

$$^{N}C_{r} = \frac{N!}{(N-r)!\ r!}$$

$$^{N}C_{r} = \frac{^{N}P_{r}}{r!}$$

Properties

$${}^{N}C_{1} = \frac{N!}{(N-1)! \times 1!} \longrightarrow N$$

Select 1 from N

$${}^{N}C_{0} = \frac{N!}{(N-0)! \times 0!} \longrightarrow 1$$

Not selecting anything

Quiz 11

$${}^{N}C_{0} + {}^{N}C_{1} + {}^{N}C_{2} + {}^{N}C_{3} + \cdots\cdots + {}^{N}C_{N} \rightarrow$$ Count of all possible subsets. 2^N

- ${}^{N}C_{0}$ → Not selecting anything
- ${}^{N}C_{1}$ → ways of selecting 1 from N
- ${}^{N}C_{2}$ → ways of selecting 2 from N
- ${}^{N}C_{3}$ → ways of selecting 3 from N
- ${}^{N}C_{N}$ → ways of selecting N from N

$\{1, 2, 3\}$

${}^{3}C_{0} = 1$: $\{\}$

${}^{3}C_{1} = 3$: $\{1\}$ $\{2\}$ $\{3\}$

${}^{3}C_{2} = 3$: $\{1,2\}$ $\{1,3\}$, $\{2,3\}$

${}^{3}C_{3} = 1$: $\{1, 2, 3\}$

All possible subsets.

Q Given 5 players, count no of ways of select 2 players.

P_1 P_2 P_3 P_4 P_5

P_1 P_2 P_3 P_4 P_5
P_1 P_3 P_2 P_4 P_5
P_1 P_4 P_2 P_3 P_5
P_1 P_5 P_2 P_3 P_4

P_2 P_3 P_1 P_4 P_5
P_2 P_4 P_1 P_3 P_5
P_2 P_5 P_1 P_4 P_3

P_3 P_4 P_1 P_2 P_5
P_3 P_5 P_1 P_2 P_4
P_4 P_5 P_1 P_2 P_3

$$^5C_2 = \frac{5!}{3!\,2!} = \frac{5 \times 4}{2} = 10$$

$$^5C_3 = \frac{5!}{2!\,3!} = \frac{5 \times 4}{2} = 10$$

$$^NC_r = {}^NC_{N-r}$$

^NC_r → Select r from N

$^NC_{N-r}$ → Select $N-r$ from (N)

Given N items → Select r from them

$a_1, a_2, a_3, \ldots, a_N$

Include in Selected r (Pick) OR Exclude from Selected r (Don't Pick)

$$^{N-1}C_{r-1} + {}^{N-1}C_r$$

$$^{N}C_{r} = {}^{N-1}C_{r-1} + {}^{N-1}C_{r}$$

$$\frac{(N-1)!}{(r-1)!\,(N-r)!} + \frac{(N-1)!}{r! \times (N-1-r)!}$$

$$(N-1)!\left(\frac{1}{(r-1)!\,(N-r)\times(N-r-1)!} + \frac{1}{r\times(r-1)!\times(N-1-r)!}\right)$$

$$(N-1)!\left(\frac{r + N-r}{r!\times(N-r)!}\right)$$

$$\frac{N\times(N-1)!}{r!\times(N-r)!}$$

$$\frac{N!}{r!\times(N-r)!}$$

$$^{N}C_{r}$$

Q Given N, r & p (p Prime no. 10^9+7) TC

Calculate ${}^{N}C_{r} \% p$ $\boxed{N, r < p}$

$${}^{N}C_{r} = \frac{N!}{r! \times (N-r)!}$$

$${}^{N}C_{r} \% p = \left(\frac{N!}{r! \times (N-r)!}\right) \% p \qquad \left(\frac{a}{b}\right) \% M = \left(\frac{a \% M}{b \% M}\right) \% M$$

$$= (N! \% p) \times \underline{\left(r! \times (N-r)!\right)^{-1} \% p}$$

$$\underline{(r!)^{-1} \% p} \times \underline{(N-r)!^{-1} \% p}$$

$$\left((N-r)!\right)^{-1} \% p$$

$$a^{-1} \% p \longrightarrow a^{p-2} \% p$$

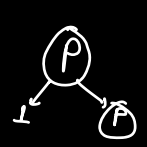
- p has to be a prime no.
- a & p have to co-prime ($\gcd(a, p) = 1$)

if $N, r < p$

$(N-r) < p$

$(N-r)! = \underline{1 \times 2 \times 3 \cdots (N-r)}$.

$\gcd(p, (N-r)!) = \underline{1}$



$((N-r)!)^{P-2} \% P$

$Pow(a, N, P) \Rightarrow a^N \% P$

$a \; ((N-r)! \% P)^{P-2} \% P \Rightarrow Pow((N-r)! \% P, P-2, P)$

$[0, P-1]$

$[0, P-1]$

Q Calculate ${}^{N}C_r \% p$

(P may not be prime)

$${}^{N}C_r \% P = \left(\frac{N!}{r!\,(N-r)!}\right) \% P$$

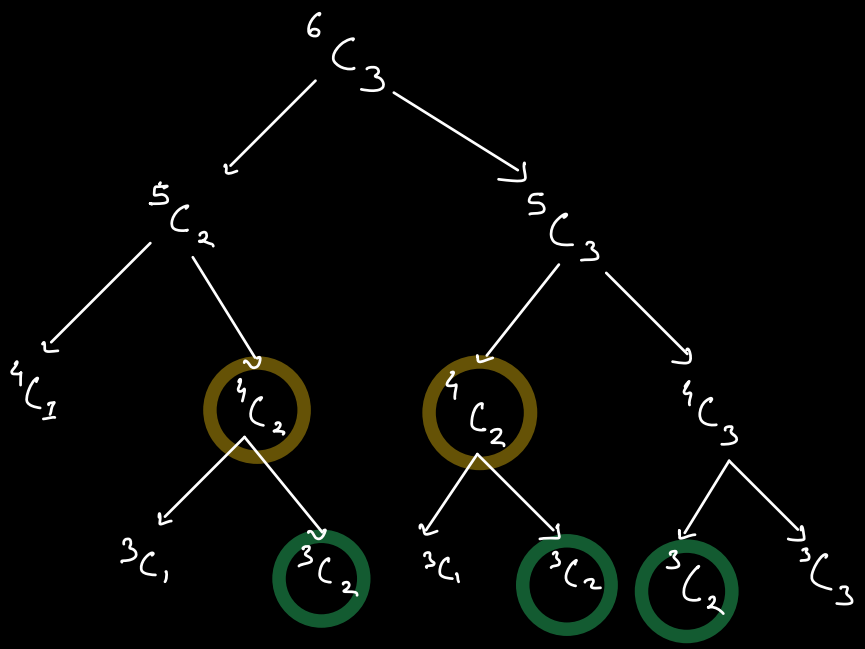
$$(a+b) \% M = (a \% M + b \% M) \% M$$

$${}^{N}C_r \% P = ({}^{N-1}C_{r-1} \% P + {}^{N-1}C_r \% P) \% P$$

$r \le n$

```
int nCr (n, r, p) {
    if (r==1) return n % p;
    if (r==n || r==0) ret 1;
    ret (nCr(n-1, r-1, p) + nCr(n-1, r, p)) % p
}
```

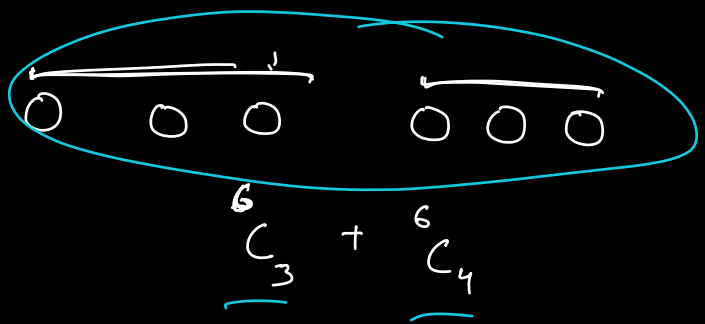
2^r



Doubts

Pick 4 from 7 i.e. $^7C_4$ OR $^6C_3$ OR $^6C_4$

$$^nC_r = {}^{(n-1)}C_{(r-1)} + {}^{n-1}C_r$$



$$^{6}C_{3} + {}^{6}C_{4}$$

$$a \times b = 1$$

$$b = 1/a = a^{-1}$$

$$(a \times b) \% p = 1$$

$$(a \% p \times (b \% p)) \% p = 1$$

$[0, p-1]$