Maths - GCD

GCD ? Greatest Common Divisor

↳ HCF → Highest Common Factor.

GCD(a,b) : largest number which divides both a & b.

↓

$x \Rightarrow a \% x = 0 \;\&\&\; b \% x = 0$

gcd (12, 20)

⇒ 4

12
1
2
3
4
6
12

20
1
2
4
5
10
20

gcd (0, 8) = 8

% ⇒ undefined

0	8
1	1
2	2
3	4
4	8
5	
↓ all	

gcd (0, x) = x ⇒ (iff x != 0)

gcd (1, 8) = 1

1	8

1 | 1
2
4
8

$gcd(1, x) = 1$.

Properties of GCD

① $gcd(a, b) = gcd(b, a)$ ⇒ commutative

② $gcd(0, a) = a$ & $gcd(1, a) = 1$.

③ $gcd(a, b, c) = gcd(a, gcd(b, c))$

⇙ associative

$= gcd(b, gcd(a, c))$

$= gcd(c, gcd(a, b))$

Q.1 Given 2 numbers → a, b, find $gcd(a, b)$?

Brute: loop over all the numbers & check if it divides a & b.

```
for ( i = 1 ; i ≤ min(a,b) , i++)
{
    if (a % i == 0 && b % i == 0)
    {
        ans = (i) ⟷ max(ans, i) !
    }
}
```

min(a,b) , i >= 1 , i--

T.C => O(N)

S.C => O(1)

Euclidean Algo

Proof: if a, b, (a > b).

gcd(a, b) = x => a % x = 0 && b % x = 0

gcd(a - b, b) = x ?

(a - b) % x ?= 0

(a % x - b % x + x) % x

= 0

gcd(a, b) = gcd(a - b, b)

given that (a > b)

if (b > a) -> swap(a, b)

gcd(b - a, a)

gcd(a, b) = p

1. a % p = 0
2. b % p = 0
3. (a - b) % p = 0

subtract

gcd(a - b, b) = q

1. (a - b) % q = 0
5. b % q = 0

add

(a - b + b) % q = 0

6.

a%q = 0

q → a , b , (a − b) →

& gcd (a − b , b) = q

q ≥ p

p → a , b , (a − b)

& gcd (a , b) = p

p ≥ q

p = q

(a, b) Find gcd (a, b)!

for (1 → min(a, b)

gcd(a,b)

```
{
  While (a>0 && b>0)
  {
    if (a > b)
    {  a = a - b
    }
    else
    {  b = b - a
    }
  }
  return max(a,b)
}
```

a = 10 , b = 6

gcd (10, 6)

= gcd (10 − 6, 6)

⇒ gcd (4, 6)

= gcd (6, 4)

= gcd (6 − 4, 4)

= gcd (2, 4)

= gcd (4, 2)

= gcd (4 − 2, 2)

= gcd (2, 2)

```
int gcd (a, b)
{
  //base case
  if (a==0) return b
  if (b==0) return a.
```

```
( if( b == 0)
// main logic
if(a<b)
{
  swap(a,b)
}
gcd (a-b, b)
}
```

= gcd(0, 2)

2

gcd (12, 8)

gcd (4, 8)

gcd (8-4, 4)

gcd (0, 4) → 4

gcd(a, b) = gcd (a-b, b)

if (a-b > b)

= gcd(a-b-b, b)

= gcd(a-b-b-a, b)

x time

gcd(a - xb, b)

a % b

gcd(12, 3) → gcd(9, 3) → gcd(6, 3) → gcd(3, 3)

gcd(0, 3)

gcd(18, 4) → gcd(14, 4) → gcd(10, 4)

gcd(6,4) ← gcd(24,4)

18%4

gcd(27,7) → gcd(20,7) → gcd(13,7)

gcd(6,7)

27%7

if (a>b)

$$gcd(a,b) = gcd(a\%b, b)$$

if (a<b)

→ a%b = ? a

a%b < b.

$$gcd(a,b) = gcd(b, a\%b)$$

```
int gcd(a,b)
{ //base case
   if(b==0) return a;
  //main logic
   return gcd(b, a%b)
}
```

gcd(a,17) = 1

⇓

gcd(17,9)

⇓

gcd(9,8)

⇓

gcd(8,1)

⇓

gcd(1,0)

T.C. = ?

$$gcd(a,b) = gcd(a\%b, b)$$

Given $(a > b)$

$b \le \frac{a}{2}$

$b > \frac{a}{2}$

$b > \frac{a}{2}$

$b + \frac{a}{2} > a$

$\frac{a}{2} > a - b$

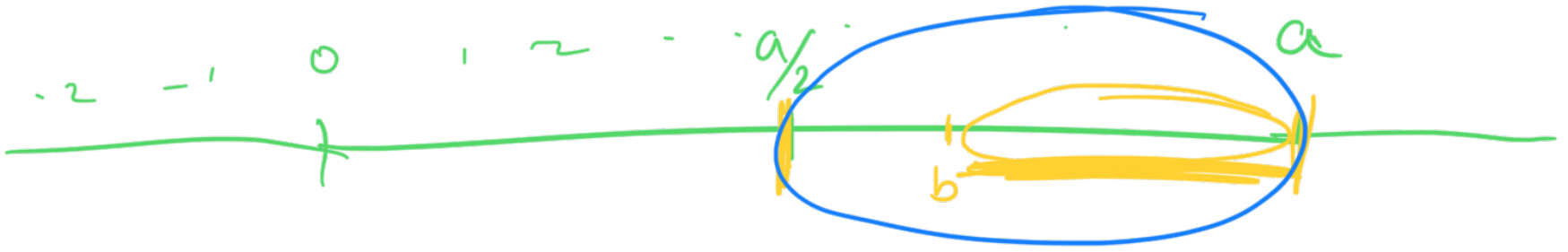
$a\%b < b \le \frac{a}{2}$

$a\%b = a - b < \frac{a}{2}$

$a\%b < \frac{a}{2}$

$a\%b < a/2$

$a\%b = a - b - b - b \ldots$



$a\%b$

$a > b$

$gcd(a, b) = gcd(a\%b, b)$

$< a/2$

$a/4$

$a/8$

$a/16$

T.C $\Rightarrow O(\log(\max(a,b)))$

$gcd(a\%b, b) = gcd(b, a\%b)$

Questions

(2) Given N array elements,

Amazon

Find gcd of the factorial of all elements;

→ ans % (10^9+7)

eg. A = [4 , 3 , 8 , 6]

gcd(4! , 3! , 8! , 6!)

4! = 1 x 2 x 3 x 4

3! = 1 x 2 x 3

8! = 1 x 2 x 3 x 4 x 5 x 6 x 7 x 8

6! = 1 x 2 x 3 x 4 x 5 x 6

gcd :- 1 * 2 * 3 = 3!

Ans => (min number)!

T.C => ? O(N + O(Min_num))

→ ① → Find min in array → O(N)

② → Find factor → a[i]! => O(min num)

→ for loop % (10^9+7)

a[i] $\leq 10^6$.

S.C => O(1)

H.W Given array,

eg. A = [14, 21, 42, 28]

Find gcd of the

gcd (a, b, c) => gcd(gcd((a,b), c)

T.C => N * log(elm)

Break : 10:18

subsequence → we

2, 7, 3, 9, 4, 14

2, 7, 4 → subsequence

Q.3 Given an array, find if there is any subsequence such that

gcd (subsequence) = 1

True/False.

eg. [4, 6, 3, 8] ans = True.

gcd(4, 3, 8) = 1 => gcd(4, 3) = 1

eg. [4, 16, 24] ans = False.

eg. [30, 10, 15, 6] ans = True.

No prime

gcd(10, 6) = 3.

[4, 6, 10]

gcd(10, 15, 6) = 1

obs: ① Assume there a subseq → gcd = 1

if(gcd (entire array) = 1) → true

else return False

Q.4. Facebook Tower Reserch

Given N positive distinct elements,

Choose a[i], a[j] such that i ≠ j

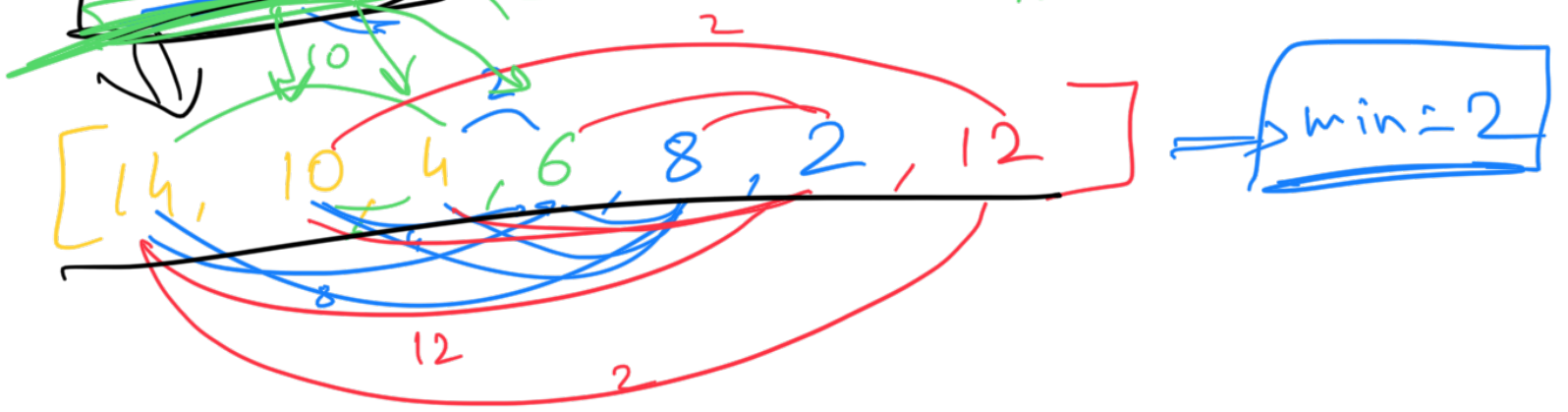
if |a[i] - a[j]| is not present in array, insert |a[i] - a[j]| to array.

Repeat till no more element can be added. Find the min element in the final array.

eg. [6, 2, 12, 8]

[6, 2, 12, 8, 4, 10] → min = 2

eg. [14 10]



min = 2

14, 10 ⇒ 14 - 10

10 - 4 ⇒ 6 ⇒ 6 - 4 = 2

repeated

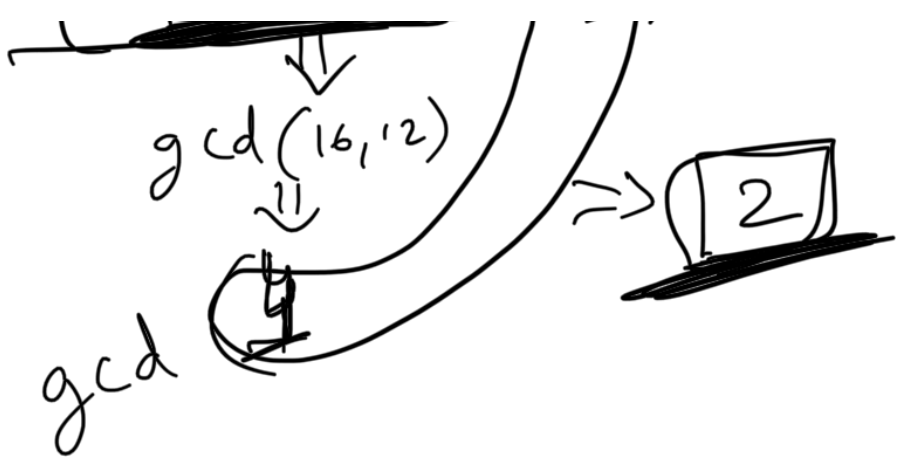
gcd (14, 10)

(14, 10) = 14 % 10 = 4

g(10, 4) 10 % 4 = 2

(4, 2) ⇒ 4 % 2 = 0

[16, 12, 6] ⇒ ans = 2



eg. [7, 9, 5]

find array [7, 9, 5, 2, 4, 3, 1, 6, 8],

min = 1

ans = gcd (all elements)

T.C ⇒ N * log (max elem)

Doubts

gcd (a, b)

T.C. ⇒ log (min(a, b))