MIE 566: Decision Making Under Uncertainty

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Project Part3

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Problem Statement

This game models a pricing competition between two agents: the **Decision Maker (DM)** and the **Opponent**. Both players aim to maximize their outcomes: the DM seeks to optimize market share, while the Opponent is one of two types: Market Share Focused or Profit Focused. Each agent has two pricing strategies to choose from, and decisions are made simultaneously without knowing the other's choice.

Decision Alternatives

- **DM:** Choose between:
    - **Low Price (L):** Focus on attracting more customers at the cost of lower profit margins.
    - **High Price (H):** Prioritize higher profit margins, sacrificing market share.
- **Opponent:** Choose between:
    - **Low Price (l):** Aim to grab a larger market share.
    - **High Price (h):** Target higher revenue per unit by setting a higher price.

Payoff Matrix

The payoffs for each decision combination reflect the trade-offs between market share, profitability, and revenue. The DM's payoffs represent their balance of market share and profits, while the Opponent's payoffs represent their total revenue.

Type 1 Opponent: Market Share Focused

DM's Price	Opponent's Price (l)	Opponent's Price (h)
L	(10, 10)	(15, 5)
H	(5, 15)	(12, 12)

Type 2 Opponent: Profit Focused

DM's Price	Opponent's Price (l)	Opponent's Price (h)
L	(10, 8)	(15, 10)
H	(5, 12)	(12, 15)

Utility

The utility functions are constructed to capture the trade-offs between market share, profitability, and revenue for two competing agents: the Decision Maker (DM) and the Opponent. For the Type 1 Opponent, who prioritizes market share, lower pricing strategies yield higher payoffs as they align with their focus on capturing customer volume. Conversely, for the Type 2 Opponent, which favors profit, higher pricing strategies result in better outcomes, reflecting their preference for maximizing revenue per unit. When the agents set different prices, the one setting the lower price gains a larger market share and achieves a higher payoff. However, when both agents set similar prices, the payoffs depend on the Opponent's type. For a Type 1 Opponent, the payoffs are nearly balanced when prices are the same: (10,10) if both choose low prices and (12,12) if both choose high prices, with the slight preference for high pricing reflecting the general advantage of market stability. For a Type 2 Opponent, when prices are similar, the payoffs show a clear distinction: (10,8) when both choose low prices, indicating the DM gains a higher relative

utility due to Type 2's preference for higher margins, and (12,15) when both choose high prices, where the Opponent's focus on profit maximization results in a greater utility for them.

Construction of Utility Function (Elicited by the Analyst)

If an agent is market share focused, his utility function = **4M+2P**

M stands for Market Capitalization in millions. Given that the prices of both agents are similar, it is expected to be **3** if the price is high and **2** if the price is low. If the prices are different, M is expected to be **6.5** if the price is low, and **0.75** if the price is high.

P stands for Preference which is equal to **2** if the price is low or 1 if the price is high**.**

If an agent is profit focused, his utility function **= 2M +2P**

M stands for Market Capitalization in millions. Given that the prices of both agents are similar, it is expected to be **5.5** if the price is high and **3** if the price is low. If the prices are different, M is expected to be **3** if the price is high, and **5** if the price is low.

P stands for Preference which is equal to **1** if the price is low or **2** if the price is high**.**

For the market share-focused agent, the coefficient 4 for M indicates a strong emphasis on maximizing market capitalization, reflecting their priority on capturing and maintaining a dominant position in the market. The coefficient 2 for P shows a moderate but secondary consideration for their preference (pricing strategy). Conversely, for the profit-focused agent, the equal coefficients of 2 for both M and P suggest a balanced valuation of market capitalization and preference, highlighting their dual focus on achieving reasonable market presence while optimizing profit margins through strategic pricing.

Uncertainty

The DM faces uncertainty regarding the Opponent's type:

- **Type 1 Opponent (Market Share Focused):** There is a 70% chance that the opponent prefers a Low-Price strategy, prioritizing market share.
- **Type 2 Opponent (Profit Focused):** There is a 30% chance that the opponent favors a High Price strategy, targeting revenue per unit.

The probabilities were determined based on common strategies in competitive markets. Companies often focus on gaining market share to establish dominance, especially in industries in which customer growth and long-term benefits like brand loyalty matter most. Studies, such as those by the OECD (OECD, 2023) and insights from Investopedia and Havard business review (Kotler, Rackham, & Krishnaswamy, 2006), show that aggressive pricing to capture market share is more common than focusing solely on profits. Based on this, it was reasonable to assign a 70% probability to the Opponent being market focused. This probabilistic uncertainty is central to the problem, making it suitable for a Bayesian game-theoretic approach. Both players must choose their strategies simultaneously while accounting for the possible motivations of the other.

Objective

The objective of this analysis is to determine the **Bayesian Nash Equilibrium**. This is the set of strategies where neither player can improve their payoff by unilaterally changing their decision, given the uncertainty about the Opponent's type.

The game will be modelled and solved using the Game Theory Explorer tool, with results presented in both **Normal Form** (payoff matrix) and **Extensive Form** (decision tree). The analysis will also discuss the implications of the results and potential limitations of the model.

Strategy

Strategies of DM are {LL, LH, HL, HH}

Strategies of the opponent are {ll, lh, hl, hh}

Pure strategies: {(LL,ll), (LL,lh), (LL,hl), (LL,hh), (LH,ll), (LH,lh), (LH,hl), (LH,hh), (HL,ll), (HL,lh), (HL,hl), (HL,hh), (HH,ll), (HH,lh), (HH,hl), (HH,hh)}

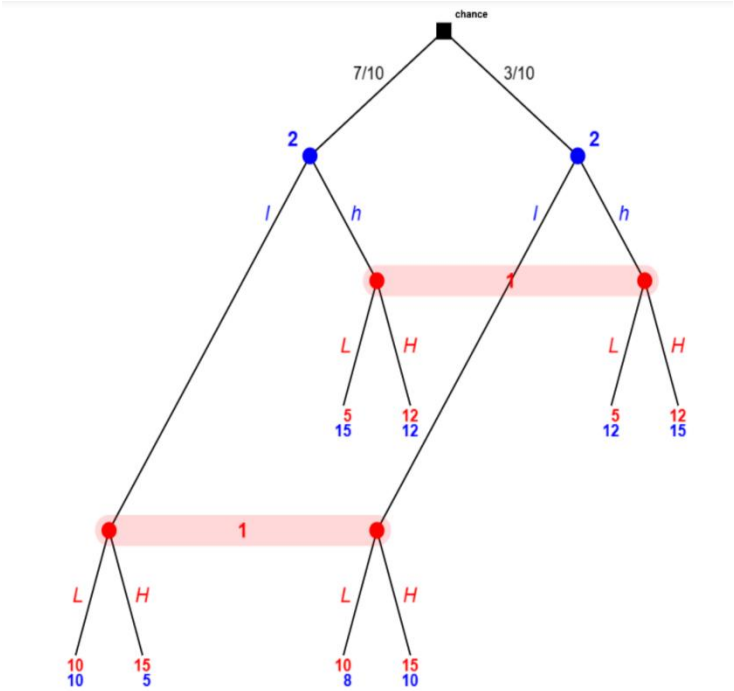
All strategies are based on the type of the Opponent. For example, (LH, hl) means if the opponent is type 1, the decision maker will set the price low, and the opponent will set the price high, while if the opponent is type 2, the decision maker will set the price high, and the opponent will set the price low.

Strategy and Opponent Types Combination

Decision Maker's Strategy	Opponent's Strategy	Payoff when the opponent is type 1 (Market Share Focused)	Payoff when the opponent is type 2 (Profit Focused)
LL	ll	(10,10)	(10,8)
LL	lh	(10,10)	(5,12)
LL	hl	(5,15)	(10,8)
LL	hh	(5,15)	(5,12)
LH	ll	(10,10)	(10,8)
LH	lh	(10,10)	(5,12)
LH	hl	(5,15)	(10,8)
LH	hh	(5,15)	(5,12)
HL	ll	(15,5)	(10,8)
HL	lh	(15,5)	(5,12)
HL	hl	(12,12)	(10,8)
HL	hh	(12,12)	(5,12)
HH	ll	(15,5)	(10,8)

HH	lh	(15,5)	(5,12)
HH	hl	(12,12)	(10,8)
HH	hh	(12,12)	(5,12)

Extensive Form



P2 denotes the

P1 denotes the Decision Maker

Chance denotes Nature

Normal Form

		Opponent			
		ll	**lh**	**hl**	**hh**
	LL	(10, 9.4)	(8.5, 10.6)	(6.5, 12.9)	(5, 14.1)

Decision Maker	**LH**	(10, 9.4)	(10.6, 11.5)	(11.4, 10.8)	(12, 12.9)
	HL	(15, 6.5)	(12, 7.1)	(8, 13.5)	(5, 14.1)
	HH	(15, 6.5)	(14.1, 8)	(12.9, 11.4)	(12, 12.9)

Nash Equilibrium

The Nash Equilibria in this game are (LH, hh) and (HH, hh) with expected payoffs of (12, 12.9). In both cases, the Opponent always chooses the strictly dominant strategy hh (High Price for both types). The DM can either choose strategies with LH, setting a low price for type 1 Opponent and a high price for the other type, or choose the weakly dominant strategy HH, setting high prices for both types.

Summary

In summary, it is clear that the strategy **hh** (High Price for both types) of the Opponent strictly dominates his other strategies, meaning it always provides a higher payoff regardless of the Decision Maker's actions. On the other hand, the strategy **HH** (High Price for both types) of the Decision Maker weakly dominates their other strategies, meaning it provides payoffs that are at least as good as other strategies and strictly better in some cases.

This model represents a **Static Bayesian Game with Incomplete Information**, as the Decision Maker is uncertain about whether the Opponent is of Type 1 (market share-focused) or Type 2 (profit-focused). The Decision Maker must rely on subjective probabilities to evaluate their strategies and make decisions based on expected payoffs, considering the incomplete information about the Opponent's type.

Limitation

- Assumptions about opponent's behavior are simplified as they can only be two types (market oriented or profit-focused) whereby in reality players have mixed strategies

which evolve as the game goes which is not captured by the binary classification therefore restricts the applicability of the model to scenarios where opponent's preferences are more dynamic and complex

- Fixed probabilities for opponent's type as they do not change during the game thus may not reflect real life situations whereby the probabilities can be updated based on observed actions by making iterative use of Bayesian inference (Bayesian Updating)
- Static Decision-making Framework which assumes the players' decisions are made simultaneously therefore taking no account of adaptive behavior whereby players can adjust their strategies based on evolving market and/or observed outcomes similar to real-world strategic decision making. Making this into a Dynamic Bayesian game can solve this limitation
- Single-Attribute Utility as payoffs (solely based on market-share and profits) are the sole utility focus thus ignoring multi-dimensional objectives, such as brand value, long-term growth, or customer retention which overlook critical factors that influence real-world decisions.
- Nash Equilibrium as the Sole Solution Concept does not consider alternative equilibrium concepts, such as correlated equilibria or mixed strategies, that might better capture nuanced strategic interactions; This might limit the scope of insights derived from the model.

References

Kotler, P., Rackham, N., & Krishnaswamy, S. (2006, July). *Ending the War Between Sales and Marketing*. Retrieved from hbr.org: https://hbr.org/2006/07/ending-the-war-between-sales-and-marketing

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