4 Convection in a 2D Annulus

This case demonstrates the application of Nek5000 to simulation of natural convection between the two concentric cylinders as considered by Grigull & Hauf [5] and presented by Van Dyke [6]. The inner cylinder (diameter $D/3$) is slightly heated with with respect to the outer one (diameter D). The Boussinesq approximation is used to formulate the equations of motion, valid in situations where density differences are small enough to be neglected everywhere except in the gravitational forcing.

Normalizing the Navier-Stokes and energy equations with D for the length scale and D/U for the time scale ($U \sim \sqrt{\beta\, g\, D\, (T_1 - T_0)}$ is the characteristic velocity in the given problem), and introducing nondimensional temperature $\theta = (T - T_0)/(T_1 - T_0)$, where T_0 and T_1 are the respective temperatures of the outer and inner cylinders, the governing nondimensional equations are

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla p + \frac{1}{\sqrt{Gr}} \nabla^2 \mathbf{u} + \theta, \qquad \nabla \mathbf{u} = 0 \\ \frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta &= \frac{1}{\sqrt{Gr}\, Pr} \nabla^2 \theta. \end{aligned} \tag{7}$$

where two nondimensional parameters were introduced

$$Gr = \frac{\beta\, g\, (T_1 - T_0)\, D^3}{\nu^2}, \qquad Pr = \frac{\alpha}{\nu}, \tag{8}$$

denoting Grashof number and Prandtl number, respectively. Here, $\mathbf{u}$ is the velocity vector, p is the pressure divided by density, ν is the kinematic viscosity, α is the thermal diffusivity, β is the volumetric thermal expansion coefficient, and g is the acceleration due to gravity.

The computational mesh for a steady-state result at Gr=120,000 and $Pr = 0.8$ is shown in Fig. 3 (left). The simulations are run until the computational time $t\,D/U \sim 1000$, by which time a steady-state solution is obtained, indicated by the change in velocity magnitude $\sim 10^{-7}$ between successive time steps. The steady-state streamlines and isotherms are visualized in Fig. 3 center and right panels. The results are in excellent agreement with the results of Grigull & Hauf that were published by Van Dyke.

[5] Grigull & Hauf, Proc. of the 3rd Int. Heat Transfer Conf. 2, p. 182–195 (1966)

[6] M. Van Dyke *An Album of Fluid Motion*, Parabolic Press, Stanford, CA, 1982

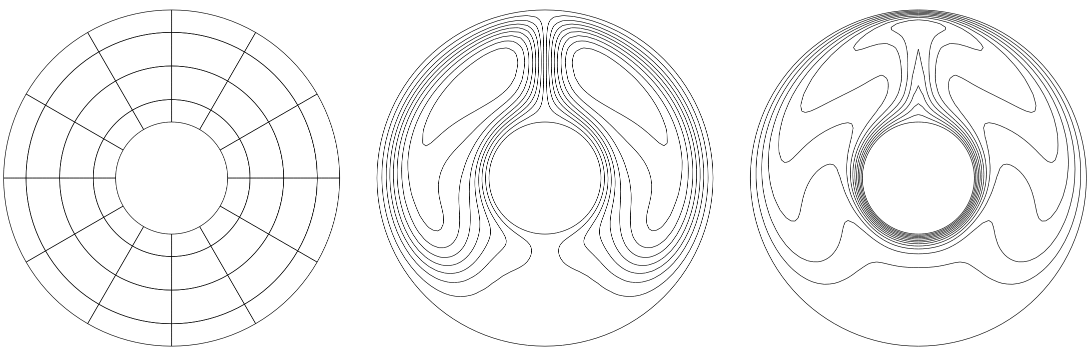

Figure 3: 2D natural convection: (left) mesh with E=32 spectral elements; (center) streamlines for Gr=120,000; and (right) isothermal lines.