7 Stability of Free-Surface Channel Flow

As a check of the free-surface ALE formulation in Nek5000, we compare the growth rate for the most unstable mode for a falling film with results from linear theory.[10] For perturbation size ϵ, one can expect $O(\epsilon)$ agreement in growth rate between linear and nonlinear models.

The nominal computational domain was taken as $\Omega = [0, 2\pi] \times [-1, 0]$, tesselated with a 6×10 array of spectral elements. A uniform element distribution was used in the streamwise direction while a stretched distribution was used in the wall-normal direction. Near the wall, an element thickness of Δy=.005 was used to resolve the boundary layer of the unstable eigenmodes. The polynomial order within each element was N = 13 and BDF3/EXT3 timestepping was used with Δt=.00125. The initial conditions corresponded to the base flow plus $\epsilon := 10^{-5}$ times the most unstable eigenmode for these particular flow conditions. The domain was stretched using an affine mapping in y to accomodate the $O(\epsilon)$ surface displacement. The eigenmodes, which are defined only on y = [-1, 0], were mapped onto the nominal domain then displaced along with the mesh. The base flow was defined as $U(y) = 1 - y^2$ over the deformed mesh. The Reynolds number was Re=30000, Weber number We=0.011269332539972, and gravitational Prandtl number Pg=.00011. The applied body force was $\mathbf{f} = (2Re^{-1}, -(Re\,Pg)^{-2})$ and the surface tension was $\sigma = We$.

Mean growth rates were computed by monitoring the L^2-norm of the wall-normal velocity v and defining $\gamma(t) := t^{-1} \ln(||v(\mathbf{x}, t)||_2/||v(\mathbf{x}, 0)||_2)$. The error is defined as $e(t) := (\gamma(t) - \gamma^*)/\gamma^*$ where γ^*=0.007984943826436 is computed using linear theory. Aside from some initial transients, the error over t = [0,200] was less than .0005.

Initial tests for this problem revealed blow-up at fairly late times ($t \sim 160$). The locality and high wavenumber content in the error indicates that the blow-up is due to lack of de-aliasing in certain nonlinear terms associated with the ALE formulation. (The convective terms were dealiased; p99=3.) As illustrated in Fig. 6, such high wavenumber errors are readily addressed with low pass filtering, here implemented with p103=.05 and p101=2. We note that the error in the predicted growth rate does increase at $t > 1000$ for reasons unknown at this point, but which might be attributable to true nonlinear effects (i.e., departure from linearity).

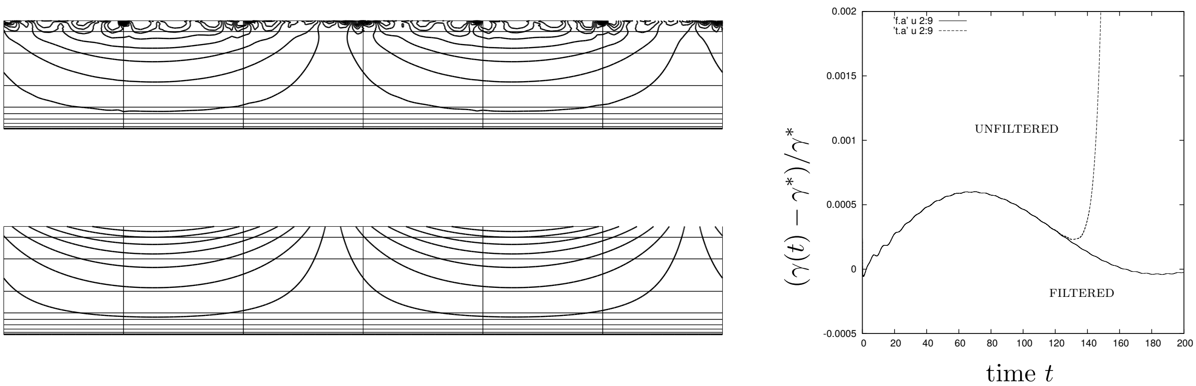

Figure 6: Eigenmodes for free-surface film flow: (left, top) contours of vertical velocity v for unfiltered and (left, bottom) filtered solution at time $t = 179.6$; (right) error in growth rate vs. t.

[10] Instabilities in free-surface Hartmann flow at low magnetic Prandtl numbers. Giannakis, D., Rosner, R., & Fischer, P.F. 2009, J. Fluid Mech., 636, 217-277