



Figure 9: Vortex breakdown results: (left) streamlines for $Re=1854$, (center) in-plane streamlines for $Re=1854$, (right) vertical velocity distributions along the centerline.

9 Vortex Breakdown

Escudier¹² studied vortex breakdown in a container with a rotating lid. The domain consists of a cylindrical container of radius R and height $H = 2R$. The top lid rotates at a constant angular velocity Ω , and the Reynolds number is $Re = \Omega^2 R / \nu$. Following the standard approach, we take $\Omega = R = \rho = 1$ and set $\nu = Re^{-1}$.

The mesh in Fig. 9 is constructed from a 2D base swept through z with element boundaries at $z=0, 0.06, 0.4, 0.8, 1.2, 1.6, 1.95$, and 2 . The mesh is concentrated near the canister walls and near the upper and lower lids. The singularity at $r=R, z=2$ is handled by shearing the side walls in the top layer of elements using a 5th-order monomial. The initial and boundary condition for $z > 1.95$ are thus $(u, v) = (-y, x) * \alpha(z)$, with $\alpha = (z - 2)^5 / \Delta_z^5$ and $\Delta_z = .05$. The simulations are time-marched to steady state at varying spatial resolutions ($N=7, 9$, and 11). Simulation times of $t_f \approx 1000$ are required to form a single bubble in the $Re = 1492$ case.

Depending on the aspect ratio and Reynolds number, one can find various steady and unsteady vortex breakdown regimes with one or more “bubbles” (reversal regions) on the axis. For $H/R=2$, Escudier documented steady-state flows with with a single bubble at $Re=1492$ and two bubbles at $Re=1854$. The streamline plots in Fig. 9 show the bubble structures for $Re=1854$. Bubble locations can be inferred from zero-crossings of axial velocity w versus z at $(x, y) = (0, 0)$, shown in the right panel. These locations are tabulated below, along with experimental results of Escudier and numerical results of Sotiropoulos and Ventikos.¹³

Locations of Vertical Velocity Reversals						
	$Re=1492$		$Re=1854$			
	z_1	z_2	z_1	z_2	z_3	z_4
$N=7$.689	.836	.427	.793	.960	1.131
$N=9$.671	.831	.420	.776	.954	1.118
$N=11$.671	.831	.420	.775	.955	1.117
Escudier	.68	.80	.42	.74	1.04(?)	1.18(?)
Sot. & Ven	.646	.774	.42	.772	.928	1.09

¹²M.P. Escudier, “Observations of the flow produced in a cylindrical container by a rotating endwall,” *Exp. Fluids* **2** 189–196 (1984).

¹³F. Sotiropoulos & Y. Ventikos, “Transition from bubble-type vortex breakdown to columnar vortex in a confined swirling flow,” *Int. J. Heat and Fluid Flow* **19** 446–458 (1998).