10.4.3.1.3

EE24BTECH11051 - Prajwal

1) Find the roots of the following quadratic equations by completing the squares.

$$4x^2 + 4\sqrt{3}x + 3 = 0 \tag{1.1}$$

Theoritical Solution-
Checking roots of equation exist or not,

$$b^2 - 4ac \geq 0 \tag{1.2}$$
$$= 48 - 4(4)(3) \tag{1.3}$$
$$= 0 \tag{1.4}$$

This means roots of equation exist and are collinear.
And its root is given by

$$4x^2 + 4\sqrt{3}x + 3 = 0 \tag{1.5}$$
$$(2x - \sqrt{3})^2 = 0 \tag{1.6}$$
$$x = -\frac{\sqrt{3}}{2} = -0.866025 \tag{1.7}$$

CODING LOGIC:-
Eigen value method

a) Characteristics polynomial is given by

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \tag{1.8}$$

where $a_n \neq 0$

b) Divide Characteristics equation by a_n

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \tag{1.9}$$
$$p(x) = x^n + \frac{a_{n-1}}{a_n}x^{n-1} + \cdots + \frac{a_1}{a_n}x + \frac{a_0}{a_n} \tag{1.10}$$

c) Companion Matrix of characteristic polynomial is given by:
Let

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix} \tag{1.11}$$

d) Finding the eigen values of the companion matrix (1.13) we will get the roots of the given quadratic equation
e) Advantages of using Eigen value approach

   i) Works for polynomials of any degree.

   ii) Handles complex roots seamlessly.