10.4.3.1.3

Prajwal
EE24BTECH11051

January 23, 2025

Problem Statement

Find the roots of the following quadratic equations by completing the squares if exits.

Theoretical Solution

Checking roots of equation exist or not,

$$b^2 - 4ac \geq 0 \tag{3.1}$$
$$= 48 - 4(4)(3) \tag{3.2}$$
$$= 0 \tag{3.3}$$

This means roots of equation exist and are coincident.
And its root is given by

$$4x^2 + 4\sqrt{3}x + 3 = 0 \tag{3.4}$$
$$(2x - \sqrt{3})^2 = 0 \tag{3.5}$$
$$x = -\frac{\sqrt{3}}{2} = -0.866025 \tag{3.6}$$

Computational logic

Eigen value method

Characteristics polynomial is given by

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \tag{4.1}$$

where $a_n \neq 0$

Divide Characteristics equation by a_n

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \tag{4.2}$$

$$p(x) = x^n + \frac{a_{n-1}}{a_n} x^{n-1} + \cdots + \frac{a_1}{a_n} x + \frac{a_0}{a_n} \tag{4.3}$$

Companion Matrix of characteristic polynomial is given by:
Let

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix} \tag{4.4}$$

QR decomposition

$$A = QR \tag{4.5}$$

Q is an $m \times n$ orthogonal matrix

R is an $n \times n$ upper triangular matrix. Given a matrix $A = [a_1, a_2, \ldots, a_n]$, where each a_i is a column vector of size $m \times 1$.

Normalize the first column of A:

$$q_1 = \frac{a_1}{\|a_1\|} \tag{4.6}$$

For each subsequent column a_i, subtract the projections of the previously obtained orthonormal vectors from a_i :

$$a_i' = a_i - \sum_{k=1}^{i-1} \langle a_i, q_k \rangle q_k \tag{4.7}$$

Normalize the result to obtain the next column of Q:

$$q_i = \frac{a_i'}{\|a_i'\|} \tag{4.8}$$

Repeat this process for all columns of A.

Finding R:-
After constructing the ortho-normal columns $q_1, q_2, \ldots, q_n$ of Q, we can compute the elements of R by taking the dot product of the original columns of A with the columns of Q:

$$r_{ij} = \langle a_j, q_i \rangle \text{ , for } i \leq j \tag{4.9}$$

QR-Algorithm

Initialization
Let $A_0 = A$, where A is the given matrix.

QR Decomposition
For each iteration $k = 0, 1, 2, \ldots:$

Compute the QR decomposition of A_k, such that:

$$A_k = Q_k R_k \tag{4.10}$$

where:

Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).

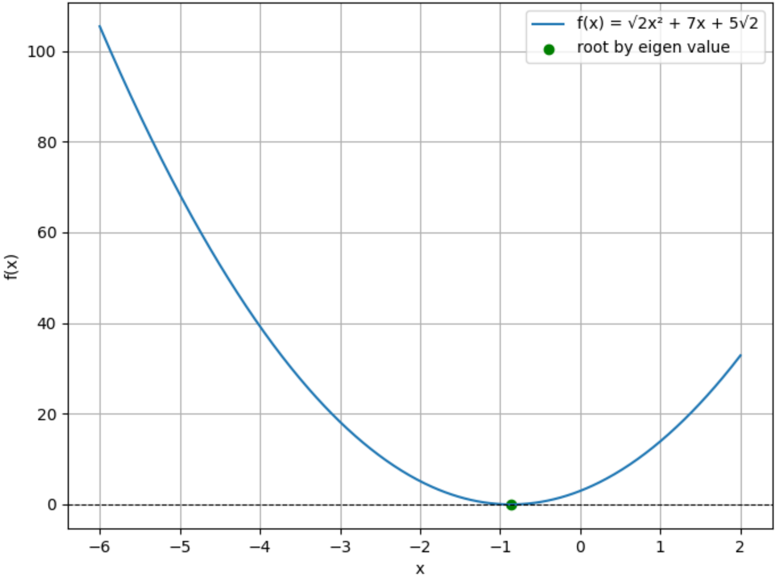
R_k is an upper triangular matrix. The decomposition ensures $A_k = Q_k R_k$.

Form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \tag{4.11}$$

Convergence
Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal entries of T are the eigenvalues of A.

The eigenvalues of matrix will be the roots of the equation.


f(x) = √2x² + 7x + 5√2
root by eigen value
100
80
60
40
20
0
f(x)
−6
−5
−4
−3
−2
−1
0
1
2
x