10.4.3.1.3

EE24BTECH11051 - Prajwal

1) Find the roots of the following quadratic equations by completing the squares.

$$4x^2 + 4\sqrt{3}x + 3 = 0 \tag{1.1}$$

Theoritical Solution-
Checking roots of equation exist or not,

$$\begin{aligned} b^2 - 4ac &\geq 0 && (1.2)\\ = 48 - 4(4)(3) && (1.3)\\ &= 0 && (1.4) \end{aligned}$$

This means roots of equation exist and are collinear.
And its root is given by

$$\begin{aligned} 4x^2 + 4\sqrt{3}x + 3 &= 0 && (1.5)\\ (2x - \sqrt{3})^2 &= 0 && (1.6)\\ x = -\frac{\sqrt{3}}{2} &= -0.866025 && (1.7) \end{aligned}$$

CODING LOGIC:-
Eigen value method

1) Characteristics polynomial is given by

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \tag{1.1}$$

where $a_n \neq 0$

2) Divide Characteristics equation by a_n

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \tag{2.1}$$

$$p(x) = x^n + \frac{a_{n-1}}{a_n}x^{n-1} + \cdots + \frac{a_1}{a_n}x + \frac{a_0}{a_n} \tag{2.2}$$

3) Companion Matrix of characteristic polynomial is given by:
Let

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix} \tag{3.1}$$

4) QR decomposition

$$A = QR \tag{4.1}$$

a) Q is an $m \times n$ orthogonal matrix
b) R is an $n \times n$ upper triangular matrix.

Given a matrix $A = [a_1, a_2, \ldots, a_n]$, where each a_i is a column vector of size $m \times 1$.

5) Normalize the first column of A:

$$q_1 = \frac{a_1}{\|a_1\|} \tag{5.1}$$

6) For each subsequent column a_i, subtract the projections of the previously obtained orthonormal vectors from a_i :

$$a_i' = a_i - \sum_{k=1}^{i-1} \langle a_i, q_k \rangle q_k \tag{6.1}$$

Normalize the result to obtain the next column of Q:

$$q_i = \frac{a_i'}{\left\|a_i'\right\|} \tag{6.2}$$

Repeat this process for all columns of A.

7) Finding R:-
After constructing the ortho-normal columns $q_1, q_2, \ldots, q_n$ of Q, we can compute the elements of R by taking the dot product of the original columns of A with the columns of Q:

$$r_{ij} = \langle a_j, q_i \rangle \text{ , for } i \leq j \tag{7.1}$$

8) **QR-Algorithm**

a) Initialization
Let $A_0 = A$, where A is the given matrix.
b) QR Decomposition
For each iteration $k = 0, 1, 2, \ldots$:
i) Compute the QR decomposition of A_k, such that:

$$A_k = Q_k R_k \tag{8.1}$$

where:
A) Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).
B) R_k is an upper triangular matrix.
The decomposition ensures $A_k = Q_k R_k$.
ii) Form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \tag{8.2}$$

c) Convergence
Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal entries of T are the eigenvalues of A.

d) The eigenvalues of matrix will be the roots of the equation.