12.8.ex12

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Problem Statement

Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

Theoretical Solution

Set up the integral:
The area under the curve can be calculated as:

$$\text{Area} = \int_a^b f(x)dx \tag{3.1}$$

Here:

$$f(x) = 3x + 2, \quad x_1 = -1, \quad x_2 = 1 \tag{3.2}$$

Check whether the line touches the x-axis in the interval $x \in (-1, 1)$

$$y = 0 = 3x + 2 \tag{3.3}$$

$$x = \frac{-2}{3} \tag{3.4}$$

$$\tag{3.5}$$

As $x = \frac{-2}{3} \in (-1, 1)$

Thus, the integral becomes:

$$\text{Area} = -\int_{-1}^{-2/3} (3x+2)\,dx + \int_{-2/3}^{1} (3x+2)\,dx \tag{3.6}$$

Compute the integral:
The integral of $3x + 2$ is:

$$\int 3x + 2\,dx = \frac{3x^2}{2} + 2x \tag{3.7}$$

Evaluate the definite integral:
Substitute the limits of integration:

$$\text{Area} = -\left[\frac{3x^2}{2} + 2x\right]_{-1}^{-2/3} + \left[\frac{3x^2}{2} + 2x\right]_{-2/3}^{1} \tag{3.8}$$

$$\text{Area} = \frac{13}{3} \tag{3.9}$$

Computational logic

Using the trapezoidal rule to get the area. The trapezoidal rule is as follows.

$$\int_a^b f\left(x\right)dx \approx \sum_{k=1}^{N} \frac{f\left(x_{k+1}\right) + f\left(x_k\right)}{2} h \tag{4.1}$$

where

$$h = \frac{b-a}{N} \tag{4.2}$$

$\therefore$The difference equation obtained is

$$A = \int_a^b f(x)\ dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{4.3}$$

$$h = \frac{b-a}{n} \tag{4.4}$$

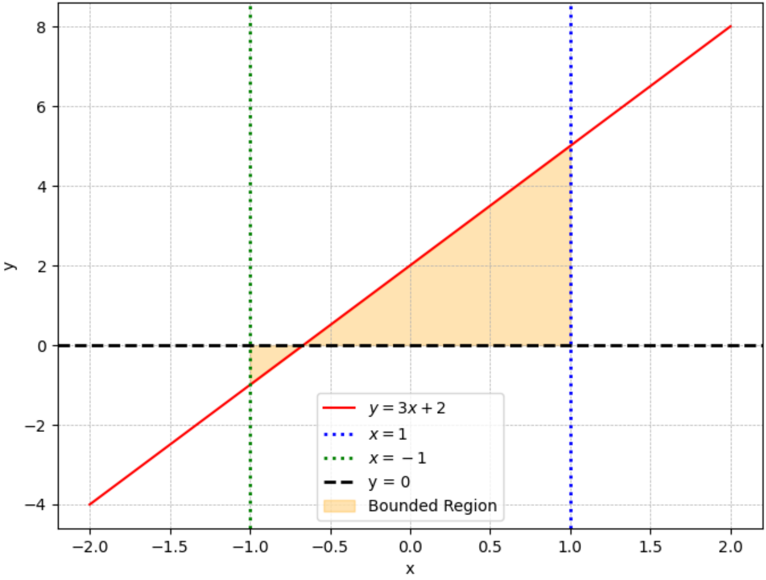
$$A = j_N,\ \text{where, } j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{4.5}$$

$$\rightarrow j_{i+1} = j_i + h\frac{(3x_{i+1} + 2 + 3x_i + 2)}{2} \tag{4.6}$$

$$x_{i+1} = x_i + h \tag{4.7}$$

$$h = 0.00001 \tag{4.8}$$

$$n = 300000 \tag{4.9}$$


8
6
4
2
0
−2
−4
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x
$y = 3x + 2$
$x = 1$
$x = -1$
y = 0
Bounded Region