10.3.2.7

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EE24BTECH11051

January 30, 2025

Problem Statement

Draw the graph of the equation $xy + 1 = 0$ and $3x + 2y12 = 0$. Determine the coordinates of the vertices of the triangle formed by the lines and x-axis.

Theoretical Solution

Given,

$$x - y + 1 = 0 \tag{3.1}$$

$$3x + 2y - 12 = 0 \tag{3.2}$$

$$y = 0 \tag{3.3}$$

lines $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ touches ‘x-axis at,

$$x = -1 \tag{3.4}$$

$$x = 4 \tag{3.5}$$

Both given line touches at,

$$x = 2 \tag{3.6}$$

$$y = 3 \tag{3.7}$$

Computational logic

Let us assume the given system of equations are consistent and we will try solving using LU decomposition
Given the system of linear equations:

$$x - y + 1 = 0 \tag{4.1}$$

$$3x + 2y - 12 = 0 \tag{4.2}$$

$$y = 0 \tag{4.3}$$

We rewrite the equations as:

$$x_1 = x, \tag{4.4}$$

$$x_2 = y, \tag{4.5}$$

giving the system:

$$x_1 - x_2 = -1 \tag{4.6}$$

$$3x_1 + 2x_2 = 12 \tag{4.7}$$

$$x_2 = 0 \tag{4.8}$$

We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{4.9}$$

where:

$$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \tag{4.10}$$

$$A_2 = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}, \tag{4.11}$$

$$A_3 = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}, \tag{4.12}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{4.13}$$

$$\mathbf{b_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \tag{4.14}$$

$$\mathbf{b_2} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}, \tag{4.15}$$

$$\mathbf{b_3} = \begin{bmatrix} -1 \\ 12 \end{bmatrix}. \tag{4.16}$$

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

LU decomposition by Doolittle's Method.

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix **A** is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where **L** is a lower triangular matrix with ones on the diagonal, and **U** is an upper triangular matrix.
For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it.

Using a code we get L,U as

$$L_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, U_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \tag{4.17}$$

$$L_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, U_2 = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \tag{4.18}$$

$$L_3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, U_3 = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \tag{4.19}$$

We solve:

$$L_1\mathbf{y_1} = \mathbf{b_1} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{4.20}$$

$$L_2\mathbf{y_2} = \mathbf{b_2} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}. \tag{4.21}$$

$$L_3\mathbf{y_3} = \mathbf{b_3} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 15 \end{bmatrix}. \tag{4.22}$$

Thus:

$$\mathbf{y_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{4.23}$$

$$\mathbf{y_2} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}. \tag{4.24}$$

$$\mathbf{y_3} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{4.25}$$

$$\tag{4.26}$$

We solve:

$$U_1\mathbf{x_1} = \mathbf{y_1} \quad \text{or} \quad \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{4.27}$$

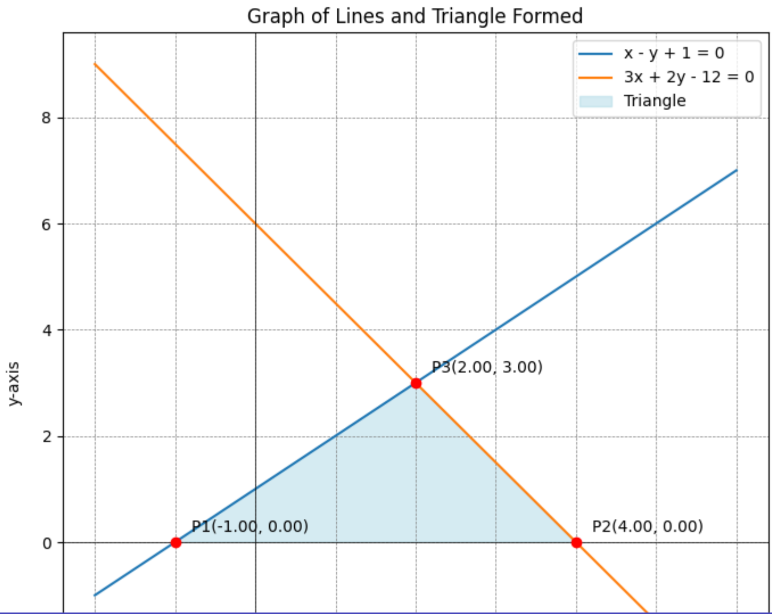
$$U_2\mathbf{x_2} = \mathbf{y_2} \quad \text{or} \quad \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}. \tag{4.28}$$

$$U_3\mathbf{x_3} = \mathbf{y_3} \quad \text{or} \quad \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{4.29}$$

$$\mathbf{x_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{4.30}$$

$$\mathbf{x_2} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}. \tag{4.31}$$

$$\mathbf{x_3} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}. \tag{4.32}$$



Graph of Lines and Triangle Formed
x - y + 1 = 0
3x + 2y - 12 = 0
Triangle
P3(2.00, 3.00)
P1(-1.00, 0.00)
P2(4.00, 0.00)
y-axis
8
6
4
2
0