Chapter-20- Vector Algebra and Three Dimensional Geometry

EE24BTECH11051 - Prajwal

I. MCQs and One Correct Answer

1) Let $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ be vectors such that $\mathbf{u}+\mathbf{v}+\mathbf{w} = 0$. If $|\mathbf{u}| = 3, |\mathbf{v}| = 4$ and $|\mathbf{w}| = 5$, then $\mathbf{u}\cdots\mathbf{v}+\mathbf{v}\cdots\mathbf{w}+\mathbf{u}\cdot\mathbf{w}$ is (1995S)

a) 47
b) -25
c) 0
d) 25

2) If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are three non-coplanar vectors then $(\mathbf{a}+\mathbf{b}+\mathbf{c})\cdots[(\mathbf{a}+\mathbf{b})\times(\mathbf{a}+\mathbf{c})]$ equals (1995S)

a) 0
b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$
c) $2\,[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$
d) $-[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

3) Let $\mathbf{a} = 2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $\mathbf{b} = \mathbf{i}+\mathbf{j}$. If $\mathbf{c}$ is a vector such that $\mathbf{a}\ldots\mathbf{c} = |\mathbf{c}|, |\mathbf{c}-\mathbf{a}| = 2\sqrt{2}$ and the angle between $(\mathbf{a}\times\mathbf{b})$ and $\mathbf{c}$ is 30°, then $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}| =$ (1999 - 2 Marks)

a) $\frac{2}{3}$
b) $\frac{3}{2}$
c) 2
d) 3

4) Let $\mathbf{a} = 2\mathbf{i}+\mathbf{j}+\mathbf{k}, \mathbf{b} = \mathbf{i}+2\mathbf{j}-\mathbf{k}$ and a unit vector $\mathbf{c}$ be coplanar. If $\mathbf{c}$ is perpendicular to $\mathbf{a}$, then $\mathbf{c} =$ (1999 - 2 Marks)

a) $\frac{1}{\sqrt{2}}(-\mathbf{j}+\mathbf{k})$
b) $\frac{1}{\sqrt{3}}(-\mathbf{i}-\mathbf{j}-\mathbf{k})$
c) $\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j})$
d) $\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}-\mathbf{k})$

5) If the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ from the sides BC, CA and AB respectively of a triangle ABC, then (2000S)

a) $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a} = 0$
b) $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{c} = \mathbf{c}\ldots\mathbf{a}$
c) $\mathbf{a}\times\mathbf{b} = \mathbf{b}\times\mathbf{c} = \mathbf{c}\times\mathbf{a}$
d) $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a} = 0$

6) Let the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ be such that $(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = 0$. Let A and B be planes determined by the pairs of vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}, \mathbf{d}$ respectively. Then the angle between A and B is (2000S)

a) 0
b) $\frac{\pi}{4}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{2}$

7) If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit coplanar vectors, then the scalar triple product $[2\mathbf{a}-\mathbf{b}, 2\mathbf{b}-\mathbf{c}, 2\mathbf{c}-\mathbf{a}] =$ (2000S)

a) 0
b) 1
c) $-\sqrt{3}$
d) $\sqrt{3}$

8) Let $\mathbf{a} = \mathbf{i}-\mathbf{k}, \mathbf{b} = x\mathbf{i}+\mathbf{j}+(1-x)\mathbf{k}$ and $\mathbf{c} = y\mathbf{i}+x\mathbf{j}+(1+x-y)\mathbf{k}$. Then $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ depends on (2001S)

a) only x
b) only y
c) Neither x Nor y
d) both x and y

9) If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors, then $|\mathbf{a}-\mathbf{b}|^2+|\mathbf{b}-\mathbf{c}|^2+|\mathbf{a}-\mathbf{b}|^2$ does not exceed (2001S)

a) 4
b) 9
c) 8
d) 6

10) If $\mathbf{a}$ and $\mathbf{b}$ are two unit vectors such that $\mathbf{a}+2\mathbf{b}$ and $5\mathbf{a}-4\mathbf{b}$ are perpendicular to each other then the angle between $\mathbf{a}$ and $\mathbf{b}$ is (2002S)

a) $45°$
b) $60°$
c) $\arccos \frac{1}{3}$
d) $\arccos \frac{2}{7}$

11) Let $\mathbf{V} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{W} = \mathbf{i} + 3\mathbf{k}$. If $\mathbf{U}$ is a unit vector, then the maximum value of the scalar triple product $(U\ V\ W)$ is (2002S)

a) -1
b) $\sqrt{10} + \sqrt{6}$
c) $\sqrt{59}$
d) $\sqrt{60}$

12) The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is (2003S)

a) 7
b) -7
c) no real value
d) 4

13) The value of 'a' so that the volume of parallelopiped formed by $\mathbf{i} + a\mathbf{j} + \mathbf{k}, \mathbf{j} + a\mathbf{k}$ and $a\mathbf{i} + \mathbf{k}$ becomes minimum is (2003S)

a) -3 b) 3 c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{3}$

14) If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$. Then $\mathbf{b}$ is (2004S)

a) $\mathbf{i} - \mathbf{j} + \mathbf{k}$
b) $2\mathbf{j} - \mathbf{k}$
c) $\mathbf{i}$
d) $2\mathbf{i}$

15) If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is (2005S)

a) $\frac{3}{2}$ b) $\frac{9}{2}$ c) $\frac{2}{9}$ d) $\frac{-3}{2}$