

9-9.2-25

EE24BTECH11051 - Prajwal

Question:

Using integration, find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

Solution: The parameters of the conics are

Variable	Description
V_1, u_1, f_1	Parameters of Parabola
V_2, u_2, f_2	Parameters of Parabola
P_1, P_2	Points of intersection
A	Area between the conics

TABLE 0: Variables Used

$$V_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, u_1 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f_1 = 0 \quad (0.1)$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, u_2 = \begin{pmatrix} 0 \\ -2 \end{pmatrix}, f_2 = 0 \quad (0.2)$$

The intersection of two conics with parameters $V_i, u_i, f_i, i = 1, 2$ is defined as

$$x^T (V_1 + \mu V_2) x + 2 (u_1 + \mu u_2)^T x + (f_1 + \mu f_2) = 0 \quad (0.3)$$

Solving this the points of intersection are

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (0.4)$$

Area between the curves is,

$$\int_0^4 \left(\sqrt{4x} - \frac{x^2}{4} \right) dx \quad (0.5)$$

By solving the integration, we get area is equal to 5.33 sq.units

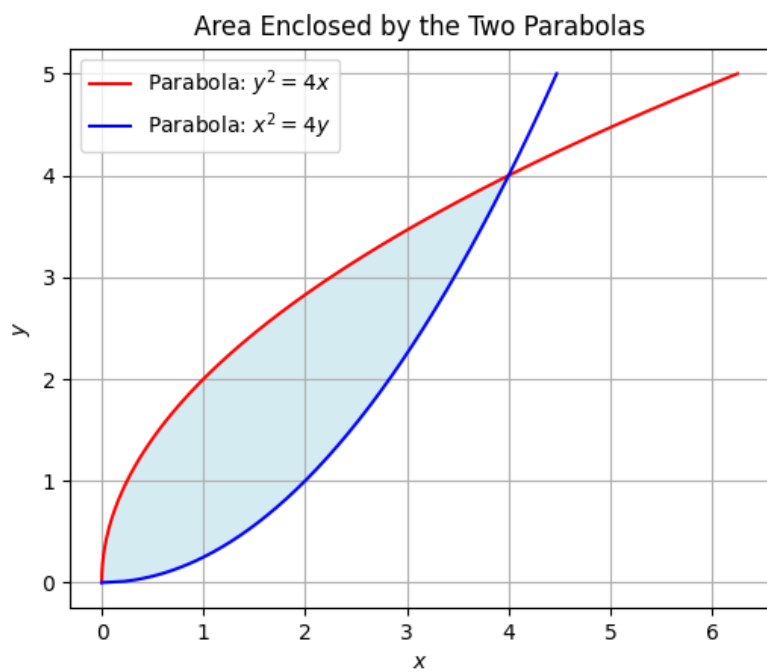


Fig. 0.1