

presentation

using beamer

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Question

Find the product of perpendiculars drawn from the foci of ellipse $(x^2/9) + (y^2/25) = 1$ upon the tangent to it at point $(3/2, 5\sqrt{3}/2)$.

Matrix Interpretation

Given equation of ellipse can be written in matrices as

$$x^T \begin{bmatrix} 25 & 0 \\ 0 & 9 \end{bmatrix} x = 225 \dots (1)$$

A conic can be expressed in the form of

$$x^T V x + 2u^T x + F = 0 \dots (2)$$

By comparing (1) and (2), we can say

$$V = \begin{bmatrix} 25 & 0 \\ 0 & 9 \end{bmatrix}, u = 0, F = -225 \dots (3)$$

If x is the some vector then

$$(p^T V + u^T)x + p^T u + F = 0 \dots (4)$$

is the Tangent vector equation

Theoretical Computations

$$([3/2 \quad 5\sqrt{3}/2] \begin{bmatrix} 25 & 0 \\ 0 & 9 \end{bmatrix})x + 0 = 225$$

$$([75/2 \quad 45\sqrt{3}/2])x = 225$$

$$\Rightarrow T : n^T x = 225$$

$$n = \begin{bmatrix} 5/2 \\ 3\sqrt{3}/2 \end{bmatrix}$$

$$e = \sqrt{1 - a^2/b^2}$$

$$e = 4/5$$

$$foci = (0, be), (0, -be)$$

$$\Rightarrow foci = (0, 4), (0, -4)$$

Theoretical Computations

Tangent of vector :

$$\begin{bmatrix} 10 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} - 225 = 0$$

let dist1 be the perpendicular distance from s1
and dist2 be the perpendicular distance from s2

$$A = \begin{bmatrix} 10 & 6\sqrt{3} \end{bmatrix} \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 & 6\sqrt{3} \end{bmatrix} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$C = \begin{bmatrix} 10 & 6\sqrt{3} \end{bmatrix} \begin{bmatrix} 3/2 \\ 5\sqrt{3}/2 \end{bmatrix}, D = \sqrt{a^2 + b^2}$$

Theoretical Computations

$$\text{dist1} = \frac{\|A-C\|}{D}$$

$$\text{dist2} = \frac{\|B-C\|}{D}$$

$$\text{product of distances}(D) = \text{dist1} * \text{dist2}$$

$$D = 9$$

Graphical verification

