Control Systems

G V V Sharma*

Contents

1 Compensator Designing 1

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Compensator Designing

1.1. For a unity feedback system

$$G(s) = \frac{K}{(s)(s+2)(s+4)(s+6)} \quad (1.1.1)$$

Design a lag compensator to yield a $K_v = 2$ and Phase Margin of 30°
Solution: Solution: Fig.1.1 models the equivalent of compensated closed loop system.

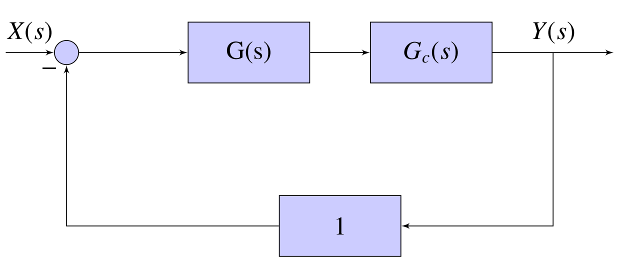


Fig. 1.1

Static Velocity Error Constant K_v is the steady-state error of a system for a unit-ramp input i.e.,

$$K_v = \lim_{s\to 0} sG(s)G_c(s) \quad (1.1.2)$$

Therefore,

$$K_v = \lim_{s\to 0} s\frac{K}{s(s+2)(s+4)(s+6)}\frac{Ts+1}{\beta Ts+1}$$
$$\implies 2 = \frac{K}{(0+2)(0+4)(0+6)}\frac{T(0)+1}{\beta T(0)+1}$$
$$\therefore K = 96 \quad (1.1.3)$$

$$G(s) = \frac{96}{s(s+2)(s+4)(s+6)} \quad (1.1.4)$$

1.2. The Magnitude and Phase response of G(s).
Solution: Substituting $s = \jmath\omega$ in (1.1.4),

$$G(\jmath\omega) = \frac{96}{(j\omega)(j\omega+2)(j\omega+4)(j\omega+6)} \quad (1.2.1)$$

$$\left|G(\jmath\omega)\right| = \frac{|96|}{\omega\sqrt{4+\omega^2}\sqrt{16+\omega^2}\sqrt{36+\omega^2}} \quad (1.2.2)$$

$$\angle G(\jmath\omega) = -90° - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) \quad (1.2.3)$$

1.3. The standard Transfer equation of Lag Compensator and its Phase and Gain
Solution:

$$G_c(s) = \frac{Ts+1}{\beta Ts+1} \quad (1.3.1)$$

$$|G_c(s)| = \frac{1}{\beta}\frac{1+\left(\frac{\omega}{T}\right)^2}{1+\left(\frac{\omega}{\beta T}\right)^2} \quad (1.3.2)$$

$$\angle G_c(s) = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T) \quad (1.3.3)$$

Where $\beta > 1$.
It can be approximated that for $\omega > \frac{1}{T}$

$$|G_c(s)| = \frac{1}{\beta} \quad (1.3.4)$$

and Phase to be very small($< 12°$).

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1.4. The Phase Margin(PM) of the Transfer function G(s)
Solution: From (1.2.2) and (1.2.2)
At Gain Crossover,

$$|G(s)| = 1 \tag{1.4.1}$$

$$\implies \omega_{gc} = 1.47 rad/sec \tag{1.4.2}$$

$$\implies \angle G\left(\jmath\omega_{gc}\right) = -160.26^\circ \tag{1.4.3}$$

$$PM = 180^\circ + \angle G\left(\jmath\omega_{gc}\right) \tag{1.4.4}$$

$$\implies PM = 19.74^\circ \tag{1.4.5}$$

The following are the Bode plots of uncompensated system

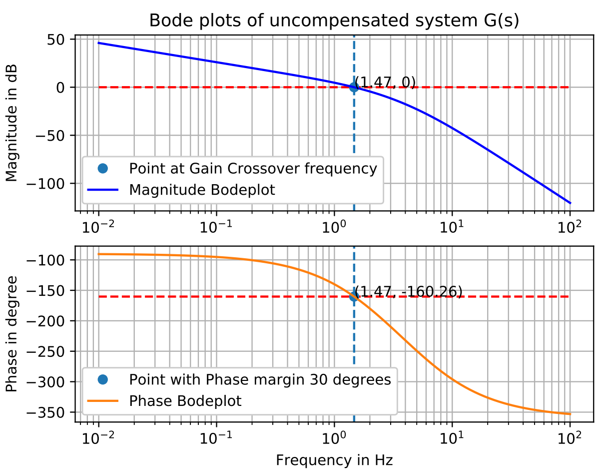


Fig. 1.4

The code for Bode plots of uncompensated system

codes/ee18btech11046_1.py

1.5. Design a lag compansator such that the Phase Margin becomes 30°
Solution: The lag compansator form is given in (1.3.1), Let

$$G'(s) = G(s)\,G_c(s) \tag{1.5.1}$$

The $PM = 30^\circ$ when $\angle G'(\jmath\omega) = -150^\circ$
Since the addition of compensator reduces Gain of system, thereby reducing Gain Crossover frequency which increases Phase Margin(PM) of system.
Since, Compansator also has small negative phase(say ϵ),let $\epsilon = 5^\circ$.i.e, $\angle G_c(s) = 5$

$$\angle G'(s) = \angle G(s) + \angle G_c(s) \tag{1.5.2}$$

$$\implies -150^\circ = \angle G(s) - 5^\circ \tag{1.5.3}$$

$$\implies \angle G(s) = -145^\circ \tag{1.5.4}$$

The value of ω where $\angle G(s) = -145^\circ$ is

$$\angle G(s) = -145^\circ \tag{1.5.5}$$

$$\implies \omega_{req} = 1.10953 rad/sec \tag{1.5.6}$$

The value $\frac{1}{T}$ is exactly 2 octaves below ω_{req} obtained in (1.5.6)

$$\frac{1}{T} = \frac{\omega_{req}}{4} \tag{1.5.7}$$

$$\implies T = 3.605 \tag{1.5.8}$$

Now we should take β such that Gain Crossover frequency occurs at ω_{req} i.e., to make $\left|G'(\jmath\omega)\right| = 1$
From (1.3.4),

$$\left|G'\left(\jmath\omega_{gc}\right)\right| = \left|G\left(\jmath\omega_{gc}\right)\right|\left|G_c\left(\jmath\omega_{gc}\right)\right| = 1 \tag{1.5.9}$$

$$\implies 1.4936 \times \frac{1}{\beta} = 1 \tag{1.5.10}$$

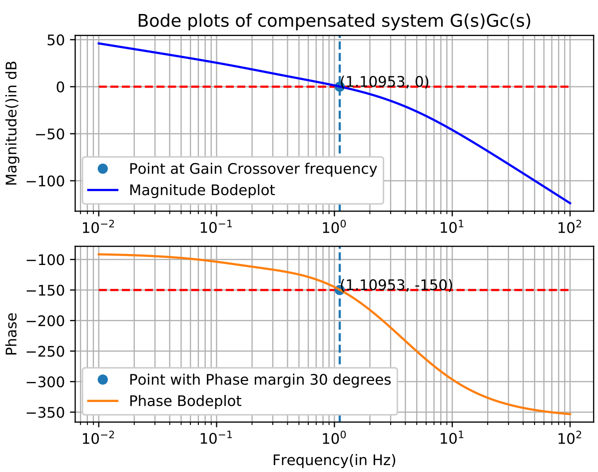
$$\implies \beta = 1.4936 \tag{1.5.11}$$

Substituting values of T and β obtained from(1.5.8) and(1.5.11) in (1.3.1) The required Compensator Transfer is

$$G_c(s) = \frac{3.605s + 1}{5.384s + 1} \tag{1.5.12}$$

The following are the Bode plots of compensated system
The code for Bode plots of compensated system

codes/ee18btech11046_2.py



Bode plots of compensated system G(s)Gc(s)
Magnitude()in dB
50
0
−50
−100
(1.10953, 0)
Point at Gain Crossover frequency
Magnitude Bodeplot
10⁻²
10⁻¹
10⁰
10¹
10²
Phase
−100
−150
−200
−250
−300
−350
(1.10953, -150)
Point with Phase margin 30 degrees
Phase Bodeplot
Frequency(in Hz)

Fig. 1.5