Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

2.3 Phase

3 Second order System

3.1 Damping

3.2 Example

3.3 Settling Time

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

4.5 Example

5 State-Space Model

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

6 Nyquist Plot

6.1 Introduction

6.2 Example

7 Compensators

7.1 Phase Lead

7.2 Example

8 Gain Margin

8.1 Introduction

8.2 Example

8.3 Example

9 Phase Margin

9.1 Intodaction

9.2 Example

10 Oscillator

10.1 Introduction

10.2 Example

11 Root Locus

11.1. The root locus of the feedback control system having the characteristic equation $s^2 + 6Ks +$

$2s+5 = 0$ where K>0, enters into the real axis at
(A) s = -1
(B) s = - $\sqrt{5}$
(C) s = -5
(D) s = $\sqrt{5}$
Solution:

11.2. Root Locus:
Root Locus is a method for plotting the locus of the poles of transfer function for different values of gain parameter K of the function from 0 to ∞.

11.3. How to draw a Root Locus
Let there be a Open-loop transfer function $KG(s)$ with unit gain negative feedback. Then the closed-loop transfer function for that particular system is:

$$\frac{N'(s)}{D'(s)} = \frac{KG(s)}{1+KG(s)H(s)} \tag{11.3.1}$$

Then the characteristic equation which gives poles is:

$$1 + KG(s)H(s) = 0 \tag{11.3.2}$$

a) Locate the open loop poles and zeros in the s-plane.
b) Find the number of root locus branches.
The root locus branches start at the open loop poles and end at open loop zeros. So, the number of root locus branches 'N' is equal to the number of finite open loop poles 'P' or the number of finite open loop zeros 'Z', whichever is greater.
c) Identify and draw the real axis root locus branches.
If the angle of the open loop transfer function at a point is an odd multiple of 180°, then that point is on the root locus. If odd number of the open loop poles and zeros exist to the left side of a point on the real axis, then that point is on the root locus branch. Therefore, the branch of points which satisfies this condition is the real axis of the root locus branch.
d) Find the centroid and the angle of asymptotes.
   - If P=Z, then all the root locus branches start at finite open loop poles and end at finite open loop zeros.
   - If P¿Z , then Z number of root locus branches start at finite open loop poles and end at finite open loop zeros and PZ number of root locus branches start at finite open loop poles and end at infinite open loop zeros.
   - If P¡Z , then P number of root locus branches start at finite open loop poles and end at finite open loop zeros and ZP number of root locus branches start at infinite open loop poles and end at finite open loop zeros

So, some of the root locus branches approach infinity. Asymptotes give the direction of these root locus branches. The intersection point of asymptotes on the real axis is known as centroid.

$$Centroid = \frac{\sum \text{Real(poles)} - \Sigma\text{Real(zeros)}}{P-Z} \tag{11.3.3}$$

The formula for angle of asymptotes θ is

$$\theta = \frac{(2q+1)180^0}{P-Z} \tag{11.3.4}$$

where q is any integer between (0,(P-Z)-1)
e) Find the intersection points of root locus branches with an imaginary axis. Whiich can be found using Routh array method
f) Find Break-away and Break-in points.
g) Find angle of departure and angle of arrival
The formula for the Angle of Departure ϕ_d is

$$\phi_d = 180^0 - \phi \tag{11.3.5}$$

The formula for the Angle of Arrival ϕ_a is

$$\phi_a = 180^0 + \phi \tag{11.3.6}$$

Where,

$$\phi = \sum \phi_p - \sum \phi_z \tag{11.3.7}$$

where ϕ_p is angle of open loop poles and ϕ_z is angle of open loop zeros.

11.4. Breakaway Point
While varying 'K', the point where the Root Locus enters real axis is called a 'Breakaway Point'.
A breakaway point is the point on a real axis segment of the root locus between two real

poles where the two real closed-loop poles meet and diverge to become complex conjugates. Since a Breakaway point corresponds to the point where root locus meets real axis. As the root locus is symmetric about real axis there will be two roots at the Breakaway point. i.e.,

$$K = -\frac{1}{G(s)} = -\frac{D(s)}{N(s)} \tag{11.4.1}$$

$$\frac{dK}{ds} = -\frac{d}{dK}\left(\frac{D(s)}{N(s)}\right) = 0 \tag{11.4.2}$$

Once a pole breaks away from the real axis, they can either travel out towards infinity (to meet an implicit zero), or they can travel to meet an explicit zero, or they can re-join the real-axis to meet a zero that is located on the real-axis.

11.5. Calculation of Breakaway point

Given,the characteristic equation:

$$s^2 + 6Ks + 2s + 5 = 0 \tag{11.5.1}$$

Dividing with s^2+2s+5 on both sides, we get

$$1 + \frac{6ks}{s^2 + 2s + 5} = 0 \tag{11.5.2}$$

This is of form $1 + KG(s) = 0$ which is closed loop characteristic equation, so that along the root locus segments on the real axis ($s = \sigma$)

$$K = -\frac{1}{G(s)} = -\frac{D(s)}{N(s)} \tag{11.5.3}$$

$$\frac{dK}{ds} = -\frac{d}{dK}\left(\frac{D(s)}{N(s)}\right) = 0 \tag{11.5.4}$$

$$K = \frac{-\left(s^2 + 2s + 5\right)}{6s} = -\frac{1}{6}\left[s + 2 + \frac{5}{s}\right] \tag{11.5.5}$$

And,

$$\frac{dk}{ds} = 0 \Rightarrow \left[1 - \frac{5}{s^2}\right] = 0 \tag{11.5.6}$$

Solving for s, we get

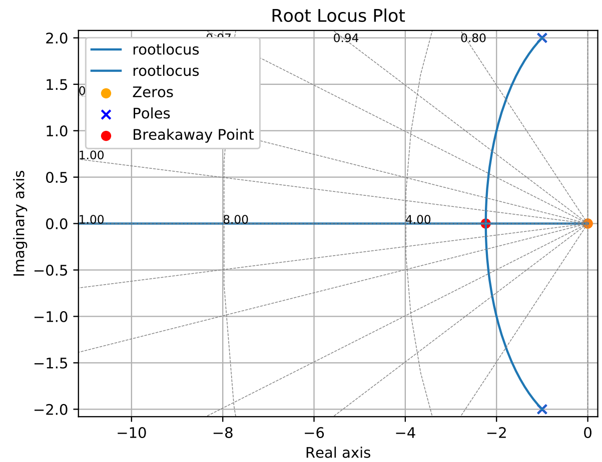
$$s^2 - 5 = 0 \Rightarrow s = \pm\sqrt{5} \tag{11.5.7}$$

Since, 's' can only be in left half of plane

$$\boxed{s = -\sqrt{5}} \tag{11.5.8}$$

Therefore the Breakaway point is at $(-\sqrt{5},0)$

11.6. Root Locus plot



codes/ee18btech11046.py