Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.2 Practical Case

3 Feedback Current Amplifier: Example

4 Feedback Transconductance Amplifier: Series-Series

5 Feedback Transresistance Amplifier: Series-Series

For the feedback transresistance amplifier in 5.1.1), use small-signal analysis to find the open-loop gain 'G', Feedback factor 'H' and Closed-loop gain 'T'. Let $R_F >> R_L$ and $r_o >> R_L$. Find the value of T for $R_L = 10K\Omega$, $R_F = 100K\Omega$ and the transistor current gain $\beta = 100$.

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5.1. Draw the equivalent control system for the feedback Transresistance amplifier shown in 5.1.1

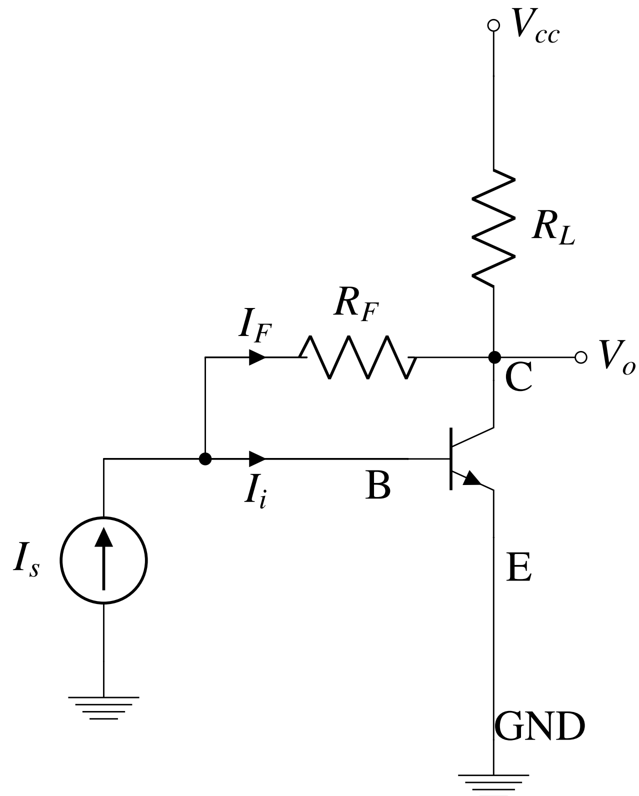

Fig. 5.1.1

Solution: see Fig. 5.1.2

5.2. For the feedback Transresistance amplifier shown in 5.1.1, Draw its small signal model. Early effect in Transistor is neglected.

Solution: see Fig. 5.2
While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis

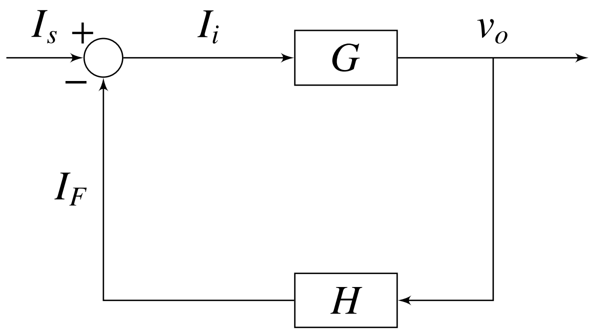

Fig. 5.1.2

of the circuit. Neglecting Early effect, in SmallSignal Analysis a npn-Transistor is modelled as a Current Source with value of current equal to $g_m V_{be}$ flowing from Collextor to Emitter. Whereas a pnp-Transistor is modelled as a Current Source with value of current equal to $g_m V_{be}$ flowing from Emitter to Collector.

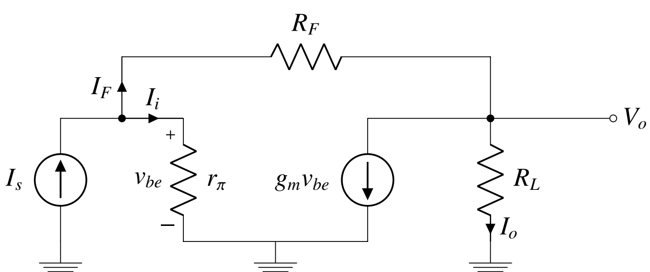

Fig. 5.2: Small Signal Model

5.3. Find small signal parameters g_m and v_{be} using DC analysis
Solution: small signal parameters of bjt are given in (5.3.1) and (5.3.2)

$$g_m = \frac{I_C}{V_T} \tag{5.3.1}$$

$$r_\pi = \frac{V_T}{I_B} \tag{5.3.2}$$

The Large signal model of circuit becomes as shown in figure 5.3
Where $V_T = 25m$volts

$$V_{BE} = 0.7volts \implies V_B = 0.7volts \tag{5.3.3}$$

$$I_E = I_B + I_C \tag{5.3.4}$$

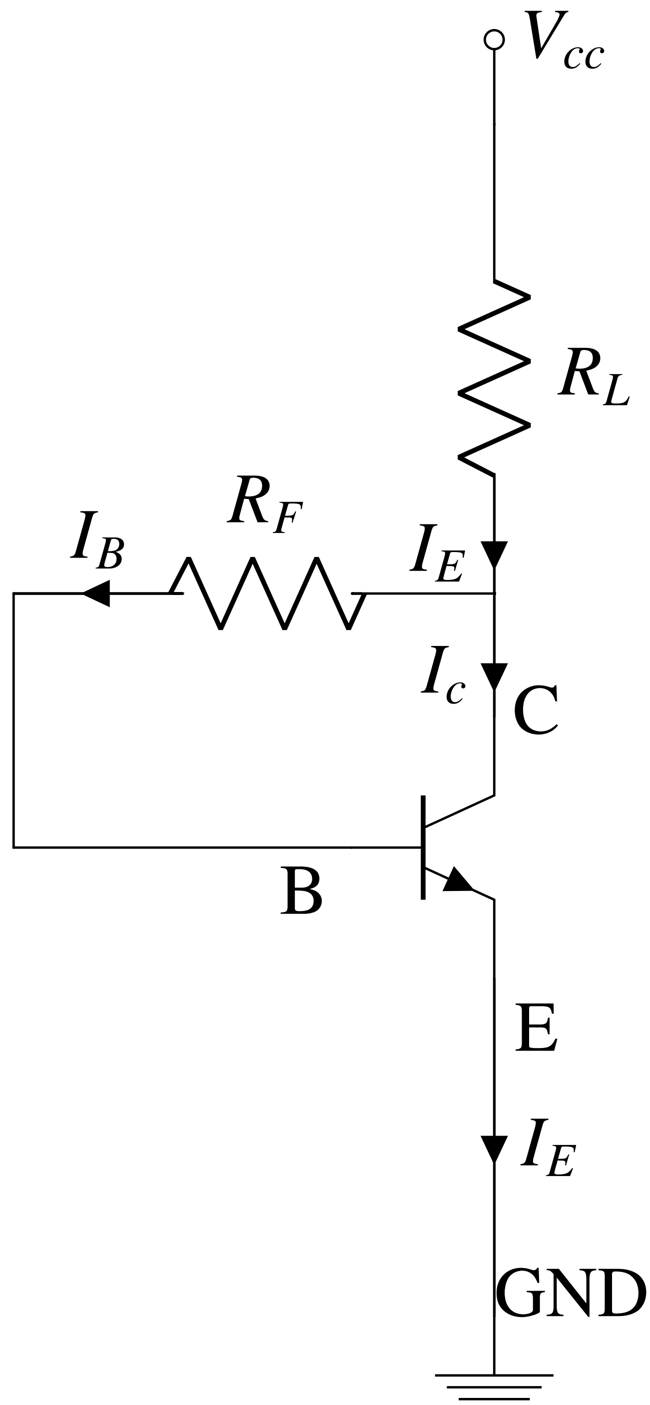

Fig. 5.3: Large signal model

$$I_C = \beta I_B \tag{5.3.5}$$

From applying KVL and KCL on Fig.

$$V_{cc} - I_E R_L - I_B R_F - 0.7 = 0 \tag{5.3.6}$$

$$\implies V_{cc} - (\beta + 1) I_B R_L - I_B R_F - 0.7 = 0 \tag{5.3.7}$$

$$I_B = \frac{V_{cc} - 0.7}{(\beta + 1) R_L + R_F} \tag{5.3.8}$$

$$I_C = \beta \frac{V_{cc} - 0.7}{(\beta + 1) R_L + R_F} \tag{5.3.9}$$

from (5.3.1), (5.3.2),I_B and I_C

$$g_m = \frac{\beta}{V_T} \frac{V_{cc} - 0.7}{(\beta + 1) R_L + R_F} \tag{5.3.10}$$

$$r_\pi = V_T \frac{(\beta + 1) R_L + R_F}{V_{cc} - 0.7} \tag{5.3.11}$$

5.4. Write all node/loop equations of Small-Signal model using KCL/KVL.Given that $R_F >> R_L$ **Solution:**

$$v_{be} = I_i r_\pi \tag{5.4.1}$$

$$v_{be} - I_F R_F = V_o \tag{5.4.2}$$

$$V_o = (I_F - g_m v_{be}) R_L \tag{5.4.3}$$

5.5. Find the expression for feedback factor H. **Solution:**

$$H = \frac{I_F}{V_o} \tag{5.5.1}$$

substituting (5.4.2) in (5.4.3)

$$V_o = (I_F - g_m V_o - g_m I_F R_F) R_L \tag{5.5.2}$$

$$\implies (1 + g_m R_L) V_o = I_F (R_L - g_m R_F R_L) \tag{5.5.3}$$

$$H = \frac{I_F}{V_o} = \frac{1 + g_m R_L}{R_L (1 - g_m R_F)} \tag{5.5.4}$$

$$\implies H \approx -\frac{1}{R_F} \tag{5.5.5}$$

5.6. Find the expression for Open loop Gain G. **Solution:**

$$G = \frac{V_o}{I_i} \tag{5.6.1}$$

Substituting (5.4.1) in (5.4.2) and subisitituting I_F from (5.5.4)

$$I_i r_\pi - \left(\frac{1 + g_m R_L}{R_L (1 - 1 + g_m R_F)} \right) R_F V_o = V_o \tag{5.6.2}$$

$$\implies G = \frac{V_o}{I_i} = \frac{r_\pi R_L (1 - g_m R_F)}{R_F + R_L} \tag{5.6.3}$$

Upon approximating since $R_F >> R_L$

$$G = -g_m r_\pi R_L \tag{5.6.4}$$

5.7. Find the expression for Closed Loop Gain $T = \frac{V_o}{I_s}$ We know that Closed Loop Gain

$$T = \frac{G}{1 + GH} \tag{5.7.1}$$

Substituting expressions from (5.5.5) and (5.6.3)

$$T = -\frac{g_m r_\pi R_L}{1 + \left(\frac{g_m r_\pi R_L}{R_F} \right)} \tag{5.7.2}$$

5.8. For the parameters given in table 5.8 . Find G,H and T. **Solution:** Substituting the parameters in

Parameters	**Value**
V_{cc}	5V
I_s	1μ
R_F	$100K\Omega$
R_L	$10K\Omega$
β	100

TABLE 5.8

(5.3.10) and (5.3.11) gives,

$$r_\pi = 6.6667 \times 10^3 \Omega \tag{5.8.1}$$

$$g_m = 0.015S \tag{5.8.2}$$

Substituting g_m, r_π obtained in (5.5.5)

$$H = -10^{-5} \tag{5.8.3}$$

Substituting g_m, r_π obtained in (5.6.4)

$$G = -10^6 \tag{5.8.4}$$

Substituting g_m, r_π obtained in (5.7.2)

$$T = -90909.09 \tag{5.8.5}$$

5.9. Draw the block diagram and circuit diagram for H.
Solution: see figs 5.9.5 and 5.9.6

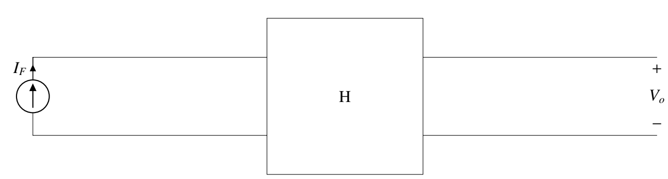

Fig. 5.9.5: Feedback block diagram

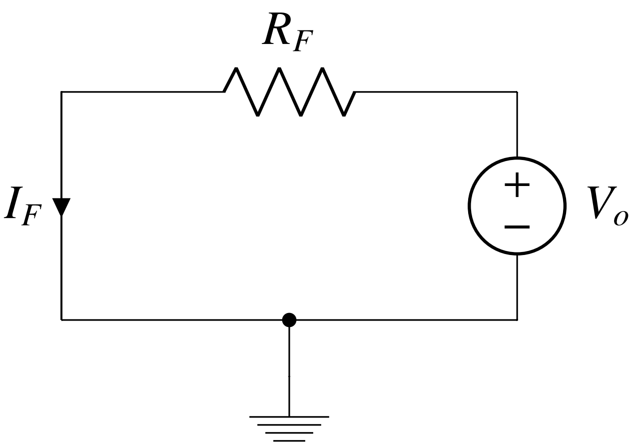

Fig. 5.9.6: Feedback circuit

From KVl on 5.9.6 we can see that

$$H = \frac{I_F}{V_o} = -\frac{1}{R_F} \tag{5.9.1}$$

5.10. Find the input and output resistances of the feedback network.
Solution: From the feedback amplifier circuit fig.5.9.6 To find the input resistance R_{11} short the output node V_o to ground.

$$R_{11} = R_F \tag{5.10.1}$$

To find the output resistance R_{22} rempve the current source and short input terminals.

$$R_{22} = R_F \tag{5.10.2}$$

5.11. Draw the block diagram and circuit diagram for G.
Solution: see figs 5.11.7 and 5.11.8

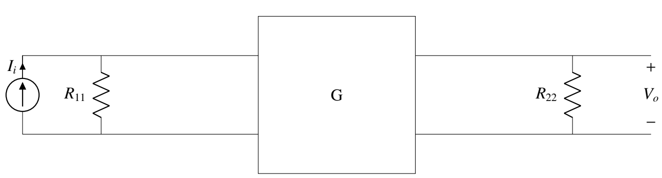

Fig. 5.11.7: Open loop block diagram

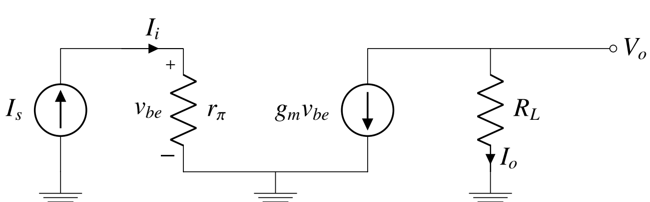

Fig. 5.11.8: Open loop block circuit diagram

5.12. Find G
Solution: From fig.5.11.8,

$$V_{be} = I_i r_\pi \tag{5.12.1}$$

From KCL at node V_o,

$$I_o = -g_m I_i r_\pi \tag{5.12.2}$$

$$V_o = -g_m I_i r_\pi R_L \tag{5.12.3}$$

Therefore,

$$G = \frac{V_o}{I_i} = -g_m r_\pi R_L \tag{5.12.4}$$

5.13. Simulate the circuit using ngspice
Solution: The following file gives instructions on how to simulate the circuit.

```
codes/ee18btech11046/spice/README
```

The following netlist simulates the feedback amplifier using parameters in table 5.8.

```
codes/ee18btech11046/spice/
    ee18btech11046_bjt.net
```

The Output Voltage obtained from spice is plotted in fig.5.13.9

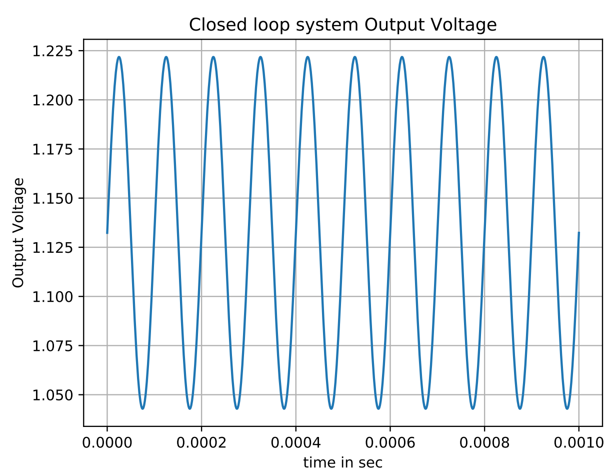

Fig. 5.13.9: Output Voltage

```
codes/ee18btech11046/spice/ee18btech11046.
    py
```

We can observe that V_o is sum of sine wave amplified by a factor of 89500 for small signal input and large signal output V_C which is close to the calculated values.