

Control Systems

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1 Compensator Designing 1

Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

1 COMPENSATOR DESIGNING

1.1. For a unity feedback system

$$G(s) = \frac{K}{(s)(s+2)(s+4)(s+6)} \quad (1.1.1)$$

Design a lag compensator to yield a $K_v = 2$ and Phase Margin of 30°

Solution: **Solution:** Fig.1.1 models the equivalent of compensated closed loop system.

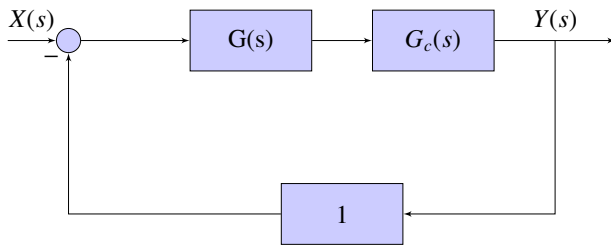


Fig. 1.1

Static Velocity Error Constant K_v is the steady-state error of a system for a unit-ramp input i.e.,

$$K_v = \lim_{s \rightarrow 0} sG(s)G_c(s) \quad (1.1.2)$$

Therefore,

$$\begin{aligned} K_v &= \lim_{s \rightarrow 0} s \frac{K}{s(s+2)(s+4)(s+6)} \frac{Ts+1}{\beta Ts+1} \\ \Rightarrow 2 &= \frac{K}{(0+2)(0+4)(0+6)} \frac{T(0)+1}{\beta T(0)+1} \\ \therefore K &= 96 \quad (1.1.3) \end{aligned}$$

$$G(s) = \frac{96}{s(s+2)(s+4)(s+6)} \quad (1.1.4)$$

1.2. The Magnitude and Phase response of $G(s)$.

Solution: Substituting $s = j\omega$ in (1.1.4),

$$G(j\omega) = \frac{96}{(j\omega)(j\omega+2)(j\omega+4)(j\omega+6)} \quad (1.2.1)$$

$$|G(j\omega)| = \frac{|96|}{\omega \sqrt{4+\omega^2} \sqrt{16+\omega^2} \sqrt{36+\omega^2}} \quad (1.2.2)$$

$$\angle G(j\omega) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{6}\right) \quad (1.2.3)$$

1.3. The standard Transfer equation of Lag Compensator and its Phase and Gain

Solution:

$$G_c(s) = \frac{Ts+1}{\beta Ts+1} \quad (1.3.1)$$

$$|G_c(s)| = \frac{1}{\beta} \frac{1 + \left(\frac{\omega}{T}\right)^2}{1 + \left(\frac{\omega}{\beta T}\right)^2} \quad (1.3.2)$$

$$\angle G_c(s) = \tan^{-1}(\omega T) - \tan^{-1}(\omega \beta T) \quad (1.3.3)$$

Where $\beta > 1$.

It can be approximated that for $\omega > \frac{1}{T}$

$$|G_c(s)| = \frac{1}{\beta} \quad (1.3.4)$$

and Phase to be very small ($< 12^\circ$).

1.4. The Phase Margin(PM) of the Transfer func-

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tion $G(s)$

Solution: From (1.2.2) and (1.2.2)

At Gain Crossover,

$$|G(s)| = 1 \quad (1.4.1)$$

$$\Rightarrow \omega_{gc} = 1.47 \text{ rad/sec} \quad (1.4.2)$$

$$\Rightarrow \angle G(j\omega_{gc}) = -160.26^\circ \quad (1.4.3)$$

$$PM = 180^\circ + \angle G(j\omega_{gc}) \quad (1.4.4)$$

$$\Rightarrow PM = 19.74^\circ \quad (1.4.5)$$

The following are the Bode plots of uncompensated system

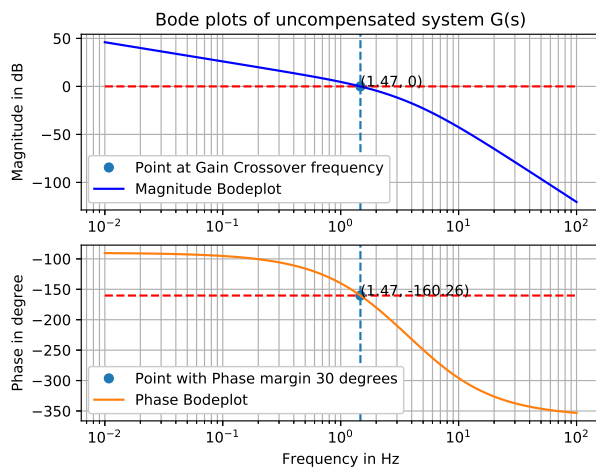


Fig. 1.4

The code for Bode plots of uncompensated system

```
codes/ee18btech11046_1.py
```

1.5. Design a lag compensator such that the Phase Margin becomes 30°

Solution: The lag compensator form is given in (1.3.1), Let

$$G'(s) = G(s)G_c(s) \quad (1.5.1)$$

The $PM = 30^\circ$ when $\angle G'(j\omega) = -150^\circ$

Since the addition of compensator reduces Gain of system, thereby reducing Gain Crossover frequency which increases Phase Margin(PM) of system.

Since, Compensator also has small negative

phase(say ϵ), let $\epsilon = 5^\circ$. i.e., $\angle G_c(s) = 5$

$$\angle G'(s) = \angle G(s) + \angle G_c(s) \quad (1.5.2)$$

$$\Rightarrow -150^\circ = \angle G(s) - 5^\circ \quad (1.5.3)$$

$$\Rightarrow \angle G(s) = -145^\circ \quad (1.5.4)$$

The value of ω where $\angle G(s) = -145^\circ$ is

$$\angle G(s) = -145^\circ \quad (1.5.5)$$

$$\Rightarrow \omega_{req} = 1.10953 \text{ rad/sec} \quad (1.5.6)$$

The value $\frac{1}{T}$ is exactly 2 octaves below ω_{req} obtained in (1.5.6)

$$\frac{1}{T} = \frac{\omega_{req}}{4} \quad (1.5.7)$$

$$\Rightarrow T = 3.605 \quad (1.5.8)$$

Now we should take β such that Gain Crossover frequency occurs at ω_{req} i.e., to make $|G'(j\omega)| = 1$
From (1.3.4),

$$|G'(j\omega_{gc})| = |G(j\omega_{gc})| |G_c(j\omega_{gc})| = 1 \quad (1.5.9)$$

$$\Rightarrow 1.4936 \times \frac{1}{\beta} = 1 \quad (1.5.10)$$

$$\Rightarrow \beta = 1.4936 \quad (1.5.11)$$

Substituting values of T and β obtained from (1.5.8) and (1.5.11) in (1.3.1) The required Compensator Transfer is

$$G_c(s) = \frac{3.605s + 1}{5.384s + 1} \quad (1.5.12)$$

The following are the Bode plots of compensated system

The code for Bode plots of compensated system

```
codes/ee18btech11046_2.py
```

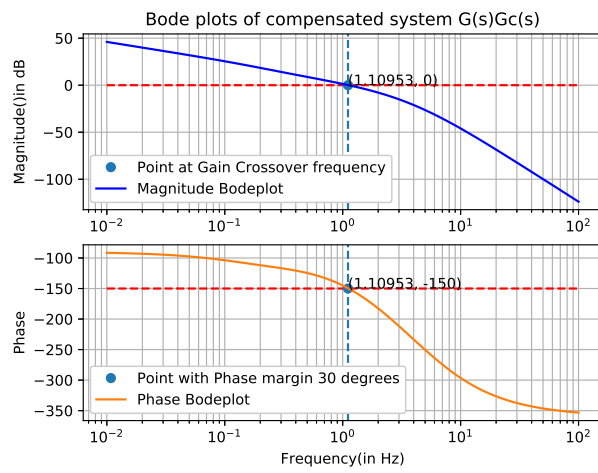


Fig. 1.5