Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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11 Root Locus

11.1. The root locus of the feedback control system having the characteristic equation $s^2 + 6Ks +$

$2s+5=0$ where K>0, enters into the real axis at
(A) s = -1
(B) s = - $\sqrt{5}$
(C) s = -5
(D) s = $\sqrt{5}$
Solution:

11.2. Root Locus:
Root Locus is a method for plotting the locus of the poles of transfer function for different values of gain parameter K of the function from 0 to ∞.

11.3. How to draw a Root Locus
Let there be a Open-loop transfer function $KG(s)$ with unit gain negative feedback. Then the closed-loop transfer function for that particular system is:

$$\frac{N'(s)}{D'(s)} = \frac{KG(s)}{1+KG(s)H(s)} \tag{11.3.1}$$

Then the characteristic equation which gives poles is:

$$1+KG(s)H(s) = 0 \tag{11.3.2}$$

a) Locate the open loop poles and zeros in the s-plane.
b) Find the number of root locus branches.
The root locus branches start at the open loop poles and end at open loop zeros. So, the number of root locus branches 'N' is equal to the number of finite open loop poles 'P' or the number of finite open loop zeros 'Z', whichever is greater.
c) Identify and draw the real axis root locus branches.
If the angle of the open loop transfer function at a point is an odd multiple of 180°, then that point is on the root locus. If odd number of the open loop poles and zeros exist to the left side of a point on the real axis, then that point is on the root locus branch. Therefore, the branch of points which satisfies this condition is the real axis of the root locus branch.
d) Find the centroid and the angle of asymptotes.
   - If P=Z, then all the root locus branches start at finite open loop poles and end at finite open loop zeros.
   - If P>Z , then Z number of root locus branches start at finite open loop poles and end at finite open loop zeros and P-Z number of root locus branches start at finite open loop poles and end at infinite open loop zeros.
   - If P<Z , then P number of root locus branches start at finite open loop poles and end at finite open loop zeros and Z-P number of root locus branches start at infinite open loop poles and end at finite open loop zeros

So, some of the root locus branches approach infinity. Asymptotes give the direction of these root locus branches. The intersection point of asymptotes on the real axis is known as centroid.

$$Centroid = \frac{\sum \text{Real(poles)} - \Sigma\text{Real(zeros)}}{P-Z} \tag{11.3.3}$$

The formula for angle of asymptotes θ is

$$\theta = \frac{(2q+1)180^0}{P-Z} \tag{11.3.4}$$

where q is any integer between (0,(P-Z)-1)
e) Find the intersection points of root locus branches with an imaginary axis. Whiich can be found using Routh array method
f) Find Break-away and Break-in points.
g) Find angle of departure and angle of arrival
The formula for the Angle of Departure ϕ_d is

$$\phi_d = 180^0 - \phi \tag{11.3.5}$$

The formula for the Angle of Arrival ϕ_a is

$$\phi_a = 180^0 + \phi \tag{11.3.6}$$

Where,

$$\phi = \sum \phi_p - \sum \phi_z \tag{11.3.7}$$

where ϕ_p is angle of open loop poles and ϕ_z is angle of open loop zeros.
The Angle of Departure exists only if there are complex poles and Angle of Arrival exists only if there are complex zeros.

11.4. Breakaway Point
While varying 'K', the point where the Root Locus enters real axis is called a 'Breakaway

Point'.

A breakaway point is the point on a real axis segment of the root locus between two real poles where the two real closed-loop poles meet and diverge to become complex conjugates. Since a Breakaway point corresponds to the point where root locus meets real axis. As the root locus is symmetric about real axis there will be two roots at the Breakaway point. i.e.,

$$K = -\frac{1}{G(s)} = -\frac{D(s)}{N(s)} \tag{11.4.1}$$

$$\frac{dK}{ds} = -\frac{d}{dK}\left(\frac{D(s)}{N(s)}\right) = 0 \tag{11.4.2}$$

Once a pole breaks away from the real axis, they can either travel out towards infinity (to meet an implicit zero), or they can travel to meet an explicit zero.

11.5. Calculating Root Locus Parameters

Given,the characteristic equation:

$$s^2 + 6Ks + 2s + 5 = 0 \tag{11.5.1}$$

Dividing with s^2+2s+5 on both sides, we get

$$1 + \frac{6ks}{s^2 + 2s + 5} = 0 \tag{11.5.2}$$

This is of form $1 + KG(s) = 0$ which is closed loop characteristic equation. Therefore,

$$G(s) = \frac{6s}{s^2 + 2s + 5} \tag{11.5.3}$$

Calculating the the poles of G(s) are $-1 + 2i$ and $-1-2i$ and zero is at $(0,0)$. i.e, P=2, Z=1. Since $P > Z$ the number of branches are 2. i.e., One branch originates at each pole and ends at zero and the other branch starts from other pole and goes to infinity.

$$Centroid = \frac{(-1-1)-0}{1} \Rightarrow Centroid = -2 \tag{11.5.4}$$

Therefore Centroid is at (-2,0)

The angle of asymptotes θ is

$$\theta = \frac{(2(0)+1)180^0}{2-1} \Rightarrow \theta = 180^0 \tag{11.5.5}$$

One branch meets Real axis at Breakaway point then goes to zero at origin and other goes to infinity following Asymptote($y = 0$).

Angle of Departure(ϕ_d) with repespect to pole(-1+2i) is

$$\phi_p = angle(-1+2i-(-1-2i)) \Rightarrow \phi_p = \frac{\pi}{2} \tag{11.5.6}$$

$$\phi_z = angle(-1+2i-(0)) \Rightarrow \phi_z = Tan^{-1}(-2) \tag{11.5.7}$$

Therefore

$$\phi_d = \pi - (\phi_p - \phi_z) \Rightarrow \phi_d = \frac{\pi}{2} + Tan^{-1}(-2) \tag{11.5.8}$$

Since there are no complex zeros there is no Angle of Arrival.

11.6. Calculation of Breakaway point

From the characteristic equation:

$$K = -\frac{1}{G(s)} = -\frac{D(s)}{N(s)} \tag{11.6.1}$$

$$\frac{dK}{ds} = -\frac{d}{dK}\left(\frac{D(s)}{N(s)}\right) = 0 \tag{11.6.2}$$

$$K = \frac{-\left(s^2+2s+5\right)}{6s} = -\frac{1}{6}\left[s+2+\frac{5}{s}\right] \tag{11.6.3}$$

And,

$$\frac{dk}{ds} = 0 \Rightarrow \left[1 - \frac{5}{s^2}\right] = 0 \tag{11.6.4}$$

Solving for s, we get

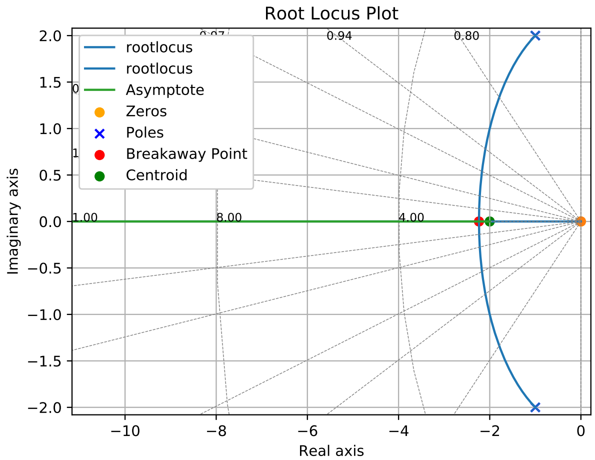
$$s^2 - 5 = 0 \Rightarrow s = \pm\sqrt{5} \tag{11.6.5}$$

Since, 's' can only be in left half of plane

$$\boxed{s = -\sqrt{5}} \tag{11.6.6}$$

Therefore the Breakaway point is at ($-\sqrt{5}$,0)

11.7. Root Locus plot



codes/ee18btech11046.py