Assignment 5

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Download all python codes from

```
https://github.com/sachinomdubey/Matrix−theory/
    Assignment5/codes
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and latex-tikz codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment5
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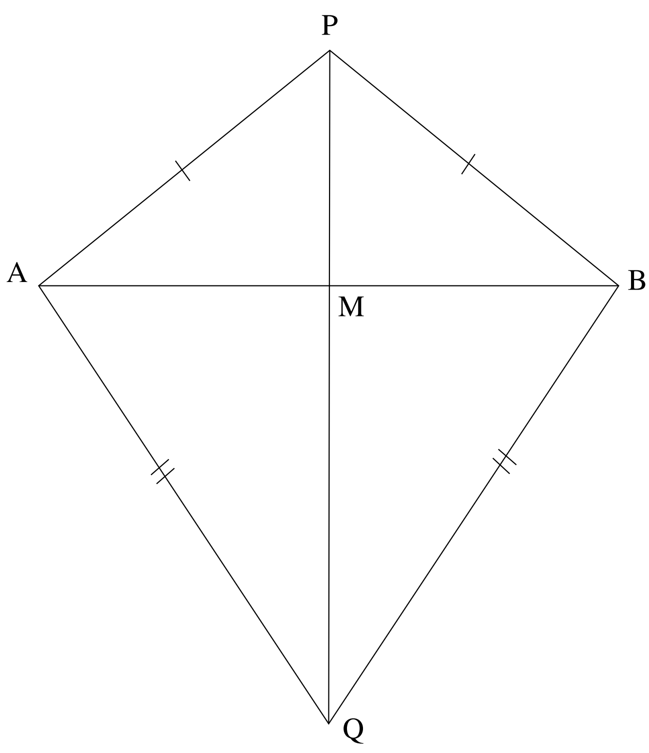
0.1 Problem

(Geolin 1.9) AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B. Show that the line PQ is the perpendicular bisector of AB.



0.2 Explanation

In order to prove that line PQ is the perpendicular bisector of AB, two conditions need to be met:

1) $PQ \perp AB$
2) AM=BM

These conditions can be proved as follow:

0.3 Solution

It is given that the points $\mathbf{P}$ and $\mathbf{Q}$ are equidistant from the points $\mathbf{A}$ and $\mathbf{B}$. Thus we can write:

$$\|\mathbf{P}-\mathbf{A}\| = \|\mathbf{P}-\mathbf{B}\| \tag{0.3.1}$$

$$\|\mathbf{Q}-\mathbf{A}\| = \|\mathbf{Q}-\mathbf{B}\| \tag{0.3.2}$$

Squaring both sides of equations 0.3.1 and expanding further, we can write:

$$(\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = (\mathbf{P}-\mathbf{B})^T(\mathbf{P}-\mathbf{B}) \tag{0.3.3}$$

$$\mathbf{P}^T\mathbf{P}-\mathbf{P}^T\mathbf{A}-\mathbf{A}^T\mathbf{P}+\mathbf{A}^T\mathbf{A} = \mathbf{P}^T\mathbf{P}-\mathbf{P}^T\mathbf{B}-\mathbf{B}^T\mathbf{P}+\mathbf{B}^T\mathbf{B} \tag{0.3.4}$$

$$\therefore \mathbf{A}^T\mathbf{A}-\mathbf{B}^T\mathbf{B} = -2\mathbf{P}^T\mathbf{B}+2\mathbf{P}^T\mathbf{A} \tag{0.3.5}$$

Similarly, Squaring both sides of equations 0.3.2 and expanding further, we can write:

$$(\mathbf{Q}-\mathbf{A})^T(\mathbf{Q}-\mathbf{A}) = (\mathbf{Q}-\mathbf{B})^T(\mathbf{Q}-\mathbf{B}) \tag{0.3.6}$$

$$\mathbf{Q}^T\mathbf{Q}-\mathbf{Q}^T\mathbf{A}-\mathbf{A}^T\mathbf{Q}+\mathbf{A}^T\mathbf{A} = \mathbf{Q}^T\mathbf{Q}-\mathbf{Q}^T\mathbf{B}-\mathbf{B}^T\mathbf{Q}+\mathbf{B}^T\mathbf{B} \tag{0.3.7}$$

$$\therefore \mathbf{A}^T\mathbf{A}-\mathbf{B}^T\mathbf{B} = -2\mathbf{Q}^T\mathbf{B}+2\mathbf{Q}^T\mathbf{A} \tag{0.3.8}$$

From equations 0.3.5 and 0.3.8, we can write:

$$2\mathbf{P}^T(\mathbf{A}-\mathbf{B}) = 2\mathbf{Q}^T(\mathbf{A}-\mathbf{B}) \tag{0.3.9}$$

$$\mathbf{P}^T(\mathbf{A}-\mathbf{B}) - \mathbf{Q}^T(\mathbf{A}-\mathbf{B}) = 0 \tag{0.3.10}$$

$$(\mathbf{P}-\mathbf{Q})^T(\mathbf{A}-\mathbf{B}) = 0 \tag{0.3.11}$$

Thus, Segment PQ is perpendicular to segment AB ($PQ \perp AB$).

From the figure, equations 0.3.1 can also be written as:

$$\|(\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{A})\| = \|(\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{B})\| \tag{0.3.12}$$

Squaring and expanding both the sides, we get:

$$((\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{A}))^T((\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{A})) = ((\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{B}))^T((\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{B})) \tag{0.3.13}$$

$$\begin{aligned}(\mathbf{P}-\mathbf{M})^T(\mathbf{P}-\mathbf{M})+(\mathbf{P}-\mathbf{M})^T(\mathbf{M}-\mathbf{A})+\\(\mathbf{M}-\mathbf{A})^T(\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{A})^T(\mathbf{M}-\mathbf{A})=\\(\mathbf{P}-\mathbf{M})^T(\mathbf{P}-\mathbf{M})+(\mathbf{P}-\mathbf{M})^T(\mathbf{M}-\mathbf{B})+\\(\mathbf{M}-\mathbf{B})^T(\mathbf{P}-\mathbf{M})+(\mathbf{M}-\mathbf{B})^T(\mathbf{M}-\mathbf{B})\end{aligned} \tag{0.3.14}$$

$$\begin{aligned}\|(\mathbf{M}-\mathbf{A})\|^2+2(\mathbf{M}-\mathbf{A})^T(\mathbf{P}-\mathbf{M})=\\\|(\mathbf{M}-\mathbf{B})\|^2+2(\mathbf{M}-\mathbf{B})^T(\mathbf{P}-\mathbf{M})\end{aligned} \tag{0.3.15}$$

Since, $PQ \perp AB$. Hence, we can write:

$$(\mathbf{M}-\mathbf{A})^T(\mathbf{P}-\mathbf{M})=(\mathbf{M}-\mathbf{B})^T(\mathbf{P}-\mathbf{M})=0 \tag{0.3.16}$$

From equation 0.3.15 and 0.3.16, we get:

$$\|(\mathbf{M}-\mathbf{A})\|=\|(\mathbf{M}-\mathbf{B})\| \tag{0.3.17}$$

Thus, M is the midpoint of segment AB ($AM = BM$). Thus, Segment PQ is perpendicular bisector of segment AB.