Assignment 5

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Download all python codes from

```
https://github.com/sachinomdubey/Matrix−theory/
    Assignment5/codes
```

and latex-tikz codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment5
```

0.1 Problem

(Geolin 1.9) AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B. Show that the line PQ is the perpendicular bisector of AB.

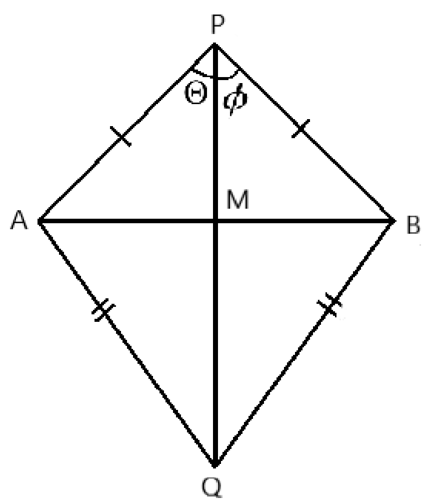

Fig. 0

0.2 Explanation

In order to prove that line PQ is the perpendicular bisector of AB, two conditions need to be met:

1) AM=BM
2) $PM \perp AB$

These conditions can be proved using cosine law in vector form and some vector theory. The solution is as follow:

0.3 Solution

It is given that the points $\mathbf{P}$ and $\mathbf{Q}$ are equidistant from the points $\mathbf{A}$ and $\mathbf{B}$. Thus we can write:

$$||\mathbf{PA}|| = ||\mathbf{PB}|| \tag{0.3.1}$$

$$||\mathbf{QA}|| = ||\mathbf{QB}|| \tag{0.3.2}$$

Using the law of cosine, we can write:

$$||\mathbf{AQ}||^2 = ||\mathbf{PA}||^2 + ||\mathbf{PQ}||^2 - 2\,||\mathbf{PA}||\,||\mathbf{PQ}||\,cos\Theta \tag{0.3.3}$$

$$||\mathbf{BQ}||^2 = ||\mathbf{PB}||^2 + ||\mathbf{PQ}||^2 - 2\,||\mathbf{PB}||\,||\mathbf{PQ}||\,cos\phi \tag{0.3.4}$$

From equations 0.3.1 0.3.2 0.3.3 and 0.3.4, we can write:

$$cos\Theta = cos\phi \tag{0.3.5}$$

$$\therefore \implies \Theta = \phi \tag{0.3.6}$$

Again using the law of cosine, we can write:

$$||\mathbf{AM}||^2 = ||\mathbf{PA}||^2 + ||\mathbf{PM}||^2 - 2\,||\mathbf{PA}||\,||\mathbf{PM}||\,cos\Theta \tag{0.3.7}$$

$$||\mathbf{BM}||^2 = ||\mathbf{PB}||^2 + ||\mathbf{PM}||^2 - 2\,||\mathbf{PB}||\,||\mathbf{PM}||\,cos\phi \tag{0.3.8}$$

From equations 0.3.1 and 0.3.6, we can write:

$$||\mathbf{BM}||^2 = ||\mathbf{PA}||^2 + ||\mathbf{PM}||^2 - 2\,||\mathbf{PA}||\,||\mathbf{PM}||\,cos\Theta \tag{0.3.9}$$

From equations 0.3.7 and 0.3.9, we can write:

$$||\mathbf{AM}||^2 = ||\mathbf{BM}||^2 \tag{0.3.10}$$

$$\therefore ||\mathbf{AM}|| = ||\mathbf{BM}|| \tag{0.3.11}$$

Thus, Segment PQ bisects segment AB.
From the figure, the vector $\mathbf{AB}$ can be written as:

$$\mathbf{AB} = \mathbf{AP} + \mathbf{PB} \tag{0.3.12}$$

Also, the vector **PM** can be written as:

$$\mathbf{PM} = \mathbf{PA} + \mathbf{AM} \tag{0.3.13}$$

The point M is the midpoint of the segment AB, therefore the vector **AM** is :

$$\mathbf{AM} = \frac{1}{2}\mathbf{AB} \tag{0.3.14}$$

$$\therefore \mathbf{PM} = \mathbf{PA} + \frac{1}{2}\mathbf{AB} \tag{0.3.15}$$

From equation 0.3.12:

$$\mathbf{PM} = \mathbf{PA} + \frac{1}{2}(\mathbf{AP} + \mathbf{PB}) \tag{0.3.16}$$

$$\mathbf{PM} = \mathbf{PA} + \frac{1}{2}\mathbf{AP} + \frac{1}{2}\mathbf{PB} \tag{0.3.17}$$

$$\mathbf{PM} = \mathbf{PA} - \frac{1}{2}\mathbf{PA} + \frac{1}{2}\mathbf{PB} \tag{0.3.18}$$

$$\mathbf{PM} = \frac{1}{2}(\mathbf{PA} + \mathbf{PB}) \tag{0.3.19}$$

For $PM \perp AB$, The dot product of vectors **AB** and **PM** should be zero. Using equations 0.3.12 and 0.3.19:

$$\mathbf{AC}^T\mathbf{PM} = (\mathbf{AP} + \mathbf{PB})^T\frac{1}{2}(\mathbf{PA} + \mathbf{PB}) \tag{0.3.20}$$

$$\mathbf{AC}^T\mathbf{PM} = \frac{1}{2}(\mathbf{AP}^T + \mathbf{PB}^T)(\mathbf{PA} + \mathbf{PB}) \tag{0.3.21}$$

$$\mathbf{AC}^T\mathbf{PM} = \frac{1}{2}(-\mathbf{PA}^T + \mathbf{PB}^T)(\mathbf{PA} + \mathbf{PB}) \tag{0.3.22}$$

$$\mathbf{AC}^T\mathbf{PM} = \frac{1}{2}(\mathbf{PB}^T\mathbf{PB} - \mathbf{PA}^T\mathbf{PA}) \tag{0.3.23}$$

$$\mathbf{AC}^T\mathbf{PM} = \frac{1}{2}(||\mathbf{PB}||^2 - ||\mathbf{PA}||^2) \tag{0.3.24}$$

$$\mathbf{AC}^T\mathbf{PM} = 0 \tag{0.3.25}$$

Thus, Segment PQ is perpendicular bisector of segment AB.