# Assignment 7

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Download all python codes from

https://github.com/sachinomdubey/Matrix−theory/Assignment7/codes

and latex-tikz codes from

https://github.com/sachinomdubey/Matrix−theory/Assignment7

## 1 Problem

(Rams 3.4.1) find the QR decomposition of the following:

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1.0.1}$$

## 2 Solution

Here, let the column vectors of $\mathbf{V}$ be $\alpha$ and $\beta$:

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{2.0.2}$$

To find $\mathbf{Q} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix}$, we will orthonormalise the columns of $\mathbf{V}$ using Gram Schmit method:

$$\mathbf{u}_1 = \frac{\alpha}{k_1} \tag{2.0.3}$$

$$k_1 = \|\alpha\| = \sqrt{1^2 + 0^2} = 1 \tag{2.0.4}$$

$$\implies \mathbf{u}_1 = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} = \frac{\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix}}{\sqrt{0^2 + (-1)^2}} = 0 \tag{2.0.7}$$

$$\implies \mathbf{u}_2 = \frac{\beta}{\|\beta\|} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{2.0.8}$$

$$\text{Also,} \quad k_2 = \mathbf{u}_2^T \beta = \begin{pmatrix} 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = 1 \tag{2.0.9}$$

The QR decomposition is given as:

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix}\begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

Where,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \tag{2.0.11}$$

$$\mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.12}$$

Putting the values of $\mathbf{u}_1$, $\mathbf{u}_2$, $k_1$, $k_2$ and $r_1$ in equation (2.0.10):

$$\boxed{\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \tag{2.0.13}$$

Where,

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.15}$$

**NOTE:** Here, Matrix $\mathbf{V}$ already had orthonormal column vectors. Hence, The obtained $\mathbf{Q}$ by Gram schmit method is same as $\mathbf{V}$ and the obtained $\mathbf{R}$ is an identity matrix.