#### 1

# Assignment 11

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Download the latex-tikz codes from

https://github.com/sachinomdubey/Matrix-theory/Assignment11

### 1 Problem

(UGC-dec2017,105):

Consider a Markov chain with five states {1, 2, 3, 2, 5} and transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix}$$
(1.0.1)

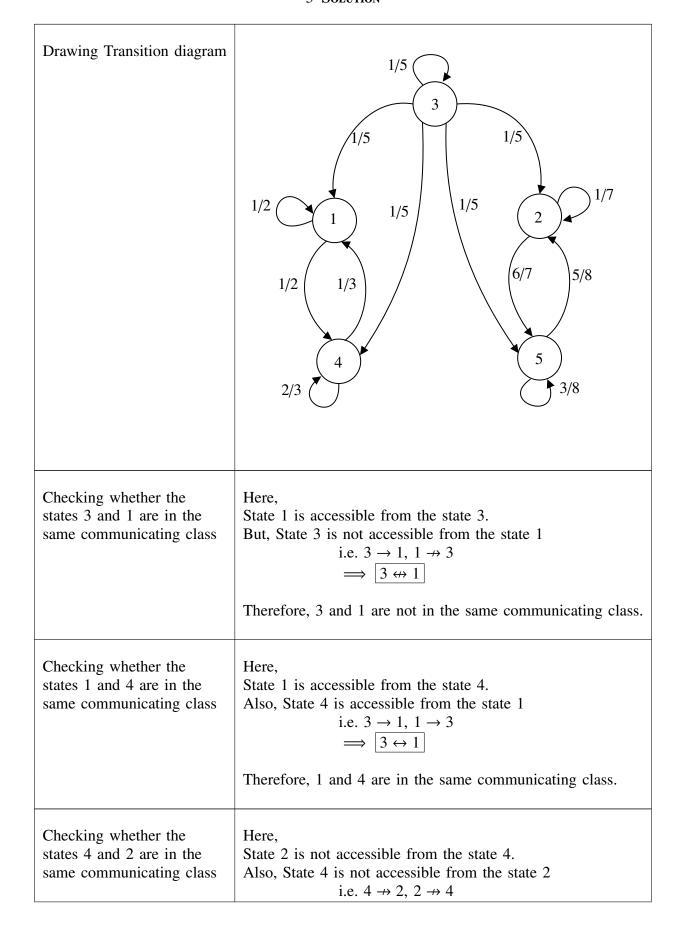
Which of the following are true?

- 1) 3 and 1 are in the same communicating class
- 2) 1 and 4 are in the same communicating class
- 3) 4 and 2 are in the same communicating class
- 4) 2 and 5 are in the same communicating class

### 2 DEFINITION AND RESULT USED

Accessibility of states in Markov's chain	We say that state $j$ is accessible from state $i$ , written as $i \to j$ , if $p_{ij}^{(n)} > 0$ for some n. Every state is accessible from itself since $p_{ii}^{(0)} = 1$
Communication between states	Two states $i$ and $j$ are said to communicate, written as $i \leftrightarrow j$ , if they are accessible from each other. In other words, $i \leftrightarrow j \text{ means } i \to j \text{ and } j \to i.$
Communicating class	For each Markov chain, there exists a unique decomposition of the state space $S$ into a sequence of disjoint subsets $C_1, C_2,,$ $S = \bigcup_{i=1}^{\infty} C_i$ in which each subset has the property that all states within it communicate. Each such subset is called a communication class of the Markov chain.

#### 3 Solution



	$\implies \boxed{4 \leftrightarrow 2}$ Therefore, 4 and 2 are not in the same communicating class.
Checking whether the states 2 and 5 are in the same communicating class	Here, State 2 is accessible from the state 5. Also, State 5 is accessible from the state 2 i.e. $5 \rightarrow 2$ , $2 \rightarrow 5$ $\Rightarrow 2 \leftrightarrow 5$ Therefore, 2 and 5 are in the same communicating class.
Conclusion	Communication classes are: $S = \{1, 4\} \cup \{3\} \cup \{2, 5\}$ Option 2) and 4) are true.