Assignment 6

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Download all python codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment6/codes
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and latex-tikz codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment6
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1 Problem

(Rams 3.4.1) find the Asymptotes of the following:

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\mathbf{x} - \begin{pmatrix}4 & 6\end{pmatrix}\mathbf{x} - 6 = 0 \tag{1.0.1}$$

2 Explanation

The following process is followed to get the equations of the Asymptotes:

1) Move the origin to the center of the given equation to get equation of the form:
$$\implies \mathbf{x}^T\mathbf{V}\mathbf{x} = 1 \tag{2.0.1}$$
2) The equations of the Asymptotes of (2.0.1) can be obtained by factoring the RHS and equating each factors to zero.
3) Obtain the equations of the Asymptotes of the original equation by restoring the origin, which is done by replacing $\mathbf{x}$ with $(\mathbf{x}+\mathbf{c})$ in the equations of the Asymptotes obtained in step2.

3 Solution

Given,

$$\mathbf{x}^T\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-2 & -3\end{pmatrix}\mathbf{x} - 6 = 0 \tag{3.0.1}$$

where,

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \tag{3.0.2}$$

$$\mathbf{u} = \begin{pmatrix}-2\\-3\end{pmatrix} \tag{3.0.3}$$

Here, $\det(\mathbf{V}) = -1$. Since $\det(\mathbf{V}) < 0$ the given equation represents a hyperbola with center:

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix}2\\-3\end{pmatrix} \tag{3.0.4}$$

Moving the origin to the center $\mathbf{c}$, The above equation (3.0.1) can be modified as

$$(\mathbf{x}+\mathbf{c})^T\mathbf{V}(\mathbf{x}+\mathbf{c}) + 2\mathbf{u}^T(\mathbf{x}+\mathbf{c}) - 6 = 0 \tag{3.0.5}$$

From equation (3.0.5) consider,

$$\implies (\mathbf{x}+\mathbf{c})^T\mathbf{V}(\mathbf{x}+\mathbf{c}) \tag{3.0.6}$$

$$\implies \mathbf{x}^T\mathbf{V}\mathbf{x} + \mathbf{c}^T\mathbf{V}\mathbf{x} + \mathbf{x}^T\mathbf{V}\mathbf{c} + \mathbf{c}^T\mathbf{V}\mathbf{c} \tag{3.0.7}$$

we know that

$$\mathbf{x}^T\mathbf{V}\mathbf{c} = \mathbf{c}^T\mathbf{V}\mathbf{x} \tag{3.0.8}$$

Substituting equation (3.0.8) in equation (3.0.7)

$$\implies \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{c}^T\mathbf{V}\mathbf{x} + \mathbf{c}^T\mathbf{V}\mathbf{c} \tag{3.0.9}$$

$$\mathbf{c}^T\mathbf{V}\mathbf{x} = \begin{pmatrix}2 & -3\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} \tag{3.0.10}$$

$$\mathbf{c}^T\mathbf{V}\mathbf{c} = \begin{pmatrix}2 & -3\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}2\\-3\end{pmatrix} = -5 \tag{3.0.11}$$

Substituting the equations (3.0.10), (3.0.11) in equation (3.0.9) we get

$$\implies \mathbf{x}^T\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} - 5 \tag{3.0.12}$$

From equation (3.0.5) consider,

$$\implies 2\mathbf{u}^T(\mathbf{x}+\mathbf{c}) \tag{3.0.13}$$

$$\implies 2\begin{pmatrix}-2 & -3\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-2 & -3\end{pmatrix}\begin{pmatrix}2\\-3\end{pmatrix} \tag{3.0.14}$$

$$\implies -2\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} + 10 \tag{3.0.15}$$

Substituting equations (3.0.12), (3.0.15) in equation

(3.0.5) we get

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} - 2\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} + 10 - 11 = 0 \tag{3.0.16}$$

$$\implies \mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} - 1 = 0 \tag{3.0.17}$$

$$\implies \mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 1 \tag{3.0.18}$$

Factoring the RHS of the equation 3.0.18:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} \tag{3.0.19}$$

$$\implies (x^2 - y^2) \tag{3.0.20}$$

$$\implies (x - y)(x + y) \tag{3.0.21}$$

Equating the factors to zero to obatin the equations of the Asymptotes of 3.0.5 (hyperbola with center at origin):

$$(x - y) = 0 \implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.22}$$

$$(x + y) = 0 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.23}$$

The equation of the Asymptotes of the original hyperbola with center at $\mathbf{c}$ can be obtained as:

$$\begin{pmatrix} 1 & -1 \end{pmatrix} (\mathbf{x} + \mathbf{c}) = 0 \tag{3.0.24}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} (\mathbf{x} + \mathbf{c}) = 0 \tag{3.0.25}$$

Putting value of $\mathbf{c} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, we get:

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \left(\mathbf{x} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right) = 0 \tag{3.0.26}$$

$$\implies \boxed{\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 5} \tag{3.0.27}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \left(\mathbf{x} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} \right) = 0 \tag{3.0.28}$$

$$\implies \boxed{\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = -1} \tag{3.0.29}$$

The equations 3.0.27 and 3.0.29 represent the equations of the Asymptotes of the original hyperbola with center at $\mathbf{c}$.

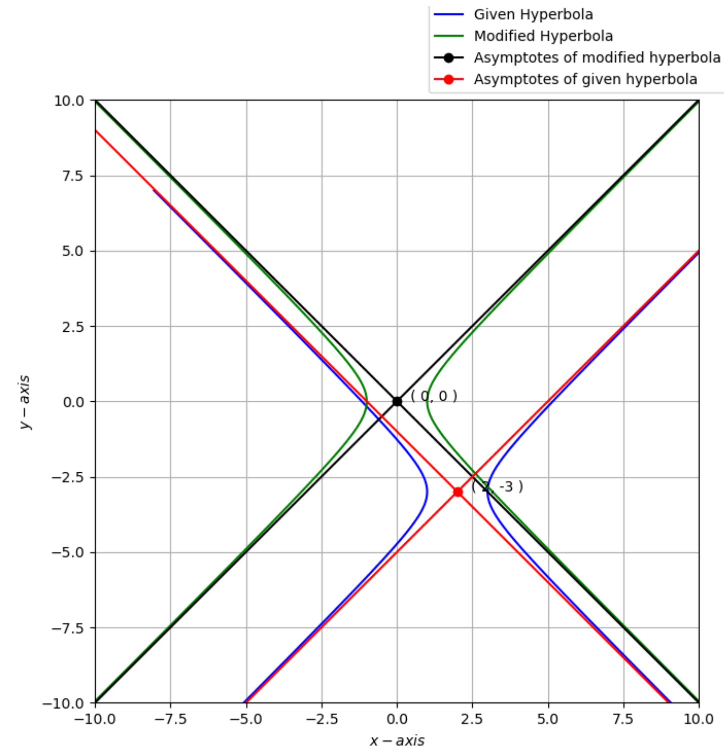

Fig. 3: Plot of the Asymtotes.