Assignment 8

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Download all python codes from

```
https://github.com/sachinomdubey/Matrix−theory/
    Assignment8/codes
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and latex-tikz codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment8
```

1 Problem

(Dresden/Page80/Example1/D)
Determine the distances of the point $D(-1, 2, -4)$ from the plane:

$$3x + 2y - 6z - 2 = 0 \tag{1.0.1}$$

2 Solution

Equation of plane can be written in the form:

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.1}$$

Writing the given plane equation (1.0.1) in the form (2.0.1):

$$\begin{pmatrix} 3 & 2 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2 \tag{2.0.2}$$

Where,

$$\mathbf{n} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad c = 2 \tag{2.0.4}$$

A vector from the plane to the point $D(-1, 2, -4)$ is given by:

$$\mathbf{w} = \mathbf{D} - \mathbf{x} \tag{2.0.5}$$

Where,

$$\mathbf{D} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{2.0.7}$$

The projection of $\mathbf{w}$ onto the normal vector $\mathbf{n}$ can be written as:

$$\text{proj}_{\mathbf{n}}\mathbf{w} = \frac{\mathbf{n}^T\mathbf{w}}{\mathbf{n}^T\mathbf{n}} \cdot \mathbf{n} \tag{2.0.8}$$

$$\text{proj}_{\mathbf{n}}\mathbf{w} = \frac{\mathbf{n}^T(\mathbf{D} - \mathbf{x})}{\mathbf{n}^T\mathbf{n}} \cdot \mathbf{n} \tag{2.0.9}$$

$$\text{proj}_{\mathbf{n}}\mathbf{w} = \frac{\mathbf{n}^T\mathbf{D} - \mathbf{n}^T\mathbf{x}}{\mathbf{n}^T\mathbf{n}} \cdot \mathbf{n} \tag{2.0.10}$$

$$\tag{2.0.11}$$

From equation (2.0.1),

$$\text{proj}_{\mathbf{n}}\mathbf{w} = \frac{\mathbf{n}^T\mathbf{D} - c}{\mathbf{n}^T\mathbf{n}} \cdot \mathbf{n} \tag{2.0.12}$$

$$\tag{2.0.13}$$

Putting the values of $\mathbf{n}$, $\mathbf{D}$ and c, we get:

$$\text{proj}_{\mathbf{n}}\mathbf{w} = \frac{\begin{pmatrix} 3 & 2 & -6 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} - 2}{\begin{pmatrix} 3 & 2 & -6 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}} \cdot \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \tag{2.0.14}$$

$$\text{proj}_{\mathbf{n}}\mathbf{w} = \frac{23}{49} \cdot \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \tag{2.0.15}$$

The distance d_{min} between point $D(-1, 2, -4)$ and the given plane is obtained as:

$$d_{min} = \left\|\text{proj}_{\mathbf{n}}\mathbf{w}\right\| \tag{2.0.16}$$

$$d_{min} = \frac{23}{49} \cdot \left\|\begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}\right\| \tag{2.0.17}$$

$$\therefore d_{min} = \frac{23}{49} \times \sqrt{(3)^2 + (2)^2 + (-6)^2} \tag{2.0.18}$$

$$\therefore d_{min} = \frac{23}{49} \times 7 \tag{2.0.19}$$

$$\implies \boxed{d_{min} = \frac{23}{7} = 3.2857} \tag{2.0.20}$$

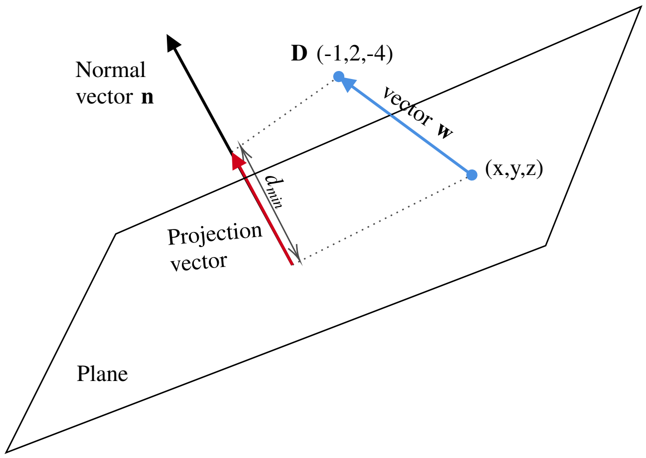

Fig. 0: Distance between a point and a plane