Assignment 2

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Download all python codes from

```
https://github.com/sachinomdubey/Matrix−theory/
    Assignment2/codes
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and latex-tikz codes from

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https://github.com/sachinomdubey/Matrix−theory/
    Assignment2
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Q no. 73. Find the angle between the following pair of lines:

1)

$$L_1 : \quad \mathbf{x} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \tag{0.0.1}$$

$$L_2 : \quad \mathbf{x} = \begin{pmatrix} 7 \\ -6 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \tag{0.0.2}$$

2)

$$L_1 : \quad \mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \tag{0.0.3}$$

$$L_2 : \quad \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ -56 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} \tag{0.0.4}$$

Solution:

1) The direction vectors of the lines are $\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$.
Thus, the angle θ between two vectors is given by

$$\cos\theta = \frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} \tag{0.0.5}$$

$$= \frac{19}{3 \times 7} \tag{0.0.6}$$

$$\implies \theta = 25.21^\circ \tag{0.0.7}$$

2) The direction vectors of the lines are $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix}$.
Thus, the angle θ between two vectors is given by

$$\cos\theta = \frac{\mathbf{a}^T\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} \tag{0.0.8}$$

$$= \frac{16}{\sqrt{6} \times \sqrt{50}} \tag{0.0.9}$$

$$\implies \theta = 22.52^\circ \tag{0.0.10}$$

Note : In both problems, the respective pair of lines do not intersect each other (called skew lines), The obtained angle is the angle between the normal vectors of the lines. The proof that the pair of lines do not intersect is as follows:

Problem 1 : Equating the x, y and z components of both lines and forming equation in the augmented matrix form. The Matrix is row reduced as follows:

$$\begin{pmatrix} 3 & -1 & 5 \\ 2 & -2 & -1 \\ 6 & -2 & -1 \end{pmatrix} \tag{0.0.11}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 3 & -1/3 & 5/3 \\ 2 & -2 & -1 \\ 6 & -2 & -1 \end{pmatrix} \tag{0.0.12}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 3 & -1/3 & 5/3 \\ 0 & -4/3 & -13/3 \\ 6 & -2 & -1 \end{pmatrix} \tag{0.0.13}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 6R_1} \begin{pmatrix} 3 & -1/3 & 5/3 \\ 0 & -4/3 & -13/3 \\ 0 & 0 & -11 \end{pmatrix} \tag{0.0.14}$$

Here, Rank(A) $\neq$ Rank(A|B). Hence, these three equations are inconsistent, which proves that the two lines do not intersect in the 3D plane. (Where A is the coefficient matrix and A|B is the augmented matrix.)

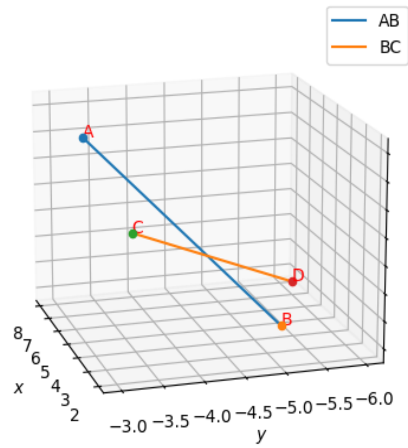

Fig. 2: Problem 1 : Lines crossing each other, but not intersecting

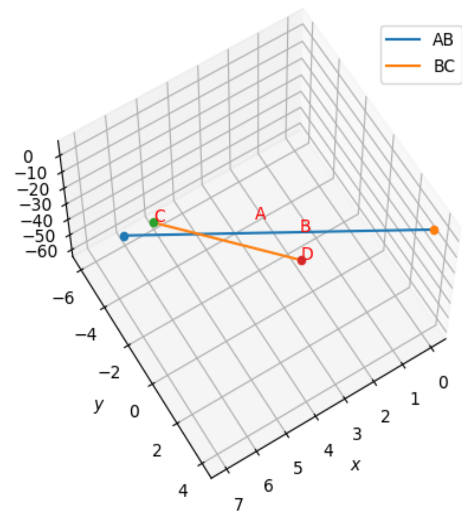

Fig. 2: Problem 2 : Lines crossing each other, but not intersecting

Problem 2 : Equating the x, y and z components of both lines and forming equation in the augmented matrix forms. The Matrix is row reduced as follows:

$$\begin{pmatrix} 1 & -3 & -1 \\ -1 & 5 & -2 \\ -2 & 4 & -54 \end{pmatrix} \tag{0.0.15}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 2 & -3 \\ -2 & 4 & -54 \end{pmatrix} \tag{0.0.16}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 2R_1} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 2 & -3 \\ 0 & -2 & -56 \end{pmatrix} \tag{0.0.17}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 / 2} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & -3/2 \\ 0 & -2 & -56 \end{pmatrix} \tag{0.0.18}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 2R_2} \begin{pmatrix} 1 & -3 & -1 \\ 0 & 1 & -3/2 \\ 0 & 0 & -59 \end{pmatrix} \tag{0.0.19}$$

Here, Rank(A) $\neq$ Rank(A|B).
Hence, the equations are inconsistent, which proves that the two lines do not intersect in the 3D plane. (Where A is the coefficient matrix and A|B is the augmented matrix.)