

Assignment 11

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Download the latex-tikz codes from

<https://github.com/sachinomdubey/Matrix-theory/Assignment11>

1 PROBLEM

(UGC-dec2017,105) :

Consider a Markov chain with five states $\{1, 2, 3, 4, 5\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix} \quad (1.0.1)$$

Which of the following are true?

- 1) 3 and 1 are in the same communicating class
- 2) 1 and 4 are in the same communicating class
- 3) 4 and 2 are in the same communicating class
- 4) 2 and 5 are in the same communicating class

2 DEFINITION AND RESULT USED

Accessibility of states in Markov's chain	We say that state j is accessible from state i , written as $i \rightarrow j$, if $p_{ij}^{(n)} > 0$ for some n . Every state is accessible from itself since $p_{ii}^{(0)} = 1$
Communication between states	Two states i and j are said to communicate, written as $i \leftrightarrow j$, if they are accessible from each other. In other words, $i \leftrightarrow j$ means $i \rightarrow j$ and $j \rightarrow i$.
Communicating class	For each Markov chain, there exists a unique decomposition of the state space S into a sequence of disjoint subsets C_1, C_2, \dots , $S = \bigcup_{i=1}^{\infty} C_i$ in which each subset has the property that all states within it communicate. Each such subset is called a communication class of the Markov chain.

3 SOLUTION

Drawing Transition diagram	
Checking whether the states 3 and 1 are in the same communicating class	<p>Here, State 1 is accessible from the state 3. But, State 3 is not accessible from the state 1 i.e. $3 \rightarrow 1, 1 \nrightarrow 3$ $\Rightarrow \boxed{3 \leftrightarrow 1}$</p> <p>Therefore, 3 and 1 are not in the same communicating class.</p>
Checking whether the states 1 and 4 are in the same communicating class	<p>Here, State 1 is accessible from the state 4. Also, State 4 is accessible from the state 1 i.e. $3 \rightarrow 1, 1 \rightarrow 3$ $\Rightarrow \boxed{3 \leftrightarrow 1}$</p> <p>Therefore, 1 and 4 are in the same communicating class.</p>
Checking whether the states 4 and 2 are in the same communicating class	<p>Here, State 2 is not accessible from the state 4. Also, State 4 is not accessible from the state 2 i.e. $4 \nrightarrow 2, 2 \nrightarrow 4$</p>

	$\Rightarrow 4 \leftrightarrow 2$ <p>Therefore, 4 and 2 are not in the same communicating class.</p>
Checking whether the states 2 and 5 are in the same communicating class	<p>Here, State 2 is accessible from the state 5. Also, State 5 is accessible from the state 2 i.e. $5 \rightarrow 2, 2 \rightarrow 5$ $\Rightarrow 2 \leftrightarrow 5$</p> <p>Therefore, 2 and 5 are in the same communicating class.</p>
Conclusion	<p>Communication classes are:</p> $S = \{1, 4\} \cup \{3\} \cup \{2, 5\}$ <p>Option 2) and 4) are true.</p>