

Assignment 5

Sachinkumar Dubey - EE20MTECH11009

Download all python codes from

<https://github.com/sachinomdubey/Matrix-theory/Assignment5/codes>

and latex-tikz codes from

<https://github.com/sachinomdubey/Matrix-theory/Assignment5>

0.1 Problem

(Geolin 1.9) AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B . Show that the line PQ is the perpendicular bisector of AB .

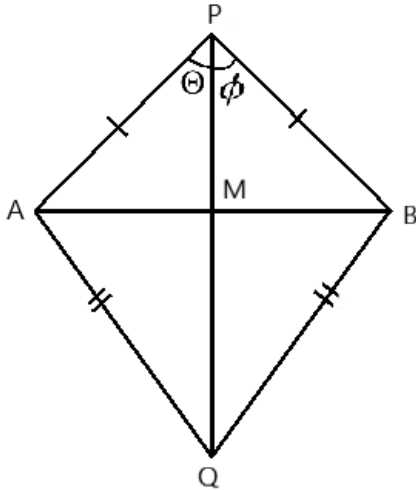


Fig. 0

0.2 Explanation

In order to prove that line PQ is the perpendicular bisector of AB , two conditions need to be met:

- 1) $PQ \perp AB$
- 2) $AM = BM$

These conditions can be proved as follow:

0.3 Solution

It is given that the points P and Q are equidistant from the points A and B . Thus we can write:

$$\|P - A\| = \|P - B\| \quad (0.3.1)$$

$$\|Q - A\| = \|Q - B\| \quad (0.3.2)$$

Squaring both sides of equations 0.3.1 and expanding further, we can write:

$$(P - A)^T (P - A) = (P - B)^T (P - B) \quad (0.3.3)$$

$$P^T P - P^T A - A^T P + A^T A =$$

$$P^T P - P^T B - B^T P + B^T B \quad (0.3.4)$$

$$\therefore A^T A - B^T B = -2P^T B + 2P^T A \quad (0.3.5)$$

Similarly, Squaring both sides of equations 0.3.2 and expanding further, we can write:

$$(Q - A)^T (Q - A) = (Q - B)^T (Q - B) \quad (0.3.6)$$

$$Q^T Q - Q^T A - A^T Q + A^T A =$$

$$Q^T Q - Q^T B - B^T Q + B^T B \quad (0.3.7)$$

$$\therefore A^T A - B^T B = -2Q^T B + 2Q^T A \quad (0.3.8)$$

From equations 0.3.5 and 0.3.8, we can write:

$$2P^T (A - B) = 2Q^T (A - B) \quad (0.3.9)$$

$$P^T (A - B) - Q^T (A - B) = 0 \quad (0.3.10)$$

$$(P - Q)^T (A - B) = 0 \quad (0.3.11)$$

Thus, Segment PQ is perpendicular to segment AB ($PQ \perp AB$).

From the figure, equations 0.3.1 can also be written as:

$$\|(P - M) + (M - A)\| = \|(P - M) + (M - B)\| \quad (0.3.12)$$

Squaring and expanding both the sides, we get:

$$\begin{aligned} ((P - M) + (M - A))^T ((P - M) + (M - A)) &= \\ ((P - M) + (M - B))^T ((P - M) + (M - B)) & \end{aligned} \quad (0.3.13)$$

$$\begin{aligned}
& (\mathbf{P} - \mathbf{M})^T (\mathbf{P} - \mathbf{M}) + (\mathbf{P} - \mathbf{M})^T (\mathbf{M} - \mathbf{A}) + \\
& (\mathbf{M} - \mathbf{A})^T (\mathbf{P} - \mathbf{M}) + (\mathbf{M} - \mathbf{A})^T (\mathbf{M} - \mathbf{A}) = \\
& (\mathbf{P} - \mathbf{M})^T (\mathbf{P} - \mathbf{M}) + (\mathbf{P} - \mathbf{M})^T (\mathbf{M} - \mathbf{B}) + \\
& (\mathbf{M} - \mathbf{B})^T (\mathbf{P} - \mathbf{M}) + (\mathbf{M} - \mathbf{B})^T (\mathbf{M} - \mathbf{B})
\end{aligned} \tag{0.3.14}$$

$$\begin{aligned}
& \|(\mathbf{M} - \mathbf{A})\|^2 + 2 (\mathbf{M} - \mathbf{A})^T (\mathbf{P} - \mathbf{M}) = \\
& \|(\mathbf{M} - \mathbf{B})\|^2 + 2 (\mathbf{M} - \mathbf{B})^T (\mathbf{P} - \mathbf{M})
\end{aligned} \tag{0.3.15}$$

Since, $PQ \perp AB$. Hence, we can write:

$$(\mathbf{M} - \mathbf{A})^T (\mathbf{P} - \mathbf{M}) = (\mathbf{M} - \mathbf{B})^T (\mathbf{P} - \mathbf{M}) = 0 \tag{0.3.16}$$

From equation 0.3.15 and 0.3.16, we get:

$$\|(\mathbf{M} - \mathbf{A})\| = \|(\mathbf{M} - \mathbf{B})\| \tag{0.3.17}$$

Thus, M is the midpoint of segment AB ($AM = BM$). Thus, Segment PQ is perpendicular bisector of segment AB .