INTERFERENCE

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Introduction

→ The interference phenomana provide direct evidence for the wave nature of light.

→ The theory of interference was developed by Thomas Young in 1801.

→ Many observations in day to day life are due to interference of light.

Ex: Multiple Colours on Soap bubble as well as on thin layer of floating oil when viewed under Sun light.

→ Interference phenomena is based on the principle of Superposition of waves.

Superposition principle :

→ When two (or) more waves acting simultaneously at a point in the medium, then the resultant displacement of a particle at that point at any instant of time is the algebraic sum of the displacements of the same particle due to individual waves.

→ If y_1 is the displacement of the particle due to one wave at any instant in the absence of the other wave and y_2 is the displacement of the particle due to other wave at the same instant in the absence of the first wave, then the resultant displacement of the particle is given by

$$\boxed{y = y_1 + y_2}$$

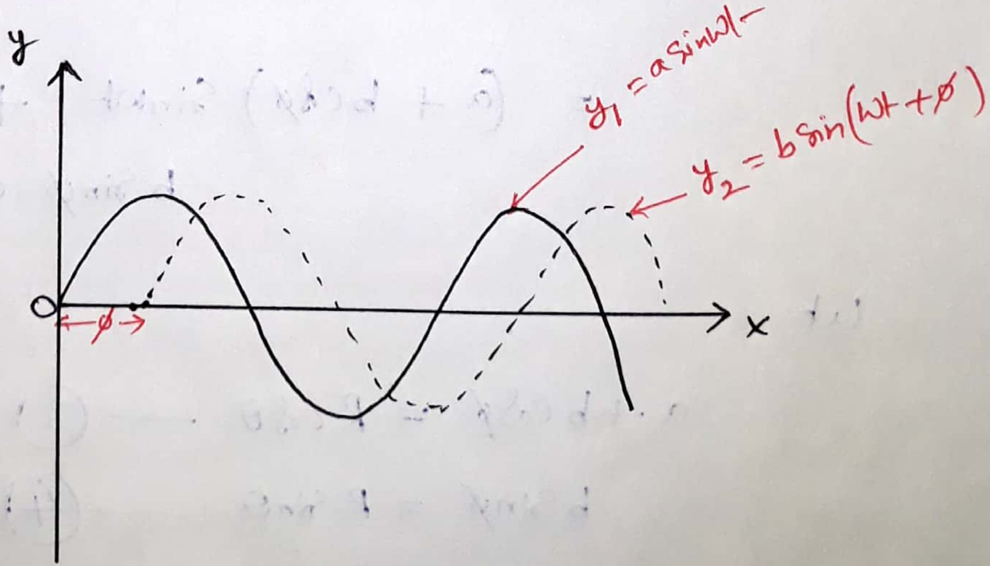
Interference :

→ When two (or) more waves having same frequency and constant phase difference superimpose each other, the intensity of light should be modified.

→ The modification of intensity of light due to the superposition of waves is called interference.

Resultant Intensity due to interference of two Coherent Waves:

→ Consider two waves having same frequency and constant phase difference "ϕ" travelling in the same direction as shown in the fig.



→ If y_1 and y_2 are the displacements due to two waves, then

$$y_1 = a \sin \omega t \quad \text{--- (1)}$$

$$y_2 = b \sin(\omega t + \phi) \quad \text{--- (2)}$$

where "a" and "b" are amplitudes.

→ According to Superposition principle, the resultant displacement is

$$y = y_1 + y_2$$

$$= a \sin \omega t + b \sin(\omega t + \phi)$$

$$= a \sin \omega t + b \sin \omega t \cdot \cos \phi + b \cos \omega t \cdot \sin \phi$$

$$= (a + b \cos \phi) \sin \omega t + b \sin \phi \cdot \cos \omega t$$

Let

$$a + b \cos \phi = R \cos \theta \quad \text{--- (3)}$$

$$b \sin \phi = R \sin \theta \quad \text{--- (4)}$$

$$\therefore\ y = R \sin\omega t . \cos\theta + R \cos\omega t . \sin\theta$$

$$= R \sin(\omega t + \theta) \quad \text{—— (5)}$$

Where R is the resultant amplitude

Resultant amplitude R :

Squaring and adding equations (3) and (4), we get

$$R^2 (\cos^2\theta + \sin^2\theta) = (a + b\cos\phi)^2 + (b \sin\phi)^2$$

$$R^2 = a^2 + b^2\cos^2\phi + 2ab\cos\phi + b^2\sin^2\phi$$

$$= a^2 + b^2(\sin^2\phi + \cos^2\phi) + 2ab\cos\phi$$

$$R^2 = a^2 + b^2 + 2ab\cos\phi$$

$$\therefore\ R = \sqrt{a^2 + b^2 + 2ab\cos\phi} \quad \text{—— (6)}$$

The resultant amplitude R depends on the amplitudes of individual waves and phase difference bet/. the two waves.

→ If amplitudes of two waves are equal, then

$$R = \sqrt{a^2 + a^2 + 2a^2\cos\phi}$$

$$R = \sqrt{2a^2(1+\cos\phi)}$$

∴ Resultant Intensity $I = R^2$

$$= 2a^2(1+\cos\phi)$$

$$\boxed{I = 4a^2\cos^2\phi/2} \quad (7)$$

Conditions for Maxima and Minima intensity

(1) Maxima Intensity (Constructive interference)

If $\phi = 2n\pi$, then the $(n = 0, 1, 2, \ldots)$ resultant intensity is maximum.

$$I_{max} = 4a^2$$

$$= 4I_0 \quad (\text{where } I_0 = a^2)$$

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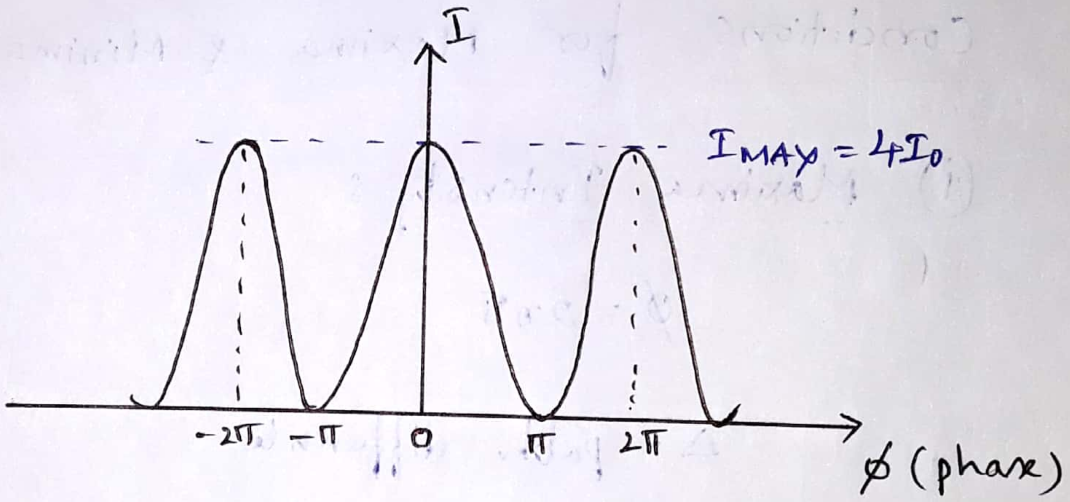
(2) Minima Intensity : (Destructive Interference)

If $\phi = (2n+1)\pi$, then the resultant
$(n = 0, 1, 2, \ldots)$
intensity is minimum.

$$I_{Min} = 0$$

Resultant Intensity Distribution Curve



→ The resultant Intensity distribution consists of alternate bright and dark bands called interference fringes.

Relation bet/. phase and path difference

→ If the phase difference between two waves is "ϕ", then the corresponding path difference Δ is given by

$$\boxed{\Delta = \frac{\lambda}{2\pi} \times \text{phase difference} (\phi)}$$

Conditions for Maxima & Minima

(1) Maxima Intensity :

$$\phi = 2n\pi$$

Δ = path difference

$$= \frac{\lambda}{\cancel{2\pi}} \times \cancel{2}n\cancel{\pi}$$

$\boxed{\Delta = n\lambda}$ Where $n = 0, 1, 2, \ldots$

(2) Minima Intensity :

$$\phi = (2n+1)\pi$$

$$\Delta = \text{Path difference} = \frac{\lambda}{2\cancel{\pi}} \times (2n+1)\cancel{\pi}$$

$$\boxed{\Delta = (2n+1)\lambda/2}$$

Where $n = 0, 1, 2, \ldots$

Conditions for sustained Interference

The conditions for observing Sustained interference pattern with good contrast are as follows.

1. The two Sources should be Coherent.
2. The distance between two Sources should be small.
3. The distance between Sources and Screen should be large.
4. The amplitudes of interfering waves should be equal
5. The sources must be narrow.
6. The light should be monochromatic.
7. If the interfering beams are polarized, they must be in the same state of polarization.

Types of Interference

The phenomena of interference grouped into two Categories.

1. Division of wave front
2. Division of amplitude

Division of wave front:

→ In this case the single wave front is divided into two parts to produce two coherent sources for interference.

Ex: 1. Young's Double Slit experiment.

2. Fresnel's biprism expt.

Division of amplitude:

→ In this case the amplitude of the incident light wave is divided into two parts to produce two

Coherent Sources for interference.

Ex: ① Newton's rings experiment

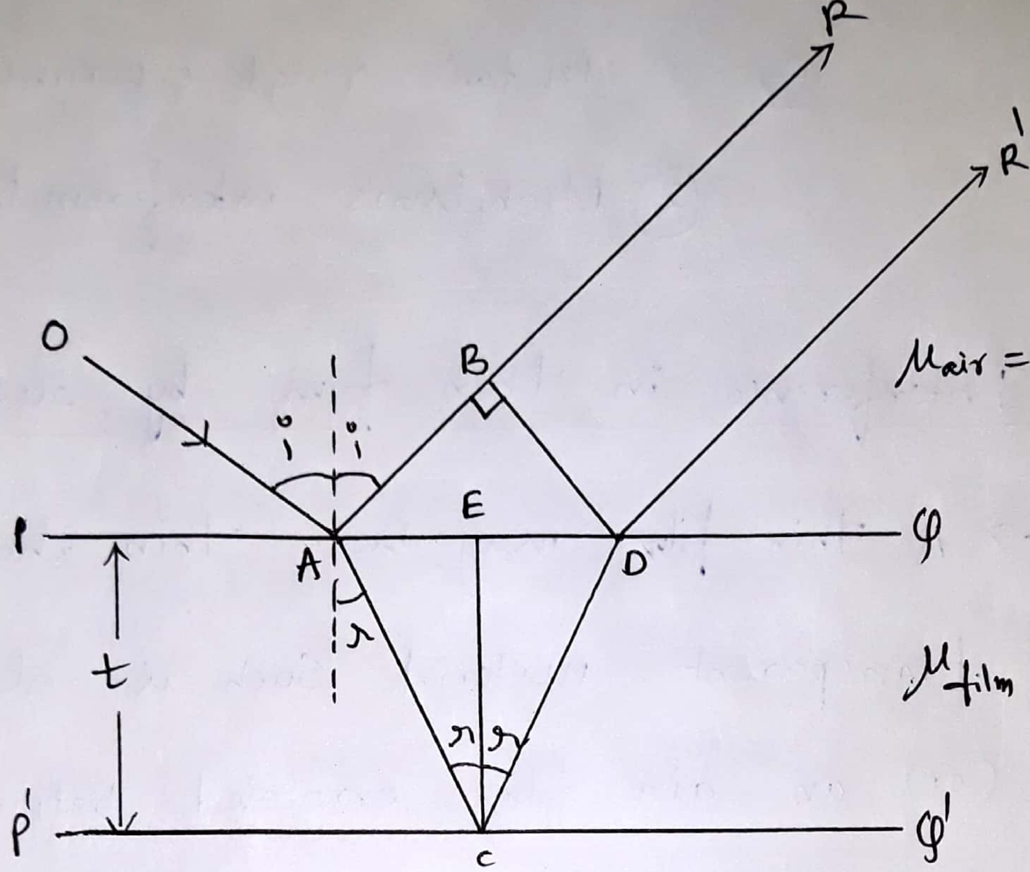
② Michelson's interferometer expt.

Interference in thin films by reflection

→ A thin film may be a thin sheet of transparent material such as glass, mica (or) an air film enclosed between two transparent plates (or) a soap bubble.

→ In thin films interference is due to superposition of light reflected from the top and the bottom surface of a thin film.

→ Let us consider a thin film of uniform thickness t and refractive index μ.



→ If a ray of light OA is incident on the upper surface of the film. with an angle of incidence "i", then the part of the light is reflected along AR and the remaining part of the light is transmitted into the thin film.

→ The transmitted beam AC is reflected at the lower surface and emerges out of the film along DR'.

→ The rays travelling along the paths AR and ACDR' are derived from a single incident ray OA. Hence they are coherent and can produce interference pattern.

→ The optical path difference between two reflected rays = Path (AC+CD) in medium − path AB in air

$$\Delta = \mu(AC+CD) - AB \quad (\because \text{for Air } \mu = 1) \quad \text{--- (1)}$$

→ From

ΔAEC,

$$\cos r = \frac{CE}{AC}$$

$$\Rightarrow AC = \frac{CE}{\cos r} = \frac{t}{\cos r} \quad (\because CE = t)$$

→ from ΔCED,

$$\cos r = \frac{CE}{CD}$$

$$\Rightarrow CD = \frac{CE}{\cos r} = \frac{t}{\cos r}$$

$$\therefore AC + CD = \frac{t}{\cos r} + \frac{t}{\cos r}$$

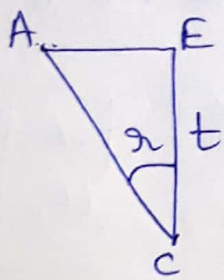
$$= \frac{2t}{\cos r} \quad \text{—— (2)}$$

→ From ΔABD,

$$\sin i = \frac{AB}{AD}$$

$$\Rightarrow AB = AD \sin i \quad \text{—— (3)}$$

But $AD = AE + ED$ —— (4)



From ΔAEC, $\tan r = \frac{AE}{CE}$

$$\Rightarrow AE = CE \tan r$$

$$= t \tan r$$

Ily

From ΔCED, $\tan r = \frac{ED}{CE}$

$$\Rightarrow ED = CE \tan r$$

$$= t \tan r$$

from eq. (4),

$$AD = AE + ED$$

$$= t \tan r + t \tan r$$

$$= 2t \tan r$$

$\therefore$ from eq (3),

$$AB = AD \sin i$$

$$= 2t \tan r \, \underline{\sin i} \quad \text{—— (5)}$$

$\rightarrow$ from Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \sin i = \mu \sin r$$

$\therefore$ from eq. (5),

$$AB = 2t \tan r \, \mu \sin r$$

$$= 2t \cdot \frac{\sin r}{\cos r} \cdot \mu \sin r$$

$$= \frac{2\mu t \sin^2 r}{\cos r} \quad \text{—— (6)}$$

$\therefore$ The optical path difference between ther reflected rays $= \Delta$

$$\Delta = \mu(AC+CD) - AB$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1-\sin^2 r)$$

$$= \frac{2\mu t}{\cancel{\cos r}} \cos^{\cancel{2}} r$$

$$\Delta = 2\mu t \cos r \quad \text{--- (7)}$$

→ Since the incident ray OA is reflected at the optically denser medium, an additional phase change π (or) path difference $\lambda/2$ occurs.

→ $\therefore$ The effective path difference between two reflected rays $= 2\mu t \cos r + \lambda/2$ — (8)

Conditions :

① **Bright film** : (Constructive interference)

The film appears bright if the path difference $\Delta = n\lambda$

$$2\mu t\cos r + \lambda/2 = n\lambda$$

$$2\mu t\cos r = (2n-1)\lambda/2 \quad \text{—— (9)}$$

where $n = 1, 2, 3, \ldots$

② **Dark film** : (Destructive interference)

The film appears dark if the path difference $\Delta = (2n+1)\lambda/2$

$$2\mu t\cos r + \lambda/2 = (2n+1)\lambda/2$$

$$2\mu t\cos r = n\lambda \quad \text{—— (10)}$$

where $n = 0, 1, 2, \ldots$

Colours of thin films

→ When white light is incident on the thin film (oil film floating on water (or) soap bubble), the different points on the thin film satisfy the condition for constructive interference for different wavelengths and hence appear multicoloured.

→ No dark band is seen because if a particular point satisfies condition for constructive interference for a particular wavelength, the same point may satisfy condition for constructive interference for different wavelength and hence throughout the film multicolours appear.

→ If monochromatic light is incident on the thin film, then those points at which the film satisfies the condition for constructive interference will appear bright and the remaining regions appear dark. Hence the bright and dark bands of that particular colour appear.

Newton's Rings

→ When a plano - convex lens of large radius of curvature is placed on a plane glass plate such that convex surface is in contact with plane surface of the glass plate, an air film of gradually increasing thickness is formed between them.

→ The thickness of air film at the point of Contact is zero.

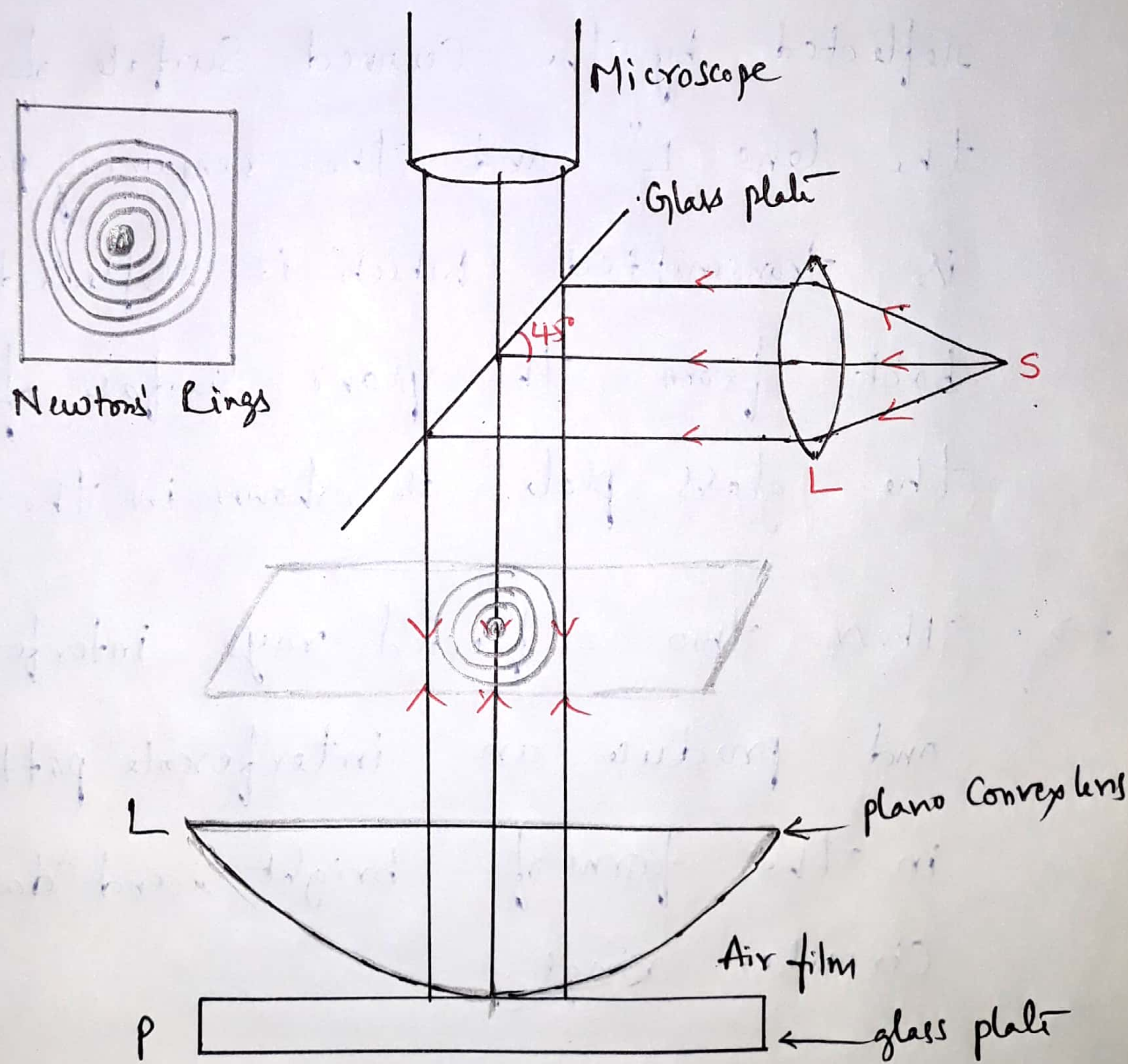
→ If a monochromatic light is allowed to fall normally and the film is viewed in the reflected light, then alternate dark and bright Concentric Circular rings are observed around the point of Contact. These rings are called "Newton's rings".

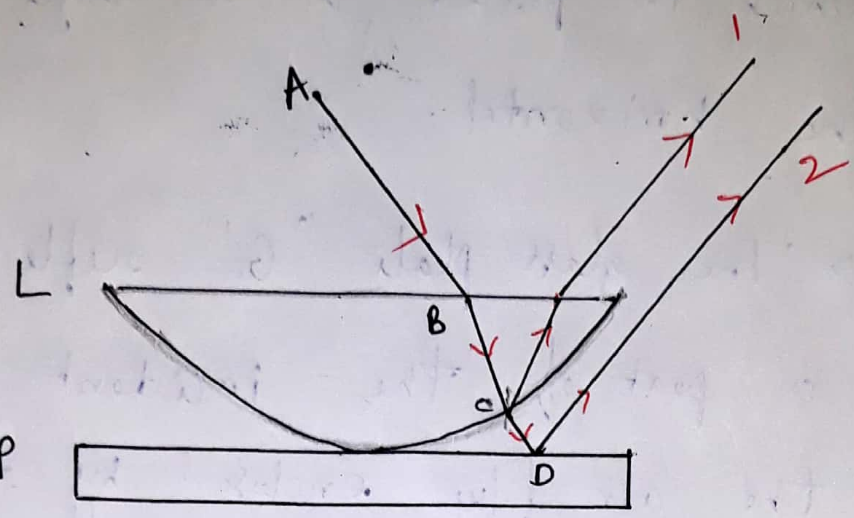
Experimental arrangement :

→ The experimental arrangement Consists of a plano-convex lens "L" of large radius of Curvature and is placed on a plane glass plate P.

→ The light from a monochromatic source incident on a glass plate "G" which is placed at an angle of 45° with the horizontal.

→ The glass plate "G" reflects normally a part of the incident light towards the air film enclosed by the lens and glass plate.



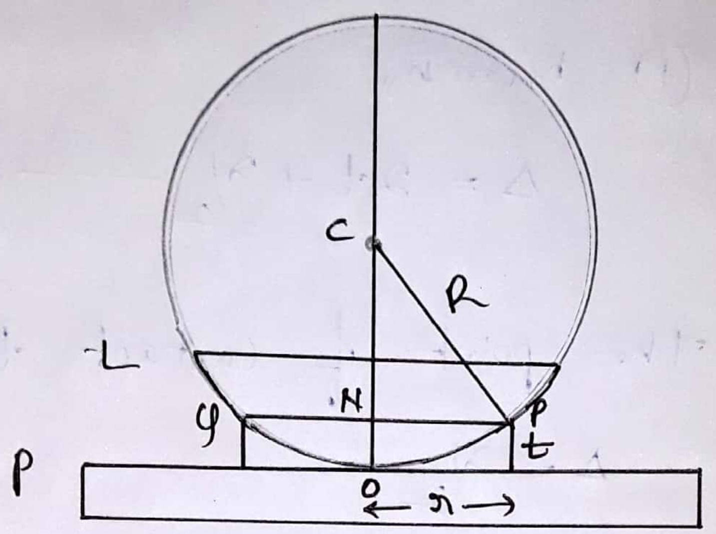


→ A part of the incident light is reflected by the curved surface of the lens "L" and the remaining part is transmitted which is reflected back from the plane surface of the glass plate as shown in the fig.

→ These two reflected rays interfere and produce an interference pattern in the form of bright and dark circular rings.

→ These rings can be viewed with microscope "M" focussed on the film.

Diameters of Bright and Dark rings :



→ Let "R" be the radius of Curvature of plano Convex lens.

→ Let "r" be the radius of Newton's ring Corresponding to the air film of thickness t.

→ The optical path difference between the rays reflected from Convex and plane Surface of the glass plate is given by

$$\Delta = 2\mu t \cos r + \lambda/2 \quad \text{--- (1)}$$

For normal incidence $r=0$ and for air film $\mu=1$.

$\therefore$ Eq. (1) becomes,

$$\Delta = 2t + \lambda/2 \quad \text{---} \quad (2)$$

→ At the point of contact $t=0$,

$$\Delta = \lambda/2$$

This is the condition for Minimum intensity. Hence the dark spot is formed at the center.

Conditions :

(1) Bright ring (Constructive interference).

The ring appears bright if the path difference $\Delta = n\lambda$

$n=0,1,2,\ldots$

$$\therefore 2\mu t + \lambda/2 = n\lambda$$

$$2t = (2n-1)\lambda/2 \quad \text{---} \quad (3)$$

$n=1,2,\ldots$

② Dark Ring (Destructive Interference)

The ring appears dark if the path difference $\Delta = (2n+1)\lambda/2$

$$\therefore \quad 2t + \lambda/2 = (2n+1)\lambda/2$$

$$2t = n\lambda \quad \text{—— (4)}$$

$$n = 0, 1, 2, \dots$$

→ Now let us find the thickness of the air film in terms of radius of Newton's ring and radius of curvature of the plano convex lens.

From ΔCNP,

$$CN^2 + NP^2 = CP^2$$

$$(R-t)^2 + r^2 = R^2$$

$$\cancel{R^2} + t^2 - 2Rt + r^2 = \cancel{R^2}$$

$$r^2 = 2Rt - t^2$$

As $t << R$, t^2 can be neglected

$$\therefore \cancel{t}^2 = 2Rt$$

$$\Rightarrow t = \frac{r^2}{2R} \quad \text{--- (5)}$$

Diameters of Bright Rings

From eq (3),

$$2t = (2n-1)\lambda/2$$

$$\cancel{2}\frac{r^2}{\cancel{2}R} = (2n-1)\lambda/2 \qquad \left(\text{from eq (5)} \quad t = \frac{r^2}{2R}\right)$$

$$r_n^2 = \frac{(2n-1)\lambda R}{2}$$

r_n = radius of n^{th} Bright ring

If D_n = Diameter of n^{th} Bright ring

$$= 2r_n \qquad \Rightarrow \frac{D_n}{2} = r_n$$

$$\therefore \frac{D_n^2 \text{ (Bright)}}{\cancel{4}\,2} = \frac{(2n-1)\lambda R}{\cancel{2}}$$

$$D_n^2 = 2(2n-1)\lambda R$$

$$\therefore \boxed{D_n = \sqrt{2(2n-1)\lambda R}} \quad \text{--- (6)}$$

$$D_n \propto \sqrt{2n-1}$$

Hence the diameters of bright rings are proportional to the square root of the odd natural numbers.

Diameters of dark Rings :

from eq (4),

$$2t = n\lambda$$

$$\cancel{2} \cdot \frac{r^2}{\cancel{2}R} = n\lambda$$

$$r_n^2 = n\lambda R$$

r_n = Radius of n^{th} dark ring

If D_n (dark) = Diameter of n^{th} dark ring

$$= 2r_n$$

$$\Rightarrow r_n = D_n/2$$

$$\frac{D_n^2 \text{ (Dark)}}{4} = n\lambda R$$

$$D_n^2 \text{ (Dark)} = 4n\lambda R$$

$$\therefore \boxed{D_n = \sqrt{4n\lambda R}} \quad \text{—— (7)}$$

$$\Rightarrow D_n \propto \sqrt{n}$$

Hence the diameter of dark rings are proportional to the square root of natural numbers.

* Applications of Newton's rings :

1. Determination of wavelength of given source of light

We know that, the diameter of n^{th} dark ring is $D_n = \sqrt{4n\lambda R}$

$$\Rightarrow D_n^2 = 4n\lambda R \quad \text{—— (1)}$$

If $D_{(n+p)}$ is the diameter of the $(n+p)^{th}$ dark ring, then $D_{n+p} = \sqrt{4(n+p)\lambda R}$

$$\Rightarrow D^2_{n+p} = 4(n+p)\lambda R \quad \text{--- (2)}$$

(2) − (1) $\Rightarrow$

$$D^2_{n+p} - D^2_n = 4p\lambda R$$

$$\therefore \boxed{\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}} \quad \text{--- (3)}$$

2. Determination of refractive index of liquid:

→ If D_n and D_{n+p} are diameters of dark rings when the air film is placed between the lens and glass plate, then

$$D^2_n = 4n\lambda R$$

$$D^2_{n+p} = 4(n+p)\lambda R$$

$$D^2_{n+p} - D^2_n = 4p\lambda R \quad \text{--- (1)}$$

→ If D'_n and D'_{n+p} are the diameters of dark rings when the liquid is

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placed between the lens and the glass plate,

then

$$D_n'^2 = 4n\frac{\lambda}{\mu}R$$

$$D_{n+p}'^2 = 4(n+p)\frac{\lambda}{\mu}R$$

where μ = Refractive index of liquid

$$D_{n+p}'^2 - D_n'^2 = \frac{4p\lambda R}{\mu} \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{D_{n+p}^2 - D_n^2}{D_{n+p}'^2 - D_n'^2} = \frac{\cancel{4p\lambda R}}{\frac{\cancel{4p\lambda R}}{\mu}} = \mu$$

$$\therefore \boxed{\mu = \frac{D_{n+p}^2 - D_n^2}{D_{n+p}'^2 - D_n'^2}} \quad \frac{\text{(Air)}}{\text{(liquid)}} \quad \text{--- (3)}$$

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Applications of thin film interference

(1) Anti-reflection Coating :

→ A thin transparent film coated on a surface in order to suppress the reflections from the surface is called an anti-reflection film.

→ The optical thickness of AR coating is one-quarter of the wavelength.

→ The lenses of cameras and telescopes are coated with AR film to reduce reflections.

→ We can increase the efficiency of solar cells by coating solar panels with AR film of suitable refractive index