DIFFRACTION

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Introduction

→ The wave nature of light is further confirmed by the phenomenon of diffraction

→ **Diffraction :**

"The bending of waves around the edges of obstacles (or) openings if the dimensions of the obstacles (or) openings are comparable to the wavelength of the waves is called diffraction".

→ Since the wavelength of light is much smaller than the dimensions of most obstacles, we don't encounter diffraction effects of light in everyday observations.

→ The Correct interpretation of diffraction phenomenon was provided by Fresnel. According to him, the diffraction is due to interference of Secondary waves originating from different points of the wavefront incident on the obstacle (or) opening.

→ The finite resolution of our eye (or) optical instruments such as telescopes, Microscopes is limited due to the phenomenon of diffraction.

→ According to Richard Feynman

"No-one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage and there is no specific, important difference between them.

He suggested that when there are only two sources, say two, we call it interference as in young's slits, but with a large number of sources, the process is labelled as diffraction.

Examples:

→ ① Diffraction creates the "rainbow" colours reflected from a CD (or) DVD.

② Colourful rings around light sources like candle flames (or) street lights in the fog are due to Diffraction. These colourful rings are called Diffraction Coronas.

Differences between Fresnel and Fraunhofer's Diffraction :

Fresnel Diffraction	Fraunhofer Diffraction.
1. The source and the screen are placed at finite distance from the source.	1. The source and the screen are placed at infinite distance from the source
2. No lens is used.	2. Converging lens is used
3. The wavefront undergoing diffraction is either spherical (or) cylindrical	3. The wavefront undergoing diffraction is plane wavefront.
4. Mathematical analysis is complicated	4. Mathematical analysis is simple

Differences between Interference and Diffraction :

Interference	Diffraction.
1. The Superposition of two Separate wavefronts originating from the Separate coherent Sources.	1. The Superposition of Secondary waves originating from the different points of the Same wavefront.
2. All the bright fringes have the Same intensity.	2. The bright fringes are of varying intensity.
3. The regions of Minimum intensity are perfectly dark.	3. The regions of Minimum are not perfectly dark.
4. The fringe width may (or) may not be equal	4. The fringe width of Various fringes are never equal

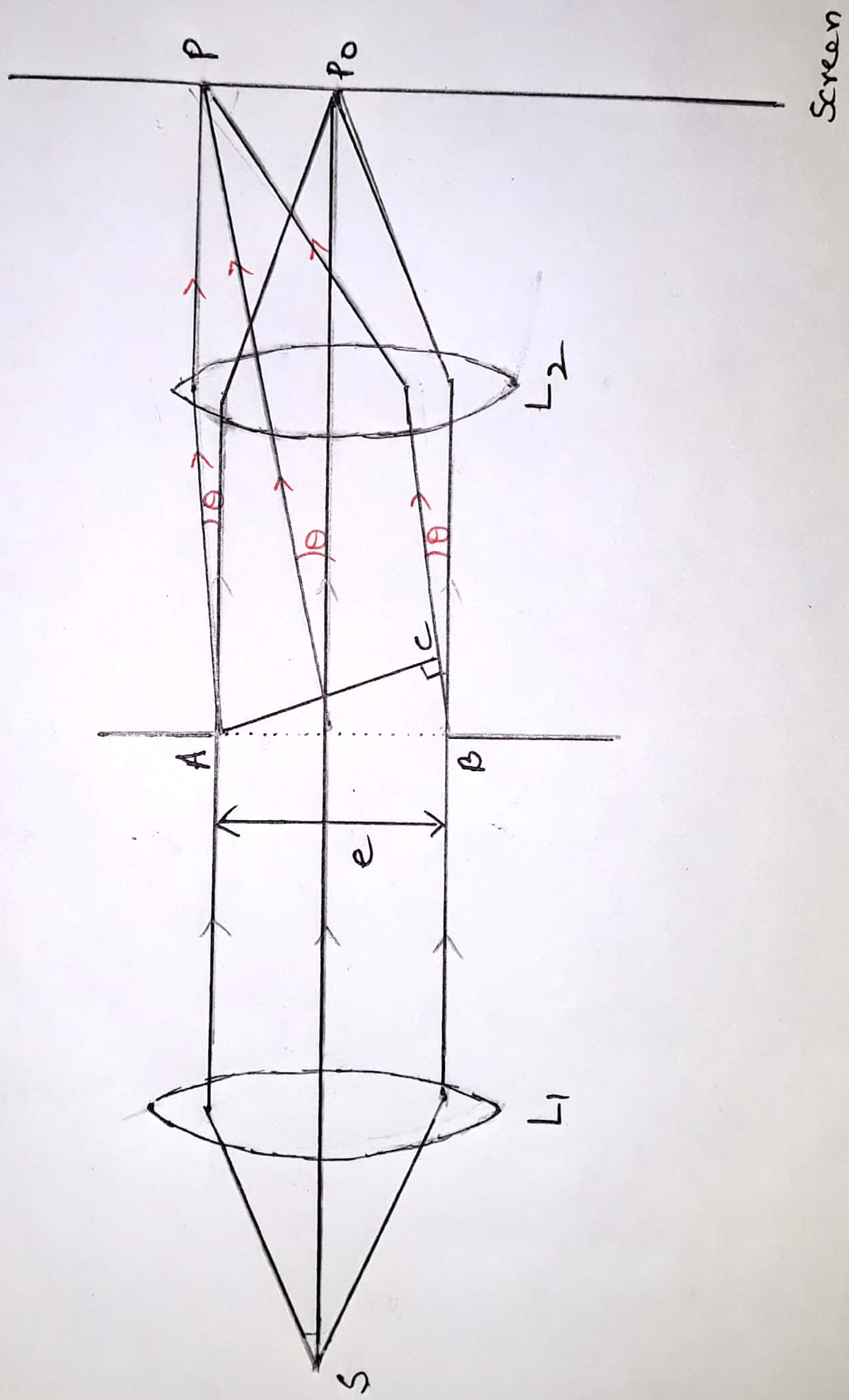
Fraunhofer Diffraction due to Single Slit

→ Consider a slit AB of width 'e' and S is a source of monochromatic light of wave length λ.

→ The source S is placed at the focal point of convex lens L_1. Hence the source is effectively at infinite distance from the slit.

→ As the source is effectively at infinite distance, the wave front incident on the slit is plane wavefront.

→ The diffracted light from the slit is focussed on the screen by using another convex lens L_2. The screen is placed at focal point of convex lens L_2. Hence the screen is effectively at infinite distance from the slit.


P
Po
Screen
L2
L1
A
B
C
e
S
θ
θ
θ

→ According to geometrical optics a bright image having uniform illumination with sharp edges should be formed on the screen.

→ But the diffraction pattern on the screen consisting of a wide central bright band surrounded by alternate bright and dark bands of varying intensity are observed.

Positions of Central Maximum, Minima and Secondary Maxima:

→ According to Huygen's wave theory, each point on AB sends out secondary waves in all directions.

→ The secondary waves travelling in a direction parallel to the direction of incident beam are focussed at point P_0 on the screen.

→ As the path difference between secondary waves reaching the point P_0 is zero, the point P_0 appears as bright.

→ Now consider secondary waves diffracted at angle θ. These secondary waves are focussed at point P on the screen.

→ The intensity at point P depends on the path difference between secondary waves emitted from the slit.

→ Let us assume that width of slit is divided into 'n' equal parts and the amplitude of each wave from each part is 'a'.

→ The path difference between the Secondary waves from A and B in The direction 'θ' is given by,

$$BC = AB \sin\theta$$

$$= e \sin\theta \quad \text{——— (1)}$$

→ The Corresponding phase difference is $\phi = \frac{2\pi}{\lambda} e \sin\theta$ ——— (2)

∴ The phase difference between Secondary waves from two Consecutive Slits is

$$d = \frac{\phi}{n} = \frac{2\pi e \sin\theta}{\lambda n} \quad \text{——— (3)}$$

→ using the method of vector addition of amplitudes, the resultant amplitude at P is given by

$$\boxed{R = \frac{a \sin \frac{nd}{2}}{\sin \frac{d}{2}}} \quad \text{——— (4)}$$

$$R = \frac{a \sin \cancel{n}\left(\frac{\cancel{2}\pi e \sin\theta}{\cancel{2}\lambda \cancel{n}}\right)}{\sin \frac{\cancel{2}\pi e \sin\theta}{\cancel{2}\lambda n}}$$

$$= \frac{a \sin \frac{\pi e \sin\theta}{\lambda}}{\sin \frac{\pi e \sin\theta}{\lambda n}}$$

$$= \frac{a \sin \alpha}{\sin \frac{\alpha}{n}}$$

Where $\alpha = \frac{\pi e \sin\theta}{\lambda}$

$\because$ n is very Large, $\frac{\alpha}{n}$ is very Small

$\therefore$ $\sin \frac{\alpha}{n} = \frac{\alpha}{n}$ (From Small Angle approximation)

$\therefore$ Resultant amplitude $R = \frac{a \sin \alpha}{\frac{\alpha}{n}}$

$$= \frac{n a \sin \alpha}{\alpha}$$

$$\boxed{R = \frac{A \sin\alpha}{\alpha}} \quad \text{--- (5)}$$

where $A = na$

$\therefore$ Resultant Intensity $I = R^2$

$$= \frac{A^2 \sin^2\alpha}{\alpha^2}$$

$$\boxed{I = \frac{I_0 \sin^2\alpha}{\alpha^2}} \quad \text{--- (6)}$$

where $I_0 = A^2$

From eq (6), the intensity at P depends on the value of "α" i.e., on the value of "θ".

Position of Central Maximum:

$$R = \frac{A \sin\alpha}{\alpha}$$

If $\theta \to 0$, then $\alpha \to 0$

$$\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1 \ (\text{MAX})$$

Hence the central maximum is observed

in the direction $\theta = 0$.

$\therefore$ The Intensity of Central Maximum

$$I = A^2 = I_0.$$

Positions of Minima :

$$R = \frac{A \sin\alpha}{\alpha}$$

The intensity is minimum if $\sin\alpha = 0$

$$\Rightarrow \boxed{\alpha = \pm m\pi} \quad \text{—— (7)}$$

$m = 1, 2, 3, \ldots$

$m \neq 0$ because $m = 0$ corresponds to Central maximum.

But $\alpha = \dfrac{\pi e \sin\theta}{\lambda}$

From eq (7),

$$\frac{\cancel{\pi} e \sin\theta}{\lambda} = \pm m \cancel{\pi}$$

$$\therefore \boxed{e \sin\theta = \pm m\lambda} \quad \text{—— (8)}$$

If $m=1$, then $\theta_1 = \sin^{-1}\left(\frac{\lambda}{e}\right)$.

If $m=2$, then $\theta_2 = \sin^{-1}\left(\frac{2\lambda}{e}\right)$

$\vdots$

In this way we can find the directions of minima by putting $m = 1, 2, 3, \ldots$ in the eq. (8).

Positions of Secondary Maxima :

→ In addition to the central maximum at $\alpha = 0$, there are secondary maxima between equally spaced minima.

→ The positions of secondary maxima can be obtained by differentiating eq. (6) and equating to zero.

i.e., $\frac{dI}{d\alpha} = 0$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left(\frac{I_0 \sin^2\alpha}{\alpha^2}\right) = 0$$

$$\Rightarrow I_0 \left(\frac{2\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

$\sin\alpha = 0$ (or) $\alpha\cos\alpha - \sin\alpha = 0$

$\because \sin\alpha \neq 0$ ($\sin\alpha = 0$ corresponds to Minima)

Hence the condition for the maxima is

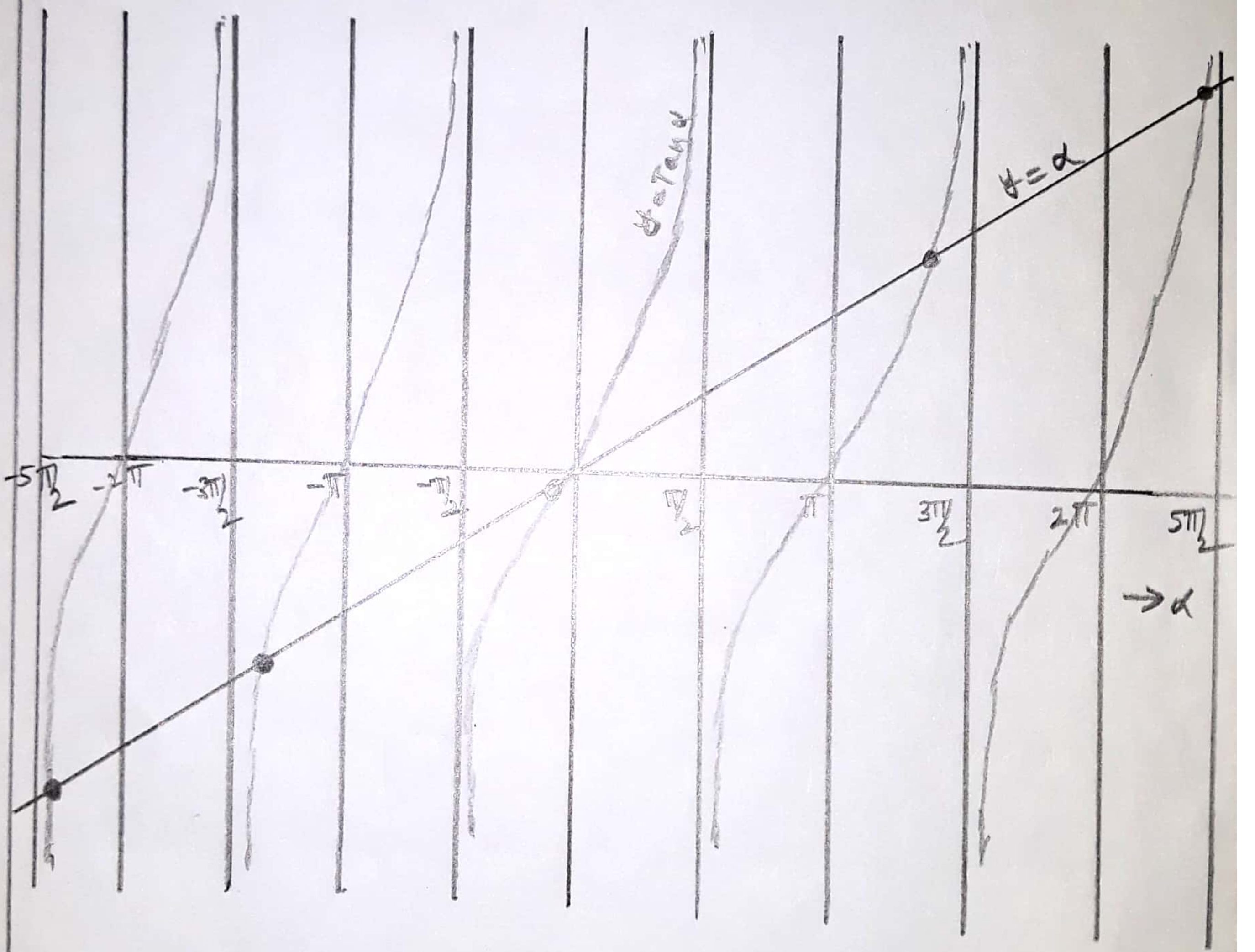
$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\boxed{\alpha = \tan\alpha} \quad \text{———} \quad 9$$

The above eq. is called transcendental equation.

The roots (or) solutions (or) the values of α satisfying the above equation are obtained graphically.

→ If we draw the graph between $y = \alpha$ and $y = \tan\alpha$, then the points of intersection of these two curves are



y = Tan α
y = α
-5π/2
-2π
-3π/2
-π
-π/2
0
π/2
π
3π/2
2π
5π/2
→ α

the solutions of eq. 9.

→ From the graph, the points of interaction are $\alpha = 3\pi/2, 5\pi/2, \ldots$

∴ The intensity of first secondary maxima is $I_1 = I_0\left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2$

$$= \frac{4 I_0}{9\pi^2} = 0.04\, I_0 \quad \text{—— (10)}$$

i.e., I_1 has 4% of I_0.

The intensity of second secondary maxima is $I_2 = I_0\left(\frac{\sin \frac{5\pi}{2}}{5\pi/2}\right)^2$

$$= \frac{4}{25\pi^2} I_0 = 0.016\, I_0$$

$$= 1.6\% \text{ of } I_0 \quad \text{—— (11)}$$

The intensity of third Secondary Maximum

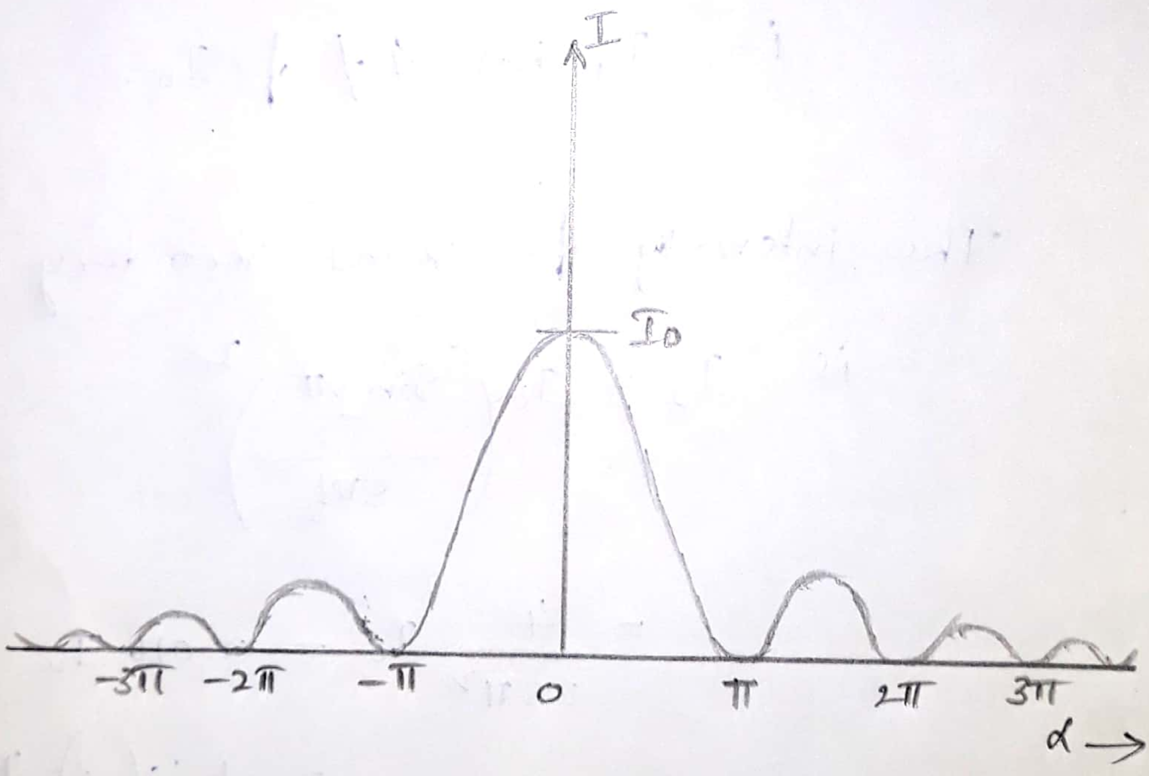
is $I_3 = I_0 \left(\frac{\sin 7\pi/2}{7\pi/2} \right)^2$

$= \frac{4}{49\pi^2} I_0 = 0.0083\, I_0$

$= 0.8\%$ of I_0.

—x—

Intensity Distribution Curve:



Fraunhofer Diffraction due to Double Slit

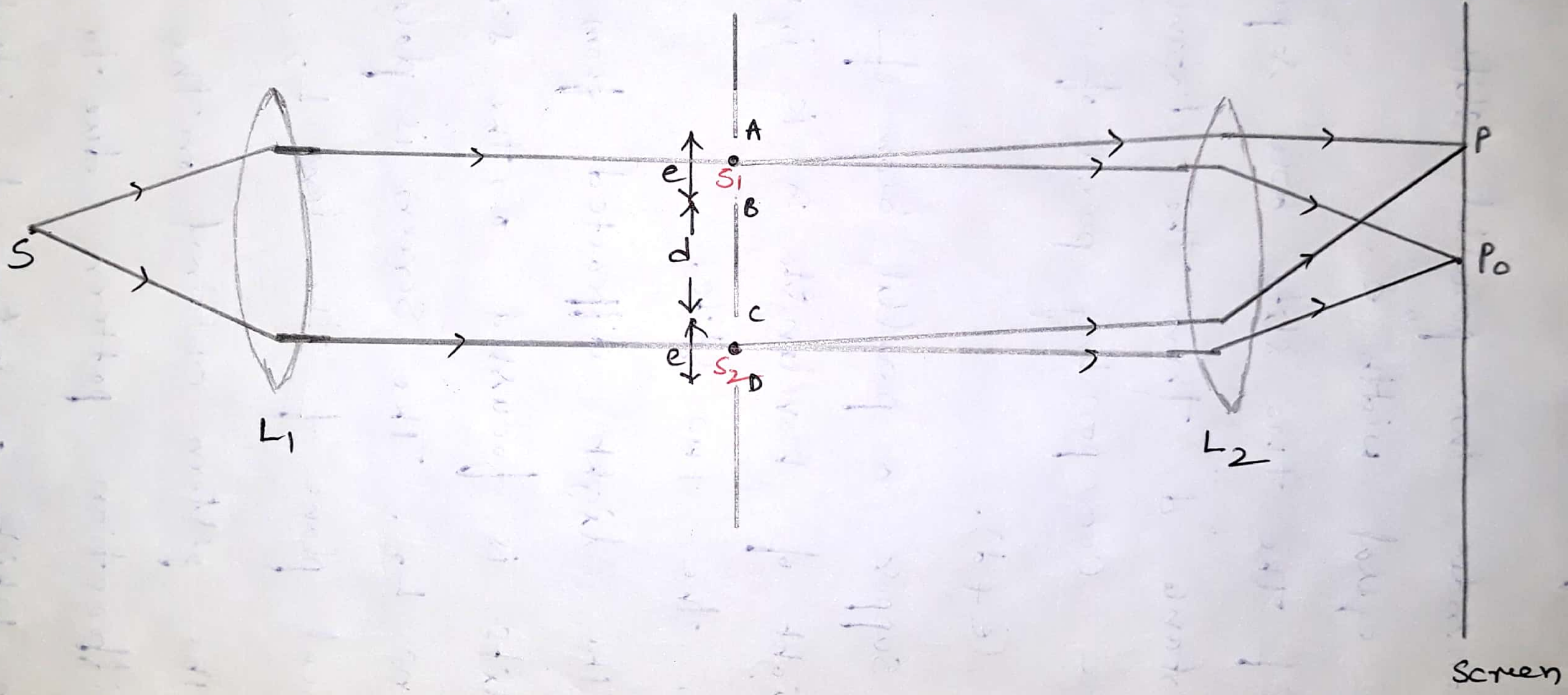
→ Consider two parallel slits AB and CD of equal width "e".

→ If the two slits are separated by a distance "d", then the distance between the corresponding points of the two slits is (e+d).

→ Suppose a parallel beam of monochromatic light of wavelength λ is incident normally on the two slits.

→ The light diffracted from these two slits is focussed on the screen by a lens L_2. The screen is placed at the focal plane of the lens L_2.

→ The pattern obtained on the screen is the diffraction pattern due to a single slit on which a system of interference fringes are superimposed.



S
L_1
A
S_1
B
C
S_2
D
e
d
e
L_2
P
P_0
Screen

→ From the theory of diffraction at a single slit, the resultant amplitude due to secondary waves diffracted from each slit in a direction "θ" is

$$R = \frac{A \sin\alpha}{\alpha} \quad \text{—(1)} \quad \text{where } \alpha = \frac{\pi e \sin\theta}{\lambda}$$

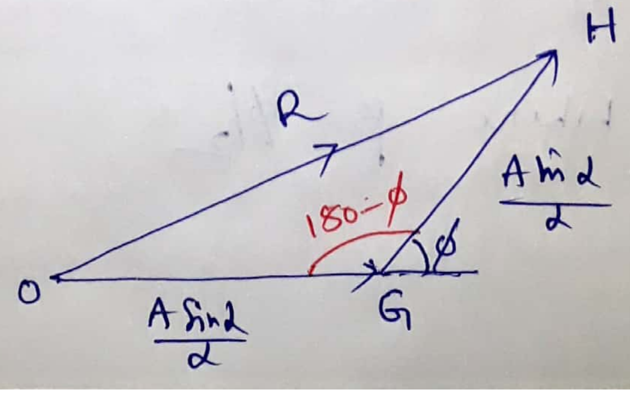
→ Let S_1 and S_2 be the mid points of two slits and each source send a wave of amplitude $\left(\frac{A \sin\alpha}{\alpha}\right)$ in a direction "θ".

→ The path difference between the waves from S_1 and S_2 in the direction θ is

$$\Delta = (e+d)\sin\theta \quad \text{—— (2)}$$

→ The corresponding phase difference is

$$\phi = \frac{2\pi}{\lambda}(e+d)\sin\theta \quad \text{—— (3)}$$



From the cosine rule for the triangle OGH,

$$OH^2 = OG^2 + GH^2 - 2 \times OG \times GH \times \cos(180 - \phi)$$

$$\Rightarrow R^2 = \left(\frac{A \sin\alpha}{\alpha}\right)^2 + \left(\frac{A \sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos\phi$$

$$= \left(\frac{A \sin\alpha}{\alpha}\right)^2 (1 + \cos\phi).$$

$\therefore$ Resultant Intensity $I = R^2$.

$$= \left(\frac{A \sin\alpha}{\alpha}\right)^2 \left(4 \cos^2 \phi/2\right)$$

$$I = 4 \left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2 \phi/2$$

$$\boxed{I = 4 \left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2 \beta} \quad \text{—— } (4)$$

where $\beta = \phi/2$

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From eq (4), the resultant intensity depends on two factors.

(1) $\left(A \frac{\sin\alpha}{\alpha}\right)^2$ gives the diffraction due to single slit

(2) $\cos^2\beta$ gives the interference pattern due to waves of same amplitude from the two 2 slits.

Conditions Corresponds to Diffraction

(1) **Central Maximum :**

The diffraction central maximum observed in the direction $\theta = 0$ i.e, $\boxed{\alpha = 0}$

(2) **Minima :**

The diffraction minima are observed if $\boxed{\alpha = \pm m\pi}$ $m = 1, 2, 3, \ldots$
$m \neq 0$

$e \sin\theta = \pm m\lambda$

(3) Secondary Maxima :

The secondary Maxima are obtained in the direction $\alpha = (2n+1)\pi/2$

$n = 1, 2, \ldots$

$\alpha = 3\pi/2, 5\pi/2, 7\pi/2, \ldots$

Conditions Corresponds to interference

1. **Maxima :** $\cos^2\beta = 1$

The bright fringes are obtained in the direction $\beta = \pm n\pi$

But $\beta = \dfrac{\pi(e+d)\sin\theta}{\lambda}$.

$$\therefore \frac{\cancel{\pi}(e+d)\sin\theta}{\lambda} = \pm n\cancel{\pi}$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda}$$

where $n = 0, 1, 2, \ldots$

$n=0 \longrightarrow$ Zero order Maxima

$n=1 \longrightarrow$ First order Maxima.

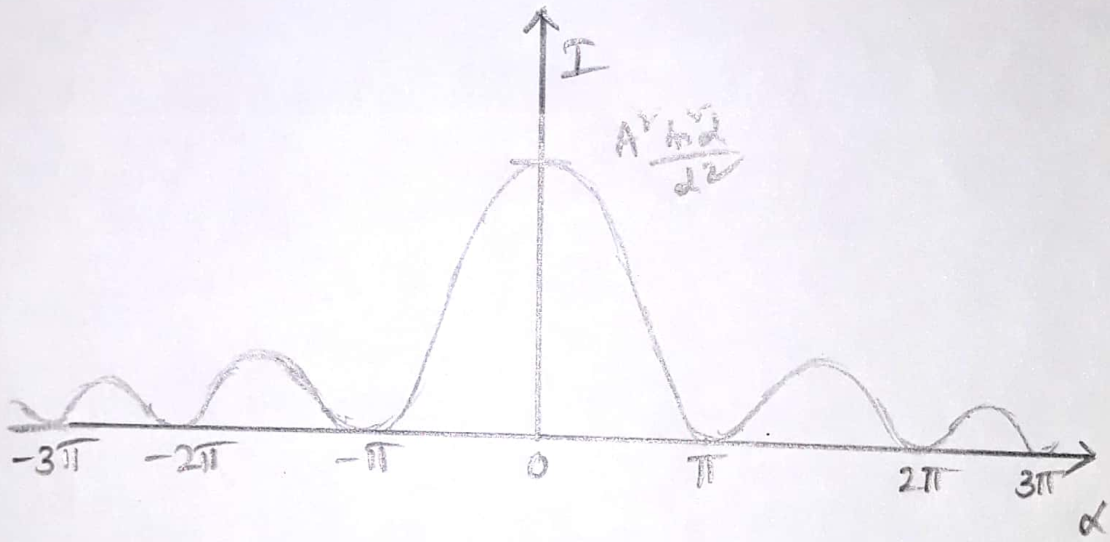
2. Minima: $\cos^2\beta = 0$

The dark fringes are obtained in the direction $\beta = (2m+1)\pi/2$

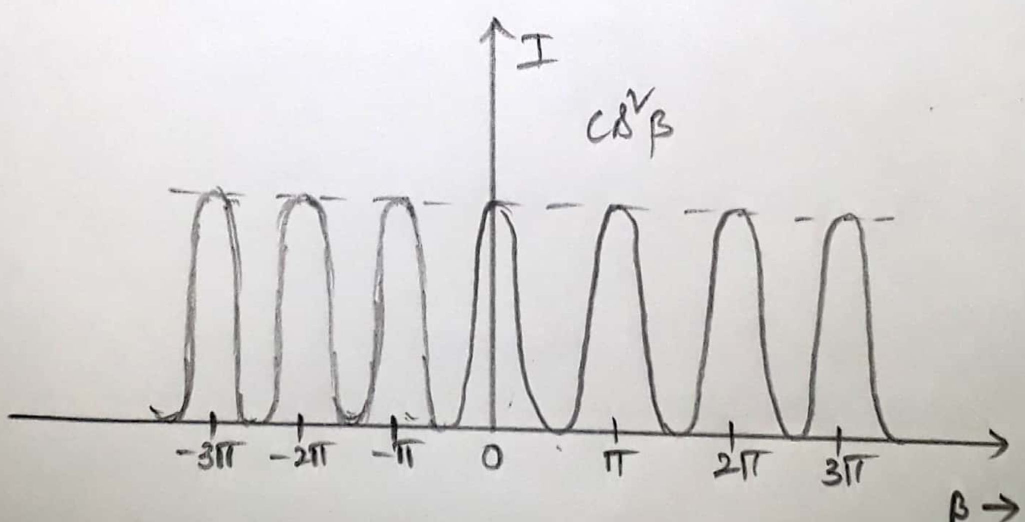
$m = 0, 1, 2, \ldots$

$= \pi/2, 3\pi/2, \ldots$

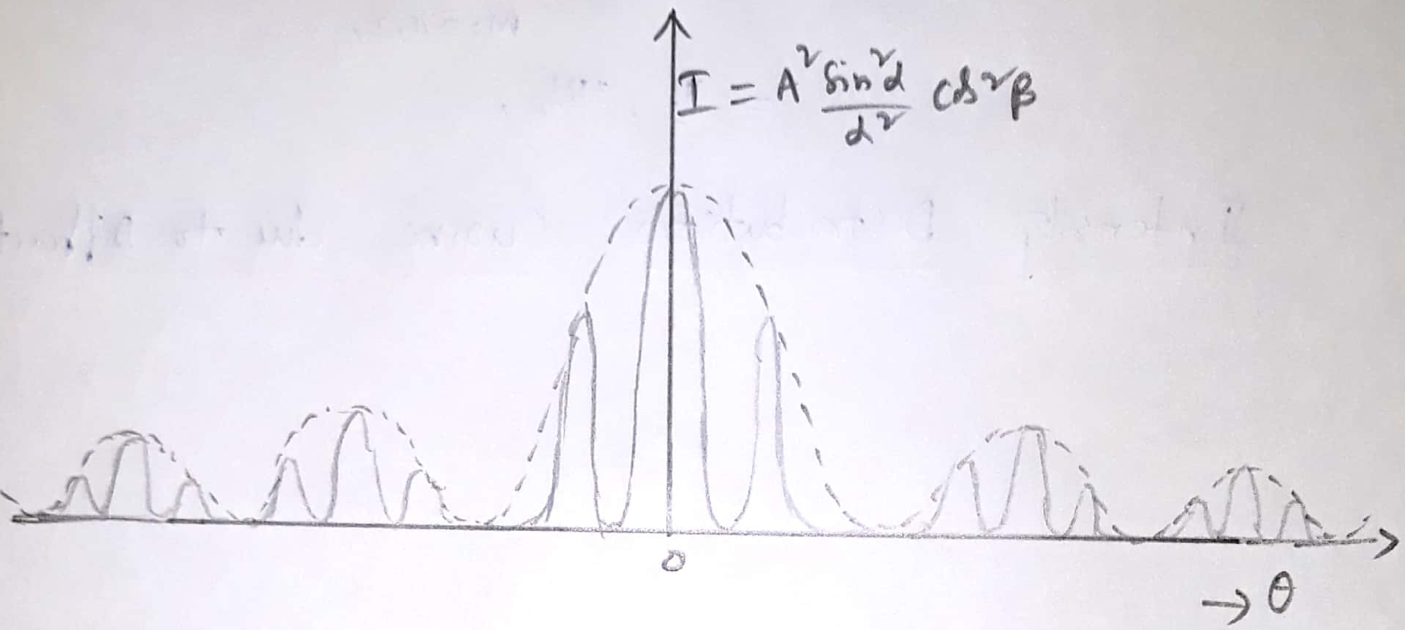
Intensity Distribution Curve due to Diffraction



Intensity Distribution Curve due to interference



Resultant Intensity Distribution Curve due to interference and Diffraction.



Fraunhofer Diffraction due to N-slits

→ **Diffracting Grating :**

"An arrangement consisting of a large number of equidistant parallel slits on a plane glass plate is called a diffraction grating."

→ It can be made by drawing a large number of equidistant and parallel lines on an optically plane glass plate with the help of a sharp diamond point. The rulings are opaque and space between the rulings act as slits.

Grating

- plane Reflection Grating
- plane Transmission Grating

Reflection Grating : A reflection Grating Consists of a Series of fine parallel grooves on a flat Metallic Surface.

Transmission Grating : A transmission Grating Consists of a Series of parallel rulings drawn by a diamond point on a glass plate.

→ If the distance between two Consecutive lines is "d" and the width of the each slit (spacing between the two consecutive lines) is "e", then (e+d) is called grating element.

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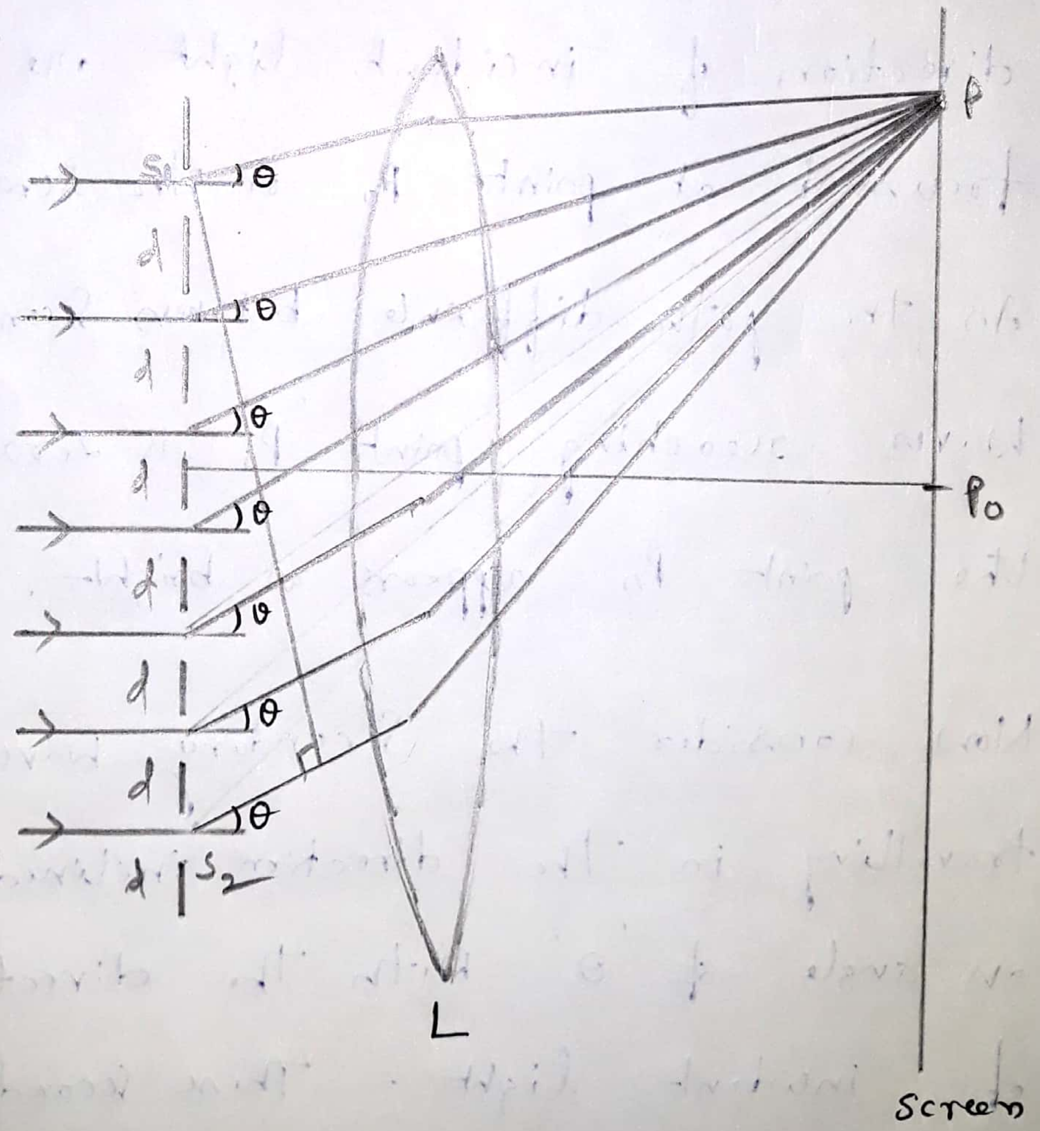
→ Let us discuss the Diffraction due to N- slits.

→ Consider a section AB of the grating. Let (e+d) be the grating element. Any points on successive slits separated by a distance (e+d) are called Corresponding points.



→ Suppose a plane wave of monochromatic light of wavelength "λ" is incident normally on the grating.

→ According to Huygen's theory, each point in all slits sends secondary waves in all directions.

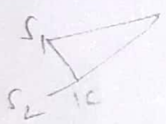
→ The secondary waves travelling in the direction of incident light are focussed at point P_0 on the screen. As the path difference between secondary waves reaching point 'P_0' is zero, the point P_0 appears as bright.

→ Now consider the secondary waves travelling in the direction inclined at an angle of 'θ' with the direction of incident light. These secondary waves are focussed at the point P.

→ According to the theory of fraunhofer Diffraction at a single slit, the secondary waves from all points in a slit diffracted in a direction "θ" is equivalent to a single wave of amplitude $\frac{A \sin\alpha}{\alpha}$.

Where $\alpha = \frac{\pi e \sin\theta}{\lambda}$.

→ Path difference between the rays from the slits S_1 and S_2 is



$$\Delta = S_2K = S_1S_2 \sin\theta$$

$$= (e+d)\sin\theta. \quad \text{—— (1)}$$

The corresponding phase difference is

$$\phi = \frac{2\pi}{\lambda}(e+d)\sin\theta. \quad \text{—— (2)}$$

→ If there are N-slits, then the phase difference between the waves from two consecutive slits in the direction "θ" is

$$d = \frac{2\pi}{\lambda} \cdot \frac{(e+d)\sin\theta}{N} \quad — (3)$$

→ The Resultant amplitude in the direction θ is obtained from

$$R = \frac{A' \sin \frac{Nd}{2}}{\sin d/2} \quad — (4)$$

Where A' = Resultant amplitude from each slit.

→ ∴ The Resultant Amplitude

$$R = \frac{A \sin \alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin \beta}$$

Resultant Intensity $I = R^2$

$$\boxed{I = \frac{A^2 \sin^2 \alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2 \beta}} \quad — (5)$$

Where $\beta = \frac{\pi (e+d) \sin\theta}{\lambda}$

From eq ⑤, the resultant pattern is due to diffraction at single slit and interference between waves from N-slits.

In the eq ⑤,

① $\frac{A^2 \sin^2 \alpha}{\alpha^2}$ gives diffraction pattern due to a single slit.

② $\frac{\sin^2 N\beta}{\sin^2 \beta}$ gives the interference pattern due to N-slits.

Principal Maxima :

$$I = \frac{A^2 \sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

The intensity maxima are observed in the directions given by

$$\sin \beta = 0.$$

$$\boxed{\beta = \pm n\pi}$$

But $\beta = \frac{\pi(e+d)\sin\theta}{\lambda}$

$$\therefore \frac{\cancel{\pi}(e+d)\sin\theta}{\lambda} = \pm n\cancel{\pi}$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda} \quad \text{--- (6)}$$

where $n = 0, 1, 2, \ldots\ldots$

Eq (6) gives the positions of intensity maxima. These maxima are most intense and are called "principal maxima".

$n = 0 \rightarrow$ Zero order maximum

$n = 1 \rightarrow$ First order principal maximum

$n = 2 \rightarrow$ Second order principal maximum

⋮

$\rightarrow$ Eq (6) is called "grating equation".

Minima :

The intensity minima are observed in the directions given by

$$\sin N\beta = 0$$

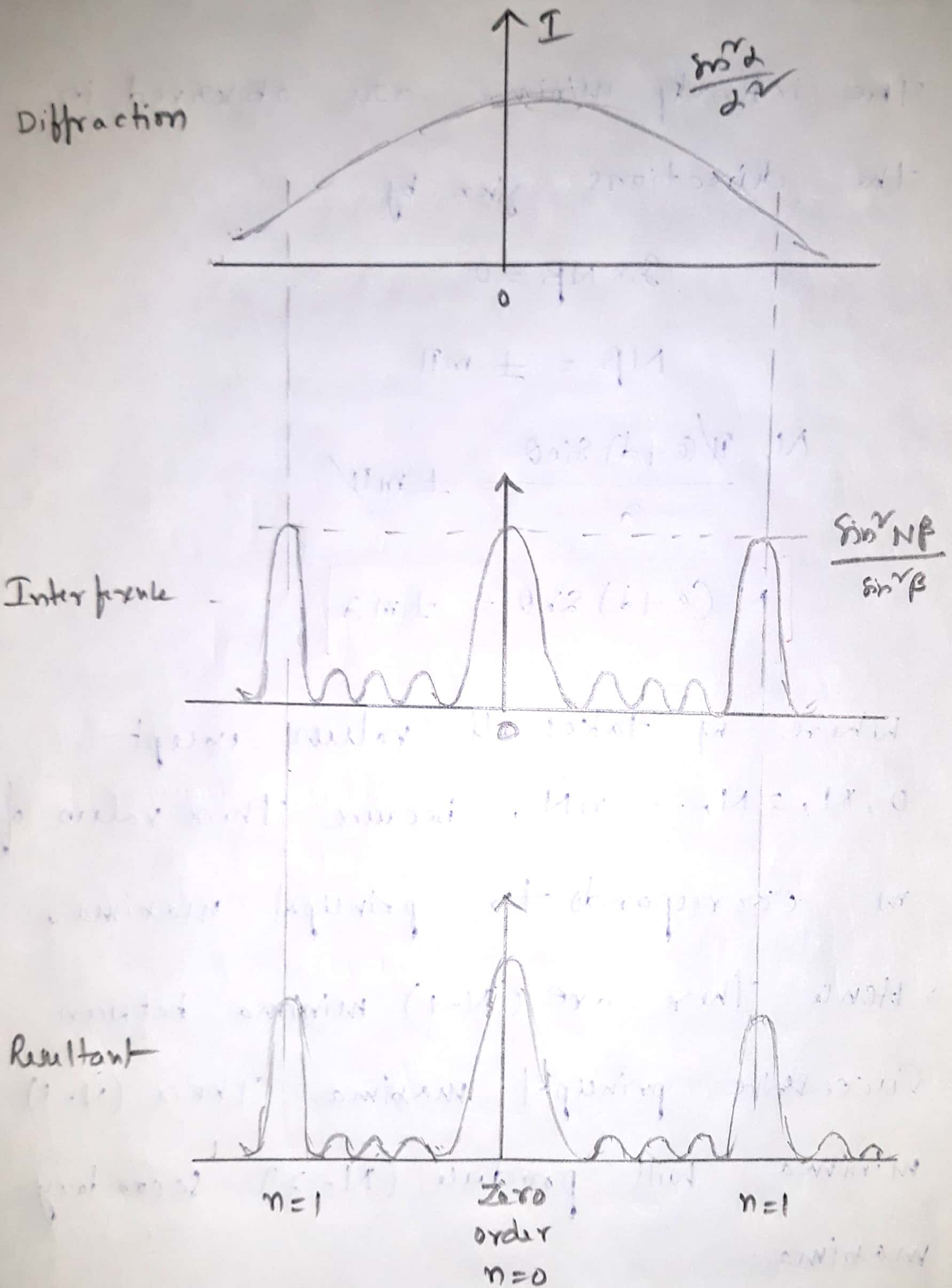
$$N\beta = \pm m\pi$$

$$N \cdot \frac{\cancel{\pi}(e+d)\sin\theta}{\lambda} = \pm m\cancel{\pi}$$

$$\boxed{N(e+d)\sin\theta = \pm m\lambda}$$

where m takes all values except 0, N, 2N, ... nN, because these values of m corresponds to principal maxima.

→ Hence there are (N−1) minima between successive principal maxima. These (N−1) minima will produce (N−2) secondary maxima.



I
$\frac{\sin^2\alpha}{\alpha^2}$
Diffraction
0
$\frac{\sin^2 N\beta}{\sin^2\beta}$
Interference
0
Resultant
n=1
Zero order n=0
n=1

Missing orders :

In the resultant pattern, certain interference maxima will be absent because they coincide with diffraction minima.

→ The principal maxima in the Grating Spectrum are obtained in the directions given by

$$(e+d)\sin\theta = n\lambda \quad \text{—} (1)$$

$n = 0, 1, 2, \ldots$

Where $(e+d)$ is the grating element and "n" is the order of Diffraction.

→ The Diffraction minima are obtained in the directions given by

$$e\sin\theta = m\lambda \quad \text{—} (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{e+d}{e} = \frac{n}{m}.$$

If $d = e$, then $n = 2m$

$= 2, 4, 6, \ldots$

(for $m = 1, 2, \ldots$)

i.e., the second order, fourth order, sixth order ...... interference maxima will be absent because they will coincide with first order, second order, third order ..... diffraction minima, respectively.

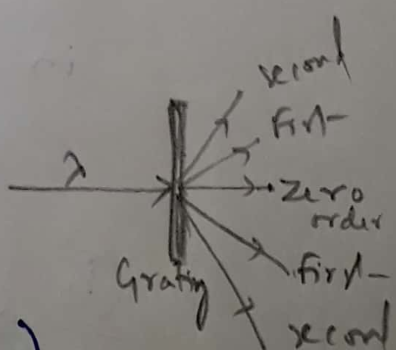
— x —

Maximum number of orders possible with a Grating:

We know that the positions of the principal maxima are given by the equation

$$\boxed{(e+d)\sin\theta = n\lambda} \qquad n = 0, 1, 2, \ldots$$

The Max. angle of Diffraction is 90°.



$$\therefore \quad n_{max} \leq \frac{(e+d)}{\lambda}$$

$$\boxed{n_{max} \leq \frac{1}{N\lambda}} \quad \left(\because \frac{1}{(e+d)} = N\right)$$

Resolving Power

"The ability of an optical instrument to produce two separate images of two closely spaced objects is called its resolving power".

Resolving Power of a Grating

→ The ability of a diffraction grating to form separate maxima of two wavelengths which are very close to each other is called the resolving power of a grating.

→ If $d\lambda$ is the difference in two wavelengths which are just resolvable by grating and λ is the mean wavelength, then the resolving power of a grating is

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

where n = order of spectrum

N = Number of lines per unit width of the grating

$= \frac{1}{(e+d)}$

where $(e+d)$ is grating element.

Dispersive power of a Grating

→ The Dispersive power of a grating is defined as the ratio of the difference in angles of diffraction of any two neighbouring spectral lines to the difference in the wavelengths between the two spectral lines.

$\therefore$ Dispersive power $= \frac{d\theta}{d\lambda}$.

We know that,

Grating eq : $(e+d) \sin\theta = n\lambda$

Differentiate w.r.t. λ, we get

$$(e+d) \cos\theta \cdot \frac{d\theta}{d\lambda} = n$$

$$\therefore \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}}$$

Hence the Dispersive power is

① Directly proportional order of the Spectrum.

② Inversely proportional to grating element (or) Directly proportional to Number of rulings on the grating

③ Inversely proportional to $\cos\theta$.