Polarization

1. Introduction.
2. Types of polarization
3. Polarization by reflection, refraction and double refraction.
4. Nicol's Prism.
5. Half wave and Quarter wave plates

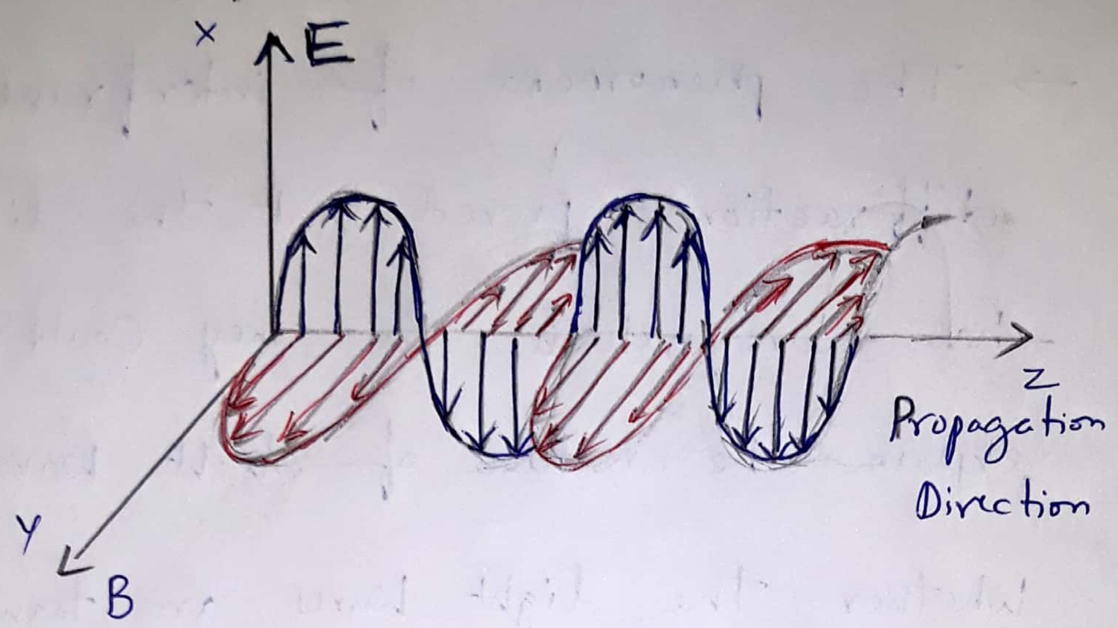
Introduction

→ The phenomena of interferance and diffraction proved that the light has wave nature. But they could not explain the nature of light waves i.e., whether the light waves are transverse (or) Longitudinal.

→ The phenomenon of polarization proved that light waves are transverse waves.

→ According to Maxwell's electromagnetic theory, the light waves are transverse waves.

→ In electromagnetic wave both electric and magnetic fields are oscillating at right angles to each other and to the direction of propagation of EM wave.



→ As the electric vector is responsible for the sensation of vision, usually the vibrations of electric vector are considered as light waves.

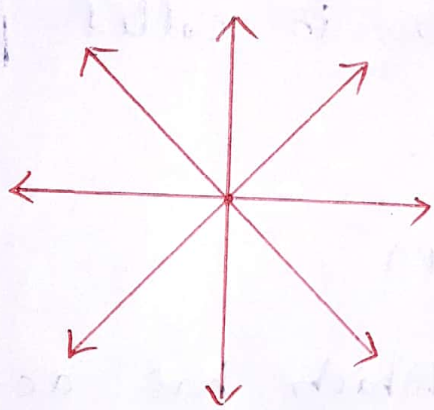
→ The natural light is unpolarized which can be transformed into different types of polarized light using simple optical devices.

→ The state of polarization can't be perceived by an unaided human eye

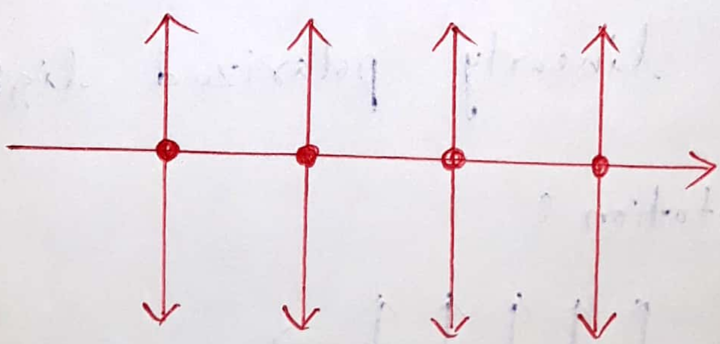
Unpolarized light :

The light in which the electric field vector vibrates in all possible directions perpendicular to the direction of propagation is called Unpolarized light.

Representation :



(OR)



• (Dot) ⟶ It represents vibrations perpendicular to the plane of the paper

$\updownarrow$ (Arrow) → It Represents vibrations along the plane of the paper.

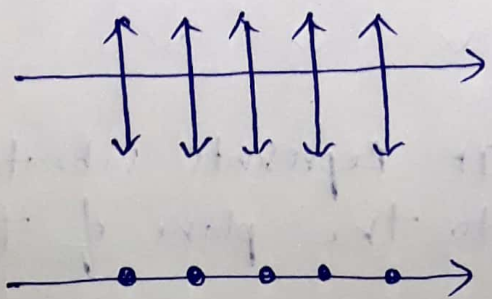
Plane polarized (or) Linearly polarized Light

The light wave in which electric field vector is vibrating along a single direction is called plane polarized light.

(OR)

The light which has acquired the property of one - sidedness is called linearly polarized light.

Representation :

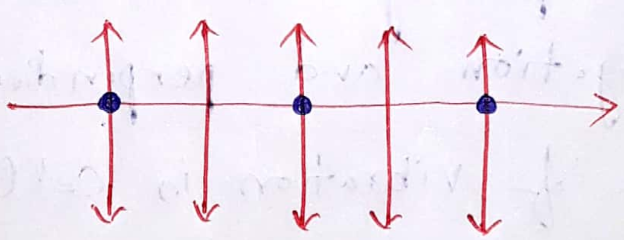


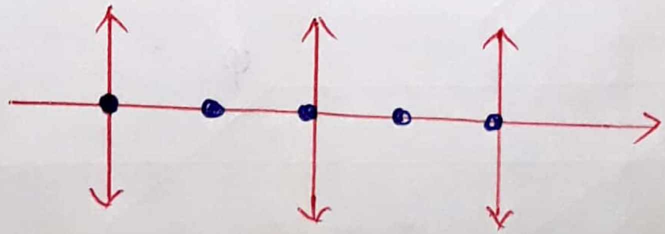
Partially Plane Polarized light

If the linearly polarized light contains small additional component of unpolarized light, then it becomes partially plane polarized light.

Representation :

It is represented by either more arrows and less dots (or) less arrows and more dots. as shown in the fig.





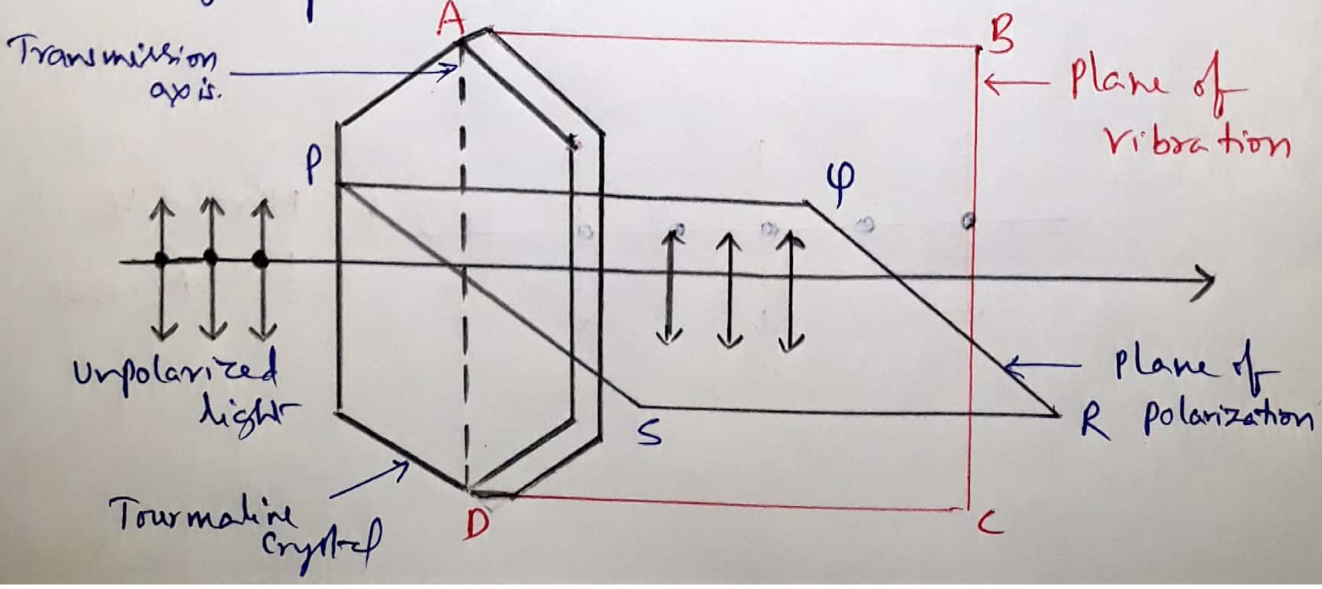
Plane of vibration and Plane of Polarization

Plane of vibration :

The plane containing the direction of vibration of electric vector and the direction of propagation of light is called plane of vibration.

Plane of Polarization :

The plane passing through the direction of propagation and perpendicular to the plane of vibration is called plane of polarization.



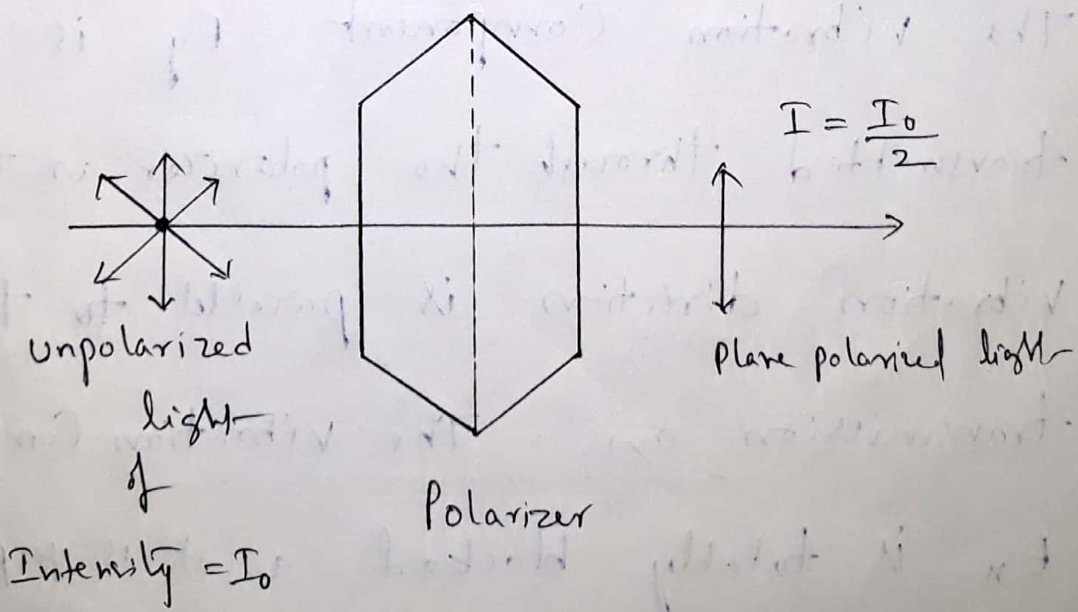
Polarization of light

The process of Converting unpolarized light into polarized light is Called polarization of light.

→ Tourmaline Crystal is used to Convert Unpolarized light into polarized light.

Intensity of polarized light:

→ Suppose an unpolarized light is incident normally on tourmaline Crystal as shown in the fig.



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→ Any vibration of amplitude 'E_0'

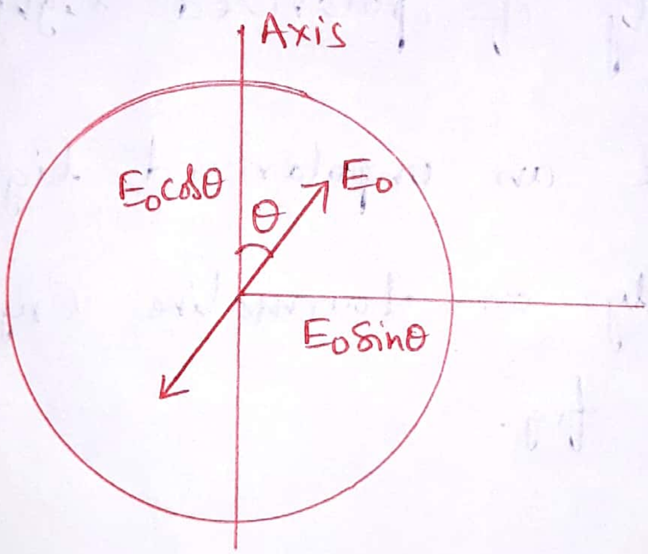
making an angle θ with the axis

of the polarizer may be resolved

into two Components.

$$E_x = E_0 \cos\theta$$

$$E_y = E_0 \sin\theta$$



→ The vibration Component E_y is transmitted through the polarizer as this vibration direction is parallel to the transmission axis. The vibration Component E_x is totally blocked as it is perpendicular to the transmission axis.

→ Hence, the intensity of the transmitted light is given by

$$I \propto E_y^2$$

$$= E_0^2 \cos^2\theta$$

$$= I_0 \cos^2\theta.$$

→ In unpolarized light, all the values of "θ" are equally probable. Therefore, the fraction of light transmitted through the polarizer is equal to the average value of $\cos^2\theta$.

$$\therefore \frac{I}{I_0} = \langle \cos^2\theta \rangle$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{1}{2}$$

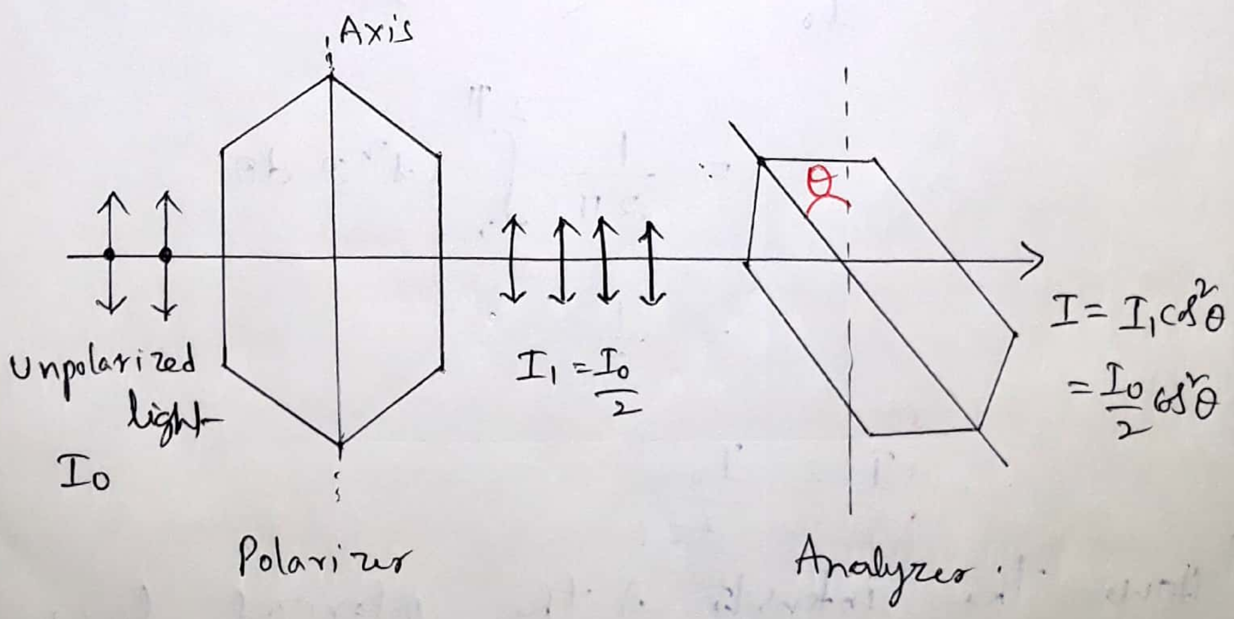
$$\therefore \boxed{I = \frac{I_0}{2}}$$

Hence the intensity of the polarized light is equal to the half of the intensity of the incident unpolarized light.

Malus Law

→ This law was discovered by E.L. Malus in 1809.

→ According to this law, the intensity of the polarized light transmitted through the analyzer varies as the square of cosine of the angle between the planes of transmission of the polarizer and analyzer.



If $\theta = 0°$ (or) $180°$, then $I = I_1$ (Maximum Intensity)

If $\theta = 90°$, then $I = 0$ (No light)

Production of Plane Polarized light -

The plane polarized light can be produced by the following Methods.

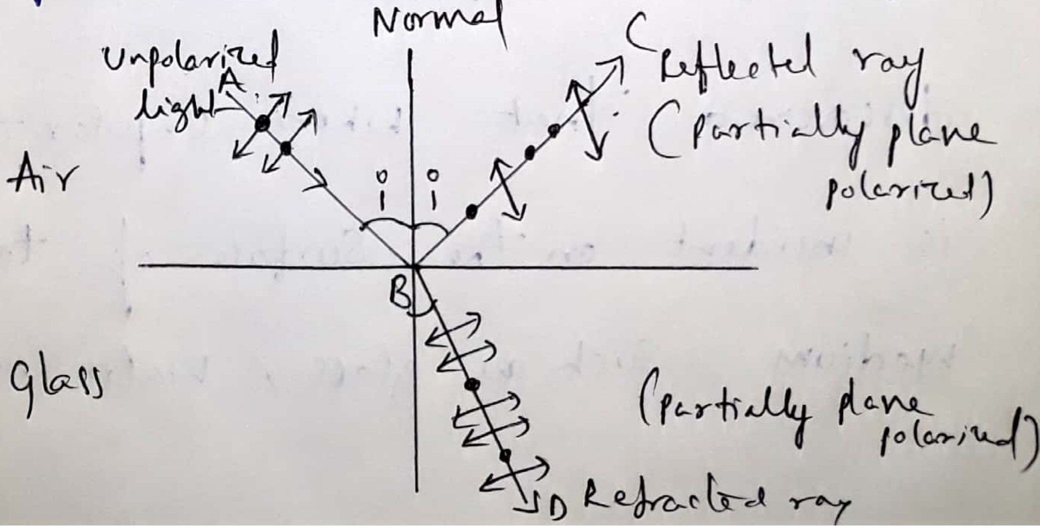
1. Polarization by reflection
2. Polarization by refraction
3. Polarization by double refraction
4. Polarization by Scattering.

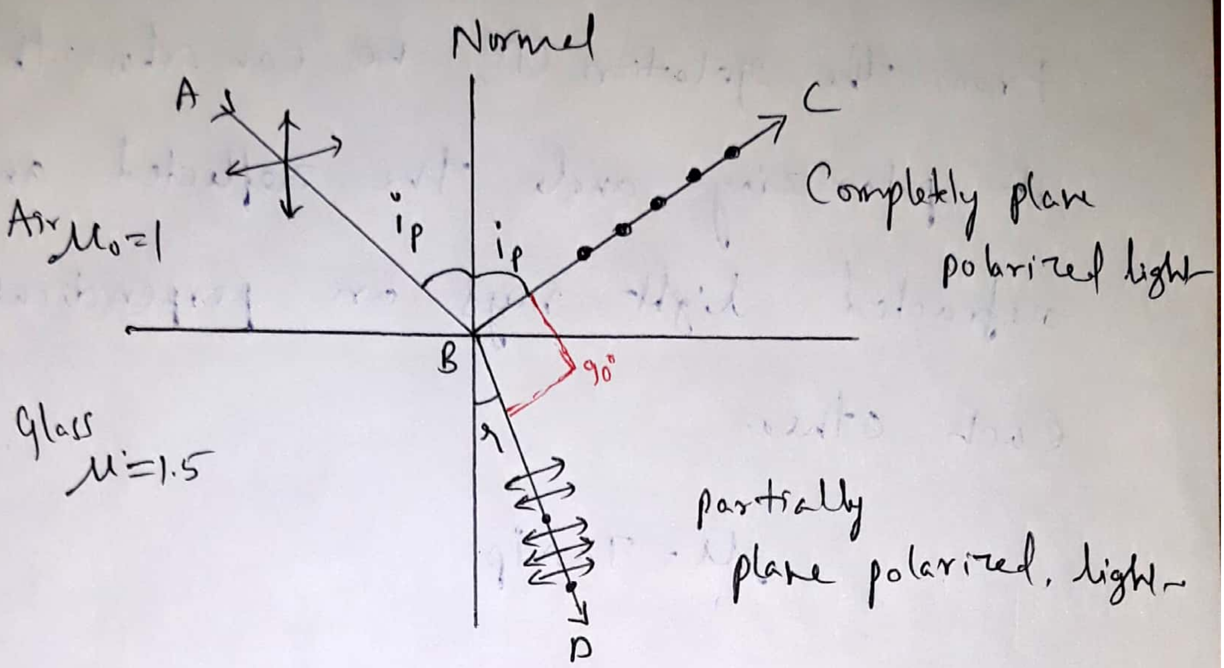
Polarization by reflection :

→ In 1808, a French Scientist E.L. Malus discovered that when unpolarized light is incident on the surface of transparent medium such as glass, water etc., then the reflected light is partially plane polarized. The degree of polarization depends on the angle of Incidence

→ At a particular angle of incidence, the reflected light is Completely plane polarized light with vibrations perpendicular to the plane of Incidence (or) parallel to the plane of reflecting surface

→ This particular angle of incidence at which the reflected light is completely plane polarized light is called angle of polarization (or) Brewster's angle.





Brewster's Law :

→ This law was discovered experimentally by Sir David Brewster.

→ According to Brewster's law, when Unpolarized light is incident on the reflecting Surface (glass, water...etc), then the reflected light is completely plane polarized provided the refractive index of the material of the reflecting Surface is equal to the tangent of the polarizing angle. i.e. $\boxed{\mu = \tan i_p}$ — (1)

From the relation ①, we can show that, at polarizing angle the reflected and refracted light rays are perpendicular to each other.

$$\mu = \tan i_p$$

$$= \frac{\sin i_p}{\cos i_p} \quad \text{--- (2)}$$

From Snell's law,

$$\mu = \frac{\sin i_p}{\sin r} \quad \text{--- (3)}$$

Compare ① & ②, we get

$$\frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r}$$

$$\therefore \sin r = \cos i_p$$

$$= \sin(90 - i_p)$$

$$\Rightarrow r = 90 - i_p$$

$$\boxed{r + i_p = 90^\circ}$$

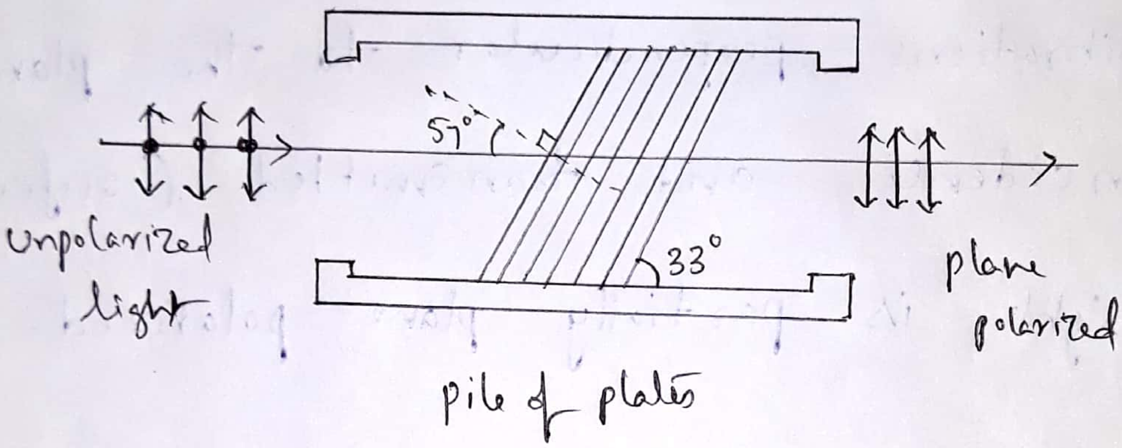
Polarization by refraction

→ If unpolarized light is incident on the smooth surface of a glass slab at polarizing angle, then the reflected light is completely plane polarized with vibrations perpendicular to the plane of incidence and transmitted (refracted) light is partially plane polarized.

→ If the unpolarized light enters a glass tube consists of number of glass plates inclined at an angle of 33° to the axis of the tube, then the successive reflections from each glass plate filters the perpendicular component from the transmitted ray.

→ Hence the light emerging from the glass tube is plane polarized with vibrations parallel to the plane of incidence.

→ Hence pile of plates acts as a polarizer.



Polarization by double refraction

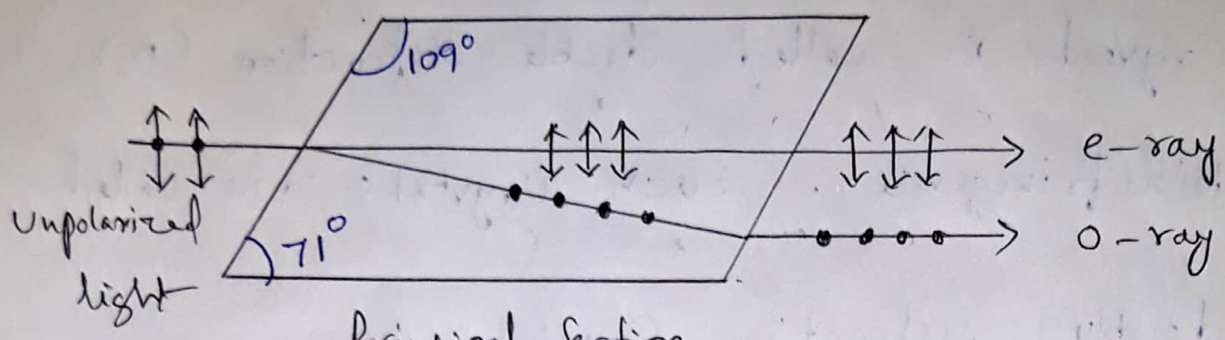
→ The double refraction phenomenon was discovered by Erasmus Bartholinus in 1669.

→ There are certain crystals which splits incident light ray into two refracted rays. This phenomenon of producing two refracted rays by a Crystal is called double refraction (or) birefringence. Such Crystals are called doubly refracting Crystals.

→ Double refraction is exhibited by all optically anisotropic materials.

→ The two refracted rays formed in double refraction are linearly polarized in mutually perpendicular directions.

→ one of the rays obey the Snell's law of refraction and is called ordinary ray (or) o-ray. The other ray does not obey the laws of refraction and is called extraordinary ray (or) e-ray.



Principal Section of Calcite crystal

Birefringent Crystals

Uniaxial

Biaxial

<u>uniaxial crystals:</u>

In these crystals, there is only a single direction known as optic axis along which two refracted rays travel with the same velocity

Ex: Calcite, Quartz, Tourmaline

Biaxial Crystals:

In these Crystals, there are two directions along which the velocity of refracted rays are same and both of the refracted rays are extra ordinary rays.

Ex: Topaz, Sugar, mica.

Uniaxial Crystals

Negative | positive

Negative Uniaxial Crystal:

1. → In negative Crystals, the velocity of e-ray is greater than that of o-ray in all directions except along optic axis. Hence

$\boxed{V_e > V_o}$ → in all directions except along optic axis

$V_e = V_o$ → along optic axis

(2)

→ If μ_o and μ_e are refractive indices of o-ray and e-ray respectively, then

$$\mu_o = \frac{c}{V_o} \quad \text{and} \quad \mu_e = \frac{c}{V_e}$$

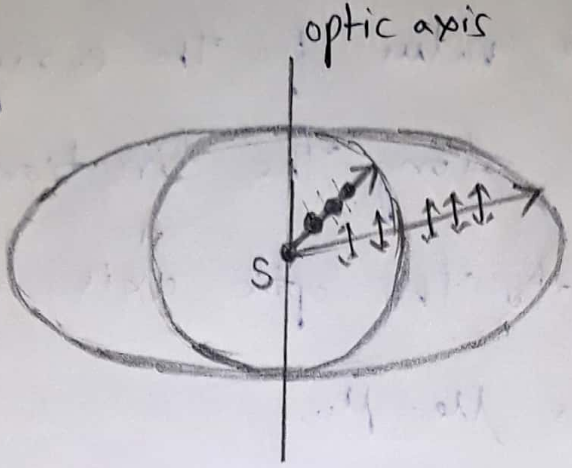
As $V_e > V_o$, the refractive index for e-ray is smaller than that for o-ray.

i.e., $\boxed{\mu_e < \mu_o}$

The e-ray refractive index has a minimum value in the direction perpendicular to the optic axis and along the optic axis it is equal to μ_o.

(3)

→ In negative crystals, the ellipsoid (e-ray wavefront) lies outside the sphere (o-ray wavefront).



④
→ Example for Negative Uniaxial crystal is Calcite

Positive Uniaxial Crystal :

①
→ In positive Crystals, the velocity of the e-ray is everywhere smaller that of o-ray except along the optic axis.

$\boxed{V_e < V_o}$ in ~~all~~ directions except along the optic axis

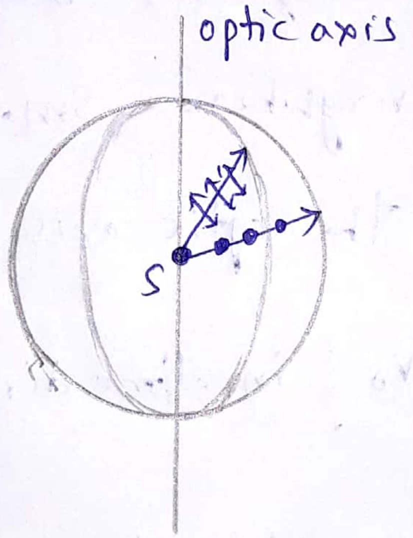
$V_e = V_o$ along the optic axis.

②
→ As $V_e < V_o$, the refractive index for e-ray is larger than that for o-ray

i.e., $\boxed{\mu_e > \mu_o}$

The maximum value for the e-ray refractive index occurs along the direction perpendicular to the optic axis and along the optic axis $\mu_e = \mu_o$.

③ → In positive Crystals, the ellipsoid Corresponding to the e-ray lies inside the sphere Corresponding to the o-ray.



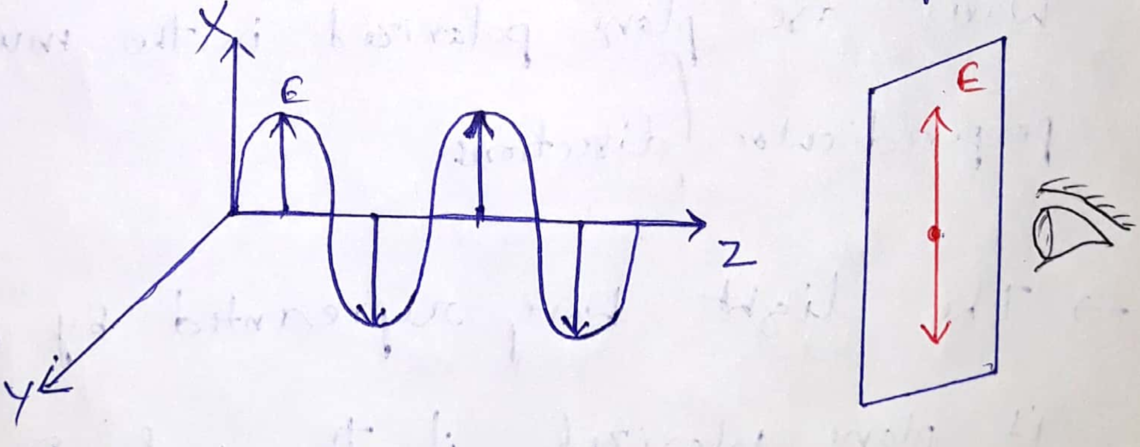
④ → Example: for Uniaxial positive Crystal is Quartz

Types of Polarized light

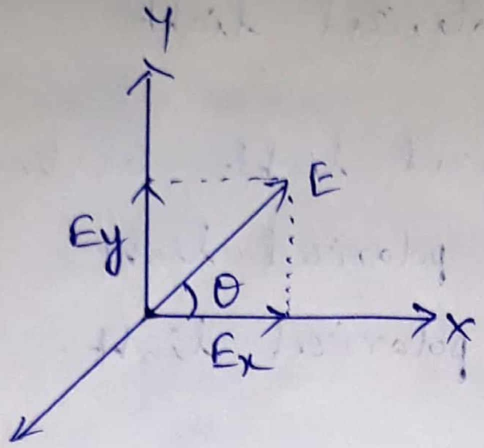
1. Plane polarized light (or) linearly polarized light
2. Elliptically polarized light
3. Circularly polarized light.

Plane polarized light :

As the light wave propagates through any medium, if the electric field vector E is vibrating in a single direction perpendicular to the direction of propagation of light, then the wave said to be plane polarized.



→ Suppose E vector is oriented at angle "θ" to the x-direction. The E vector can be resolved into its rectangular components E_x and E_y.



$$\boxed{\vec{E} = \hat{i}\bar{E}_x + \hat{j}E_y}$$

→ It implies that the original linearly polarized wave may be viewed as a superposition of two coherent waves having zero phase difference. These two coherent waves are plane polarized in two mutually perpendicular directions.

→ The light wave represented by E is plane polarized if the angle θ remains constant in time (or) if one of the components is always zero.

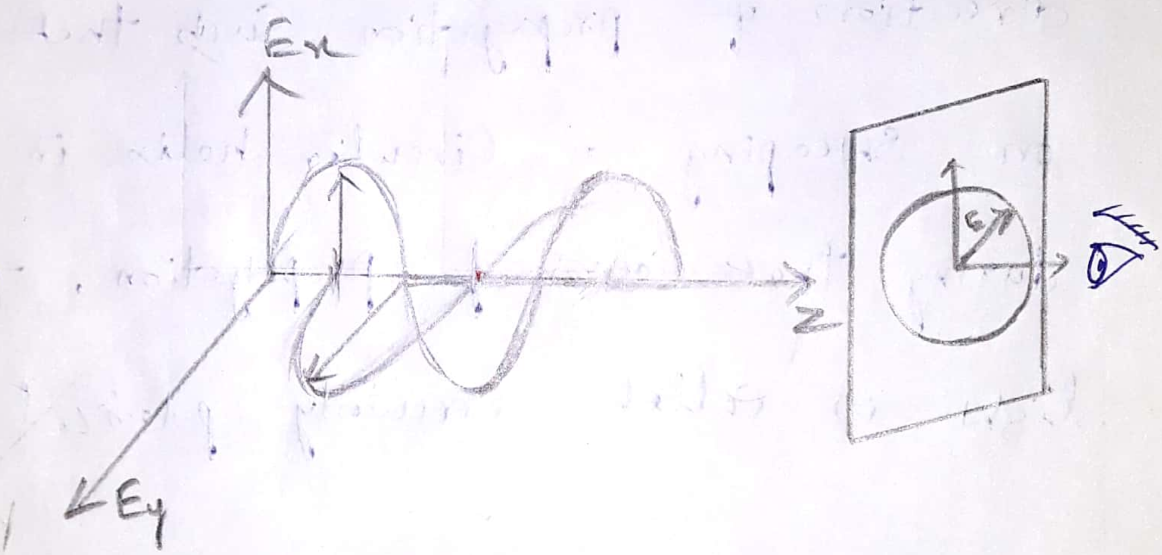
Circularly polarized light:

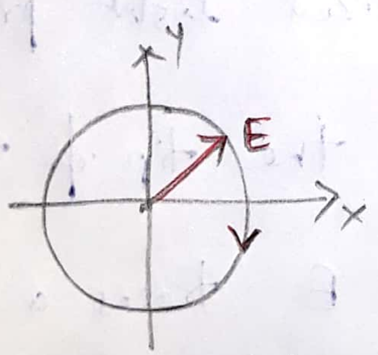
→ If two coherent light waves E_x and E_y are equal in magnitude but differ in phase by $\pi/2$ superpose, then the magnitude of the resultant vector E remains constant but rotates about the direction of propagation such that it goes on sweeping a circular helix in space during the course of propagation. This light is called circularly polarized light.

→ As circularly polarized light propagates through a medium, the tip of the electric field vector E traces a circle on the plane perpendicular to the ray direction.

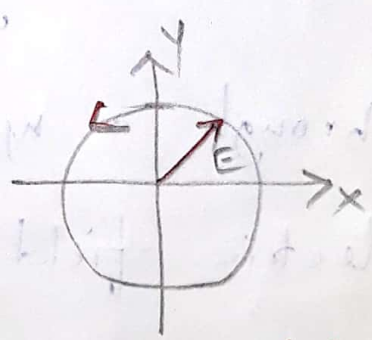
→ If the rotation of the tip is clockwise, then the light is said to be right circularly polarized. (RCPL)

If it rotates anticlockwise, then the light is said to be left circularly polarized (LCPL)





Right circularly polarized light



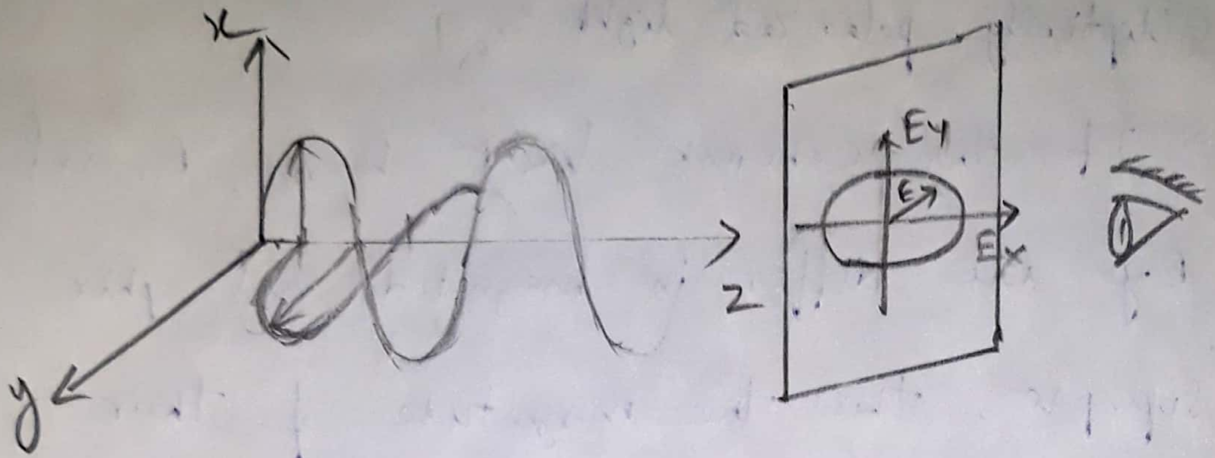
Left circularly polarized light

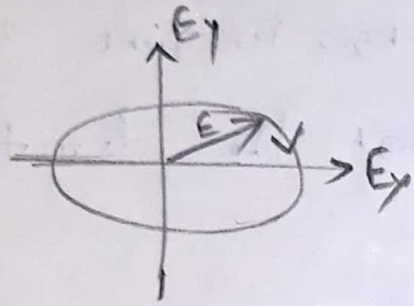
Elliptically polarized light:

→ If two coherent light waves E_x and E_y are differ in magnitude and phase superpose, then the magnitude of their resultant vector E changes in time and the vector E rotates about the direction of propagation.

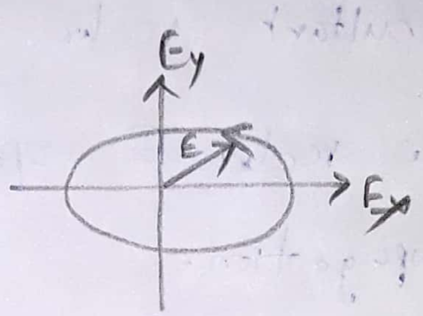
→ The tip of the E vector sweeps a flattened helix in space and traces an ellipse in a plane perpendicular to the direction of the ray. This light is called elliptically polarized light.

→ If the rotation of the E vector is in clockwise, then the light is said to be right elliptically polarized light and if the rotation is in anticlockwise, then light is left elliptically polarized light.





Right Elliptically polarized light



Left elliptically polarized light

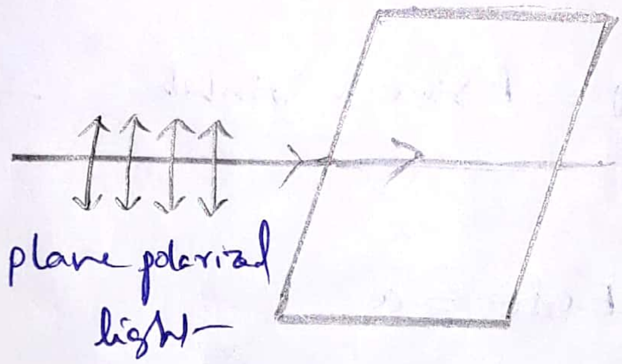
Mathematical Representation :

→ Suppose a plane polarized light is incident normally on a Calcite crystal which is cut with its optic axis parallel to its faces.

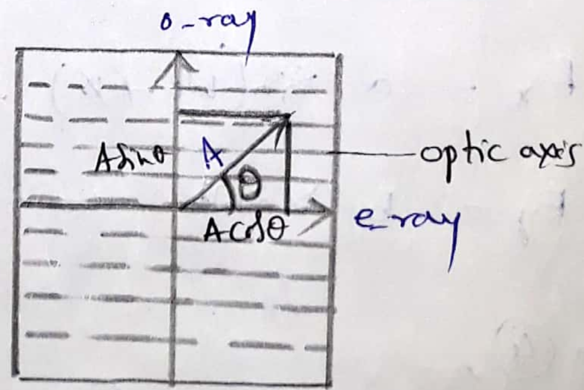
→ Let θ be the angle made by the plane of vibration of polarized light with the optic axis of the crystal.

→ Inside the Calcite Crystal the plane polarized light splits into two Components ordinary and extraordinary.

→ If A is the amplitude of the plane polarized light, then $A\cos\theta$ is the amplitude of the e-ray along the optic axis and $A\sin\theta$ is the amplitude of o-ray in a direction perpendicular to the optic axis.



Calcite crystal



→ As o-ray and e-rays travel with different velocities, they emerge out of the crystal with some phase difference

→ This phase difference ϕ depends on the thickness of the crystal.

→ The o-ray and e-rays can be expressed as

e-ray → $E_x = E\cos\theta \cdot \sin(\omega t + \phi)$ —— ①

o-ray → $E_y = E\sin\theta \cdot \sin\omega t$ —— ②

Let $E\cos\theta = a$

$E\sin\theta = b$

$\therefore$ $E_x = a \sin(\omega t + \phi)$ —— ③

$E_y = b \sin\omega t$ —— ④

from ④,

$$\sin\omega t = \frac{E_y}{b}$$

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From eq. (3),

$$\frac{E_x}{a} = \sin(\omega t + \phi)$$

$$= \sin\omega t \cdot \cos\phi + \cos\omega t \cdot \sin\phi$$

$$= \frac{E_y}{b} \cdot \cos\phi + \sqrt{1 - \sin^2\omega t} \cdot \sin\phi$$

$$= \frac{E_y}{b} \cos\phi + \sqrt{1 - \frac{E_y^2}{b^2}} \cdot \sin\phi$$

$$\Rightarrow \left(\frac{E_x}{a} - \frac{E_y}{b} \cdot \cos\phi\right) = \sqrt{1 - \frac{E_y^2}{b^2}} \cdot \sin\phi$$

Squaring on both sides, we get

$$\left(\frac{E_x}{a} - \frac{E_y}{b} \cdot \cos\phi\right)^2 = \left(1 - \frac{E_y^2}{b^2}\right) \sin^2\phi$$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} \cdot \cos^2\phi - \frac{2E_xE_y}{ab}\cos\phi$$

$$= \sin^2\phi - \frac{E_y^2}{b^2}\sin^2\phi$$

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2}(\sin^2\phi + \cos^2\phi) - \frac{2E_xE_y}{ab}\cos\phi = \sin^2\phi.$$

$$\therefore \quad \boxed{\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab}\cos\phi = \sin^2\phi} \quad -(5)$$

This is a general equation of ellipse

Case (i):

If phase difference $\phi = 0, 2\pi, \ldots$

then $\sin\phi = 0$ and $\cos\phi = 1$

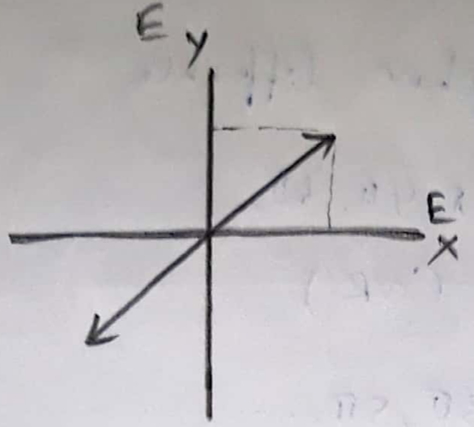
From eq (5),

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} - \frac{2E_xE_y}{ab} = 0$$

$$\left(\frac{E_x}{a} - \frac{E_y}{b}\right)^2 = 0$$

$$\boxed{E_y = \left(\frac{b}{a}\right) E_x} \quad -(6)$$

This is an equation of straight line with positive slope.



If $\phi = \pi, 3\pi, \ldots,$ then $\left.\begin{array}{l}\sin\phi = 0 \\ \cos\phi = -1\end{array}\right\}$

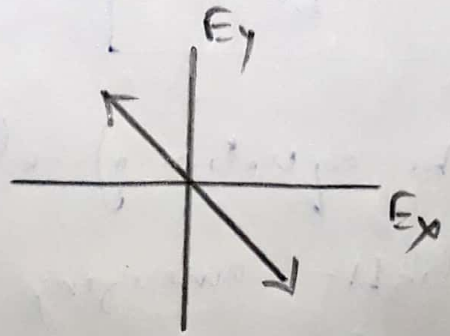
From eq. 5;

$$\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} + \frac{2E_xE_y}{ab} = 0$$

$$\left(\frac{E_x}{a} + \frac{E_y}{b}\right)^2 = 0$$

$$\therefore \boxed{E_y = \left(-\frac{b}{a}\right) E_x} \quad \text{—} (7)$$

This is an equation of straight line with negative slope.



Hence if the phase difference

$$\phi = 0, 2\pi, 4\pi, 6\pi, \ldots$$

(OR)

$$= \pi, 3\pi, 5\pi, \ldots$$

then light emerges from the crystal is plane polarized with vibrations in the same plane as in the incident light.

Case (ii):

If $\phi = (2m+1)\pi/2$

$m = 0, 1, 2, \ldots$

$$= \pi/2, 3\pi/2, 5\pi/2, \ldots$$

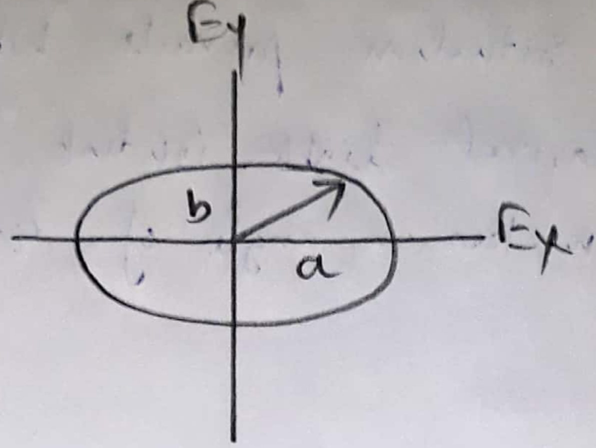
then $\cos\phi = 0$ $\sin\phi = 1$

From eq (5),

$$\boxed{\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1} \quad \text{--- (8)}$$

This is the equation of ellipse.

Hence the light emerging out of the crystal is elliptically polarized light.



Case(iii):

If $\delta = (2m+1)\pi/2$ and $a = b$,

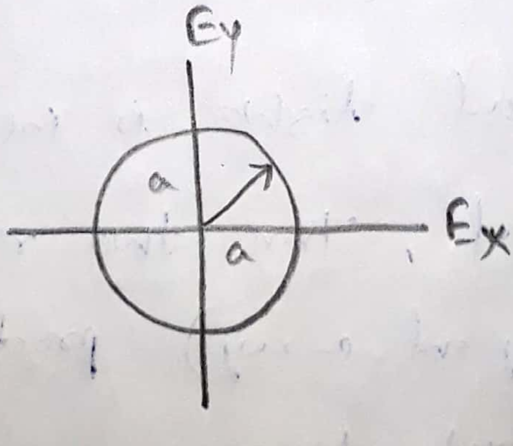
$m = 0, 1, 2, \ldots$

then from eq. 5,

$$\boxed{E_x^2 + E_y^2 = a^2} \quad \text{—— (9)}$$

This is an equation of a circle.

Here the light emerging out of the Crystal is Circularly polarized light.



→ This situation prevails when the plane polarized light incident on the calcite crystal makes an angle of 45^0 with optic axis

Proof:

$$a = b$$

$$E\cos\theta = E\sin\theta$$

$$\theta = 45^0$$

NICOL PRISM

→ Nicol prism is an optical device used for producing and analysing plane polarized light. This was invented by William Nicol in 1828.

→ It is made from Calcite crystal.

Principle:

→ If unpolarized light is incident on calcite crystal, then two refracted rays (o-ray and e-ray) produced due to double refraction

→ Nicol prism eliminates o-ray by total internal reflection and only e-ray is transmitted through the prism.

Construction:

→ A Calcite crystal whose length is three times its width is taken.

→ The end faces of crystal are grounded in such a way that the angles in the principal section becomes 68° and 112° instead of 71° and 109°.

→ Now the crystal is cut into two pieces by a plane (diagonal plane) perpendicular to the principal section and to the new end faces A'B and CD'.

→ After grinding and polishing the cut pieces to obtain optically flat surfaces, they are

Cemented together with Canada balsam.

→ Canada balsam is a transparent glue of refractive index 1.55 for Sodium light.

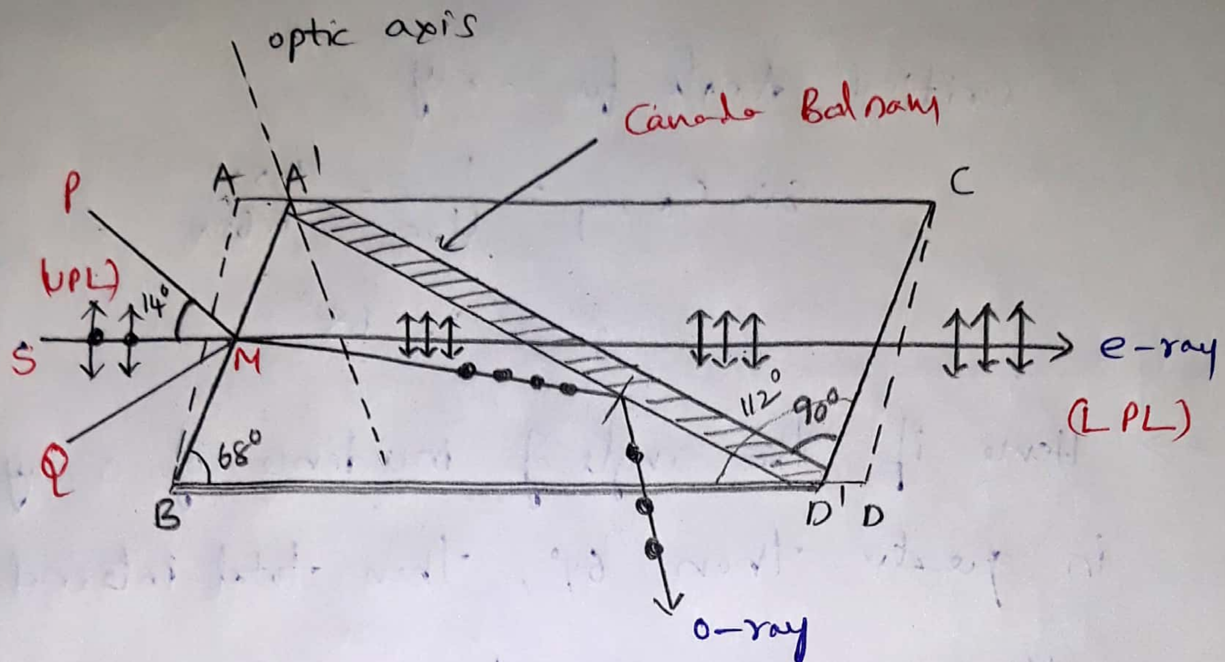
→ Canada balsam is chosen because its refractive index lies between the refractive indices for the o-ray and e-ray for calcite.

Ex: For Sodium D line,

μ_o = Refractive index of o-ray $= 1.658$

μ_c = Refractive index of Canada balsam $= 1.55$

μ_e = Refractive index of e-ray $= 1.486$



Working :

→ When unpolarized light ray which is nearly parallel to BD' incident on the face A'B, it splits up into two refracted rays o-ray and e-ray.

→ The Canada balsam acts as a rarer medium for an o-ray and denser medium for an e-ray.

→ The refractive index for o-ray with respect to that of Canada balsam is given by

$$\mu_{oc} = \frac{1.658}{1.550}$$

∴ critical Angle for o-ray

$$\sin C_{o\text{-}ray} = \frac{1}{\mu_{oc}} = \frac{1.550}{1.658}$$

$$= 68^\circ$$

→ Hence if the angle of incidence of o-ray is greater than 69°, then total internal reflection takes place for o-ray. But the e-ray does not suffer total internal reflection as it is travelling from a rarer to denser medium.

→ The face where the o-ray is incident is black so that the o-ray is completely absorbed and only the e-ray is emerging from the Nicol prism.

→ Hence the Nicol prism is able to convert unpolarized light into plane polarized light (e-ray).

Nicol prism as Polarizer and Analyzer:

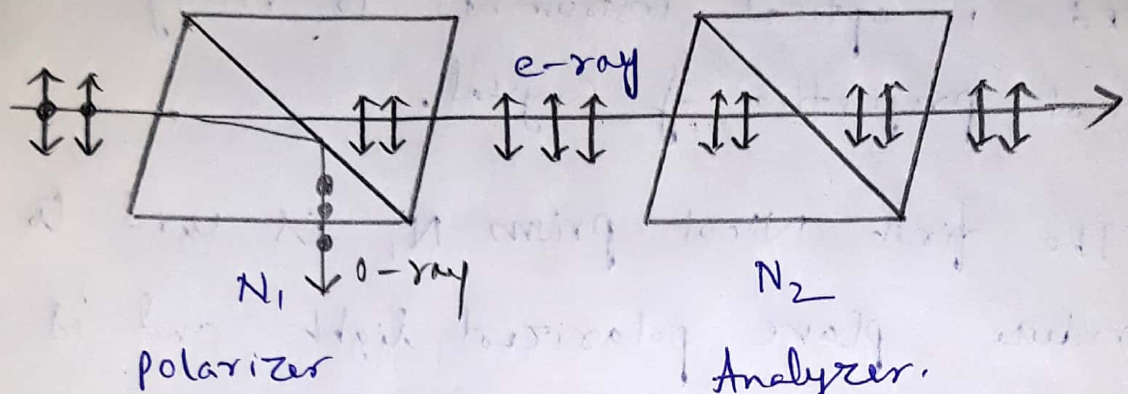
→ The combination of two Nicol prisms is used in optical instruments to study the optical properties of crystal.

→ The first Nicol prism N_1 is used to produce plane polarized light and is called polarizer.

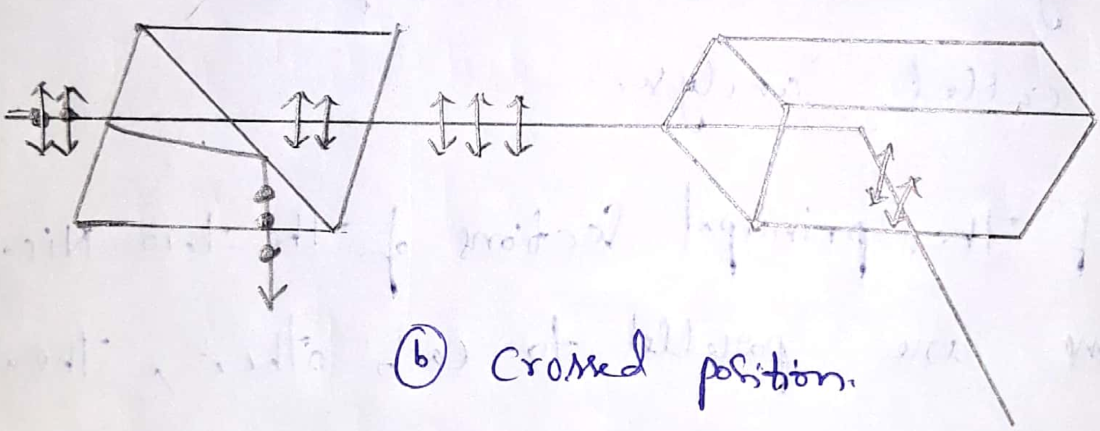
→ The second Nicol prism N_2 is used to analyse the light emerging from N_1 and is called analyser.

→ If the principal sections of the two Nicol prisms are parallel to each other, then the e-ray emerging from N_1 passes through analyzer.

→ If the principal sections of the two Nicol prisms are perpendicular to each other, then the e-ray emerging from N_1 is totally reflected by N_2 and no light is emerging from analyzer.



(a) Parallel position



(b) Crossed position.

Retardation plates

Quarter wave plate | Half wave plate

Quarter wave plate : (QWP)

A quarter wave plate is a thin plate of birefringent crystal having its refracting faces parallel to the direction of optic axis and its thickness adjusted such that it introduces a path difference of $\lambda/4$ (or) phase difference of $\pi/2$ between the o-ray and e-rays. emerging from the crystal.

→ If t is the thickness of the quarter wave plate, then the optical path difference between ordinary and extraordinary rays is

$$\Delta = \mu_o t \sim \mu_e t$$
$$= (\mu_o \sim \mu_e) t$$

For QWP, $\Delta = \lambda/4$

$$\therefore \quad (\mu_o \sim \mu_e)t = \lambda/4$$

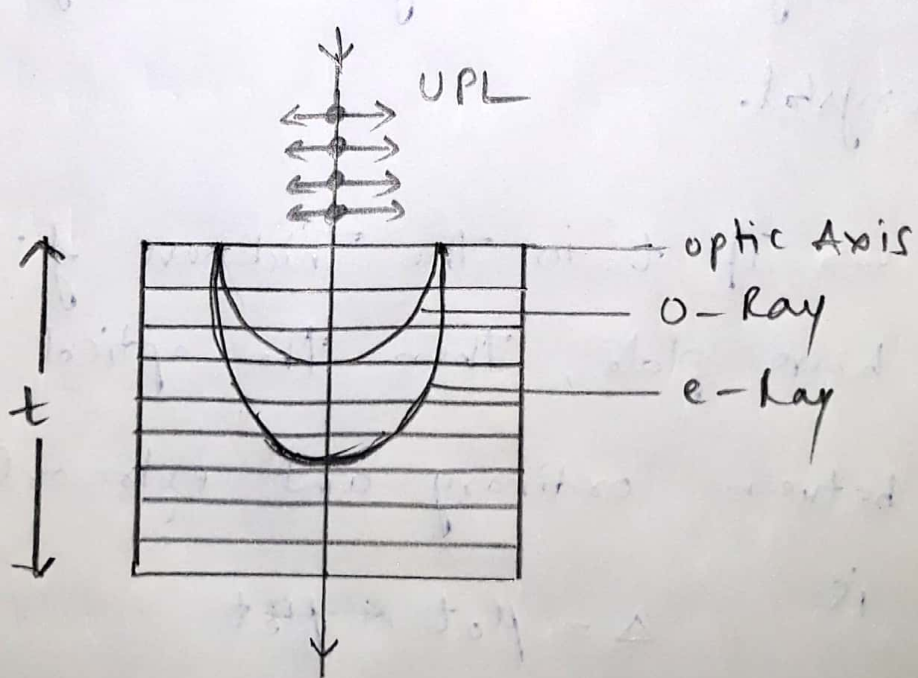
$$\Rightarrow \quad t = \frac{\lambda}{4(\mu_o \sim \mu_e)}$$

For positive crystal, $\mu_e > \mu_o$

$$\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$$

For Negative Crystal, $\mu_o > \mu_e$

$$\boxed{t = \frac{\lambda}{4(\mu_o - \mu_e)}}$$



Half wave plate : (HWP)

A half wave plate is a thin plate of birefringent crystal having its refracting faces parallel to the direction of optic axis and its thickness adjusted such that it introduces a path difference of $\lambda/2$ (or) phase difference of π between o-ray and e-rays emerging from the crystal.

→ The optical path difference between ordinary and extraordinary rays is

$$\Delta = \mu_o t \sim \mu_e t$$

$$= (\mu_o \sim \mu_e) t$$

For HWP, $\Delta = \lambda/2$

$$\therefore (\mu_o \sim \mu_e) t = \lambda/2$$

$$t = \frac{\lambda}{2(\mu_o \sim \mu_e)}$$

For positive Crystal, $\mu_e > \mu_o$

$$\boxed{t = \frac{\lambda}{2(\mu_e - \mu_o)}}$$

For Negative Crystal, $\mu_o > \mu_e$

$$\boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}}$$

→ The action of a quarter wave plate on the plane polarized light is to Convert it into elliptically (or) Circularly polarized light depending upon the angle of incident light vector with the direction of the optic axis of the QWP.

→ The action of a quarter wave plate on Circularly (or) elliptically polarized light waves is to Convert it into plane polarized light.

→ The action of HWP on plane polarized light is to produce plane polarized light with plane of vibration is rotated through an angle of 2θ with respect to the direction of vibration of the incident light.

→ The action of HWP on circularly (or) elliptically polarized light is to change LCP into RCP (or) RCP into LCP.