MODERN THEORY OF GENERALISED RELATIVITY

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Abstract:

We consider a (4+D)–dimensional Friedmann–Robertson–Walker type universe having complex scale factor R + iR_I, where R is the scale factor corresponding to the usual 4–dimensional Universe while iR_I is that of D–dimensional space. It is then compared with (4+D)–dimensional Kaluza–Klein Cosmology having two scale factors R and a(= iR_I). It is shown that the rate of compactification of higher dimension depends on extra dimension 'D'. The Wheeler–DeWitt equation is constructed and general solution is obtained. It is found that for D = 6 (i.e. in 10 dimension), the Wheeler–DeWitt equation is symmetric under the exchange of $R_I \leftrightarrow R$.

I. Introduction:

In 1915 Einstein published the general theory of relativity. He expected the universe to be 'closed' and to be filled with matter. Now, if we go out-side the gravitating sphere, we see the gravitation would be weaker and weaker. According to Einstein's theory of general relativity, the matter-space-time cannot be separated by any cost. Thus, out-side the Einstein's universe, where real time cannot be defined, the corresponding space (although, the matter belongs to another phase) must be measured as imaginary. Thus the space-time of the universe is actually a complex space-time. Here we consider the real space-time (unfolded space-time) for Einstein and imaginary space-time (folded space-time) for us. We found a relation between folded and unfolded space-time of the universe by using Wheeler De-Witt equation. The generalized solution for the Einstein field equations for a homogeneous universe was first presented by Alexander Friedmann. The Friedmann equation for the evolution of the cosmic scale factor R(t) which represents the size of the universe, is

$$\left[\frac{\dot{R}(t)}{R(t)}\right]^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R^2(t)}, \text{ i.e } \quad \dot{R}^2(t) = \frac{8\pi G}{3}\left[\rho(t).R^3(t)\right]\frac{1}{R(t)} - kc^2$$

Differentiating the above equation with respect to time t and since the total matter in a given expanding volume is unchanged, that means $\left[\rho(t).R^3(t)\right] =$ constant. We have,

$$2\dot{R}(t)\ddot{R}(t) = -\frac{8\pi G}{3}\left[\rho(t)R^3(t)\right]\frac{\dot{R}(t)}{R^2(t)}$$

i.e

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3}\rho(t)$$

Since $\ddot{R}$ is always negative, at a finite time in the past R must have been equal to zero. Then, according to these models, the contents of all the galaxies must have once been squeezed together in a small volume where the temperature would have been immensely high. The radiation left over from this fireball must still be around today, although cooled to a much lower temperature due to expansion of the universe. So as the size 'R' of the Einstein matter universe numerically squeezed in a zero volume, then we may assume the Einstein's matter universe is then changed to iR_I in other phase which can be measured classically, because the energy never die at all.

So we may consider the scale factor 'R' of the Einstein universe is only the real part and there may exist an imaginary part (pseudo-space) 'iR_I' and hence the size of the universe is actually written as in the form of R + iR_I.

If we consider the (4 + D)–dimensional Kaluza–Klein cosmology with a Robertson–Walker type metric having two scale factors 'a' and 'R', corresponding to 'D'–dimensional internal so called folded space and 4–dimensional unfolded space, respectively. In the expansion of 'R'– universe, the internal space 'a' decreases. Avoiding 'R'– universe squeezed in a zero volume, we may assume that the physical universe must be changed into another phase of energy group SU(11) as explained in my previous articles by the phase transition system with the help of latent energy group SU(6) [see my articles 2014, 2017].

Thus, it is then compared with (4+D)–dimensional Kaluza–Klein Cosmology having two scale factors with the Einstein's 4–dimensional space of the scale factor R and the D–dimensional internal space 'a' (= iR_I). The knowledge about the quantum state space for the gravity system and gravity matter system are very limited and the definition of the inner-product in quantum state space has not been found. We consider a (4+D)–dimensional Friedmann–Robertson–Walker type universe having complex scale factor R + iR_I, where R is the scale factor corresponding to the usual 4–dimensional Universe while iR_I is that of D–dimensional space. It is shown that the rate of compactification of higher dimension depends on extra dimension 'D'. The Wheeler–DeWitt equation is constructed and general solution is obtained. It is found that for D = 6 (i.e. in 10 dimension), the Wheeler–DeWitt equation is symmetric under the exchange $R_I \leftrightarrow R$.

II. Cosmology with complex scale factor:

Consider the metric in which the space–time is assumed to be of Robertson–Walker type having a complex scale factor $R + iR_I$, where the scale factor R stands for (3 +1) – dimensional space–time and iR_I (= a) is that for the internal space with dimension D. Avoiding the imaginary term, the line element can be expressed in the form

$$ds^2 = -N^2(t)dt^2 + R^2(t)\frac{dx^i dx^j}{\left(1+\frac{kr^2}{4}\right)^2} + a^2(t)\frac{dy^a dy^b}{\left(1+k^{/}\rho^2\right)^2} \qquad (1)$$

where N(t) is the lapse function, $r^2 = x^i x^i$ (i = 1, 2, 3), $\rho^2 = y^a y^a$ (a = 1, 2, 3,......, D) and k, $k^{/} = 0, \pm 1$, for flat, closed or open type of 4–dimensional universe and D–dimensional space. For simplicity, we assume the internal space to be flat i.e. $k^{/} = 0$. The form of energy–momentum tensor is chosen as $T_{AB} = (-\rho, p, p, p, p_D, p_D, \ldots \ldots, p_D)$.............(2). Now, we examine the case for which the pressure along with all the extra–dimensions vanishes, namely, $p_D = 0$. In doing so, we are motivated by the brane world scenarios where the matter is to be confined to the 4–dimensional universe, i.e. auxiliary hyper–space, so that all components of T_{AB} is set to zero but the space–time components and it means no matter escapes through the extra dimensions. We assume the energy–momentum tensor $T_{\mu\nu}$ to be that of an exotic χ fluid with the equation of state

$$p_\chi = \left(\frac{m}{3} - 1\right)\rho_\chi \qquad (3)$$

Where p_χ and ρ_χ are the pressure and density of the fluid, respectively and the parameter m is restricted to the range $0 \le m \le 2$ so that there is violation of strong energy condition and the universe experiences accelerated expansion. The scalar curvature corresponding to the metric (1) has the expression

$$\mathcal{R} = \frac{-6\ddot{R}R_I N R + 6RR_I \dot{N}\dot{R} - 2R^2\dot{R}\dot{R}_I N + 2R^2\dot{N}\dot{R}_I - 2R_I N^3 k + R_I N^3 k^2 r^2 - 6R_I N\dot{R}^2 - 6R\dot{R}\dot{R}_I N}{R^2 N^3 R_I} \qquad (4)$$

Now substituting it into the dimensionally extended Einstein–Hilbert action (without higher dimensional cosmological term) including a matter term indicating the above mentioned exotic fluid the effective Lagrangian becomes

$$L = i^D\left[\frac{1}{2N}R\,R_I^D\dot{R}^2 + \frac{D(D-1)}{12N}R\,R_I^{D-2}\dot{R}_I^2 + \frac{D}{2N}R^2R_I^{D-1}\dot{R}\,\dot{R}_I - \frac{1}{2}kNR\,R_I^D + \frac{1}{6}N\rho_\chi R^3 R_I^D\right] \qquad (5)$$

The continuity equation, by using the contracted Bianchi identity in (4+D) dimensions, namely

$$\nabla_M^{G^{MN}} = \nabla_M^{T^{MN}} = 0$$

Together with the assumption that the matter is confined to (3 + 1) dimensional space–time gives $\nabla_i T^{ij} = 0$

i.e $\dot{\rho}_\chi R + 3(p_\chi + \rho_\chi)\dot{R} = 0$ .... (6). Using the continuity equation (6), the energy density in a closed (k = 1) Friedmann–Robertson–Walker universe is $\rho_\chi(R) = \rho_\chi(R_o)\left(\frac{R_o}{R}\right)^m$............ (7), where R_o is the value of R at an arbitrary reference time t_o.

Again, if we believe that the cosmological term plays an important role in vacuum energy density, then we may the cosmological term as $\Lambda \equiv \rho_\chi(R)$.

i.e
$$L = i^D\left[\frac{1}{2N}R\,R_I^D\dot{R}^2 + \frac{D(D-1)}{12N}R^3R_I^{D-2}\dot{R}_I^2 + \frac{D}{2N}R^2R_I^{D-1}\dot{R}\,\dot{R}_I - \frac{1}{2}N\,R\,R_I^D + \frac{1}{6}N\Lambda R^3 R_I^D\right] \qquad (8)$$

With $$\Lambda(R)=\Lambda(R_o)\left(\frac{R_o}{R}\right)^m \qquad (9)$$

Taking m = 2 and $\Lambda(R)=\frac{3}{R^2}$ we have $$\Lambda(R_o)R_o^2=3 \qquad (10)$$

The lapse function N(t), is an arbitrary function of time due to the fact that Einstein's general relativity is a reparametrization invariant theory. We therefore, take the gauge

$$N(t)=R^3(t)\,R_I^D(t\,)\,i^D \qquad (11)$$

Then Lagrangian (8) becomes

$$L=\frac{1}{2}\frac{\dot{R}^2}{R^2}+\frac{D(D-1)}{12}\frac{\dot{R}_I^2}{R_I^2}+\frac{D}{2}\frac{\dot{R}}{R}\frac{\dot{R}_I}{R_I} \qquad (12)$$

We now define the new variables X = log R, Y = log R_I (13) Then the equation (12) becomes

$$L=\frac{1}{2}\dot{X}^2+\frac{D(D-1)}{12}\dot{Y}^2+\frac{D}{2}\dot{X}\,\dot{Y} \qquad (14)$$

The equation of motion are obtained

$$\ddot{X}+\frac{D}{2}\ddot{Y}=0 \qquad (15)$$

$$\ddot{X}+\frac{D-1}{3}\ddot{Y}=0 \qquad (16)$$

Solving equations (15) and (16) we get,

$$\ddot{X}=0 \;\ldots\ldots\ldots\ldots\ldots\ldots(17) \quad \text{and} \quad \ddot{Y}=0 \qquad (18)$$

The solutions for X and Y from equations (17) and (18) are

$$X=At+\gamma\ , \qquad Y=Bt+\delta$$

$$\therefore \quad R(t)=A'e^{\alpha t} \quad \text{and} \quad R_I(t)=B'e^{\beta t}$$

Where the constants A, B, γ and δ or A', B', α & β should be obtained from the initial conditions. Consider the size of all spatial dimensions be the same at t = 0 and assumed that this size would be the Planck size ℓ_p in accordance with quantum cosmological considerations. So we take $R(0) = R_I(0) = \ell_p$ so that $A' = B' = \ell_p$. Thus $R(t) = \ell_p\, e^{\alpha t}$ and $R_I(t) = \ell_p\, e^{\beta t}$ It is important to note that the constants α, β are not independent and a relation may be obtained between them. This is done by imposing the zero energy condition H = 0 which is the well–known result in cosmology due to the existence of arbitrary laps function N(t) in the theory. The Hamiltonian constraint is obtained through the legender transformation of the Lagrangian (14)

$$H=\frac{1}{2}\dot{X}^2+\frac{D(D-1)}{12}\dot{Y}^2+\frac{D}{2}\dot{X}\dot{Y}=0$$

$$\text{i.e.}\quad H=\frac{1}{2}\alpha^2+\frac{D(D-1)}{12}\beta^2+\frac{D}{2}\alpha\beta=0 \qquad (19)$$

This constraint is satisfied only for $\alpha \le 0$, $\beta \ge 0$ or $\alpha \ge 0$, $\beta \le 0$

To find α,β, we first obtained the Hubble parameter for R(t). Again,

$H_I=\frac{\dot{R}_I}{R_I}=\beta$, and the Hubble parameter $H_R=\frac{\dot{R}}{R}=\alpha$ by which the constant α is fixed. $R(t)=\ell_p e^{H_R t}$, $R_I(t)=\ell_p e^{-H_R t}$ (20)

for D = 1 also, if $\alpha \neq 0$, then $\alpha = -\beta$ or $\alpha + \beta = 0$, or $\dot{X}+\dot{Y}=0$, or X + Y = constant, or $\log R + \log R_I$ = constant, $\therefore$ RR_I = constant. Again, for $D \neq 1$, then either $\alpha = 0$ or $\beta = 0$

i.e $\dot{X}=0$ or $\dot{Y}=0$, $\therefore$ X = constant, Y = constant.

$R = R_I = \ell_p$ (a time independent scale factor) Therefore, for D > 1,

$$\alpha_{\pm} = \frac{D\beta}{2}\left[-1 \pm \sqrt{1-\frac{2}{3}\left(1-\frac{1}{D}\right)}\right] \text{ That is } H_{R^{\pm}} = \frac{DH_I}{2}\left[-1 \pm \sqrt{1-\frac{2}{3}\left(1-\frac{1}{D}\right)}\right] \quad (21)$$

So
$$R_I(t) = \ell_p e^{H_I t} \qquad \psi(Y) = e^{\pm \frac{\gamma\sqrt{D}\,Y}{3\sqrt{D}+\sqrt{3(D+2)}}} \quad (22)$$

$$R_{\pm}(t) = \ell_p e^{\frac{DH_I t}{2}\left[-1 \pm \sqrt{1-\frac{2}{3}\left(1-\frac{1}{D}\right)}\right]}$$

and (23)

$$R(t) = \ell_p e^{H_R t} \qquad R_I = \ell_p e^{H_I t}$$

$$R_{I_{\pm}}(t) = \ell_p e^{\frac{2H_R t}{D}\left[-1 \pm \sqrt{1-\frac{2}{3}\left(1-\frac{1}{D}\right)}\right]^{-1}} \quad (24)$$

It is easy to show that the Lagrangian (14) (or the equations of motions) is invariant under the transformation $R \to R^{-1}$, $R_I \to R_I^{-1}$.......(29) Which is consistent with the time reversal $t \to -t$ Therefore, we have dynamical symmetry between R and R_I, namely $R_I \leftrightarrow R$.

III. Quantum cosmology over complex space:

The goal of quantum cosmology by solving the WDW equation over the complex space ($R + iR_I$) is to understand the origin and evolution of the universe. In principle, it is very difficult to solve the WDW equation in the super space due to the large number of degrees of freedom. In practice, one has to *freeze out* of all but a finite number of degree of freedom of the gravitational and matter fields. This procedure is known as quantization in mini–super–space, and will be used in the following. The mini–super–space in our model is two dimensional with gravitational variables X and Y. To obtain the Wheeler–DeWitt equation, in this mini–super–space, we start with the Lagrangian (14). The conjugate momenta corresponding to X and Y are obtained as

$$P_x = \frac{\partial L}{\partial \dot{X}} = \dot{X} + \frac{D}{2}\dot{Y} \qquad \& \qquad P_y = \frac{\partial L}{\partial \dot{Y}} = \frac{D}{2}\dot{X} + \frac{D(D-1)}{6}\dot{Y} \quad (25)$$

From which we obtain

$$\dot{X} = \frac{6}{D+2}\left[P_x\left(\frac{1-D}{3}\right) + P_y\right] \;............(26) \qquad \& \qquad \dot{Y} = \frac{6}{D(D-1)}\left[P_y\frac{2(1-D)}{D+2} - P_x\frac{D(1-D)}{D+2}\right] \quad (27)$$

Substituting equations (26) & (27) into the Hamiltonian constraint (23) which obtained through the legender transformation of Lagrangian (19) as
$$H = (1-D)P_x^2 - \frac{6}{D}P_y^2 + 6P_x P_y = 0 \quad (28)$$

Now, we may use the following quantum mechanical replacements $P_x \to -i\frac{\partial}{\partial X}$, $P_y \to -i\frac{\partial}{\partial Y}$

and Wheeler–DeWitt equations takes the form
$$\left[(D-1)\frac{\partial^2}{\partial X^2} + \frac{6}{D}\frac{\partial^2}{\partial Y^2} - 6\frac{\partial}{\partial X}\frac{\partial}{\partial Y}\right]\psi(X,Y) = 0 \quad (29)$$

Where $\psi(X, Y)$ is the wave function of the universe in the (X, Y) mini–super-space. The general solution is

$$\psi_D(R,R_I) = \phi_1\left[\log\left\{R^{3D+\sqrt{3D^2+6D}}.R_I^{D(D-1)}\right\}\right] + \phi_2\left[\log\left\{R^{3D-\sqrt{3D^2+6D}}.R_I^{D(D-1)}\right\}\right] \quad (30)$$

Using separation of variables $\psi(X, Y) = \phi(X) \cdot \psi(Y)$, (31)

We obtain the following equations $\frac{\partial\phi}{\partial X} = \frac{\gamma}{D-1}\phi(X)$.......(32) & $\frac{\partial\psi}{\partial Y} = \frac{\gamma}{3+\sqrt{\frac{3(D+2)}{D}}}\psi(Y)$ (32)

We assume that $\gamma > 0$. After finding solutions of equations (31) & (32) are may write

$$\psi_D^{\pm}(R, R_I) = A^{\pm}R^{\pm\frac{\gamma}{D-1}}R_I^{\pm\frac{\gamma\sqrt{D}}{3\sqrt{D}\pm\sqrt{3(D+2)}}} \;...............(33). \text{ and } \quad \psi_D^{\pm}(R, R_I) = B^{\pm}R^{\pm\frac{\gamma}{D-1}}R_I^{\mp\frac{\gamma\sqrt{D}}{3\sqrt{D}\pm\sqrt{3(D+2)}}} \quad (34)$$

Where $A^{\pm}, B^{\pm}$ are the normalization constant, Now, for $\psi_D^{\pm}(\mathbf{R}, \mathbf{R_I}) \leftrightarrow \psi_D^{\pm}(\mathbf{R_I}, \mathbf{R})$,

Again the other solution of (29) is $\psi_D(X,Y) = \phi_1\left\{D(D-1)Y + 3DX + \sqrt{3D^2+6D}\,X\right\}$

$+\phi_2\left\{D(D-1)Y + 3DX - \sqrt{3D^2+6D}\,X\right\}$ (36)

i.e. $3D \pm \sqrt{3D^2+6D} = D(D-1)$ ....(35)

We have D = 1, 6. Thus for D > 1 i.e. for D = 6 there is a change symmetry, $\psi_D(R, R_I) \leftrightarrow \psi_D(R_I, R)$ under the exchange $R_I \leftrightarrow R$.

IV. The cosmological model with a scalar field potential:

Let us consider a flat Friedmann-Lemaitre-Robertson-Walker universe with metric,

$$dS^2 = N^2(t)dt^2 - a^2(t)dt^2 \qquad (37)$$

with the equation of state

$$p = \gamma\rho + \frac{A}{\rho^{\alpha}} \text{ (where } \gamma, A, \alpha \text{ are arbitrarily constants).} \qquad (38)$$

The Friedmann equation is

$$H^2 = \rho \qquad (39)$$

where the Hubble parameter H is as usual

$$H \equiv \frac{\dot{R}}{R} \qquad (40)$$

where "dot" means the derivative with respect to the cosmic time t. The energy conservation condition is

$$\dot{\rho} = -3H(\rho + p) \qquad (41)$$

We have from equation (37) & (38),

$$\dot{\rho} = -3\rho^{1/2}\left(\gamma\rho + \frac{A}{\rho^{\alpha}} + \rho\right) = -3A\rho^{\frac{1}{2}-\alpha} - 3(\gamma+1)\rho^{3/2}$$

i.e $$\frac{H^{2\alpha+1}dH}{A + (\gamma+1)H^{2\alpha+2}} = -\frac{3\dot{R}}{2R}dt$$

Integrating, we have

$$\rho(R) = \frac{1}{(\gamma+1)^{1/(\alpha+1)}}\left[\left(\frac{B}{R}\right)^{3(\gamma+1)(\alpha+1)} - A\right]^{\frac{1}{\alpha+1}} \qquad (42)$$

Again from (39), (40) and (42), we get

$$\frac{\dot{R}^2}{R^2} = \frac{1}{(\gamma+1)^{1/(\alpha+1)}}\left[\left(\frac{B}{R}\right)^{3(\gamma+1)(\alpha+1)} - A\right]^{\frac{1}{\alpha+1}}$$

i.e., $$\dot{R} = \frac{1}{(\gamma+1)^{\frac{1}{2(\alpha+1)}}}\left[\left(\frac{B}{R}\right)^{3(\gamma+1)(\alpha+1)} - A\right]^{\frac{1}{2(\alpha+1)}} \qquad (43)$$

When the cosmological radius tends to the critical value $R^* = \dfrac{B}{A^{\frac{1}{3(\gamma+1)(\alpha+1)}}}$, the energy density disappears and the pressure according to the equation (38) grows indefinitely. Then cosmological time $t = t_B$. Again, when the cosmological radius grows indefinitely, i.e., $R \to \infty$, then the energy density does not disappear and had some parametric value as $\dot{R} \to \infty$ (i.e., $\dot{R}_I \to \infty$, then $R_I = R_I^* = iR^*$ in other phase), then

$$\rho(R) = \frac{1}{(\gamma+1)^{1/(\alpha+1)}}\left[-A\right]^{\frac{1}{1+\alpha}} \qquad (44)$$

As well as the pressure is then

$$p=\frac{\gamma}{(\gamma+1)^{1/(\alpha+1)}}[-A]^{\frac{1}{1+\alpha}}+A.\frac{1}{(\gamma+1)^{\frac{\alpha}{\alpha+1}}}[-A]^{-\frac{\alpha}{\alpha+1}}$$

$$p(R)=\frac{A}{[-A]^{\alpha/(\alpha+1)}.(\gamma+1)^{1/(\alpha+1)}}$$ is negative for $A<0$, $\gamma>0$, $\alpha>0$. (45)

Hence, $$\rho+3p=\frac{2A}{(\gamma+1)^{1/(\alpha+1)}[-A]^{\alpha/(\alpha+1)}} \qquad (46)$$

is not real for $A>0$, $\alpha>0$, $\gamma>-1$ and $\rho+3p \not> 0$ when $A \not> 0$. If γ=0 and $A<0$, $\alpha=1$, the universe coincide with the Chaplygin gas model (and pressure is then negative. Also for $\gamma=-1$ and $A<0$, then pressure p will be $-\infty$ and ρ is then ∞. Thus the strong energy condition $\rho+3p>0$ violated due to the different value of the constant parameters, we have from the equation (43),

$$\ddot{R}=-\frac{1}{2(\gamma+1)^{\frac{1}{(\alpha+1)}}R^{3\gamma+2}}.\frac{2AR^{3(\gamma+1)(\alpha+1)}+(3\gamma+1)B^{3(\gamma+1)(\alpha+1)}}{\left[B^{3(\gamma+1)(\alpha+1)}-AR^{3(\gamma+1)(\alpha+1)}\right]^{\frac{\alpha}{\alpha+1}}}<0 \qquad (47)$$

[for, $R<R^*$]

From the expression (43) and (47), when the cosmological radius tends to a finite value R^*, then $\dot{R}=0$, and $\ddot{R}=-\infty$, i.e., the declaration is infinite. Thus, the stop of expansion occurs in a singular way that means Big-Brake Singularity.

Let us notice that the Big-Brake Singularity just like other soft singularities possesses an important property the Christoffell Symbols at the singularity are finite (or even zero). Thus the matter can pass through this singularity and then the geometry of the space-time can re-appear. It is easy to show that the expression (48) are well defined at $R>R^*$ namely, after arriving at the point of Big-Brake $R(t_B)=R^*$, which is at the same time the point of the maximal expansion of the universe, the universe begin contracting and this contraction culminates in the encounter with a Big-Crunch Singularity at $R=0$. Thus the model describes the evolution of the universe from the Big-Bang to the Big-Crunch passing through the soft Big-Brake singularity at the moment of the maximal expansion of the universe.

V. Einstein's solution in general phase without cosmological constant:

Imagine, a uniform distribution of matter filling the infinite Euclidean space in a phase other than Einstein's universe. We know that any finite distribution of pressure–free matter would tend to shrink under its own gravity.

We consider the closed surface from the very early universe as rectangular–parallelepiped after then a cubical box and then spherical space–time. The equation of the diagonal of the parallelepiped as

$$x_1^2+x_2^2+x_3^2=r_2^2 \qquad (48)$$

Consider the side of the cubical box is r_1 and also consider the diagonal remains unchanged for parallelepiped and cubical box.

Hence the diagonal of the cubical box is $$r_2=\sqrt{3}r_1 \qquad (49)$$

Now we consider the equation (49) of a 3–surface in Cartesian co–ordinates x_1, x_2, x_3, & x_4 by

$$x_1^2+x_2^2+x_3^2+x_4^2=S_1^2 \qquad (50)$$

where S_1 is the radius of the 3–surface of a 4- dimensional hyper–sphere. We consider the total mater in the cubical surface and Einstein's spherical surface remains unaltered. Hence

$$r_1^3\varepsilon_1=\frac{4}{3}\pi R^3\varepsilon \quad \text{i.e.} \quad \mathbf{r_1}=\left(\frac{4\pi}{3}\right)^{1/3}\left(\frac{\varepsilon}{\varepsilon_1}\right)^{1/3}\mathbf{R} \qquad (51)$$

[Taking only the real part]

$$\therefore \quad \mathbf{r_2}=\sqrt{3}\,\mathbf{r_1}=\sqrt{3}\left[\frac{4\pi}{3}\right]^{1/3}\left[\frac{\varepsilon}{\varepsilon_1}\right]^{1/3}\mathbf{R} \qquad (52)$$

In terms of a constant negative curvature, the equation (50) becomes

$$x_1^2+x_2^2+x_3^2-x_4^2=-S_1^2$$

Here, $\mathbf{S_1 = r_2 = \phi(t).R(t)}$ (53)

Where $\phi(t) = \sqrt{3}\left(\frac{4\pi}{3}\right)^{1/3}\left(\frac{\varepsilon}{\varepsilon_1}\right)^{1/3}$ (54)

Where, ε, ε_1 are the energy densities of the two phase respectively. Comparing with the Einstein's universe, the most general line element satisfying the Weyl-postulate and the cosmological principal is given by

$$ds^2 = c'^2 dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \mathrm{Sin}^2\theta\, d\phi^2\right)\right]$$

Where, S(t) = ϕ(t). R(t); $|\phi(t)| \geq \sqrt{3}\left(\frac{4\pi}{3}\right)^{1/3}$ (55)

And the Einstein's equations become,

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + kc'^2}{S^2} = \frac{8\pi G}{c'^2}T_1^1 = \frac{8\pi G}{c'^2}T_2^2 = \frac{8\pi G}{c'^2}T_3^3 \quad (56)$$

& $$\frac{\dot{S}^2 + kc'^2}{S^2} = \frac{8\pi G}{3c'^2}T_o^o \quad (57)$$

Where, S(t) = ϕ(t), R(t) $\dot{S} = \dot{\phi}(t)R(t) + \dot{R}(t)\phi(t)$ (58)

VI. Einstein Universe derived from wider universe:

From the equation (56) & (57) we have the relation,

$$\frac{d}{ds}(\varepsilon_1 S^3) + 3p' S^2 = 0 \quad (59)$$

When $T_1^1 = T_2^2 = T_3^3 = -p'$ and $T_o^o = \varepsilon_1$

At the very early epoch, $p' = 0$, then

$$\varepsilon_1 S^3 = \text{constant.} \quad (60)$$

By the equation (58) we have $\varepsilon_1\phi^3(t).R^3(t)$ = constant

i.e εR^3 = constant. [by the equation (54)]

Consider, when R = R_o, ε = ε_o then

$$\varepsilon R^3 = \varepsilon_o R_o^3 \quad (61)$$

Which may be found by the Einstein's universe at the early epoch when pressure p = 0, by using the Friedmann equations and with the help of Robertson–Walker line element.

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \mathrm{Sin}^2\theta\, d\phi^2\right)\right] \quad (62)$$

Again in absence of any external forces, the velocity u^i satisfies the geodesic equation

$$\frac{du^i}{ds} + \Gamma^i_{K\ell}\, u^K u_\ell = 0 \quad (63)$$

Substitute $\Gamma^i_{K\ell}$ in the equation (55) gives the result

$$u^\mu S^2 = \text{constant.} \quad (64)$$

However, u^μ measures the velocity in the co–moving (r, θ, ϕ) co–ordinates, we have

$u^\mu \phi^2(t) R^2(t)$ = constant …. (65)

Again, if we substitute Γ^i_{kl} in the equation (62) which gives the result $u^\mu R^2(t)$=constant. (66)

Now, if we compare (66) with (65), we get ϕ(t) = constant.

i.e. $\varepsilon_1(t) \propto \varepsilon(t)$ (67)

Which indicates the large or small matter energy density in the vapor phase (so called nothing) changes to the large or small matter energy density in the liquid phase (Einstein's Universe) and hence it is found, an important fact, that the existence of discrete structure in the universe, ranging from galaxies to super–clusters.

VII. The field equation in complex quantum state:

The work covered in the Einstein field equations did not tell us the important item of information about the universe is what happened, when the volume of the matter universe squeezed into zero volume and there before. To find the answer to this question it is necessary to do beyond the concept of Einstein universe. We need a new concept with the Einstein's universe to proceed any further, and Einstein's general relativity with complex space–time is one of such theory. We will consider alternative approaches to cosmology but for the present is Kantowski–Sachs universe. We have the line element to start with:

$$ds^2 = -N^2(t)dt^2 + a^2(t)dr^2 + b^2(t)(d\theta^2 + \mathrm{Sin}^2\theta d\phi^2) \qquad (68)$$

The only nontrivial Einstein equations of the above metric is then

$$T_0^0 = -\frac{c'^4}{8\pi G}\left[\frac{2\dot{a}\dot{b}}{abN^2} + \frac{\dot{b}^2}{b^2N^2} + \frac{1}{b^2}\right]$$

$$T_1^1 = -\frac{c'^4}{8\pi G}\left[\frac{2\ddot{b}}{bN^2} - \frac{\dot{b}\dot{N}}{bN^3} + \frac{\dot{b}^2}{bN^2} + \frac{1}{b^2}\right] \qquad (69)$$

$$T_2^2 = -\frac{c'^4}{8\pi G}\left[\frac{\ddot{a}}{aN^2} + \frac{\ddot{b}}{bN^2} + \frac{\dot{a}\dot{b}}{abN^2} - \frac{\dot{a}\dot{N}}{aN^3} - \frac{\dot{b}\dot{N}}{bN^3}\right] = T_3^3$$

For, considering N = 1, we have

$$\frac{\dot{\mathbf{b}}^2}{\mathbf{b}^2} + \frac{2\dot{\mathbf{a}}\dot{\mathbf{b}}}{\mathbf{ab}} + \frac{1}{\mathbf{b}^2} = -\frac{8\pi G}{c'^4}\mathbf{T}_0^0 \qquad (70)$$

$$\frac{2\ddot{\mathbf{b}}}{\mathbf{b}} + \frac{\dot{\mathbf{b}}^2}{\mathbf{b}^2} + \frac{1}{\mathbf{b}^2} = -\frac{8\pi G}{c'^4}\mathbf{T}_1^1 \qquad (71)$$

$$\frac{\ddot{\mathbf{a}}}{\mathbf{a}} + \frac{\ddot{\mathbf{b}}}{\mathbf{b}} + \frac{\dot{\mathbf{a}}\dot{\mathbf{b}}}{\mathbf{ab}} = -\frac{8\pi G}{c'^4}\mathbf{T}_2^2 = -\frac{8\pi G}{c'^4}\mathbf{T}_3^3 \qquad (72)$$

Where c' is the velocity of photon–like particle in vapor stage and $c' > c$, the velocity of photon.
Here, we consider a = R & b = iR_I, [where $i = \sqrt{-1}$].
Before we consider specific forms of T_k^i, it is worth noting that three properties must be satisfied by the energy tensor in the present framework of cosmology. The first is obviously define negative pressure by
The second T_o^o, $T_2^2 = T_3^3$. is defined the matter density and the third T_1^1 is define the latent energy density.

$$\text{If } T_2^2 = T_3^3 = 0, \text{ then } \frac{\ddot{R}}{R} + \frac{\ddot{R}_I}{R_I} + \frac{\dot{R}}{R}\frac{\dot{R}_I}{R_I} = 0 \qquad (73)$$

If $R_I \leftrightarrow R$ at 10-dimensional, then, $t = \frac{R^{\frac{3}{2}} - E}{D}$, [where D, E are integration constants].
Again, we consider the case, when

$$N(t) = R^3(t)R_I^D(t)i^D; \quad \text{then, } T_0^0 = -\frac{c'^4}{8\pi G}\left[\frac{2\dot{R}\dot{R}_I}{R^7R_I^{2D+1}i^{2D}} + \frac{\dot{R}_I^2}{R^6R_I^{2D+2}i^{2D}} - \frac{1}{R_I^2}\right]$$

$$T_1^1 = -\frac{c'^4}{8\pi G}\left[\frac{\ddot{R}}{R^7R_I^{2D}i^{2D}} + \frac{\ddot{R}_I}{R^6R_I^{2D+1}i^{2D}} - \frac{\dot{R}\dot{R}_I^2}{R^7R_I^{2D+1}i^{2D}}\right]$$

$$T_2^2 = T_3^3 = -\frac{c'^4}{8\pi G}\cdot\frac{1}{R^6R_I^{2D}i^{2D}}\left[\frac{\ddot{R}}{R} + \frac{\ddot{R}_I}{R_I} + \frac{\dot{R}\dot{R}_I}{RR_I} - \frac{3\dot{R}^2}{R^2} - \frac{D\dot{R}\dot{R}_I}{RR_I} - \frac{3\dot{R}\dot{R}_I}{RR_I} - \frac{D\dot{R}_I^2}{R_I^2}\right]$$

Again from (21), (22), (23) we get

$$\mathbf{T}_o^o = \frac{c'^4}{8\pi G}\left[\frac{e^{-2H_I t}}{\ell_p^2} - H_I^2 - 2H_I H_R\right] \qquad (74)$$

$$\mathbf{T}_1^1 = \frac{c'^4}{8\pi G}\left[\frac{e^{-2H_I t}}{\ell_p^2} - 3H_I^2\right] \qquad (75)$$

&

$$\mathbf{T}_2^2 = \mathbf{T}_3^3 = -\frac{c'^4}{8\pi G}\left[H_R^2 + H_I^2 + H_R H_I\right] \qquad (76)$$

Now, it is clear that, when $H_I = H_R$ then $R_I \propto R$.Which is possible for 6+4 =10-dimensional space–time. Then the equations (74) & (75) are identical

i.e $$T_o^o = T_1^1 = -\frac{3c'^4}{8\pi G}\left[H_R^2 - \frac{e^{-2H_R t}}{3\ell_p^2}\right] \quad (77)$$

$$T_2^2 = T_3^3 = -\frac{3c'^4}{8\pi G}[H_R^2] = -\frac{3c'^4}{8\pi G}[H_I^2] = -p \quad (78)$$

$$i.e.\ p + \epsilon = \frac{c'^4}{8\pi G \ell_p^2} e^{-2H_R t} \quad (79)$$

VIII. The quantum state of the black-hole:

A spherically symmetric black-hole with mass M and electric charge e is described by the space-time metric

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1}dr^2 - r^2(d\theta^2 + Sin^2\theta\, d\varphi^2) \quad (80)$$

Where $M^2 > e^2$ and r is a radial co-ordinate chosen to make the surface area of a sphere of radius r equal to $4\pi r^2$, as in Minkowski-Space.

Comparing the equation (68) with (80), we have

$$-N^2(t) = 1 - \frac{2M}{r} + \frac{e^2}{r^2} \quad (81)$$

$$a^2(t) = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} \quad (82)$$

$$b(t) = ir \quad (83)$$

If we choose, $N(t) = 1$, then from (81), we get

$$1 - \frac{2M}{r} + \frac{e^2}{r^2} = -1$$

$$i.e.\ r = r_\pm = \frac{1}{2}M \pm \frac{1}{2}(M^2 - 2e^2)^{1/2} \quad (84)$$

Thus from (82) & (83), we get,

$$a^2(t) = R^2(t) = 1\ and\ b(t) = iR_I = ir$$

$$i.e.\ \ R_I = r_\pm = \frac{1}{2}M \pm \frac{1}{2}(M^2 - 2e^2)^{1/2} \quad (85)$$

When $R = R_I$, at 7 – dimensional space-time, then

$$M^2 = e^2 + \frac{e^4}{4} + 1 \quad (86)$$

Thus, the metric (75) is evidently singular at

$$r = r_\pm \equiv \frac{1}{2}M \pm \frac{1}{2}(M^2 - 2e^2)^{1/2} \quad (87)$$

These are not singularities in the geometry itself but in the co-ordinate system (t, r), similar to these which occur to latitude and longitude on the surface of a sphere at the poles. In fact, the outer surface r_+corresponds to the event horizon: notice that $r_+ \to M\ \ as\ e \to 0.$ This surface has global significance, but locally an internal observer would find nothing unusual about the space-time geometry there. The inner surface r_- is another type of horizon inside the hole- itself.

XII. The Laws of Classical Black-Hole 'Thermodynamics':

There is a general tendency for self-gravitating systems to grow rather than shrink because gravity always attracts. The behavior between the black-holes and thermodynamic equilibrium systems were noted some time ago (Carter 1973). In the black-hole case, the inability for light to change from inside the event horizon precludes the escape of any material, so the horizon acts as a sort of asymmetric one-way surface: things can fall in and make the hole bigger but not come out and make it smaller that means changes to another phase. This is reminiscent of the second law of thermodynamics, in which there is an asymmetric tendency for a one-way increase in entropy. The size of the black-hole is analogous to the entropy. The above statement can be explained as, in the theory of SU(11), it is possible to change any of 30-bosons of SU(5) of the matter energy group into any of 30-bosons of SU(6) of the latent energy group and vice-versa by the exchange of J-bosons of SU(11) [see my article 2012].

The analogy is almost trivial for a spherical, electrically neutral (Schwarzschild) black-hole. In the more general case of black-holes that possess angular momentum J by SU(6) and electric charge e by U(1), the size of the black hole depends both on J and e in a rather complicated way. If the total surface area of the horizon is used as a measure of size then this is given by the formula (Samrr 1973):

$$A = 4\pi\left[2M^2 - e^2 + 2M^2\left(1 - \frac{e^2}{M^2} - \frac{J^2}{M^4}\right)^{\frac{1}{2}}\right] \quad (83)$$

[Where, $M^2 = e^2 + \frac{e^4}{4} + 1$, by the equation (81)]

Where $e^2 < M^2$ and $J^2 < M^4$ (throughout, units with G = c =1 will be used) so it is clear at a glance whether a disturbance to the black-hole which changes both e and J, as well as M, will always increase the total area.

An example due to Penrose (1969) concerns a method for extracting mass-energy from a rotating black-hole. The mechanism consists of propelling a small body into the region just outside the event horizon where (due to dragging effect on the space surrounding the black-hole caused by its rotation). Some particle trajectories possess negative energy relative to infinity. Due to the symmetry breaking of SU(11), we get mainly two fragments as SU(6) and SU(5), one of which i.e. particles of the energy

group SU(6) is placed on one of these negative energy paths, and this part changes to SU(5), so reduce the total mass M of the hole somewhat and hence the mass-energy of SU(5) thereby released by this sacrificed particles of the latent energy group SU(6) which is ejected to infinity at high speed. During this energy transfer the black-hole's rotation rate is diminished somewhat, so J also decreases. Thus from equation (83), which shows that when J decreases, the area A increases, but when M decreases, the area decreases. The change in M and J are therefore in competition, but a careful calculation shows that J always wins and the area increases.
Actually, if the class of all trajectories is studied, it is found that in general the area increases by an amount corresponding to a considerable fraction of mass energy. However, the efficiency of energy extraction can be improved by approaching closely to a limiting class of trajectories for which the event horizon area remains constant. The limiting case is therefore reversible and corresponds to an isentropic change in thermodynamics. In practice, 100% efficiency (complete reversibility) would be impossible.
This strong analogy between event horizon area and entropy led to use of the name 'second law' in connection with Hawking's area theorem, which is therefore written as:

$$dA \geq 0, \quad \text{[equality corresponding to reversibility]} \qquad (84)$$

There are also analogous of the Zeroth, first and third laws of thermodynamics. From (82), we can obtain;

$$dM = (8\pi)^{-1} k dA + \Omega dJ + \varphi de \qquad (85)$$

Where $(8\pi)^{-1} k \equiv \frac{\partial M}{\partial A}$,etc. which is really just an expression of mass-energy conservation and corresponds to the first law. If A plays the role of entropy then we see from (85), that k plays the role of temperature $(kdA \sim TdS)$. The interesting thing is that k can be shown to be constant across the event horizon surface. We thus have an expression of a 'Zeroth' law, analogous to the thermodynamic one of which says that in thermodynamic equilibrium there exists a common temperature parameter for the whole system. The quantity k is known as the surface gravity of the black-hole. Its significance lies in the fact that it determines the e-folding time which controls the rate at which the collapsing star red-shifts and approaches equilibrium. For a Schwarzschild hole $k = (4M)^{-1}$ and the constant 8π in (85) has been chosen to agree with this. The remaining terms in (85) simply describe the work done (energy extracted) from changes in angular momentum (ΩdJ) and electric charge (φde) and have a very obvious structure: **Ω** is the (magnitude of) angular velocity and **ϕ** the electric potential at the event horizon.
Finally, there is the third law. It is straightforward to show that if J^2 or e^2 become large enough such that:

$$\frac{J^2}{M^4} + \frac{e^2}{M^2} = 1 \quad \text{i.e.} \quad J^2 = \left(\frac{e^4}{4} + 1\right)\left(e^2 + \frac{e^4}{4} + 1\right) \qquad (86)$$

Then k vanishes (although A does not). This corresponds to absolute zero (though with finite entropy). A black-hole with parameters given by (86) is known as an extreme Kerr-Newman black-hole. It is the limiting case of an object which still possesses an event horizon. Should the left-hand side become even infinitesimally greater than one, then the horizon would disappear and we would left with a naked singularity, i.e. the singularity would no longer be invisible inside a black-hole but would be able to influence, and be observed by, the outside universe. This circumstance is considered so undesirable for physics that most physicists believe in the so-called cosmic censorship hypothesis due to Penrose (1969): naked singularities cannot form from gravitational collapse. Cosmic censorship implies the un-attainability of 'absolute zero', k = 0 [i.e. condition (86) for an extreme black-hole], so it plays the role of the third law.

6. Concluding Remarks:

So the space-time of the universe is actually a complex space-time and there is neither any starting point nor any ending point of the wider (measurable in quantum cosmology) universe. Only there exists the initial and ending conditions for narrower (measurable in classically) universe, which emerged from wider universe by the process of changing phase, which is a continuous process.

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