Linear Systems Time Series

STAT-S 650: Time Series Analysis

Spring 2017 – Home Work solutions

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Answer 1 Attached at the end of the document

Answer 2

Given System

$$y(t) = .9 \cdot y(t\text{-}1) - .5 \cdot y(t\text{-}2) + x(t)$$

General State Space form

$$Y(t) = H \cdot Z(t) + K$$
$$Z(t) = F \cdot Z(t\text{-}1) + G \cdot X(t)$$

where,
t is the time index {1,2,3 .. t, .. N}
F is a p x p feedback or system matrix
G is a p x q input weight matrix
H is a r x p output matrix
K is a r x 1 vector of constants
Y(t) r x 1 output vector
Z(t) p x 1 state vetor
X(t) q x 1 input vector

Any Linear Difference equation of below form

$$y(t) = \alpha_1 y(t-1) + \alpha_2 y(t-2) + \alpha_3 y(t-3) + \beta_0 x(t) + \beta_1 x(t-1) + \beta_3 x(t-3) \tag{1}$$

can be written in state space from with

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} F = \begin{bmatrix} \alpha_1 & 1 & 0 \\ \alpha_2 & 0 & 1 \\ \alpha_3 & 0 & 0 \end{bmatrix} G = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} K = 0$$

Now considering the respective values for Z,F,G,H,K

$$F = \begin{bmatrix} .9 & 1 \\ -.5 & 0 \end{bmatrix} Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} H = \begin{bmatrix} 1 & 0 \end{bmatrix} G = \begin{bmatrix} 1 \\ 0 \end{bmatrix} K = 0$$

we get

$$Y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + 0$$

This implies $y(t) = z_1(t)$ where

$z_1(t) = \alpha_1 z_1(t-1) + z_2(t-2) + \beta_0 x(t)$
$z_2(t) = \alpha_2 z_1(t-1) + \beta_1 x(t-1)$

On Subsituting values for α's and β's we get

$$z_1(t) = .9z_1(t-1) + z_2(t-1) + x(t)$$
$$z_2(t) = -.5z_1(t-1)$$

Since we know that $y(t) = z_1(t)$ we can write the below from the above equation

$$y(t) = .9y(t-1) + z_2(t-1) + x(t)$$
$$z_2(t) = -.5y(t-1)$$

from above $z_2(t-1) = -.5y(t-2)$ hence we can write $y(t) = .9y(t-1) + z_2(t-1) + x(t)$ as

$$y(t) = .9y(t-1) - .5y(t-2) + x(t)$$

Since we arrived at the equation from a state space assumption the assumption is True.
Hence the values for the state space model are

$$\text{Y(t)} = \text{Z(t)}$$

where Z(t) = F · Z(t-1) + G · X(t) and with

$$F = \begin{bmatrix} .9 & 1 \\ -.5 & 0 \end{bmatrix} Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} H = \begin{bmatrix} 1 & 0 \end{bmatrix} G = \begin{bmatrix} 1 \\ 0 \end{bmatrix} K = 0$$

Question 3

```
F = matrix(data = c(.9,1,-.5,0), nrow = 2, ncol = 2, byrow = T)
print(F)
```

```
##      [,1] [,2]
## [1,]  0.9    1
## [2,] -0.5    0
```

```
eigen_F = eigen(F)
print(eigen_F$values)
```

```
## [1] 0.45+0.5454356i 0.45-0.5454356i
```

```
abs_eigen_value = abs(eigen_F$values)
print(abs_eigen_value)
```

```
## [1] 0.7071068 0.7071068
```

There are 2 eigen values and the absolute values of both the eigen values are less than 1 which means that the system is stable

Information to be used in the 4ht problem

```
print(diag(eigen_F$values))
```

```
##                 [,1]            [,2]
## [1,] 0.45+0.5454356i 0.00+0.0000000i
## [2,] 0.00+0.0000000i 0.45-0.5454356i
```

```
print(eigen_F$vectors)
```

```
##                       [,1]                  [,2]
## [1,]  0.8164966+0.0000000i  0.8164966+0.0000000i
## [2,] -0.3674235+0.4453463i -0.3674235-0.4453463i
```

Question 4

Impulse response representation of state space formulation is shown below:

$$U(t) = \sum_{k=0}^{t} \Lambda^t \cdot X(t-k)$$

where

$$U(t) = V^{-1} \cdot Z(t)$$
$$M = V^{-1} \cdot G$$

we also know that $F = V \cdot \Lambda \cdot V^{-1}$

```r
V = eigen_F$vectors
lambda = diag(eigen_F$values)
print(V)
```

```
##                         [,1]                    [,2]
## [1,]  0.8164966+0.0000000i  0.8164966+0.0000000i
## [2,] -0.3674235+0.4453463i -0.3674235-0.4453463i
```

```r
print(lambda)
```

```
##                  [,1]              [,2]
## [1,] 0.45+0.5454356i 0.00+0.0000000i
## [2,] 0.00+0.0000000i 0.45-0.5454356i
```

Calculation for M
M = V^{-1} * G

```r
G = matrix(data = c(1,0))
print(G)
```

```
##      [,1]
## [1,]    1
## [2,]    0
```

```r
V_inv = solve(V)
M = V_inv%*%G
print(M)
```

```
##                        [,1]
## [1,] 0.6123724-0.5052248i
## [2,] 0.6123724+0.5052248i
```

Therefore the impulse response erepresentation of U(t) can be seen as below

$$U(t) = \sum_{k=0}^{t} \begin{bmatrix} 0.45 + 0.5454356i & 0.00 + 0.0000000i \\ 0.00 + 0.0000000i & 0.45 - 0.5454356i \end{bmatrix}^t \cdot \begin{bmatrix} 0.6123724 - 0.5052248i \\ 0.6123724 + 0.5052248i \end{bmatrix}^t \cdot X(t-k)$$

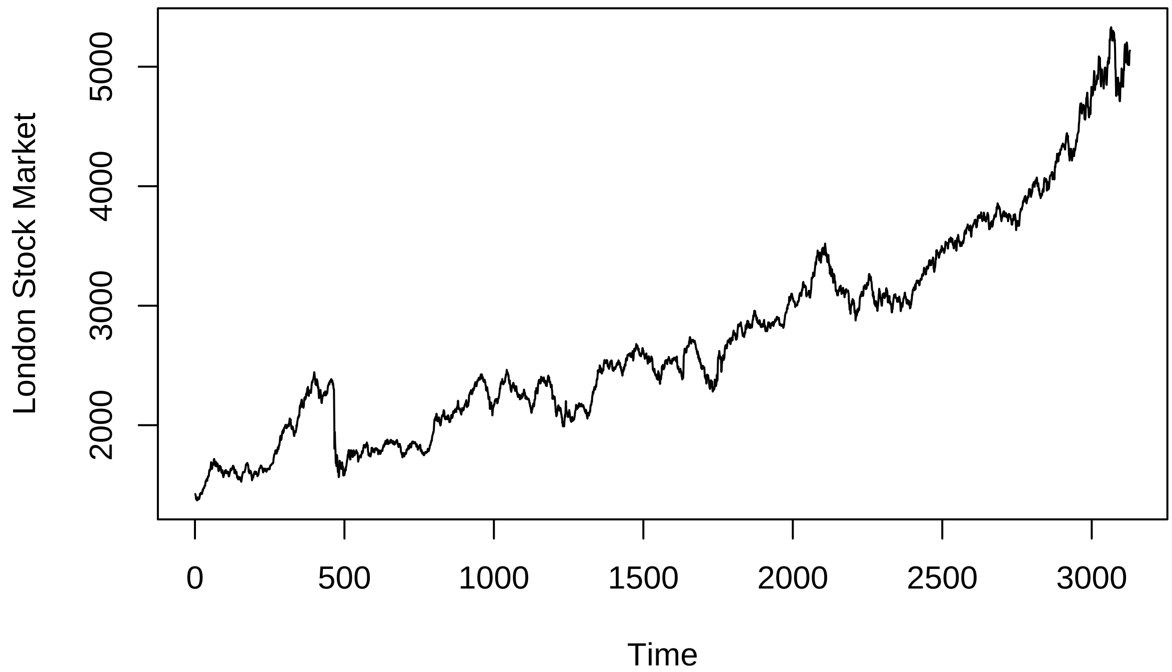
Question 5

Reading data into R Data : Stock Market Data For the purpose of this assignment I am using only apart of the entire data

```r
setwd("~/gdrive/IUBCourseWork/TimeSeries/Introductory Time Series with R-Paul S.P. Cowpertwait and Andr
stockMarketData = read.table('stockmarket.dat', header = T)
# using only a sample of the total data -- Newyork and Hongkong
SMD = stockMarketData[,c(3,7)]
london.SMD = SMD$London
newyork.SMD = SMD$NewYork
# converting the data into time series
london.SMD.ts = ts(london.SMD)
print(head(london.SMD.ts))
```

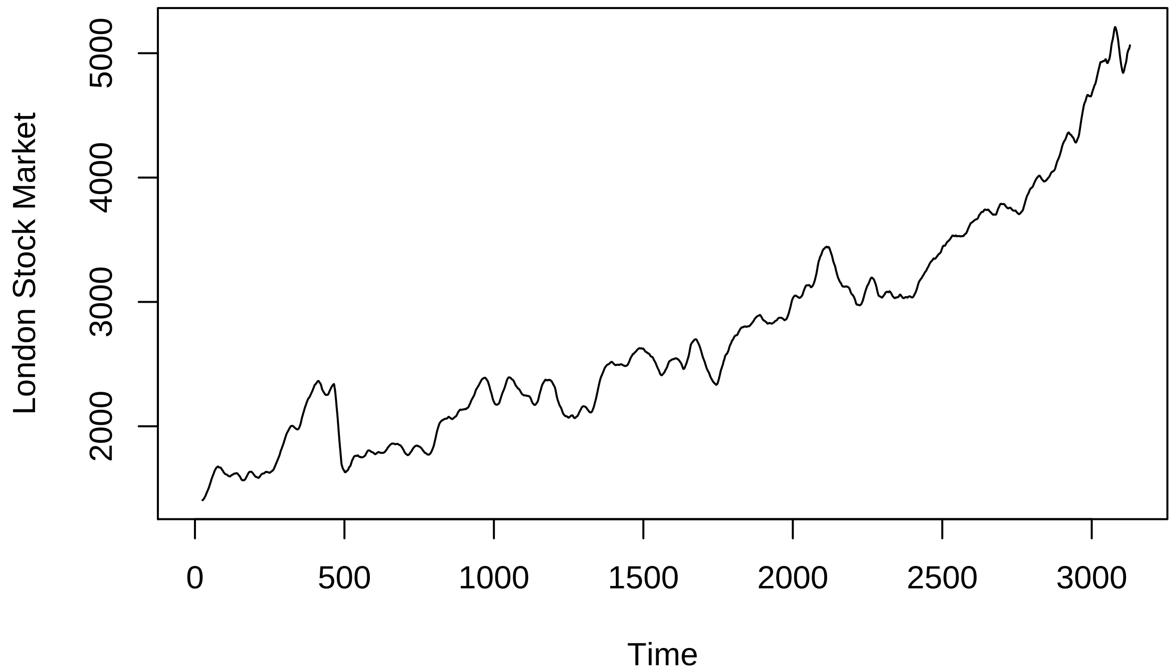
```
## [1] 1424.1 1415.2 1404.2 1379.6 1394.5 1383.2
```

```r
plot(london.SMD.ts, type='l', ylab = 'London Stock Market')
```



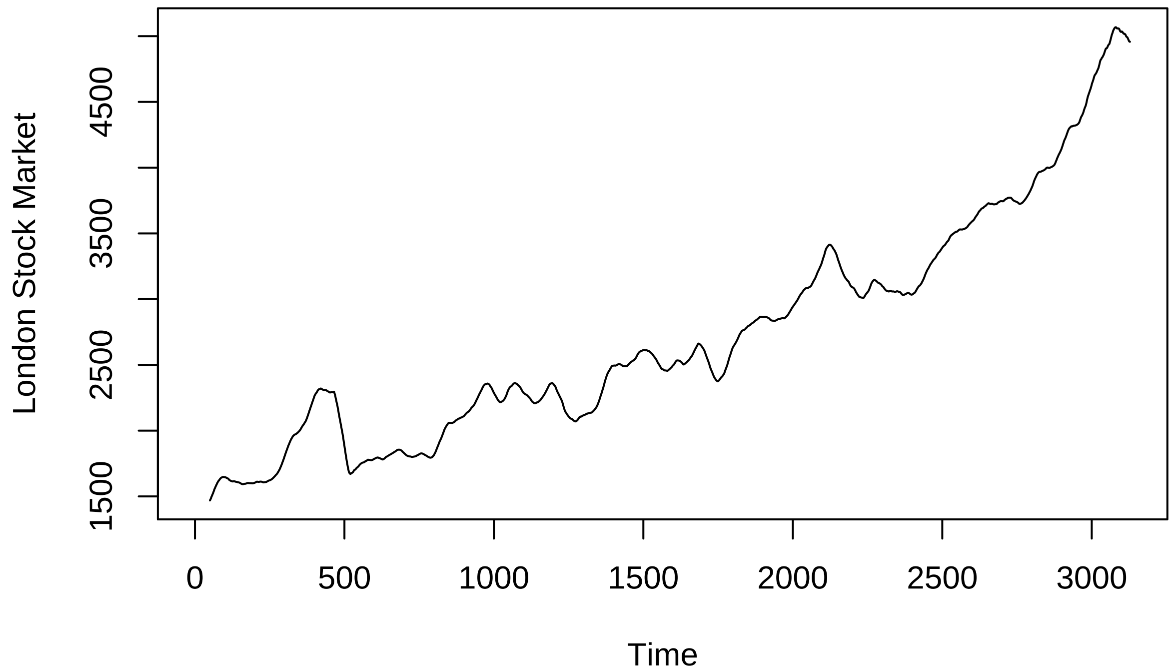
Applying Filter window size = 25

```
ws <- 25
yf <- filter(london.SMD.ts, rep(1/ws,ws), sides=1)
plot(yf, type='l', ylab = 'London Stock Market')
```



Applying Filter window size = 50

```
ws <- 50
yf <- filter(london.SMD.ts, rep(1/ws,ws), sides=1)
plot(yf, type='l', ylab = 'London Stock Market')
```

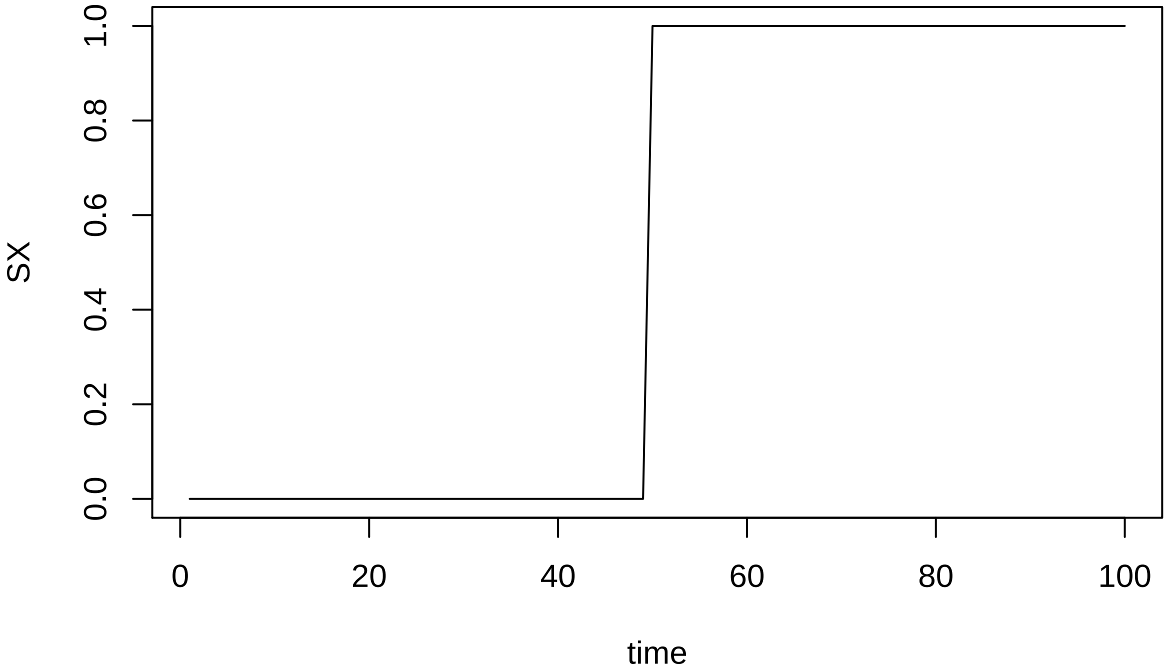


Question 7

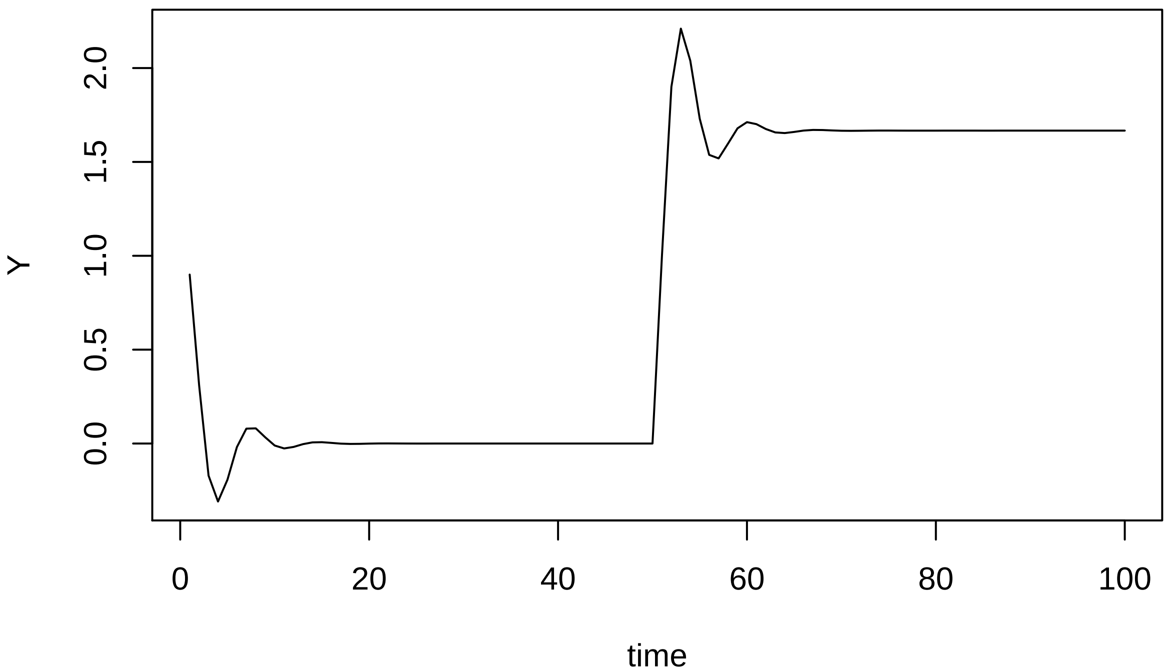
Checking for behaviour of the system

```
# Linear Difference Equation example
N=100
X = array( 0, dim=c(N,1))
X[N/2]=1
SX=array( 0, dim=c(N,1))

sum = 0
for (index in 1:N) {
  sum = sum + X[index]
  SX[index] = sum
}

X = SX;
Y=array( 0, dim=c(N,1))
y0 = 1
y_1 = 0
x0 = 0

a1 = .9
a2 = -.5
b = 1

for (index in 1:N) {
  if (index == 1) {yt = a1*y0 +a2*y_1 + b*x0}
  if (index == 2) {yt = a1*Y[index-1] + a2*y0 + b*X[index-1]}
  if (index >2 ){yt = a1*Y[index-1] + a2*Y[index-2] + b*X[index-1]}
  Y[index] = yt
```

```
}

plot(SX,type="l",xlab="time")
```




```
plot(Y,type="l",xlab="time")
```



Form the plots we can see that the systems is table.

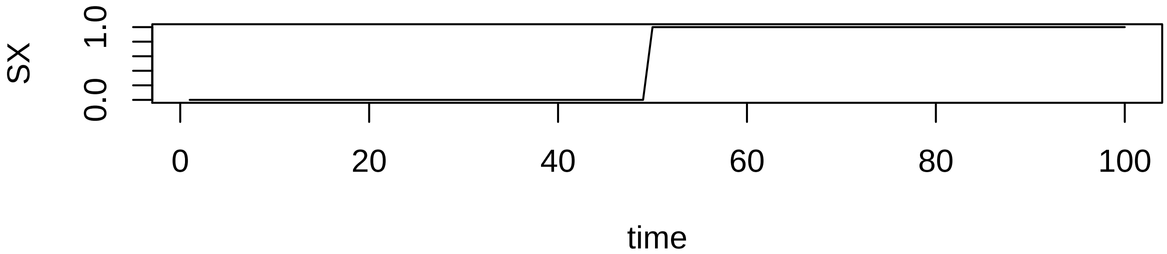
Question 8

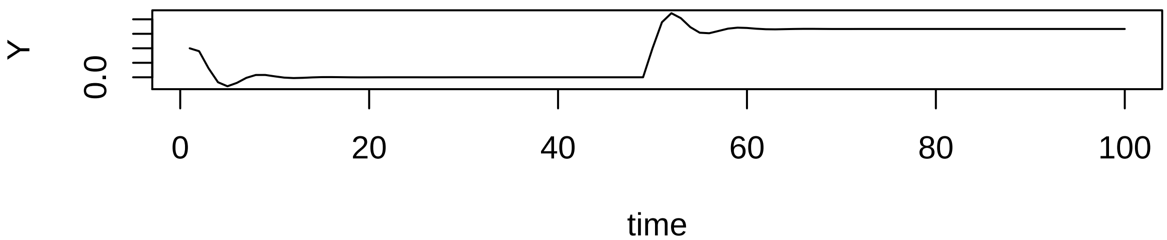
```
setwd("~/gdrive/IUBCourseWork/TimeSeries/codes/")
# State Space model
N=100
```

```r
X = array( 0, dim=c(N,1))
X[N/2]=1

SX=array( 0, dim=c(N,1))
sum = 0
for (index in 1:N) {
  sum = sum + X[index]
  SX[index] = sum
}

X = SX;
X[1,1] = 1;

F  = c(.9,-.5,1,0)
F = array( F, dim = c(2,2))
eigen(F)
```

```
## $values
## [1] 0.45+0.5454356i 0.45-0.5454356i
##
## $vectors
##                        [,1]                  [,2]
## [1,]  0.8164966+0.0000000i  0.8164966+0.0000000i
## [2,] -0.3674235+0.4453463i -0.3674235-0.4453463i
```

```r
Z = c(0,0);
Z = array( Z, dim = c(2,1))
ZM = array( 0, dim = c(2,N))
G = c(1,0)
G = array( G, dim=c(2,1))

ZM[,1] = F%*%Z + G*X[1]

for (time in 2:N) {
  LZ = ZM[,time-1]
  LZ = array( LZ, dim = c(2,1))
  Z = F%*%LZ + G%*%X[time]
  ZM[,time]=Z
}
ZM = t(ZM)

Y = ZM[,1]

layout( matrix( c(1,2), nrow=2, ncol=1, byrow="FALSE"))
plot(SX,type="l",xlab="time")
plot(Y,type="l",xlab="time")
```





The results in the problem 7 match to the results obtained here.

Question 1

P.T.O (next page)