CHAPTER-XII

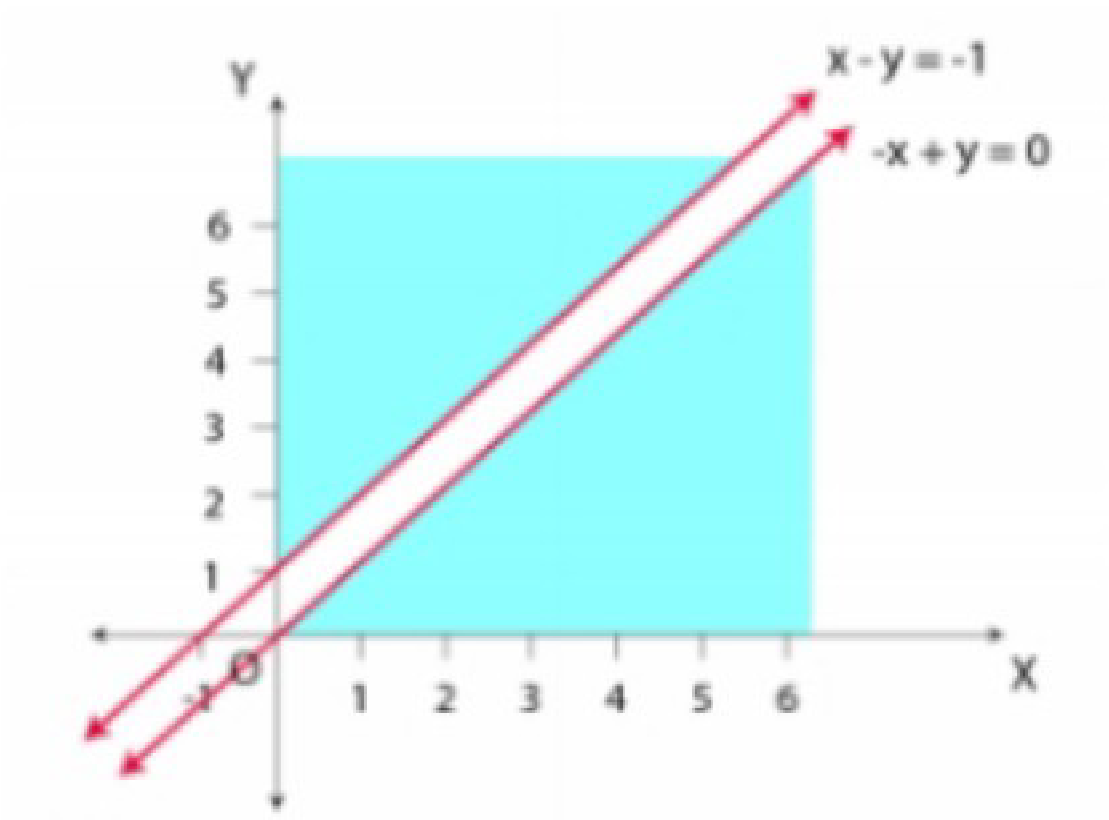
LINEAR PROGRAMMING

2 MARK QUESTIONS

1. Maximise Z = *x* + *y*, subject to $x-y\le -1,\ -x+y\le 0,\ x, y\ge 0$.

Solution:

The region determined by the constraints $x-y\le -1,\ -x+y\le 0,\ x, y\ge 0$ is given below.



There is no feasible region, and therefore, z has no maximum value.

2. How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Solution:

Let the diet contain x and y packets of foods P and Q, respectively. Hence,

$x \geq 0$ and $y \geq 0$

The mathematical formulation of the given problem is given below.

Maximise $z = 6x + 3y$ .... (i)

Subject to the constraints,

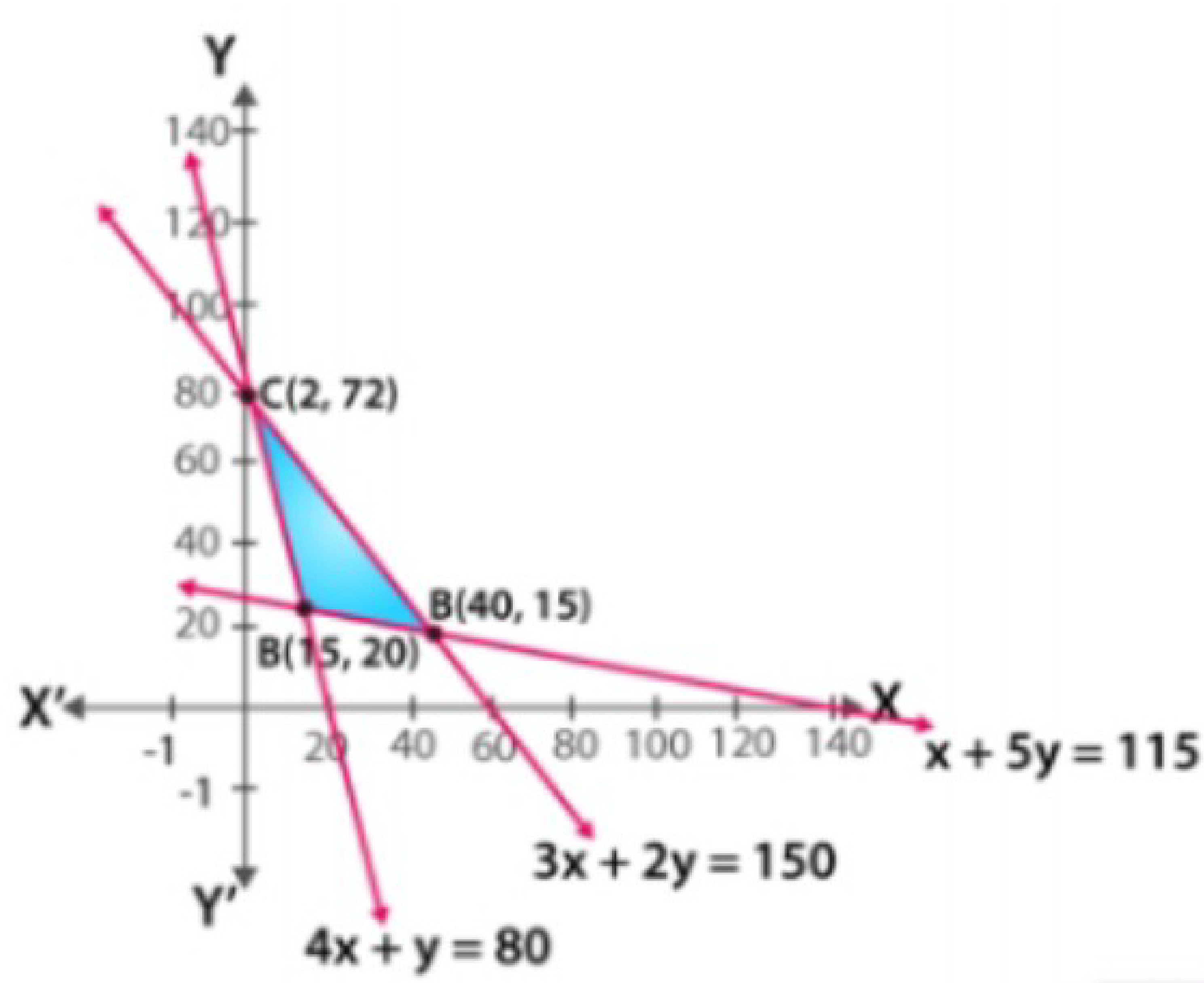
$4x + y \geq 80$ .... (ii)

$x + 5y \geq 115$ .... (iii)

$3x + 2y \leq 150$ .... (iv)

$x, y \geq 0$ .... (v)

The feasible region determined by the system of constraints is given below.



A (15, 20), B (40, 15) and C (2, 72) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 6x + 3y	
A (15, 20)	150	
B (40, 15)	285	Maximum
C (2, 72)	228	

So, the maximum value of z is 285 at (40, 15).

Hence, to maximise the amount of vitamin A in the diet, 40 packets of food P and 15 packets of food Q should be used.

The maximum amount of vitamin A in the diet is 285 units.

3. A farmer mixes two brands, P and Q, of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag, contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag. What is the minimum cost of the mixture per bag?

Solution:

Let the farmer mix x bags of brand P and y bags of brand Q, respectively.

The given information can be compiled in a table, as given below.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Vitamin C (units/kg)	Cost (Rs/kg)
Food P	3	2.5	2	250

Food Q	1.5	11.25	3	200
Requirement (units/kg)	18	45	24	

The given problem can be formulated as given below.

Minimise z = 250x + 200y ………… (i)

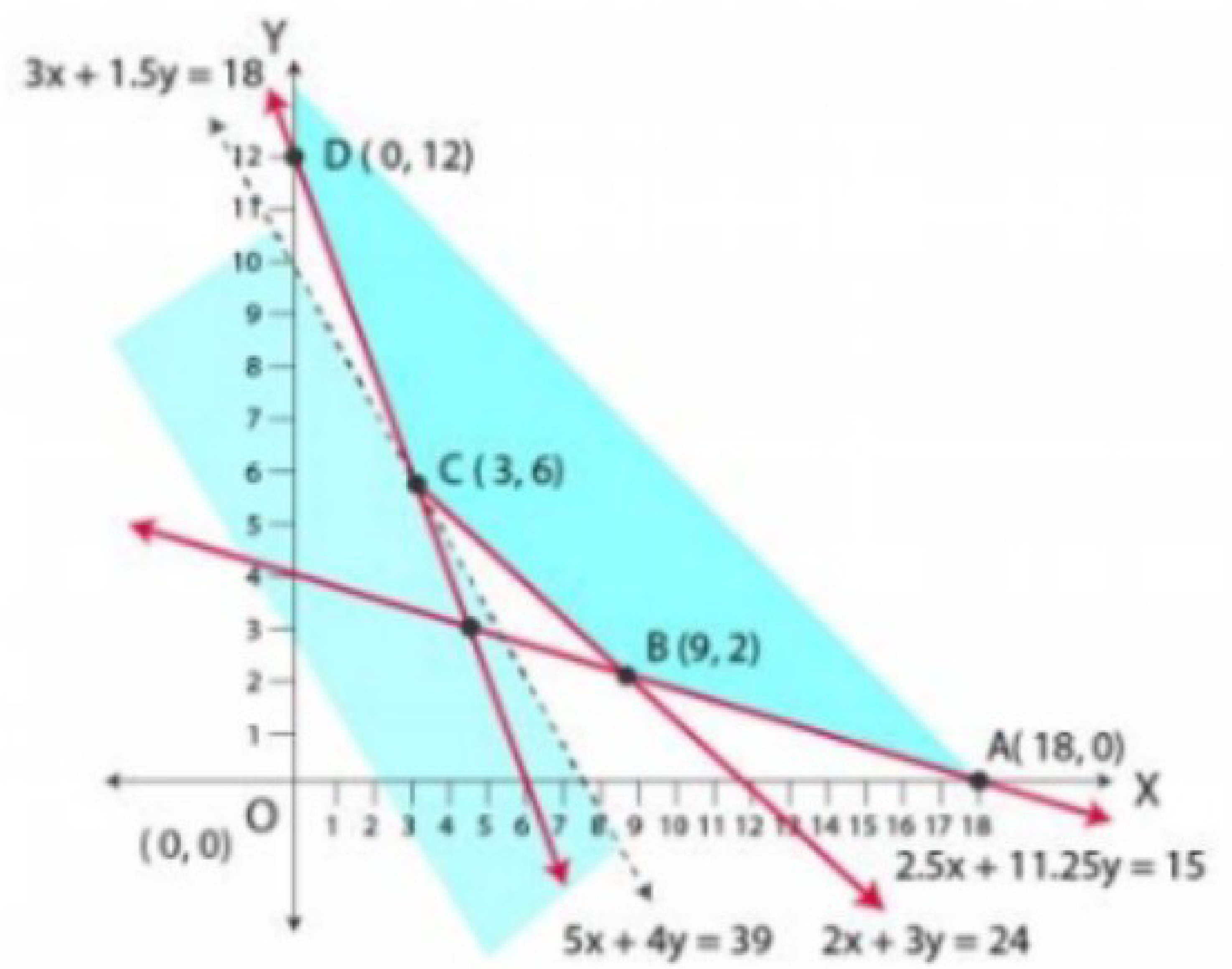
3x + 1.5y ≥ 18 …………. (ii)

2.5x + 11.25y ≥ 45 ……….. (iii)

2x + 3y ≥ 24 ………….. (iv)

x, y ≥ 0 ………. (v)

The feasible region determined by the system of constraints is given below.



A (18, 0), B (9, 2), C (3, 6) and D (0, 12) are the corner points of the feasible region.

The values of z at these corner points are given below.

Corner point	z = 250x + 200y	

A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	Minimum
D (0, 12)	2400	

Here, the feasible region is unbounded; hence, 1950 may or may not be the minimum value of z.

For this purpose, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $5x + 4y < 39$

Hence, at points (3, 6), the minimum value of z is 1950.

Therefore, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimise the cost to Rs 1950.

4.A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximise the profit? It is being given that at least one of each must be produced.

Answer:

Let number of necklaces and bracelets produced by firm per day be x and y, respectively.

Clearly, $x \geq 0, y \geq 0$

$\because$ Total number of necklaces and bracelets that the firm can handle per day is atmost 24.

$\therefore x + y \leq 24$

Since it takes one hour to make a bracelet and half an hour to make a necklace and maximum number of hours available per day is 16.

$\therefore 12x + y \leq 16$

$\Rightarrow x + 2y \leq 32$

Let Z be the profit function.

Then, $Z = 100x + 300y$

$\therefore$ The given LPP reduces to

Maximise $Z = 100x + 300y$ subject to,

$x + y \leq 24$

$x + 2y \leq 32$

and $x, y \geq 0$

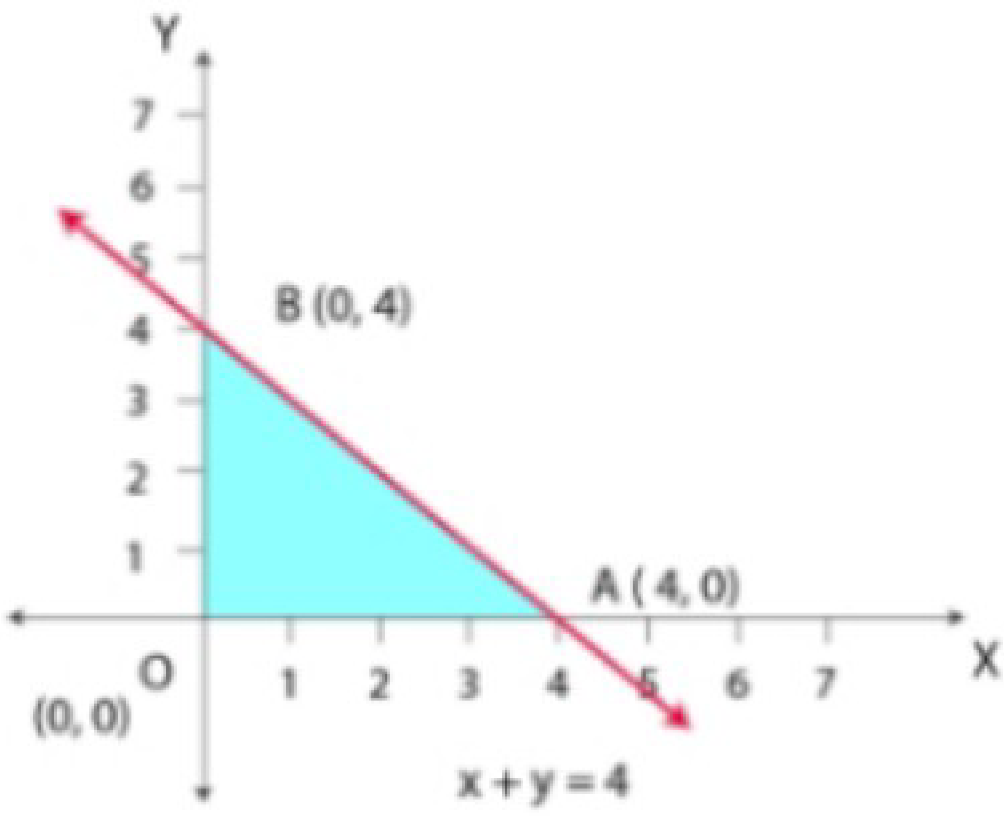
4 MARK QUESTIONS

1. Maximise Z = 3x + 4y

Subject to the constraints: $x + y \leq 4, x \geq 0, y \geq 0.$

Solution:

The feasible region determined by the constraints, $x + y \leq 4$, $x \geq 0$, $y \geq 0$, is given below.



O (0, 0), A (4, 0), and B (0, 4) are the corner points of the feasible region. The values of Z at these points are given below.

Corner point	Z = 3x + 4y	
O (0, 0)	0	
A (4, 0)	12	
B (0, 4)	16	Maximum

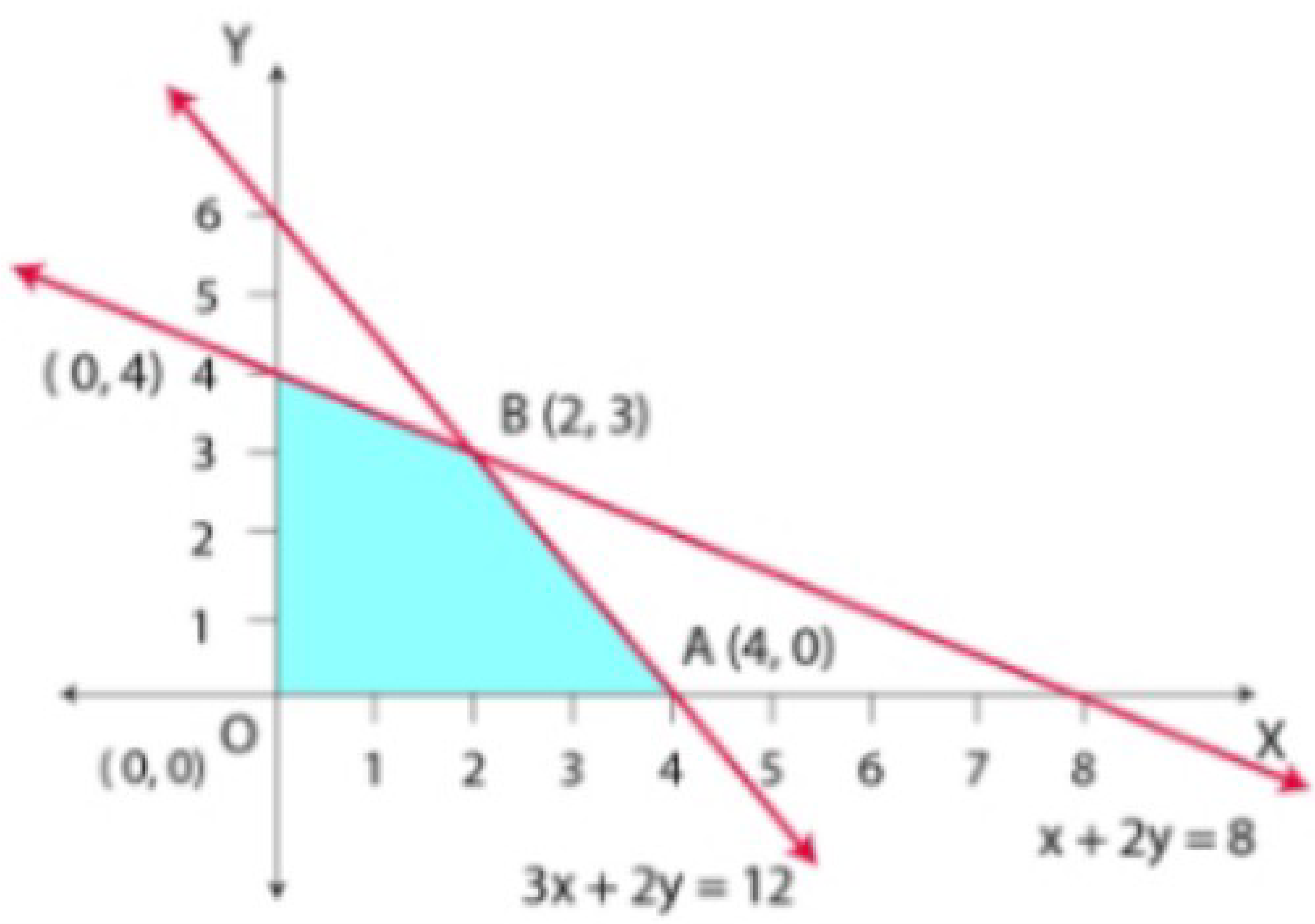
Hence, the maximum value of Z is 16 at the point B (0, 4).

2.Minimise $Z = -3x + 4y$

subject to $x+2y \le 8,\ 3x+2y \le 12,\ x \ge 0,\ y \ge 0$.

Solution:

The feasible region determined by the system of constraints, $x+2y \le 8,\ 3x+2y \le 12,\ x \ge 0,\ y \ge 0$ is given below.



O (0, 0), A (4, 0), B (2, 3) and C (0, 4) are the corner points of the feasible region.

The values of Z at these corner points are given below.

Corner point	Z = − 3x + 4y	
O (0, 0)	0	
A (4, 0)	-12	Minimum
B (2, 3)	6	
C (0, 4)	16	

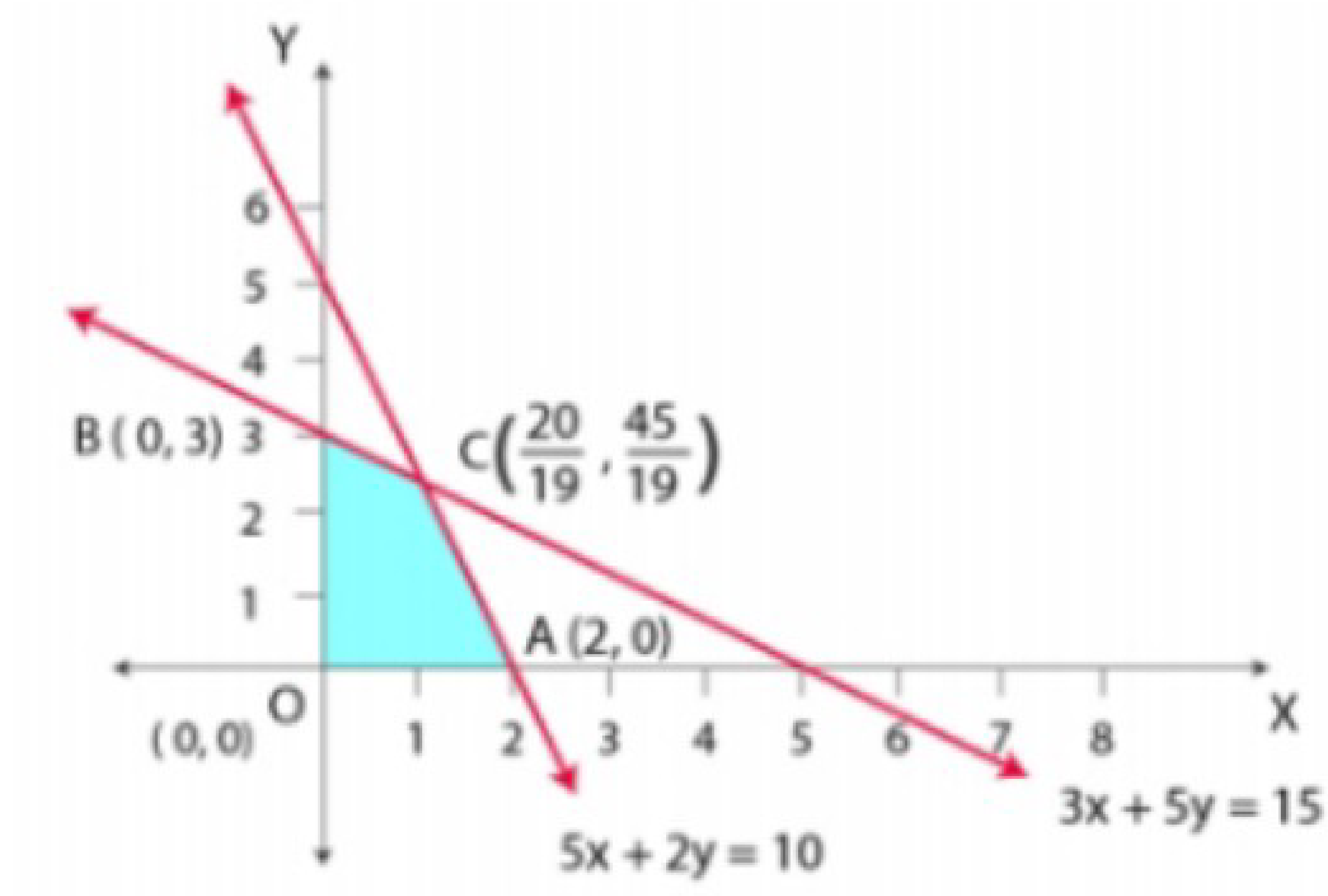
Hence, the minimum value of Z is − 12 at the point (4, 0).

3. Maximise Z = 5x + 3y

subject to $3x+5y\le 15,\ 5x+2y\le 10,\ x\ge 0,\ y\ge 0$.

Solution:

The feasible region determined by the system of constraints, $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, and $y \ge 0$, is given below.



O (0, 0), A (2, 0), B (0, 3) and C (20 / 19, 45 / 19) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = 5x + 3y	
O (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
C (20 / 19, 45 / 19)	235 / 19	Maximum

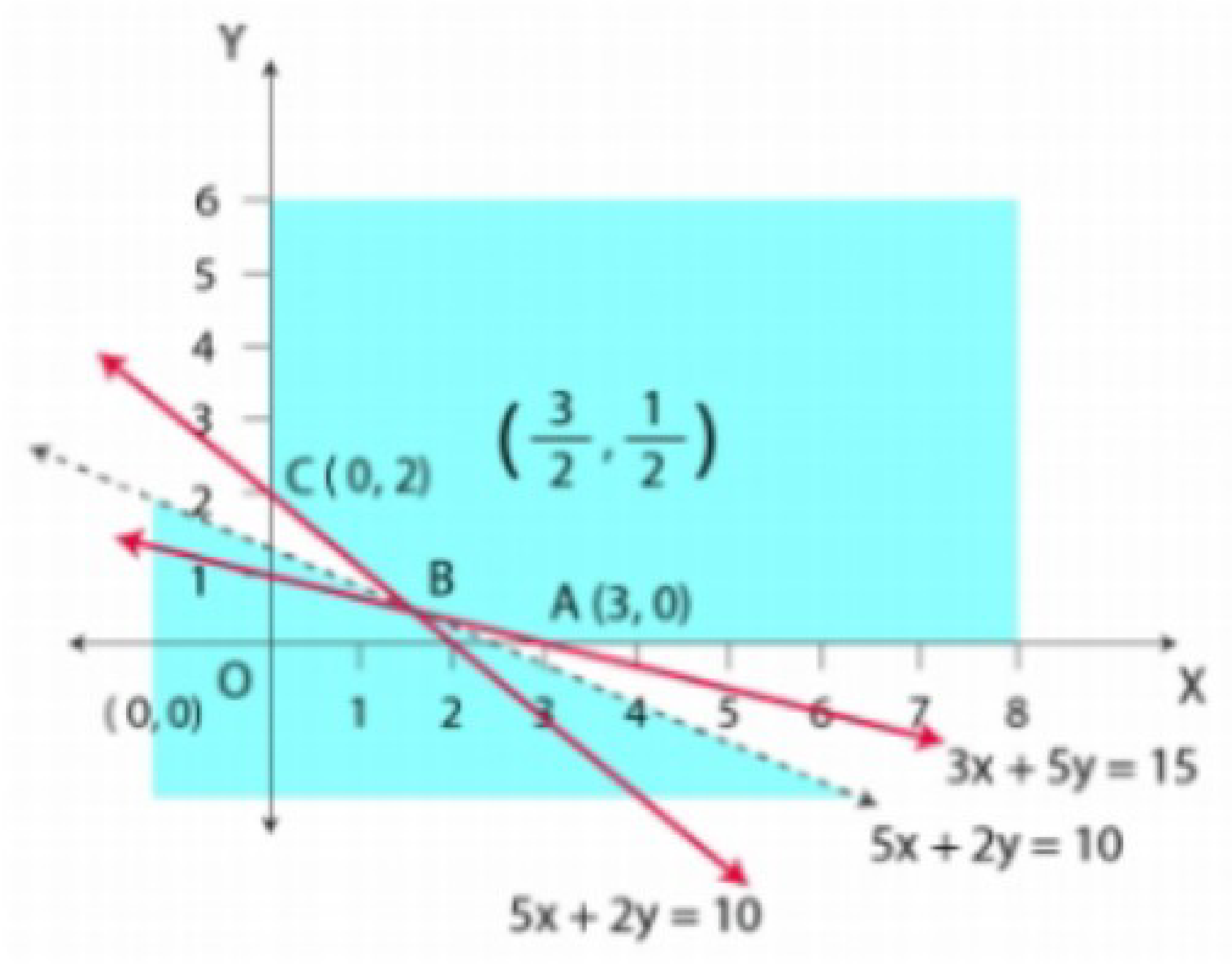
Hence, the maximum value of Z is 235 / 19 at the point (20 / 19, 45 / 19).

4. Minimise Z = 3x + 5y

such that $x+3y \geq 3,\ x+y \geq 2,\ x, y \geq 0$.

Solution:

The feasible region determined by the system of constraints, $x + 3y \geq 3$, $x + y \geq 2$, and $x, y \geq 0$, is given below.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), B (3 / 2, 1 / 2) and C (0, 2).

The values of Z at these corner points are given below.

Corner point	Z = 3x + 5y	
A (3, 0)	9	
B (3 / 2, 1 / 2)	7	Smallest
C (0, 2)	10	

7 may or may not be the minimum value of Z because the feasible region is unbounded.

For this purpose, we draw the graph of the inequality, $3x + 5y < 7$ and check whether the resulting half-plane has common points with the feasible region or not.

Hence, it can be seen that the feasible region has no common point with $3x + 5y < 7$.

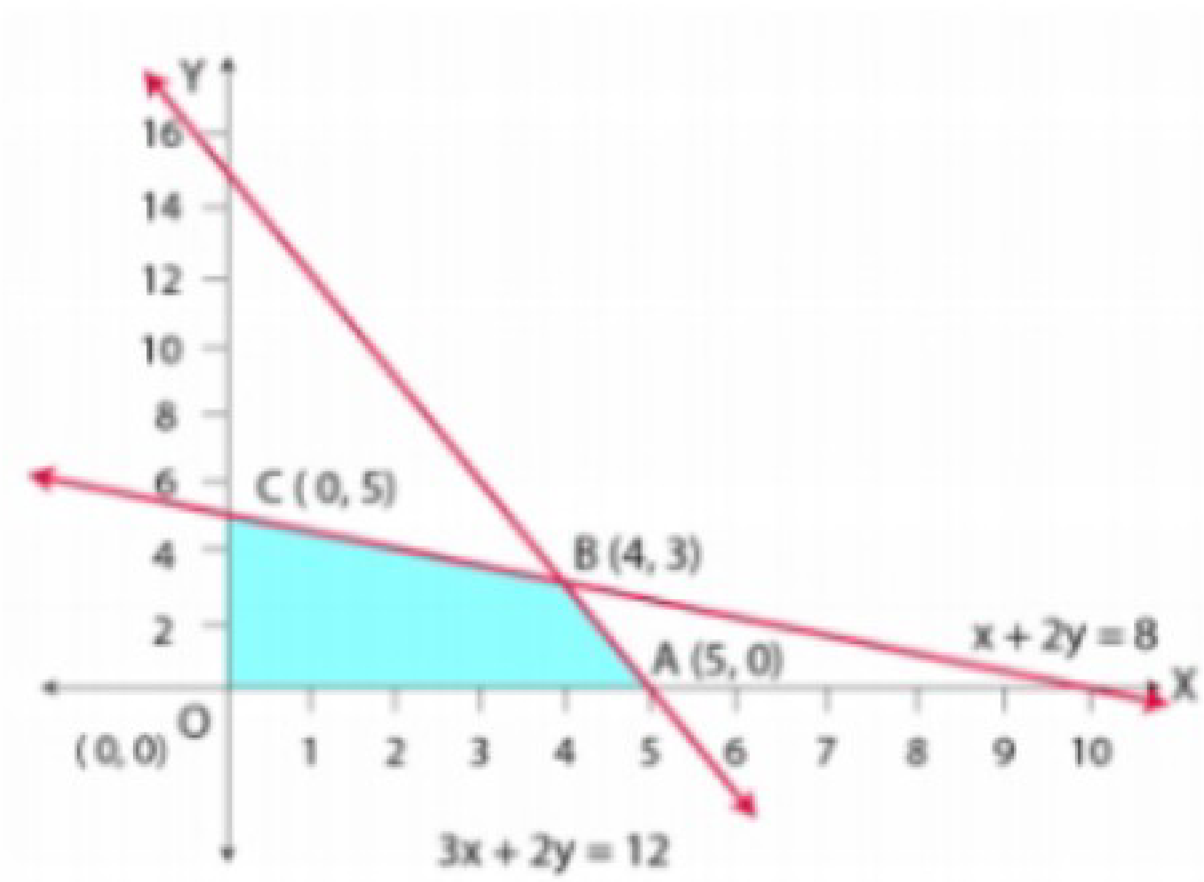
Thus, the minimum value of Z is 7 at point B (3 / 2, 1 / 2).

5. Maximise $Z = 3x + 2y$

subject to $x+2y \le 10, 3x+y \le 15, x, y \ge 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \le 10$, $3x + y \le 15$, $x \ge 0$, and $y \ge 0$, is given below.



A (5, 0), B (4, 3), C (0, 5] and D (0, 0) are the corner points of the feasible region.

The values of Z at these corner points are given below.

Corner point	Z = 3x + 2y	
A (5, 0)	15	

B (4, 3)	18	Maximum
C (0, 5)	10	

Hence, the maximum value of Z is 18 at points (4, 3).

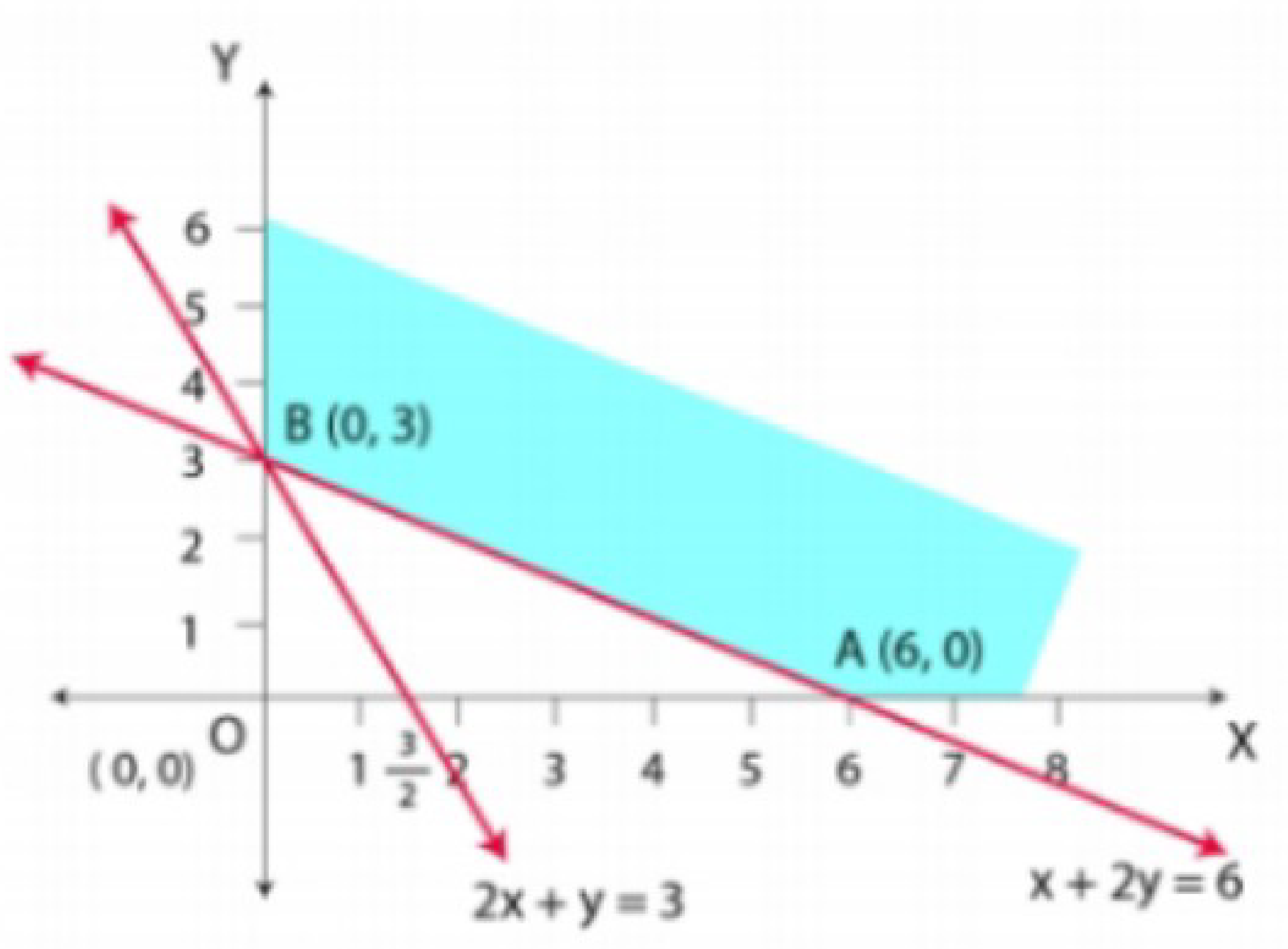
6. Minimise Z = $x + 2y$

subject to

$2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$, is given below.



A (6, 0) and B (0, 3) are the corner points of the feasible region.

The values of Z at the corner points are given below.

Corner point	Z = x + 2y

A (6, 0)	6
B (0, 3)	6

Here, the values of Z at points A and B are same. If we take any other point, such as (2, 2) on line $x + 2y = 6$, then $Z = 6$.

Hence, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line $x + 2y = 6$.

7 MARK QUESTIONS

1: Solve the following LPP graphically:

Maximise Z = 2x + 3y, subject to x + y ≤ 4, x ≥ 0, y ≥ 0

Solution:

Let us draw the graph pf x + y =4 as below.

The shaded region (OAB) in the above figure is the feasible region determined by the system of constraints x ≥ 0, y ≥ 0 and x + y ≤ 4.

The feasible region OAB is bounded and the maximum value will occur at a corner point of the feasible region.

Corner Points are O(0, 0), A (4, 0) and B (0, 4).

Evaluate Z at each of these corner points.

Corner Point	Value of Z
O(0, 0)	2 (0) + 3(0) = 0

A (4, 0)	2 (4) + 3(0) = 8
B (0, 4)	2 (0) + 3 (4)= 12 ← maximum

Hence, the maximum value of Z is 12 at the point (0, 4).

2: Solve the following linear programming problem graphically:

Minimise Z = 200 x + 500 y subject to the constraints:

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

Solution:

Given,

Minimise Z = 200 x + 500 y … (1)

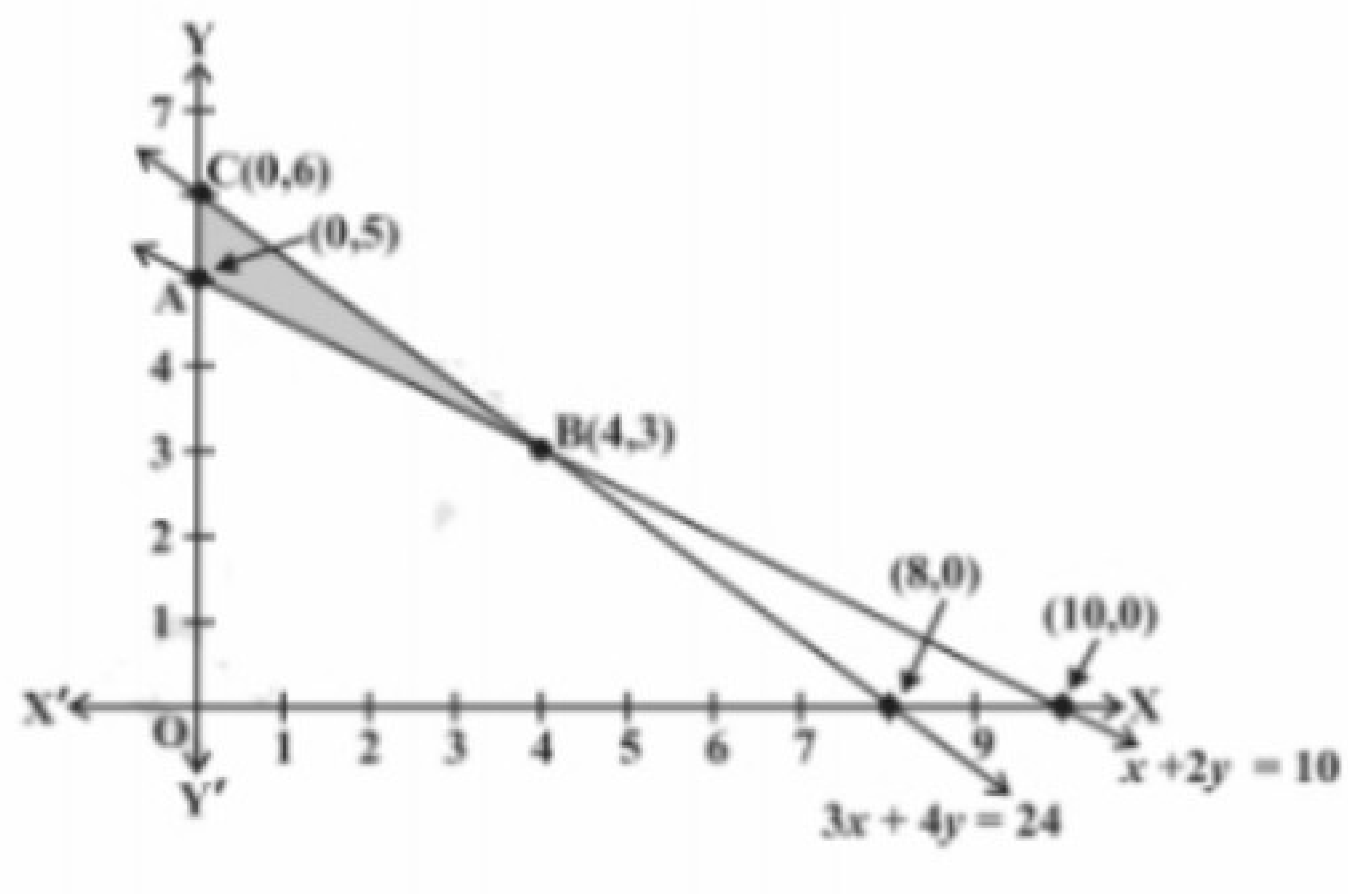
subject to the constraints:

$x + 2y \geq 10$ … (2)

$3x + 4y \leq 24$ … (3)

$x \geq 0, y \geq 0$ … (4)

Let us draw the graph of $x + 2y = 10$ and $3x + 4y = 24$ as below.



The shaded region in the above figure is the feasible region ABC determined by the

system of constraints (2) to (4), which is bounded. The coordinates of corner point A, B and C are (0,5), (4,3) and (0,6) respectively.

Calculation of Z = 200x + 500y at these points.

Corner point	Value of Z
(0, 5)	2500
(4, 3)	2300← Minimum
(0, 6)	3000

Hence, the minimum value of Z is 2300 is at the point (4, 3).

3: A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each type should be produced so that the company has a maximum profit? Formulate this problem as a LPP given that the objective is to maximise the profit.

Solution:

Let x and y denote, respectively, the number of black and white sets and coloured sets made each week.

Thus $x \geq 0, y \geq 0$

The company can make at most 300 sets a week, therefore, $x + y \leq 300$.

Weekly cost (in Rs) of manufacturing the set is $1800x + 2700y$ and the company can spend up to Rs. 648000.

Therefore, $1800x + 2700y \leq 648000$

or

$2x + 3y \leq 720$

The total profit on x black and white sets and y coloured sets is Rs $(510x + 675y)$.

Let the objective function be $Z = 510x + 675y$.

Therefore, the mathematical formulation of the problem is as follows.

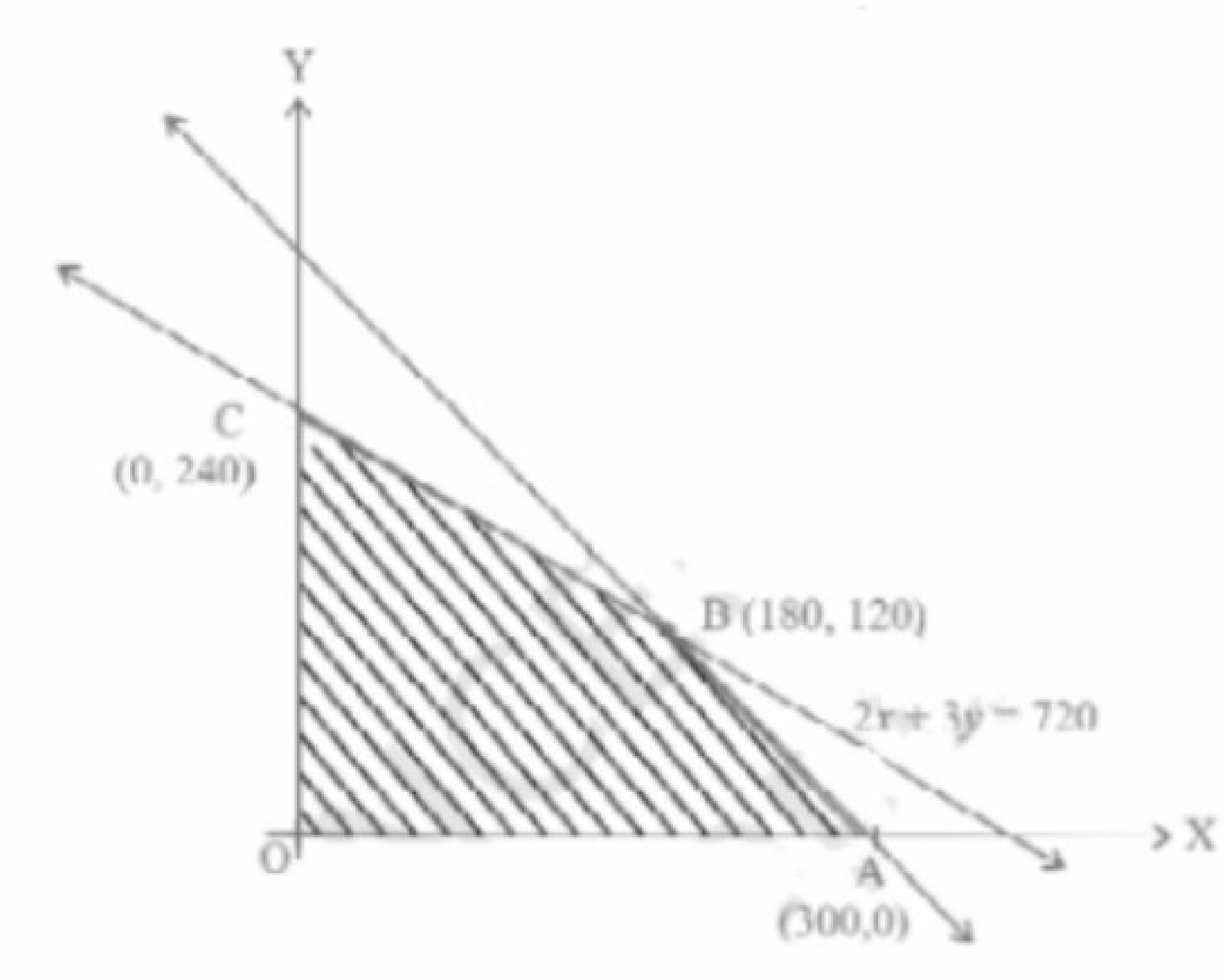
Maximise $Z = 510x + 675y$ subject to the constraints :

$x + y \leq 300$

$2x + 3y \leq 720$

$x \geq 0, y \geq 0$

The graph of x + y = 30 and 2x + 3y = 720 is given below.



Corner point	Value of Z
A(300, 0)	153000
B(180, 120)	172800 = Maximum
C(0, 240)	162000

Hence, the maximum profit will occur when 180 black & white sets and 120 coloured sets are produced.

4: A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

Solution:

Let the mixture contain x kg of Food 'I' and y kg of Food 'II'.

Clearly, $x \geq 0, y \geq 0$.

Tabulate the given data as below.

Resources	Food		Requirement
	I (x)	II (y)	
Vitamin A (units/kg)	2	1	8
Vitamin C (units/kg)	1	2	10
Cost (Rs/kg)	50	70	

Given that, the mixture must contain at least 8 units of vitamin A and 10 units of vitamin C.

Thus, the constraints are:

$2x + y \geq 8$

$x + 2y \geq 10$

Total cost Z of purchasing x kg of food 'I' and y kg of Food 'II' is $Z = 50x + 70y$

Hence, the mathematical formulation of the problem is:

Minimise Z = 50x + 70y ... (1)

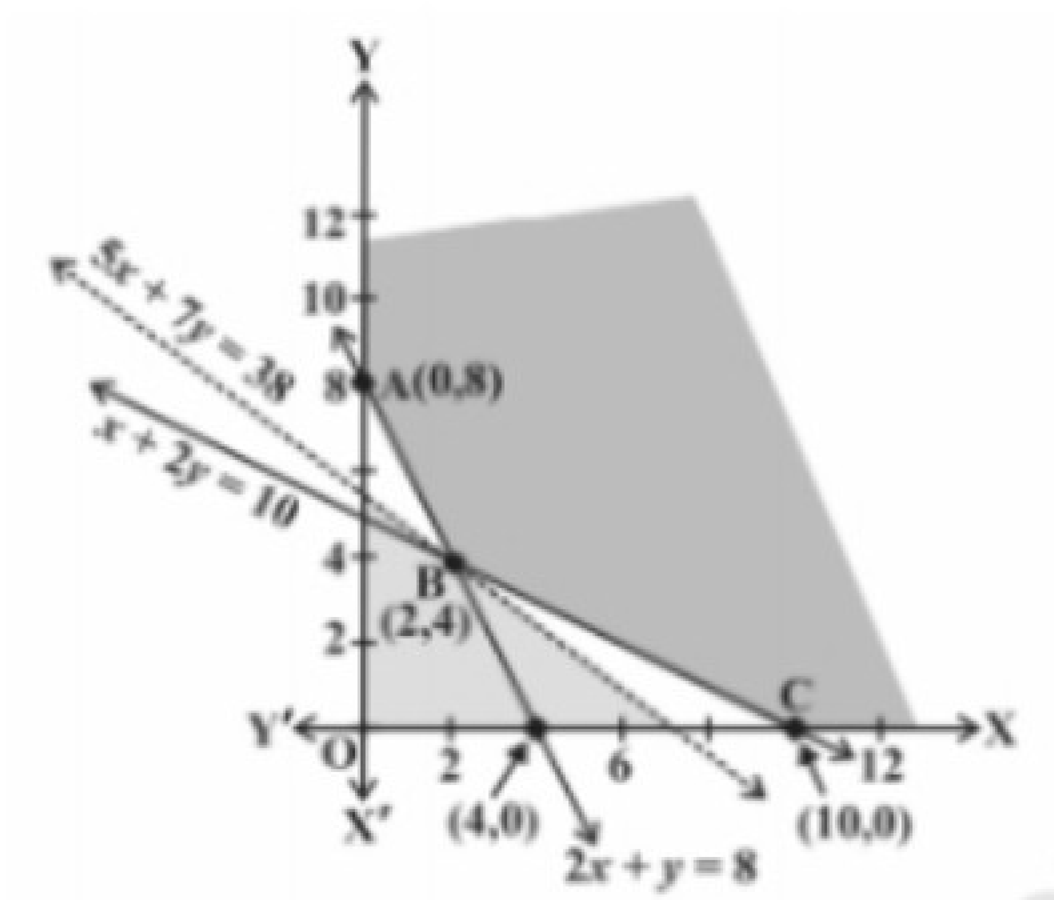
subject to the constraints:

$2x + y \geq 8$ ... (2)

$x + 2y \geq 10$ ... (3)

$x, y \geq 0$ ... (4)

Let us draw the graph of $2x + y = 8$ and $x + 2y = 10$ as given below.



Here, observe that the feasible region is unbounded.

Let us evaluate the value of Z at the corner points A(0,8), B(2,4) and C(10,0).

Corner point	Value of Z
A(0, 8)	560
B(2, 4)	380 = Minimum
C(10, 0)	500

Therefore, the minimum value of Z is 380 obtained at the point (2, 4).

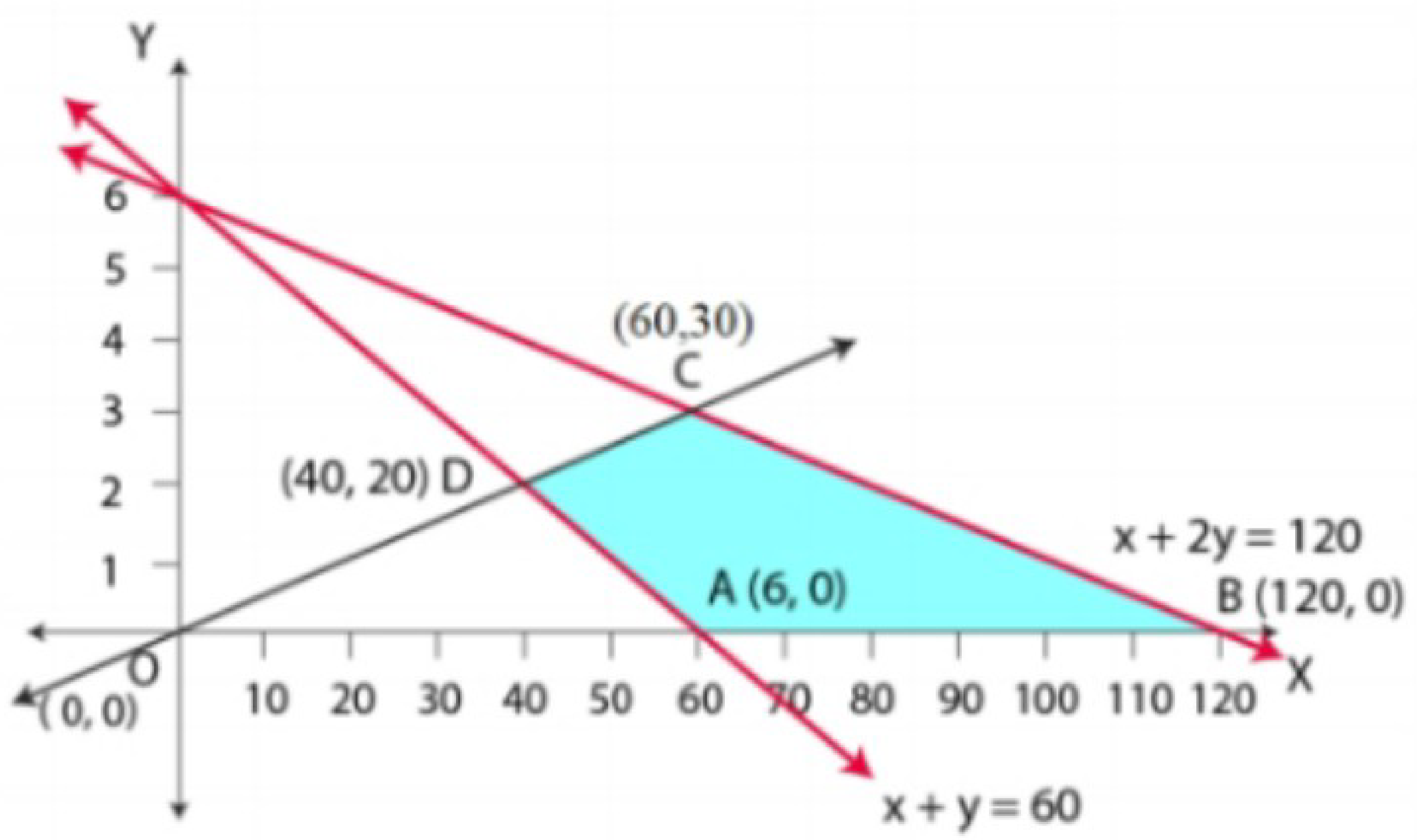
Hence, the optimal mixing strategy for the dietician would be to mix 2 kg of Food 'I' and 4 kg of Food 'II', and with this strategy, the minimum cost of the mixture will be Rs 380.

5. Minimise and Maximise Z = 5x + 10y

subject to $x+2y \le 120, x+y \ge 60, x-2y \ge 0, x, y \ge 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$, is given below.



A (60, 0), B (120, 0), C (60, 30), and D (40, 20) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = 5x + 10y	
A (60, 0)	300	Minimum
B (120, 0)	600	Maximum

156

C (60, 30)	600	Maximum
D (40, 20)	400	

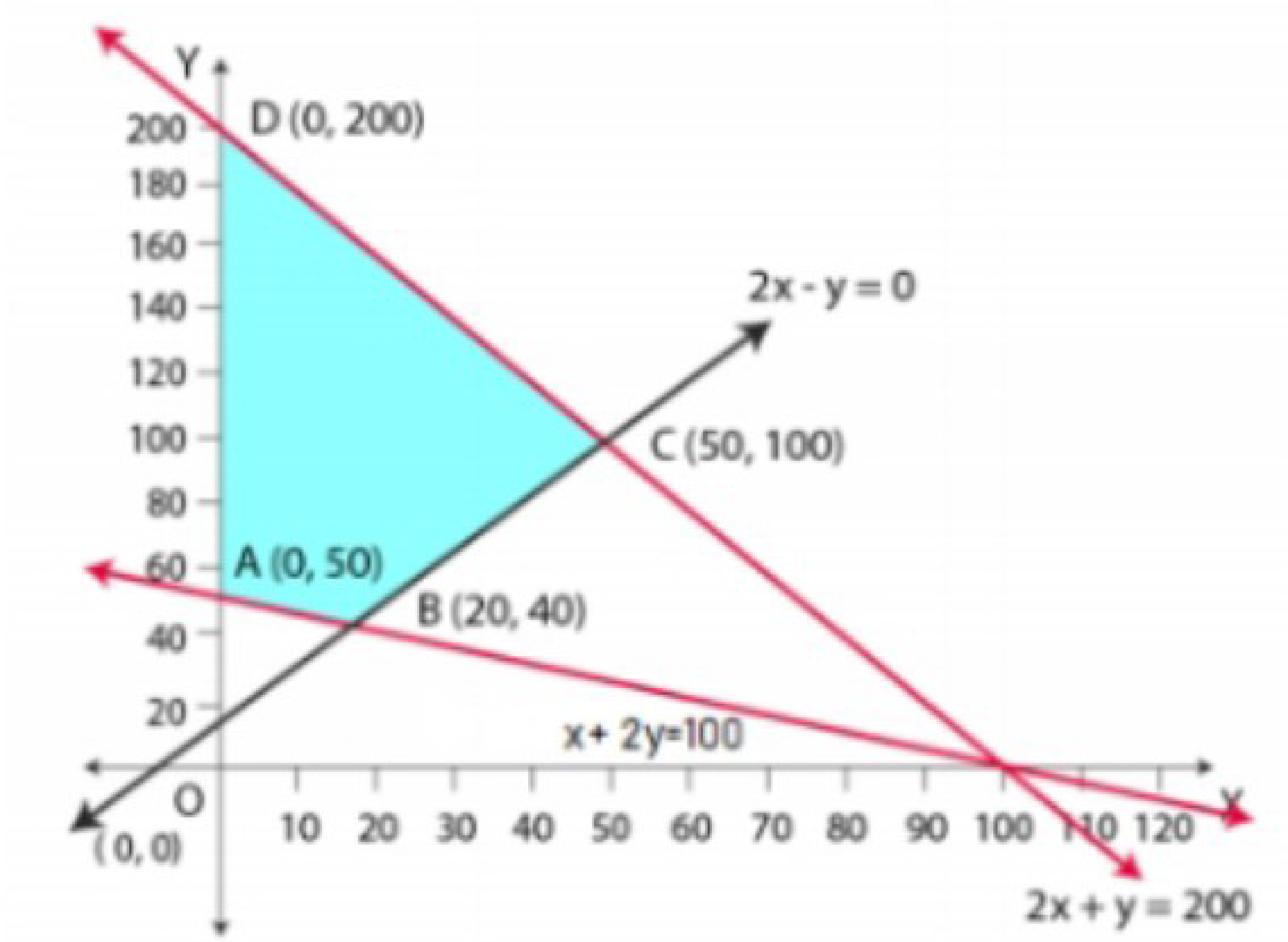
The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

6. Minimise and Maximise Z = x + 2y

subject to $x+2y \geq 100,\ 2x-y \leq 0,\ 2x+y \leq 200;\ x, y \geq 0$.

Solution:

The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is given below.



A (0, 50), B (20, 40), C (50, 100) and D (0, 200) are the corner points of the feasible region. The values of Z at these corner points are given below.

Corner point	Z = x + 2y	
A (0, 50)	100	Minimum

B (20, 40)	100	Minimum
C (50, 100)	250	
D (0, 200)	400	Maximum

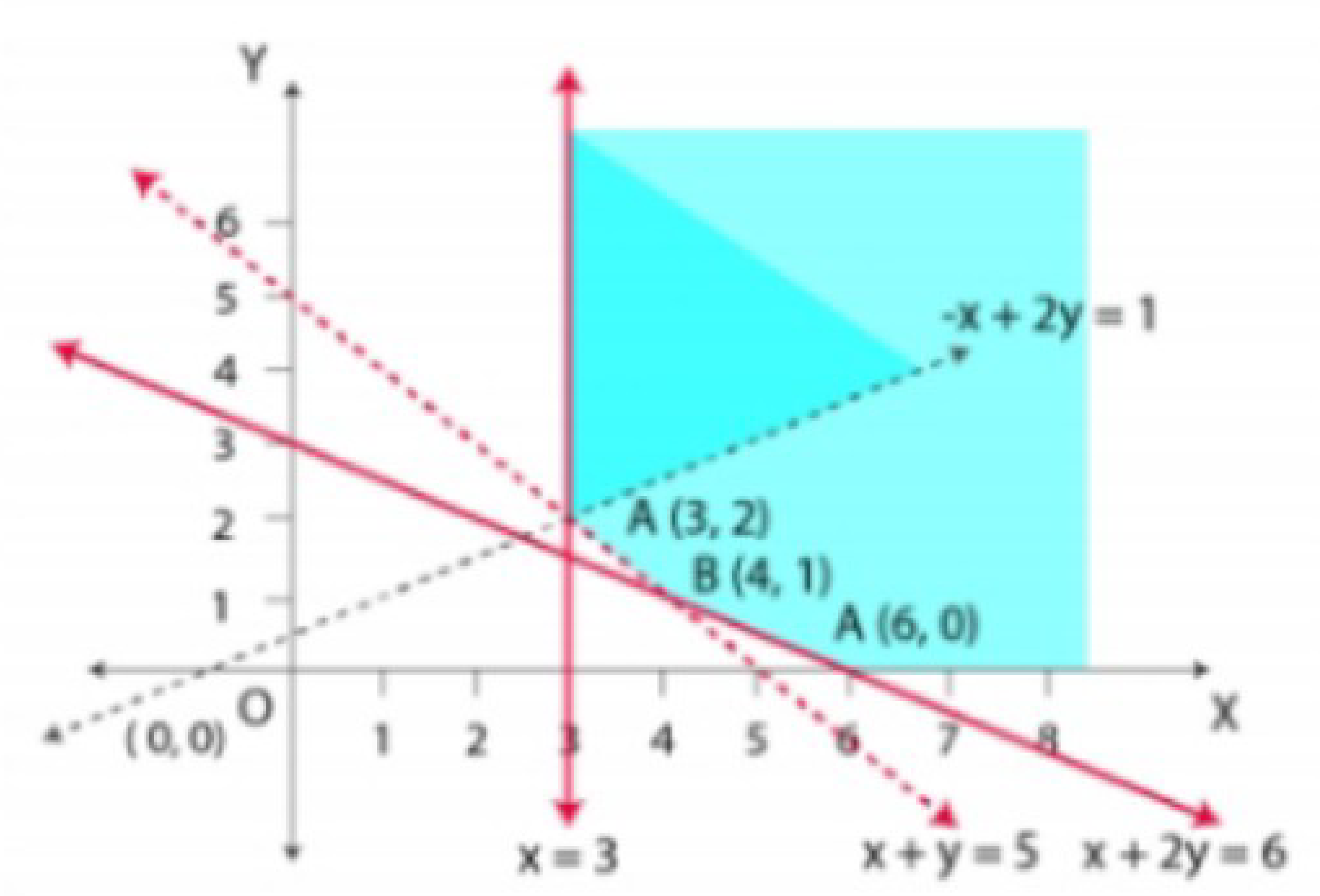
The maximum value of Z is 400 at points (0, 200), and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

7. Maximise Z = − *x* + 2*y*, subject to the constraints.

$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

Solution:

The feasible region determined by the constraints, $x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$ is given below.



Here, it can be seen that the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1) and C (3, 2) are given below.

Corner point	Z = − x + 2y

A (6, 0)	Z = − 6
B (4, 1)	Z = − 2
C (3, 2)	Z = 1

Since the feasible region is unbounded, z = 1 may or may not be the maximum value.

For this purpose, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, the resulting feasible region has points in common with the feasible region.

Hence, z = 1 is not the maximum value.

Z has no maximum value.

8.Reshma wishes to mix two types of food, P and Q, in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A, and 11 units of vitamin B. Food P costs Rs 60/kg, and Food Q costs Rs 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?

Solution:

Let the mixture contain x kg of food P and y kg of food Q.

Hence, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Vitamin A (units/kg)	Vitamin B (units/kg	Cost (Rs/kg)
Food P	3	5	60

Food Q	4	2	80
Requirement (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B. Hence, the constraints are

$3x + 4y \geq 8$

$5x + 2y \geq 11$

The total cost of purchasing food is $Z = 60x + 80y$.

So, the mathematical formulation of the given problem can be written as

Minimise $Z = 60x + 80y$ (i)

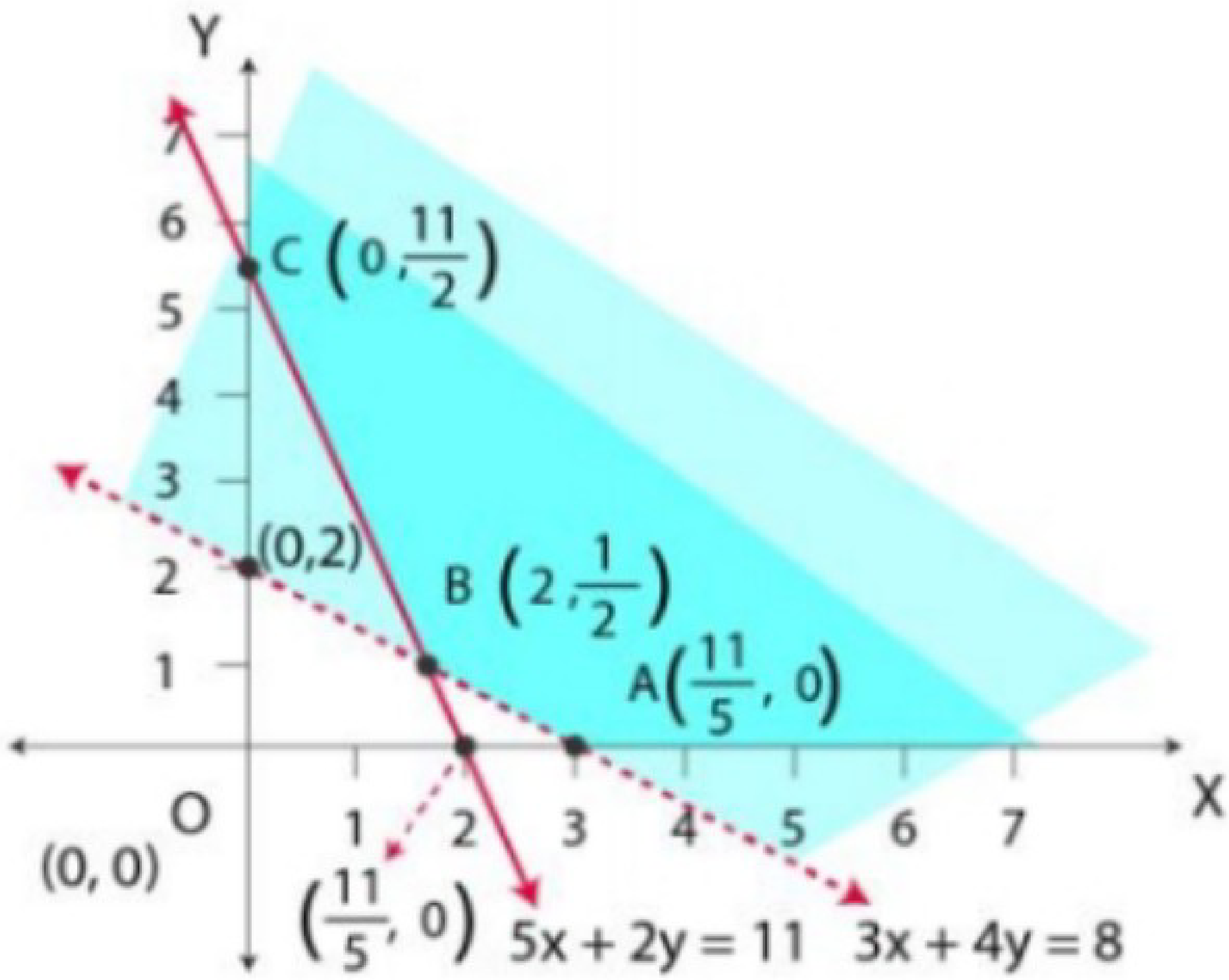
Now, subject to the constraints,

$3x + 4y \geq 8$ ... (2)

$5x + 2y \geq 11$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is given below.



Clearly, we can see that the feasible region is unbounded.

A (8 / 3, 0), B (2, 1 / 2) and C (0, 11 / 2)

The values of Z at these corner points are given below.

Corner point	Z = 60x + 80 y	
A (8 / 3, 0)	160	Minimum
B (2, 1 / 2)	160	Minimum
C (0, 11 / 2)	440	

Here, the feasible region is unbounded; therefore, 160 may or may not be the minimum value of Z.

For this purpose, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $3x + 4y < 8$.

Hence, at the line segment joining the points (8 / 3, 0) and (2, 1 / 2), the minimum cost of the mixture will be Rs 160.

9.A factory manufacturers two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.

Solution:

On each day, let the factory manufacture x screws of type A and y screws of type B.

Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	$4 \times 60 = 240$
Hand Operated Machine (min)	6	3	$4 \times 60 = 240$

The profit on a package of screws A is Rs 7 and on the package screws, B is Rs 10.

Hence, the constraints are

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem can be written as

Maximise $Z = 7x + 10y$ .... (i)

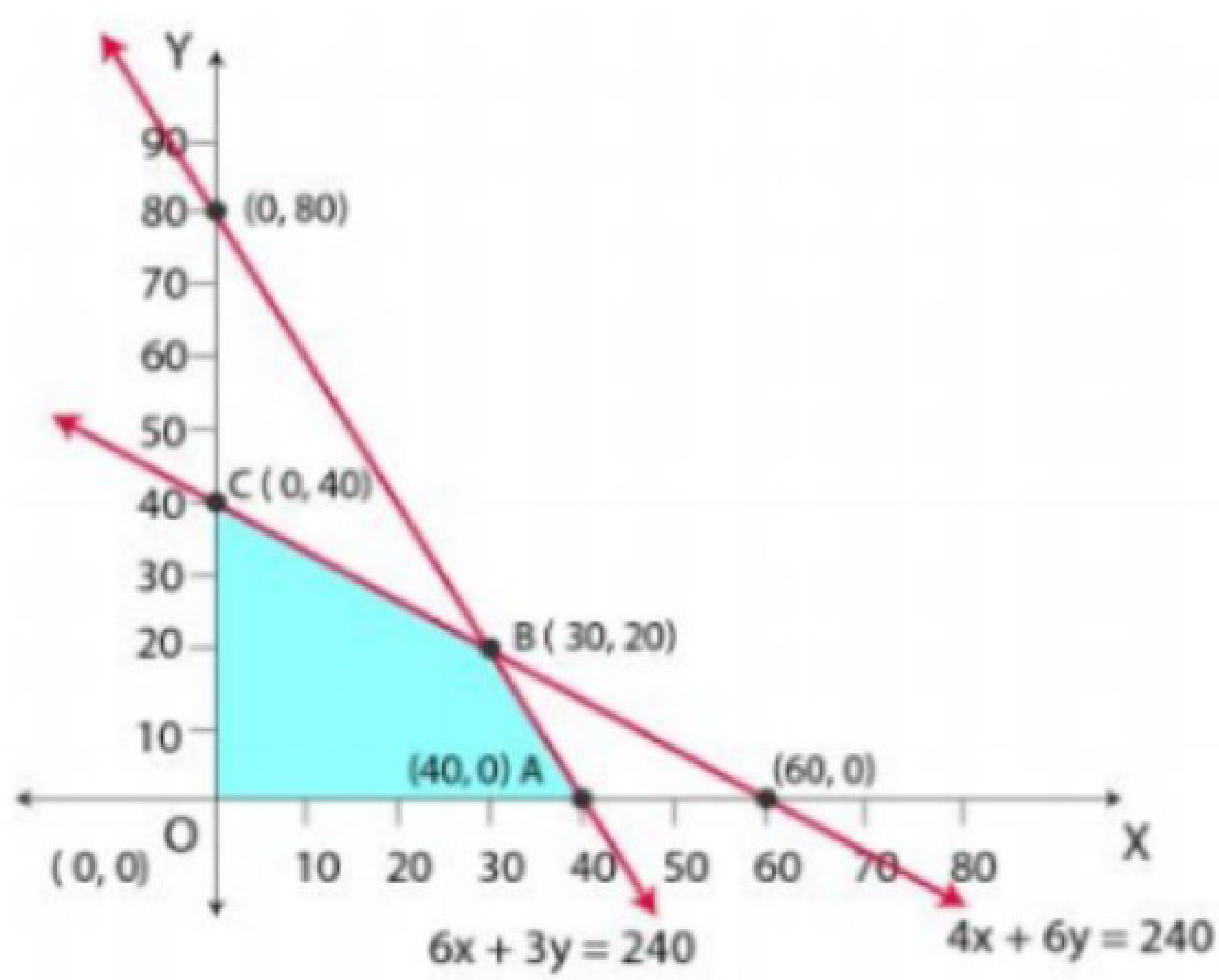
Subject to the constraints,

$4x + 6y \leq 240$ .... (ii)

$6x + 3y \leq 240$ .... (iii)

$x, y \geq 0$ .... (iv)

The feasible region determined by the system of constraints is given below.



A (40, 0), B (30, 20) and C (0, 40) are the corner points.

The value of Z at these corner points is given below.

Corner point	Z = 7x + 10y
A (40, 0)	280

B (30, 20)	410	Maximum
C (0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Hence, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

10.A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shade that he produces, how should he schedule his daily production in order to maximise his profit?

Solution:

Let the cottage industry manufacture x pedestal lamps and y wooden shades, respectively.

Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Hence, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, $Z = 5x + 3y$ ……………… (i)

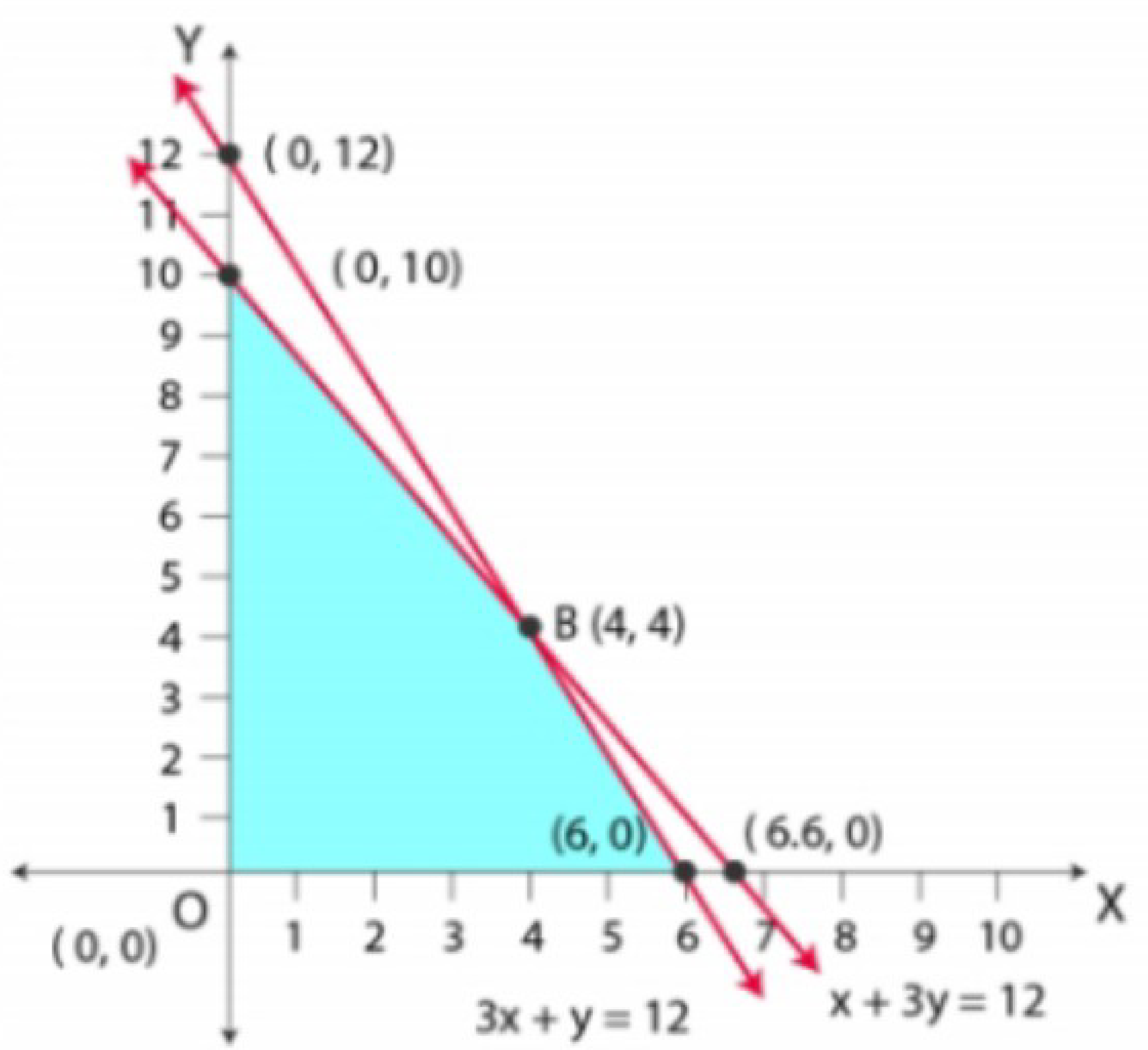
Subject to the constraints,

$2x + y \leq 12$ …………. (ii)

$3x + 2y \leq 20$ ……….. (iii)

$x, y \geq 0$ …………. (iv)

The feasible region determined by the system of constraints is given below.



A (6, 0), B (4, 4) and C (0, 10) are the corner points.

The value of Z at these corner points is given below.

Corner point	$Z = 5x + 3y$	

A (6, 0)	30	
B (4, 4)	32	Maximum
C (0, 10)	30	

The maximum value of Z is 32 at points (4, 4).

Therefore, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximise his profits.

11. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

Solution:

Let the company manufacture x souvenirs of type A and y souvenirs of type B, respectively.

Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Hence, the constraints are

$5x + 8y \le 200$

$10x + 8y \le 240$ i.e.,

$5x + 4y \le 120$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem can be written as

Maximise $Z = 5x + 6y$ …………… (i)

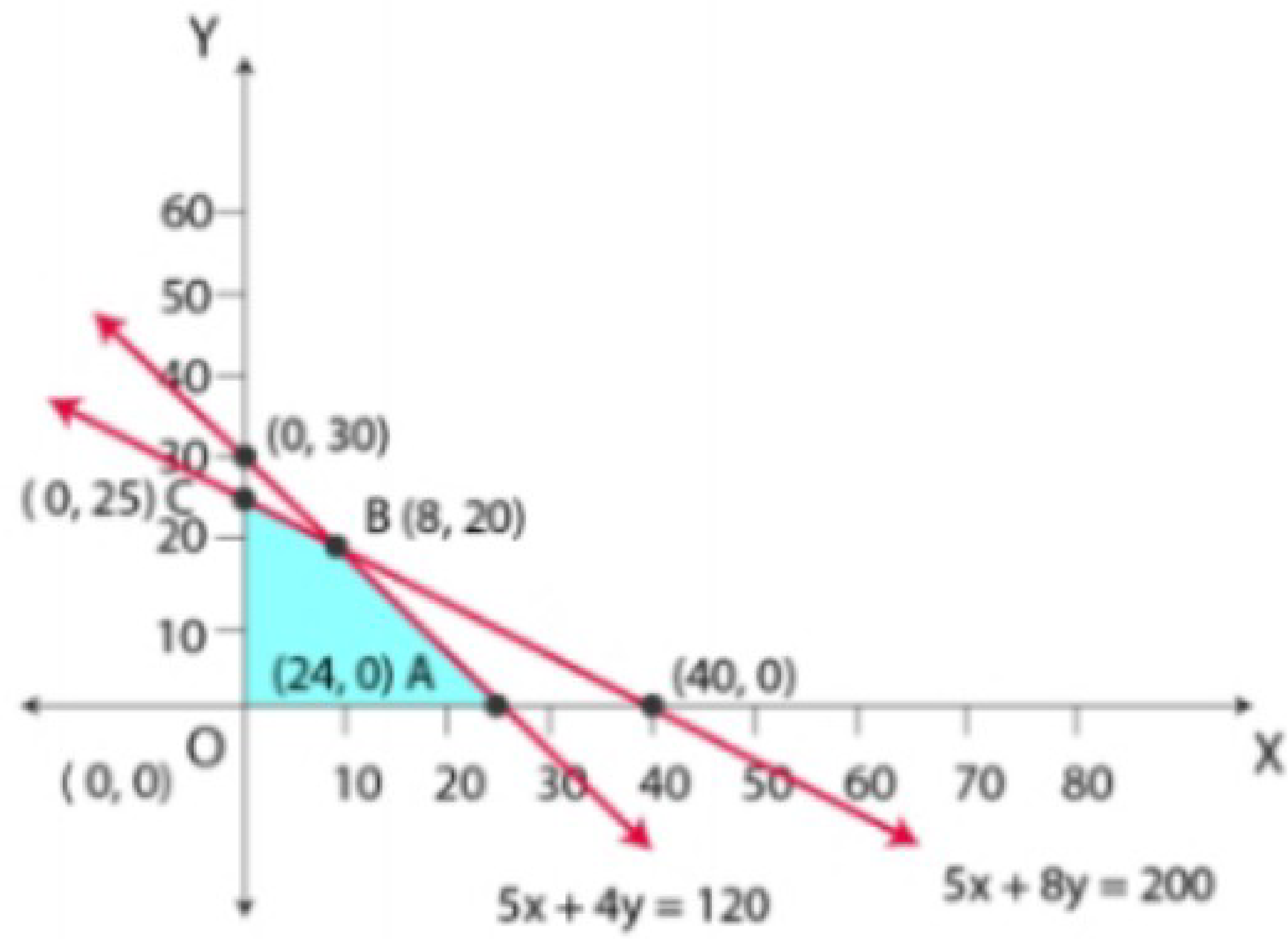
Subject to the constraints,

$5x + 8y \le 200$ ……………. (ii)

$5x + 4y \le 120$ ………….. (iii)

$x, y \ge 0$ …………… (iv)

The feasible region determined by the system of constraints is given below.



A (24, 0), B (8, 20) and C (0, 25) are the corner points.

The values of Z at these corner points are given below.

Corner point	$Z = 5x + 6y$	
A (24, 0)	120	
B (8, 20)	160	Maximum
C (0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Hence, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

12.A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25,000 and Rs 40,000, respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4,500 and on the portable model is Rs 5,000.

Solution:

Let the merchant stock x desktop models and y portable models, respectively.

Hence,

$x \geq 0$ and $y \geq 0$

Given that the cost of desktop model is Rs 25,000 and of a portable model is Rs 40,000.

However, the merchant can invest a maximum of Rs 70 lakhs.

Hence, $25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

The monthly demand of computers will not exceed 250 units.

Hence, $x + y \leq 250$

The profit on a desktop model is 4500, and the profit on a portable model is Rs 5000.

Total profit, $Z = 4500x + 5000y$

Therefore, the mathematical formulation of the given problem is

Maximum $Z = 4500x + 5000y$ ………… (i)

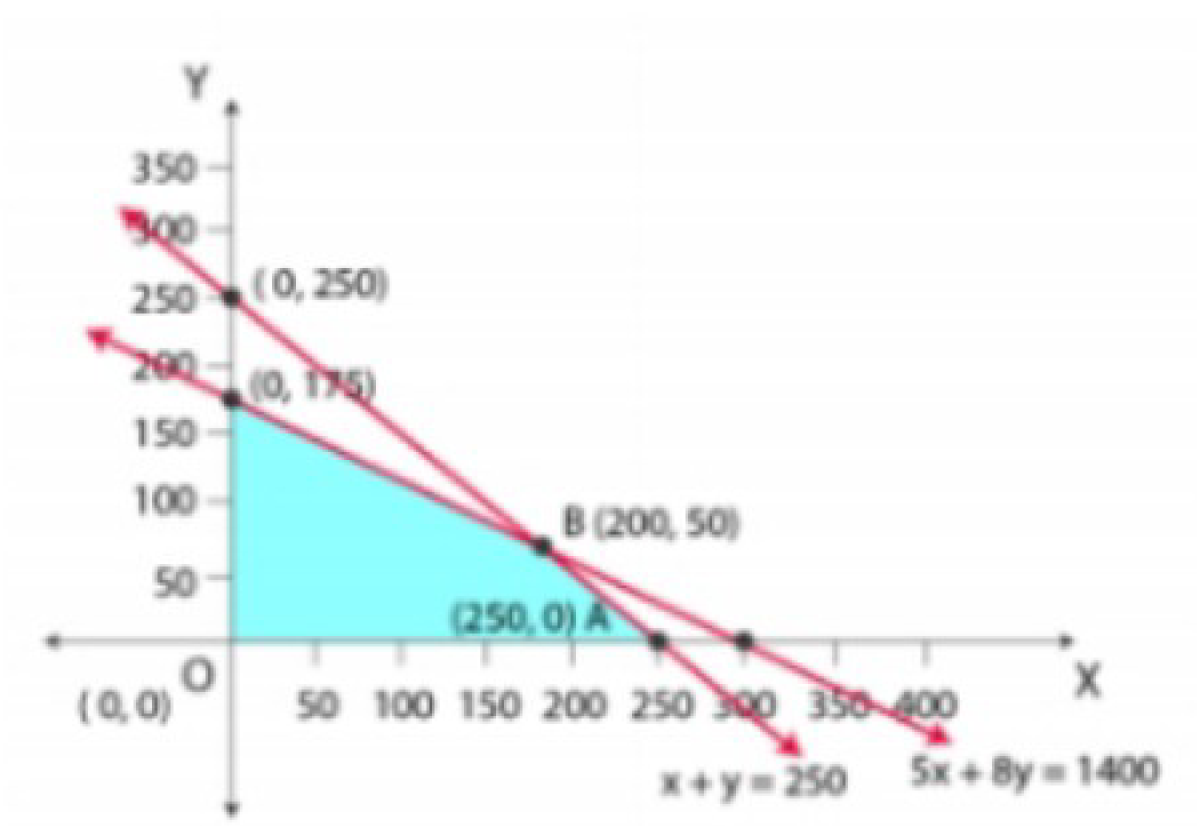
Subject to the constraints,

$5x + 8y \leq 1400$ ………… (ii)

$x + y \leq 250$ …………… (iii)

$x, y \geq 0$ ………… (iv)

The feasible region determined by the system of constraints is given below.



A (250, 0), B (200, 50) and C (0, 175) are the corner points.

The values of Z at these corner points are given below.

Corner point	$Z = 4500x + 5000y$	

A (250, 0)	1125000	
B (200, 50)	1150000	Maximum
C (0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Therefore, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 11,50,000.

13. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for the diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Solution:

Let the diet contain x units of food F_1 and y units of food F_2. Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Vitamin A (units)	Mineral (units)	Cost per unit (Rs)

Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4 per unit, and of food, F_2 is Rs 6 per unit.

Hence, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

The total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem can be written as

Minimise $Z = 4x + 6y$ …………… (i)

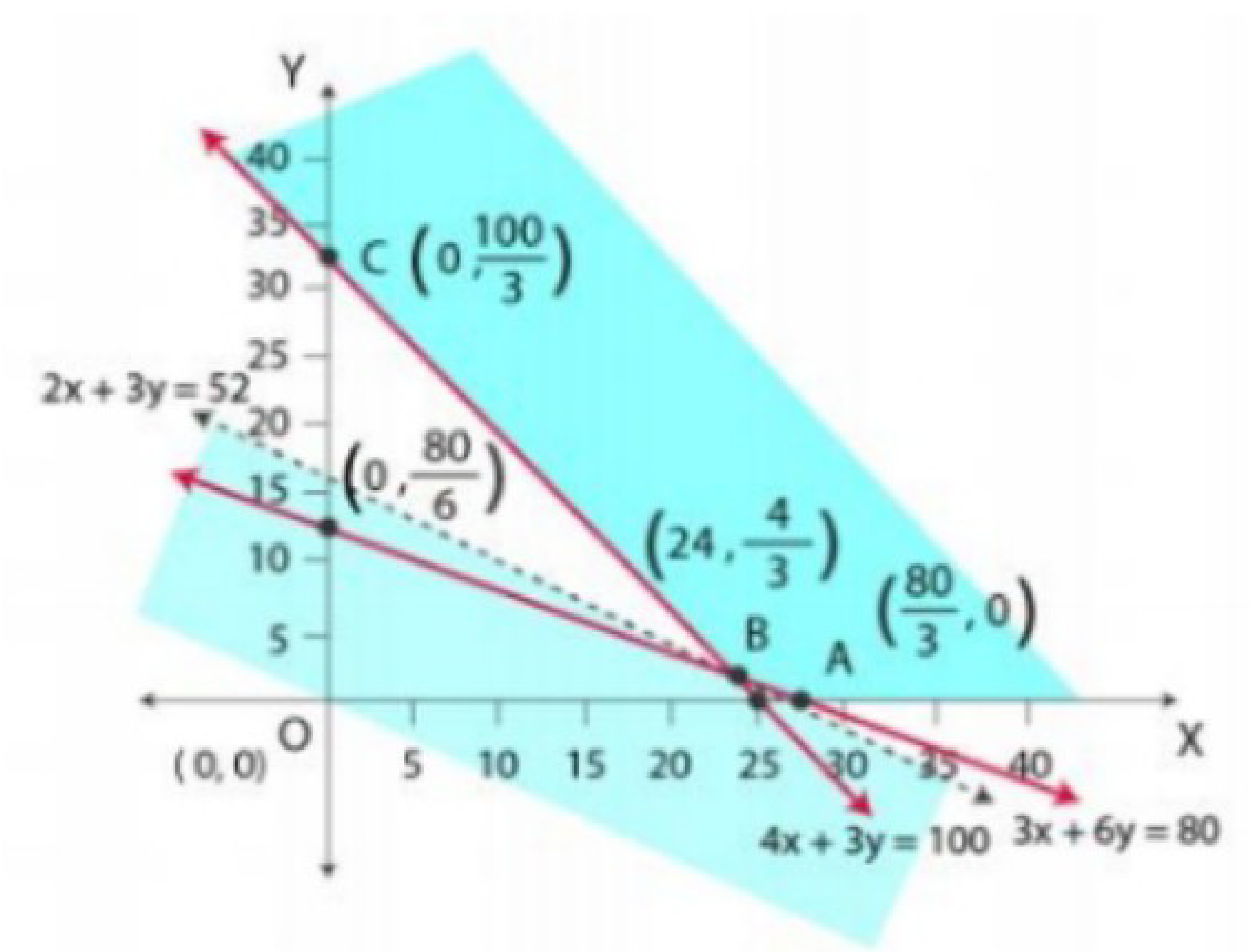
Subject to the constraints,

$3x + 6y \geq 80$ ………… (ii)

$4x + 3y \geq 100$ ………. (iii)

$x, y \geq 0$ ………… (iv)

The feasible region determined by the constraints is given below.



We can see that the feasible region is unbounded.

A (80 / 3, 0), B (24, 4 / 3), and C (0, 100 / 3) are the corner points.

The values of Z at these corner points are given below.

Corner point	Z = 4x + 6y	
A (80 / 3, 0)	320 / 3 = 106.67	
B (24, 4 / 3)	104	Minimum
C (0, 100 / 3)	200	

Here, the feasible region is unbounded, so 104 may or not be the minimum value of Z.

For this purpose, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $2x + 3y < 52$.

Hence, the minimum cost of the mixture will be Rs 104.

14. There are two types of fertilisers, F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid, and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Solution:

Let the farmer buy x kg of fertiliser F_1 and y kg of fertiliser F_2. Hence,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table, as given below.

	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs / kg)
F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen, and F_2 consists of 5% nitrogen.

However, the farmer requires at least 14 kg of nitrogen.

So, 10% of x + 5% of y $\geq$ 14

$x / 10 + y / 20 \geq 14$

By L.C.M we get

$2x + y \geq 280$

F_1 consists of 6% phosphoric acid, and F_2 consists of 10% phosphoric acid.

However, the farmer requires at least 14 kg of phosphoric acid.

So, 6% of x + 10 % of y ≥ 14

$6x / 100 + 10y / 100 \geq 14$

$3x + 5y \geq 700$

The total cost of fertilisers, $Z = 6x + 5y$

The mathematical formulation of the given problem can be written as

Minimise $Z = 6x + 5y$ .... (i)

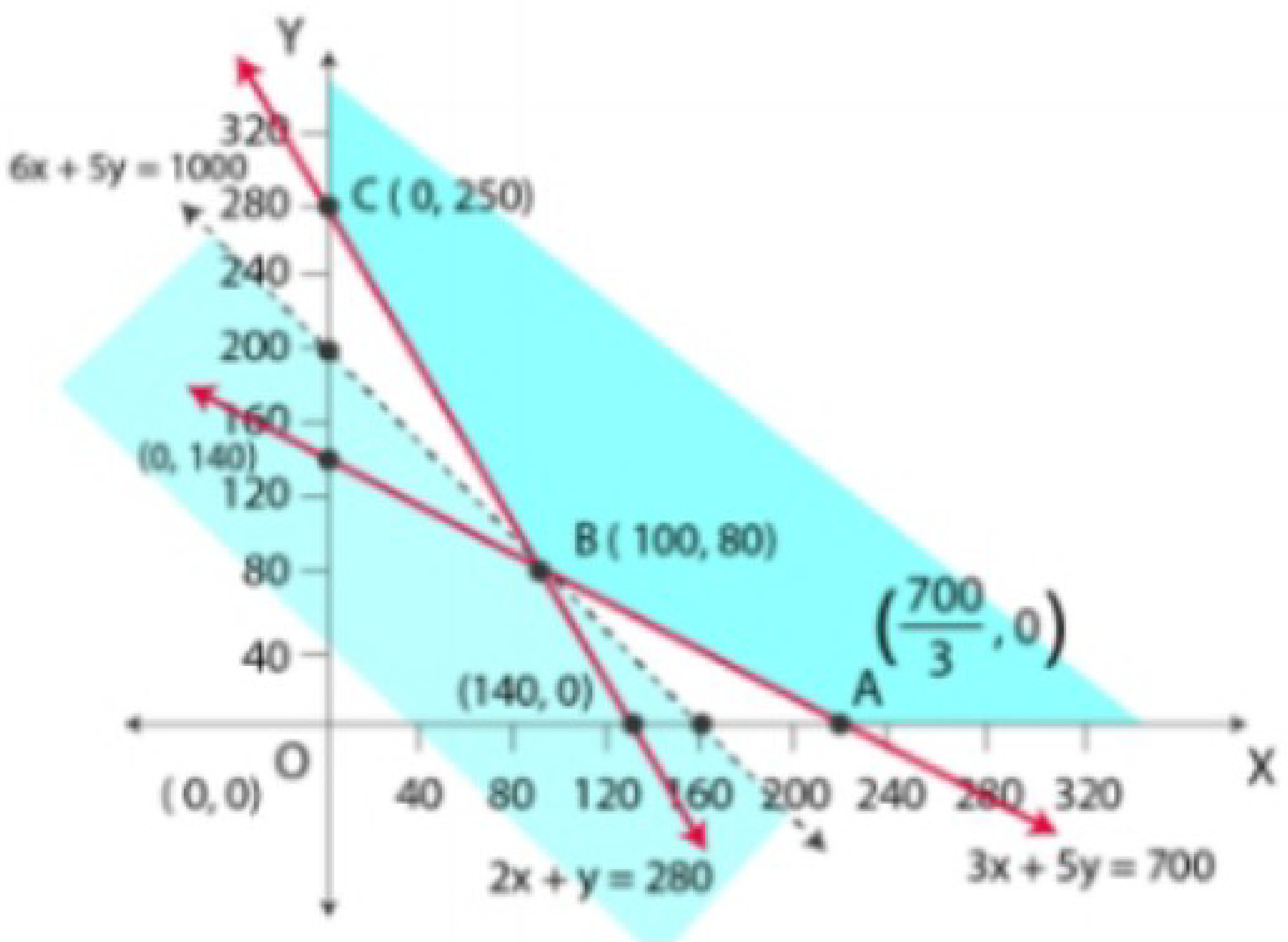
Subject to the constraints,

$2x + y \geq 280$ .... (ii)

$3x + 5y \geq 700$ .... (iii)

$x, y \geq 0$ .... (iv)

The feasible region determined by the system of constraints is given below.



Here, we can see that the feasible region is unbounded.

A (700 / 3, 0), B (100, 80) and C (0, 280) are the corner points.

The values of Z at these points are given below.

Corner point	$Z = 6x + 5y$	
A (700 / 3, 0)	1400	
B (100, 80)	1000	Minimum
C (0, 280)	1400	

Here, the feasible region is unbounded; hence, 1000 may or may not be the minimum value of Z.

For this purpose, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half-plane has points in common with the feasible region or not.

Here, it can be seen that the feasible region has no common point with $6x + 5y < 1000$

Hence, 100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 should be used to minimise the cost. The minimum cost is Rs 1000.

15. The corner points of the feasible region are determined by the following system of linear inequalities:

$2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $p = q$

(B) $p = 2q$

(C) $p = 3q$

(D) $q = 3p$

Solution:

The maximum value of Z is unique.

Here, it is given that the maximum value of Z occurs at two points, (3, 4) and (0, 5).

Value of Z at (3, 4) = Value of Z at (0, 5)

$p (3) + q (4) = p (0) + q (5)$

$3p + 4q = 5q$

$3p = 5q - 4q$

$3p = q$ or $q = 3p$

Therefore, the correct answer is option (D).