Analysis of Algorithms - Home Work 2

CSE 548 Fall '19

Prof. Jie Gao

Submission By:

Narayan Acharya

112734365

Contents

1 Question 1

Textbook [Kleinberg & Tardos] Chapter 3, page 107, problem #6.
Solution:

We know that tree T is both a DFS tree and a BFS tree. So tree T should exhibit properties of both DFS and BFS trees.

For a DFS tree for two nodes **not** to be connected by an existing edge, one must be an ancestor of the other. [See Kleinberg & Tardos, proof 3.7, page 85]

For a BFS tree for two nodes to be connected, their distance from another node w in T can differ by at most 1. [See Kleinberg & Tardos, proof 3.4, page 81]

Suppose, there exists an edge e that connects nodes u and v in our graph G but does **NOT** belong to edges E of tree T, i.e. $e = (u, v)$ and $e \notin E$.

As u and v are connected by an edge **not** in T, one of them is an ancestor to the other. Without loss of generality, assume node u is an ancestor of v. Given T is also BFS tree, distance from random node w in T to u and v can differ by a maximum of 1. Thus u has to be a direct parent of v. This implies that edge e connecting them must be a part of T.

This contradicts our initial assumption of $e \notin E$. Hence $e \in E$.

Narayan Acharya
112734365

2 Question 2

Textbook [Kleinberg & Tardos] Chapter 3, page 107, problem #8.

3 Question 3

Textbook [Kleinberg & Tardos] Chapter 3, page 107, problem #12.

4 Question 4

Textbook [Kleinberg & Tardos] Chapter 4, page 190, problem #8.

Narayan Acharya
112734365

5 Question 5

Textbook [Kleinberg & Tardos] Chapter 4, page 190, problem #21.

6 Question 6

Textbook [Kleinberg & Tardos] Chapter 4, page 190, problem #27.

References

[1] Binomial Theorem,
`https://en.wikipedia.org/wiki/Binomial_theorem`

[2] Young Tableau,
`https://en.wikipedia.org/wiki/Young_tableau`