Analysis of Algorithms - Home Work 2

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Contents

1 Question 1

Textbook [Kleinberg & Tardos] Chapter 3, page 107, problem #6.
Solution:

We know that tree T is both a DFS tree and a BFS tree. So tree T should exhibit properties of both DFS and BFS trees.

For a DFS tree for two nodes **not** to be connected by an existing edge, one must be an ancestor of the other. [See Kleinberg & Tardos, proof 3.7, page 85]

For a BFS tree for two nodes to be connected, their distance from another node w in T can differ by at most 1. [See Kleinberg & Tardos, proof 3.4, page 81]

Suppose, there exists an edge e that connects nodes u and v in our graph G but does **NOT** belong to edges E of tree T, i.e. $e = (u, v)$ and $e \notin E$.

As u and v are connected by an edge **not** in T, one of them is an ancestor to the other. Without loss of generality, assume node u is an ancestor of v. Given T is also BFS tree, distance from random node w in T to u and v can differ by a maximum of 1. Thus u has to be a direct parent of v. This implies that edge e connecting them must be a part of T.

This contradicts our initial assumption of $e \notin E$. Hence $e \in E$.

2 Question 2

Textbook [Kleinberg & Tardos] Chapter 3, page 107, problem #8.
Solution:

3 Question 3

Textbook [Kleinberg & Tardos] Chapter 3, page 107, problem #12.
Solution:

We can use directed graphs with topological ordering to judge if the information presented to the ethnographers is internally consistent or not.

For every person P_i, consider their date of birth and the date of death be denoted by b_i and d_i.

The below algorithm returns $True$ if the information presented to the ethnographers is internally consistent else returns $False$. First we construct a directed graph starting with an empty graph and adding nodes along with their edges based on information received for each person P_i. For each fact we add additional edges to our graph. If a fact is of type 1 i.e. person P_i died before person P_j, we add an edge (d_i, b_j) signifying that death of P_i precedes death of P_j. On similar lines, if fact is of type 2 i.e. lifespans of persons P_i and P_j overlapped we add edges (b_i, d_j) and (b_j, d_i) to our graph.

Finally we check if our graphs has any cycles. If our graph does have at least one cycle then there is no date that can be put first on our timeline, indicating that some information presented to us is inconsistent. If our graph does **NOT** have a cycle that means our graphs is a DAG and has topological ordering and can be considered internally consistent.

Algorithm 1 Algorithm to check consistency of time series information

procedure CHECK(List $People$, List $Facts$) ▷ Given information of of people and facts.
    Graph $G \leftarrow \phi$ ▷ Initialize empty directed graph to hold facts about people
    for P_i in $People$ **do** ▷ Store birth and death dates of People
        $b_i \leftarrow$ birth of P_i
        $d_i \leftarrow$ death of P_i
        Add nodes b_i and d_i to G
        Add edge $e \leftarrow (b_i, d_i)$ to G
    end for
    for F_i in $Facts$ **do** ▷ Extract information from facts
        if F_i about death of P_i before birth of P_j **then**
            Add edge $e \leftarrow (d_i, b_j)$ to G
        else
            if F_i life of P_i overlapped with life P_j **then**
                Add edge $e_1 \leftarrow (b_i, d_j)$ to G
                Add edge $e_2 \leftarrow (b_j, d_i)$ to G
            end if
        end if
    end for
    if G is a Directed Acyclic Graph **then return** $True$
    end if
     return $False$
end procedure

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4 Question 4

Textbook [Kleinberg & Tardos] Chapter 4, page 190, problem #8.
Solution:

5 Question 5

Textbook [Kleinberg & Tardos] Chapter 4, page 190, problem #21.
Solution:

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6 Question 6

Textbook [Kleinberg & Tardos] Chapter 4, page 190, problem #27.
Solution:

References

[1] Binomial Theorem,
https://en.wikipedia.org/wiki/Binomial_theorem