

QUANTITATIVE APTITUDE

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Time and Work
Problems on Train
Time/Distance Speed
Percentage
Average

Ratio Proportion
Profit, Loss
Discount
Simple Interest
Compound Interest

D) work from days:

If a person can do a work in n days. Then person's 1 day work. ($\frac{1}{n}$)

2) Days from work:

\Rightarrow If a person's 1 day work is equal to $\frac{1}{n}$, then the person can finish the work in ' n ' days.

3) Number of Days =
$$\frac{\text{Total work}}{\text{work done in 1 day}}$$

D) Ratio:

If A is x times as good a workman as B, then,

a) Ratio of work done by A & B in equal time: $x:1$

b) Ratio of time taken by A & B to complete the work = $1:x$ (This means that 'A' takes $(1/x)$ th time as that of 'B' to finish same amount of work).

Example:

If A is twice good a workman B, then it means that

$$1) A:B = 2:1$$

2) A finishes his work half

the time as B. i.e. if be take 5 days than

A take 2.5 days alone)

Q) Combined work :-

a) If 'A' and 'B' can finish the work in x, y day respectively, then,

$$A's \text{ one day work} = \frac{1}{x}$$

$$B's \text{ one day work} = \frac{1}{y}$$

$$\text{If } (A+B)'s \text{ one day work} = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{yx}{xy} = \frac{x+y}{xy}$$

Together finishes the work in $\left(\frac{xy}{x+y}\right)$ days.

b) If 'A', 'B', 'C' can complete the work in x, y, z day res, then,

$$(A+B+C)'s \text{ one day work} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$= \frac{yz + xz + xy}{xyz}$$

Together completed work in $\frac{xyz}{xy+yz+zx}$ days.

c) If A work in x days and if the same amount of work is done by A & B together in y days, then.

A's one day work = $\frac{1}{x}$

Q) Syntax Error
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(A+B)'s one day work = $\frac{1}{y}$

B's one day work = $\frac{1}{y} - \frac{1}{x} = \frac{x-y}{xy}$

So 'B' alone will take $\frac{xy}{x-y}$ days.

d) If A & B together perform some part of work in x days, B & C together perform work in y days, C & A together in z days then.

(A+B)'s one day work = $\frac{1}{x}$

(B+C)'s one day work = $\frac{1}{y}$

(C+A)'s one day work = $\frac{1}{z}$

$$\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = Q \text{ (A+B+C)'s one day work}$$

A+B+C's 1 day work can be written as

$$\frac{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}{Q}$$

together complete work in $\frac{Q}{\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)}$ days

If A alone works, Then deduct A's work

from the total work of B & C to find

the time taken by A alone.

A working alone, time required = A's work

less than sum work $\rightarrow (A+B+C)$'s work.

For A =
$$\frac{2xyz}{(xy+yz+zx)} \rightarrow B$$

For B =
$$\frac{2xyz}{(-xy+yz+zx)} \rightarrow C$$

For C =
$$\frac{2xyz}{(xy-yz+zx)} \rightarrow (A)$$

3) Man-Work-Hour related Problems:

Remember, $\frac{MDH}{W}$ is constant.

M - no. of men

D - no. of days

H - no. of hours

W - Amount of work done.



If man are fixed, work is proportional to time.

If work are fixed, time inversely proportional to no. of men used i.e. more no. of men less time.

$$\frac{M_1 \times T_1}{W_1} = \frac{M_2 \times T_2}{W_2}$$

1. If Neema completed a piece of work in 12 days, Seema in 18 days. If they work together, then how many days will be required to finish the work.

Neema's one day work = $\frac{1}{12}$

Seema's one day work = $\frac{1}{18}$

$$\begin{aligned} &= \frac{1}{12} + \frac{1}{18} = \frac{18+12}{18 \times 12} \\ &\Rightarrow \frac{18+12}{18 \times 12} = \frac{30}{18 \times 12} \\ &\Rightarrow \frac{30}{18 \times 12} = \frac{5}{36} \end{aligned}$$

(5.4)

- 2) If 'A' completes a piece of work in 3 days which 'B' completes it in 5 days, C taken 10 days (together)

$$\begin{aligned} &= \frac{1}{3} + \frac{1}{5} + \frac{1}{10} \\ &= \frac{50+15+30}{3 \times 5 \times 10} \Rightarrow \frac{95}{150} \end{aligned}$$

- 3) Two painters P₁ and P₂ can paint a bungalow in 3 days. If P₁ alone can paint the bungalow in 10 days. In how many days can P₂ alone complete the same paint

$$\text{work: } P_1 = \frac{1}{3} \quad P_1 + P_2 = \frac{1}{12} \quad \frac{1}{12} - \frac{1}{3} \Rightarrow \frac{1-4}{12} \Rightarrow \frac{3}{12}$$

so time part $\Rightarrow \frac{3}{12} \Rightarrow \frac{1}{4}$

(ii) If A and B can make paints in 6 days.

B and C = 10, A+B+C = 11 days than A & C.

$$A \& B = \frac{1}{6} \quad B \& C = 10 \quad A+B+C=11$$

$$(2 \times \frac{1}{4}) - (\frac{1}{6} + \frac{1}{10}) \Rightarrow \frac{2}{4} - \frac{16}{60} \Rightarrow \frac{1-4}{12} \Rightarrow \frac{1}{12}$$

$$\frac{1}{12} \times 15 = \frac{15}{12} = \frac{15-8}{30} = \frac{7}{30} \cdot 7 = \frac{49}{30} = \frac{28}{30}$$

$$\boxed{43 \left(\frac{2}{7} \right)}$$

5) Pooja is twice as efficient as Aarti and takes 90 days less than complete the job.

Find the time in which then can finish the job together.

$$\text{Pooja} = 90 \quad \text{Aarti} = 180 \quad \frac{1}{90} + \frac{1}{180} = \frac{2+1}{180} = \frac{3}{180} = \frac{1}{60} \text{ days.}$$

6) Monica twice good as sonica and together they complete a piece of work in 20 days. In how many day will monika alone will finish the work.

$$M+S = \frac{20}{3} \Rightarrow \frac{1}{20} \times \left(\frac{2+1}{2} \right)$$

$$\Rightarrow \frac{1}{20} \times 2 = \frac{1}{10}$$

- 7) 6 men can pack 12 box in 7 days by working 7 hours a day. In how many days 14 men pack 18 box if they work 9 hours per day?
- $$\frac{6 \times 7 \times 7}{12} = \frac{14 \times 9 \times d_2}{18}$$
- $$\frac{7}{2} \times \frac{189}{14 \times 9} = d_2$$
- $$\frac{7}{2}$$
- 8) 4 men and 5 boys can do piece of work in 20 days while 5 men and 4 boys can do the same work in 16 days. In how many days can 4 men and 3 boys do the same work?

1 man 1 day = x
 1 boy 1 day = y

$$(x \cdot 5) + 4y = 20 \text{ days}$$

$$(x \cdot 4) + 5y = 16 \text{ days}$$

$$20x + 25y = 100$$

$$\underline{-} 8x + 16y = 48$$

$$9y = -32$$

$$y = -\frac{32}{9}$$

16/4/25
 16/4/25

Problems On Train

Solved Examples

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Train's Scenario:

1. Two trains moving in opposite direction.
2. Two trains moving in the same direction.
3. A train crossing a stationary object of a given length like a platform or bridge.
4. A train crossing a stationary object like a pole, or man which can be considered as a point object.

Points to be remembered:

$$\frac{300}{18} =$$

$$\frac{350}{18} =$$

If length of train is P_1 and length of P_2

Q. To calculate total distance ($P+Q$)

$$R.S = V_1 - V_2$$

2) Finding relative speed.

(Object moving same direction)



$$\text{Relative speed} = V_1 + V_2$$

(Object moving opposite direction)

3) If two trains of lengths 'P' and 'Q' move in opposite direction, at V_1 m/s and V_2 m/s, then time taken by the train to cross each other

$$\text{Time taken} = \frac{P+Q}{V_1 + V_2}$$

(4) If 2 trains diff length P and Q move
in same direction at V_1 m/s and V_2 m/s,
then time taken by the trains to cross each
other, is called by a given formula

$$T = \frac{P+Q}{V_1 - V_2}$$

Quick tips and Tactics:

- 1.** Time taken by a train of length L meter
to pass a signal post or standing man =
Time taken by the train to cover the L
meter.

$$\text{Time} = \frac{L}{\text{Speed}}$$

[A signal post or a standing man is
considered to be the point object]

- 2.** Time taken by a train of length L_1 meter
to pass a stationary object of length L_2 is
basically the time taken by the train to
cover $(L_1 + L_2)$ meter.

$$\text{Time} = \frac{L_1 + L_2}{\text{Speed}}$$

3. The time taken by a train of length L_1 meter to pass a moving object of length L_2 is determined by considering the relative speed b/w the moving object.

$$\text{Time} = \frac{(L_1 + L_2)}{RS} \rightarrow V_1 + V_2 \text{ (oppo)}$$

$$(or) V_1 - V_2 \text{ (same)}$$

4. Q Trains start from two points P and Q at the same time and move towards each other. These trains take $\frac{P}{q}$ and $\frac{q}{P}$ seconds respectively to reach point P and Q respectively. The relation them is given by

$$\frac{\text{Time}_1}{\text{Time}_2} = \frac{(\text{P's speed})}{(\text{Q's speed})} = \frac{\sqrt{q}}{\sqrt{P}}$$

Conversion Units:

$$\frac{\text{Km}}{\text{hr}} = \frac{1000}{60 \times 60} = \frac{5}{18} \text{ m/s} \quad [\text{Km/hr to m/s}]$$

$$\frac{\text{m}}{\text{s}} = \frac{18}{5} \Rightarrow [\text{m/s to km/hr}]$$

6.-Types:

- Type - I. A Train crosses a stationary object on their platform. Find (a) Time taken by train (b) length of Train.

Example:

$$\textcircled{1} \text{ Length of Train} = 250\text{m} \quad \text{Speed } 70\text{km/hr}$$

What will time taken for cross any stationary object pointing at the railway station.

$$\text{Time} = \frac{\text{Length}}{\text{Speed}} = \frac{250}{70} \times \frac{5}{18} = \frac{25}{7} \times \frac{5}{18} = 18.86\text{ sec}$$

- \textcircled{2} A Train takes 10 sec to pass a signal post and covers a distance 10km in 15min. Length of train.

$$\text{Time} = \frac{L}{\text{Speed}}$$

$$10\text{sec} = \frac{L}{40\text{km/hr}}$$

$$10 \times \frac{200}{18} = \frac{1000}{9} \text{ m}$$

$$40 \times \frac{5}{18} = \frac{200}{9}$$

\textcircled{3}.

2-2 D A Train of given length crosses the platform at a given speed. Find,

a) Time taken across the platform

b) length of platform. ($L_1 + L_2$)

Example:-

3. Chandigarh express of 100m runs at a speed of 60km/hr. what will be the time taken to cross the platform of 150m long?

$$\text{time} = \frac{L_1 + L_2}{\text{Speed}}$$

$$= \frac{60 \times 5}{18} = \frac{300}{18}$$

$$= \frac{(100+150) \times 18}{300} = \frac{250 \times 18}{300} = \boxed{15}$$

4. Train running at 50 km/hr, passes

a man walking platform at 7km/hr. in same direction as that of train in 15 sec.

If this train takes 30 seconds to cross the Pf

then find the length of Train (L_1) and length of

Platform (L_2)?

Speed of Train = 50 km/hr

Time taken for cross the Pf = 30sec

Time taken for cross the man = 15sec

$$\text{Unit conversion: } 50 \text{ km/hr} = 50 \times \frac{5}{18} = 13.88 \text{ m/s}$$

$$7 \text{ km/hr} = 7 \times \frac{5}{18} = 1.94 \text{ m/s}$$

$$\text{Speed} = \frac{L_1 + L_2}{\text{Time}} \Rightarrow \left[\frac{L_1 + L_2}{30 \text{ sec}} = 11.94 \text{ m/s} \right]$$

$$\Rightarrow 11.94 \times 30 = L_1 + L_2 = 358.2 \text{ m}$$

$$L_1 \Rightarrow 11.94 \times 15 = 179.1 \text{ m}$$

$$\text{Total length } L_2 \Rightarrow 358.2 - 179.1 = 179.1 \text{ m}$$

Type-3

Find time taken by a Train to cross a person running in opposite direction at a given speed. Two speed 1) Train 2) object or person
 \rightarrow oppo \rightarrow add.

5. The chennai express of 200m runs at speed of 62km/hr and a person runs on the pf at a speed of 20km/hr in the direction oppo to that train. Find the time taken by the train to cross the running person?

Train length $L_1 = 200 \text{ m}$

$$\text{Speed} = 62 \text{ km/hr} = \frac{62 \times 5}{18} = 17.22 \text{ m/s}$$

$$\text{Speed} = 20 \text{ km/hr} = \frac{20 \times 5}{18} = \frac{50}{9} = 5.55 \text{ m/s}$$

$$\text{Speed} = (20 \text{ m/s} + 5.55) = 25.55 \text{ m/s}$$

$$\frac{200}{22.78} = 8.77 \text{ sec.}$$

Syntax Error

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6. A boy occurs opposite to that of train at speed of 20km/hr. If the relative speed b/w Train and the boy running in oppo direction 50km/hr. what is the length of Train. If it takes 20sec to cross the boy, when he is at rest?

Train speed = 30 km/hr
Speed of boy = 20 km/hr

Relative speed = 50 km/hr

Time taken to cross boy = 20 sec

$$L = S \times T$$

$$L = \left(30 + 20 \right) \times 20 \text{ sec}$$

$$L = \frac{50 \times 20}{9} \text{ m}$$

$$L = \frac{1000}{9} \text{ m}$$

Type-4.

- Find time taken by a Train to cross a person running in same direction at a given speed.

7. A boy runs on the platform of 180m at a

10km/hr in the same direction of the Train.

Find the time taken by the Train to cross the running boy if speed of the Train 11km/hr.

Platform len = 180m

$$\text{Speed of boy} = 50 \text{ km/hr} = 18 \times \frac{5}{18} = \frac{95}{18} = 0.77$$

$$\text{Train speed} = 71 \text{ km/hr} = 71 \times \frac{5}{18} = 19.72$$

Length of Train = 180m

$$\text{Relative speed} = 19.72 - 0.77 = 18.95$$

(Same direction)

$$\text{Time} = \frac{180}{18.95}$$

8. A person is walking at a speed of 5km/hr along a railway track. If he is 800m ahead of the train which is 100m long and moves at a speed of 60km/hr same direction. Then what is the time required to pass the person.

$$\text{Person speed} = 5 \text{ km/hr} = 5 \times \frac{5}{18} = \frac{25}{18} = 1.38$$

$$\text{Train speed} = 60 \text{ km/hr} = 60 \times \frac{5}{18} = \frac{300}{18} = 16.66$$

$$R.S. = 16.66 - 1.38$$

$$16.66 - \frac{70 \times 18 \times 3}{18} = 1.38$$

$$16.66 - \frac{18 \times 300}{18} = 1.38$$

$$16.66 - \frac{18 \times 100}{18} = 1.38$$

all in it 19.64

Type - 5 They are move in oppo

Time taken by 2 Trains moving oppo \rightarrow

at given speed, to cross each other.

Speed and length correspondingly added because

9. Two Trains A and B of 150m and 300m lengths at speed of 65km/hr and 80km/hr respectively in opposites \rightarrow Find the time required to cross each other after the moment they met?

$$\begin{aligned} & \text{Train A speed } 65 \times 5/18 = 18.05 \text{ m/sec} \\ & \text{Train B speed } 80 \times 5/18 = 22.22 \text{ m/sec} \\ & \text{Total speed } = 18.05 + 22.22 = 40.27 \text{ m/sec} \\ & \text{Lengths } 150 + 300 = 450 \text{ m} \\ & \text{Time } T = \frac{450}{40.27} = 11.15 \text{ sec} \end{aligned}$$

10. A passenger Train of 200m long moves at a speed of 55km/hr. A person traveling in it observes that the goods Train moving in opposite direction takes 10 sec to cross him.

Find the speed of goods Train. If Pt is 250m long.

$$\begin{aligned} P.T &= 200 \text{ m} & 15.27 \\ S &= 55 \times 5/18 = \frac{275}{18} = & \begin{array}{r} 4 \\ 95 \\ 90 \\ 1 \\ 50 \\ 18 \times 2 \\ \hline 36 \end{array} \\ & & 36 \end{aligned}$$

$$10 = \frac{200 + 250}{15.27 + V_2} = 10(15.27 + V_2) = 150$$

$$150.7 + 10V_2 = 150$$

$$10V_2 = 150.00 - 150.70$$

$$10V_2 = 297.30$$

$$V_2 = 29.73$$

Type - b:

Two Trains move at a given speed in same direction. length of Train (+) speed (-)

a) Time taken to cross each other.

b) length of Train.

Trains

i. Trains P and Q move in same direction with a speed of 85 km/hr and 70 km/hr. P = 120m Q = 210m. Time taken by Train Q to cross the Train Q?

$$T = \frac{120\text{m} + 210\text{m}}{85\text{km/hr} - 70\text{km/hr}} = \frac{330\text{m}}{15\text{km/hr}} = \frac{330 \times 18}{15 \times 1000} \text{ hr} = 3.6 \text{ hr}$$

$$1 \times 18 = 18$$

$$2 \times 18 = 36$$

$$3 \times 18 = 54$$

$$4 \times 18 = 72$$

$$5 \times 18 = 90$$

$$6 \times 18 = 108$$

$$7 \times 18 = 126$$

$$8 \times 18 = 144$$

$$9 \times 18 = 162$$

$$10 \times 18 = 180$$

$$\frac{120\text{m} + 210\text{m}}{85\text{km/hr} - 70\text{km/hr}} = \frac{330\text{m}}{15\text{km/hr}} = \frac{330 \times 18}{15 \times 1000} \text{ hr} = 3.6 \text{ hr}$$

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11/25 Time & Distance:

Syntax Error
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- Speed is defined as the distance travelled per unit time.

$$\text{Speed} = \frac{\text{distance}}{\text{Time}}$$

- If same distance x is travelled at two different speed s_1 & s_2 then avg speed is calculated.

$$\text{Avg speed}(s_a) = \frac{2s_1 s_2}{s_1 + s_2}$$

- Two bodies A and B move between two points P and Q. One starts from P and goes to Q while other starts from Q and goes to P until they meet on the way and reach their destination in time t_a and t_b respectively after meeting. Their speed s_a & s_b are given by,

$$\frac{s_a}{s_b} = \frac{\sqrt{t_b}}{\sqrt{t_a}}$$

$$4) \text{ oppo} \rightarrow s_p = s_1 + s_2$$

$$5) \text{ same} \rightarrow s_p = s_1 - s_2$$

Type 1: calculate either time, speed or distance from other given parameters.

Type 2: calculate speed of two people move b/w points A and B, in opposite directions & crossing each other on the way.

4. Two girls move in opposite direction. One from A to B and other from B to A. The girl from A reaches the destination in 16 hours and B reaches 85 hours. After having met. If former's speed is 25 km/hr, what will be the speed of latter?

$$\frac{S_a}{S_b} = \frac{\sqrt{t_b}}{\sqrt{t_a}} \Rightarrow S_b = \frac{S_a \times \sqrt{t_a}}{\sqrt{t_b}}$$

$$S_b = \frac{25 \times 11}{\sqrt{85}} = 20 \text{ km/hr}$$

5. Two bus start at ^{same} time, one from P to Q and the other Q to P. If both buses reach after 4 hours and 16 hours at Q and P respectively. after they cross each other, what would be the ratio of speed of the bus starting from P and that of the one starting from Point Q.

$$\frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2:1$$

Type 3: Finding Relative speed for two bodies moving in same (or) opposite direction.

⑥. Two towns P and Q are apart from 215 km apart. A motorcycle starts from P

towards Q at 8am. at the speed of 25 km/hr. Another rider starts from Q towards P at 9am at the speed of 20 km/hr. Find what time they will cross each other?

Assume, distance traveled by P in x hours = $25(x)$

distance traveled by Q in $(x-1)$ hours = $20(x-1)$ km/hr.

Adding:

$$25x + 20(x-1) = 275$$

$$25x + 20x - 20 = 275$$

$$45x = 275 + 20$$

to find $x = \frac{295}{45}$ hours

distance traveled = $\frac{295}{45} \times 25$ km

$\frac{295}{45} \times 25 = \frac{1475}{45} = \frac{152}{5} = 30.4$

$$x = 6.5$$

$$x-1 = 5.5$$

$$9. am + 5.5 = 8 \text{ hours } 30 \text{ min}$$

$$8 \text{ am } + 6.5 = 8 \text{ hours } 30 \text{ min}$$

Type 4: Numericals on Average speed when the same part of total distance is traveled at 2 or more different speeds.

①. A aeroplane flying 1000km covers the first 200km at the rate of 200km/hr, the second 200km at the rate of 400km/hr. The third 200km " " " " 600km/hr. last

200km " " " " 800km/hr. Avg speed.

Total time taken: $\frac{250}{200} + \frac{250}{400} + \frac{200}{600} + \frac{200}{800}$

$$\frac{800 + 400 + 200 + 200}{800} = \frac{1400}{800}$$

$$= \frac{14}{8} = \frac{7}{4} = \frac{1\frac{1}{4}}{1\frac{1}{3}} = \frac{31}{12} = \frac{25}{12}$$

$$\text{Avg speed} = \frac{\text{distance}}{\text{Time}} = \frac{1020}{\frac{31}{12}} \times 12 = 480 \text{ km/hr}$$

- ⑧. Jennifer travels first 4 hours at her journey speed of 80 miles/hr and remaining 6 hours at speed of 50 miles/hr. avg speed.

$$D_1 = 80 \times 4$$

$$= 320$$

$$D_2 = \frac{30 \times 6}{120}$$

$$\text{Speed: } \frac{\text{distance}}{\text{time}} = \frac{50}{10}$$

$$\text{Speed} = 50$$

- ⑨. Polonium-40 emits alpha particles which have a range of 10 cm. If a sample of polonium-40 emits 10 alpha particles per second, how many alpha particles will it emit in one second?

ans: 10 cm range means 10 cm distance from source, alpha particle can travel 10 cm in one second. So, if there are 10 alpha particles emitted per second, then 10 alpha particles will travel 10 cm in one second.

Percentage:-

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Points to Remember:

1) Price & R%:

1. $\frac{Y}{100}$ is expressed as $\frac{Y}{100} \%$.

2. To find percent of $\frac{x}{y} = \left[\left(\frac{x}{y} \right) \times 100 \right] \%$.

1. Price of Goods:

1) If the price of goods increase by $R\%$, then the reduction in consumption as so not to increase the expenditure can be calculated using

$$\left[\frac{R}{(100+R)} \times 100 \right] \%$$

2) If the price of goods decreased by $R\%$,

then the increase in consumption as not to decrease the expenditure

$$\left[\frac{R}{(100-R)} \times 100 \right] \%$$

2. Numerical on Population.

Population of a city at present P and it increases at the rate of $R\%$ per annum.

1) To find population after n years:

$$P \left[1 + \frac{R}{100} \right]^n$$

2) To find population n years ago:

$$\frac{P}{\left[1 + \frac{R}{100} \right]^n}$$

3. Numerical on Depreciation:

Present Value of machine is M. If it depreciates at the rate of R% per annum.

$$\text{after } n \text{ years} = P \left[1 - \frac{R}{100} \right]^n$$

$$\text{n years ago} = P \left[1 + \frac{R}{100} \right]$$

$$= \frac{P}{\left[1 - \frac{R}{100} \right]^n}$$

$$P \left[1 - \frac{R}{100} \right]^n$$

Sample Questions:- (numericals on numbers)

1. The diff b/w two numbers is 1550. If one number is 10% of other number, Then find the numbers.

$$x = \frac{10}{84}y \Rightarrow x = \frac{5}{4}y$$

$$x - y = 1550$$

$$\frac{5}{4}y - y = 1550 \Rightarrow \frac{1}{4}y = 1550 \Rightarrow y = 1550 \times 4$$

$$x = \frac{5}{4}y \Rightarrow \frac{5}{4}(6200) \Rightarrow 7750$$

2. Two numbers P and Q are such that, the sum of 2% of P and sum of 2% of Q is two-third of the sum of 2% of P and 6% of Q. Find the ratio of P & Q.

$$2\%(\text{P}) + 2\%(\text{Q}) = \frac{2}{3} (2\% \text{P} + 6\% \text{Q})$$

$$\frac{2}{100} P + \frac{2}{100} Q = \frac{2}{3} \left[\frac{2}{100} P + \frac{6}{100} Q \right]$$

$$\frac{1}{50} P + \frac{1}{50} Q = \frac{2}{3} \left[\frac{1}{50} P + \frac{6}{100} Q \right]$$

$$\frac{1}{50} P + \frac{1}{50} Q = \frac{2}{150} P + \frac{18}{300} Q$$

$$\left[\frac{1}{50} - \frac{1}{75} \right] P = \left[\frac{1}{25} - \frac{1}{50} \right] Q$$

$$\left[\frac{-15 - 30}{3750} \right] P = \left[\frac{2 - 1}{50} \right] Q \quad \boxed{3:1}$$

$$\left[\frac{25}{3750} \right] P = \frac{1}{150} Q$$

$$\frac{1}{150} P = \frac{1}{50} Q$$

$$\frac{P}{Q} = \frac{1}{\cancel{50}} \quad \cancel{150}$$

3. 50% of a number is 18 less than two third of that number. Number =?

$$(50\%) \text{ num} = \left(\frac{2}{3} \right) \text{ num} - 18$$

$$\frac{2}{3} x - \frac{1}{2} x = 18$$

$$\left(\frac{2}{3} - \frac{1}{2} \right) x = 18$$

$$\frac{3-2}{6} x = \left(\frac{1}{6} \right) x = 18$$

$$x = 18 \times 6$$

$$\boxed{x = 108}$$

4. When 35 is subtracted from a number, it reduces to its 80%. Find the fourth-fifth of that number?

$$x - 35 = \frac{80}{100} x$$

$$x - 35 = \frac{80}{100} x$$

$$x - 35 = \frac{80}{100} x$$

$$x = \frac{37.25}{\frac{35 \times 100}{20}} \Rightarrow 17.5$$

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$$\frac{4(175)}{5} = \frac{8}{35 \times 4} = 14.0$$

2. Numerical on Depreciation:

5. The value of lathe machine depreciation at the rate of 10% per annum. If the cost of machine at present is Rs. 1,60,000 then what will be its worth after 2 years.

$$= 160000 \left[1 - \frac{10}{100} \right]^2$$

$$= 160000 \left[\frac{9}{10} \right] \Rightarrow 1,60,000 (0.81)$$

$$\Rightarrow$$

b. The value of Xerox machine depreciates at the rate of 10% per annum. If the cost of machine at present is Rs. 75,000 then what was the value of machine before 2 years?

$$\frac{921600}{75000} \Rightarrow 92,600$$

3. Numerical on population:

birth = 30

death = 10

Total net growth (rate) = $30 - 10 = 20$

$$\frac{20}{100} \times 100 = 20\%$$

① The current birth rate per thousand 30
death rate per thousand 10

Find net growth rate is terms increase %

8. The total population of a city is 6500.

The number of males and females ↑ by 5% and 10% and population becomes 7000.

Find the no. of males in village

Let no. of males = x

$$\text{females} = 6500 - x$$

in first year ↑ 5% males

$$\frac{5}{100}x + \frac{10}{100}(6500-x) = 7000 - 6500$$

$$5x + 65000 - 10x = 50000$$

$$15000 = 5x$$

$$3000 = x$$

9. The present population of a country is

10 crore. If it rises 11.28 crores during

next 3 years. Then find uniform rate of growth in population.

$$11.280000 = 100000000 \left[1 + \frac{R}{100}\right]^3$$

$$\frac{11.280000}{100000000} = [R^3]$$

$$1.1 \Rightarrow 20\%$$

10. The population of different trees in

field ↑ 10% in first year, ↑ by 8% second

year and decrease 10% in third year. If at

present no. of trees is 86130, then find

the no. of trees in the beginning.

26-180

$$(1 + \frac{10}{100})(1 + \frac{8}{100})(1 + \frac{10}{100}) = 26,180 \times \frac{10}{11} \times \frac{25}{27} \times \frac{10}{9}$$

$$\approx 25000$$

4. Numerical on prices of Goods.

II. The price of diesel ↑ by 50%. And by how much percent a truck owner must reduce his consumption in order to maintain the same budget.

$$\frac{R}{100+R} \times 100 \Rightarrow \frac{10}{150} \times 100$$

$$\Rightarrow \frac{100}{150} = \frac{2}{3}$$

Excess is 10 rupees more, but

Q. The price of rice falls by 15%. By what % can increase the consumption of rice so that his overall budget does not change?

$$\frac{15}{100-15} \times 100 = \frac{15}{85} \times 100$$

$$\Rightarrow \frac{150}{85} = \frac{30}{17}$$

5. Numericals based on marks of students.

In an examination, P scored 30% marks and it failed by 15 marks. Q scored 10% marks and obtained 85 marks more than those required to pass. Find the pass percentage.

$$(\underline{30\% \text{ of } x}) + 15 = (110\% \text{ of } x) - 35$$

$$\frac{30}{100}x + 15 = \frac{10}{100}x - 35$$

$$35 + 15 = \frac{10}{100}x - \frac{30}{100}x$$

$$100x + 50 = -\left(\frac{10}{100}\right)x^2 \Rightarrow 50 = \frac{1}{10}x^2$$

Parsing mark

$$\frac{30}{100} (500) + 15 = 165$$

$$\frac{33}{100} \times 100 = 33\%$$

$$\frac{165}{500} \times 100 = 33\%$$

15 5

starts to off as a red to red

Average:

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$$\text{D Avg} = \frac{\text{s. of Quantities}}{\text{no. of Quantities}}$$

2) A person travels a distance at speed v_1 km/hr. and at v_2 km/hr. Same distance at a speed of v km/hr. His average speed in the whole journey can be determined using the formula

$$\text{Avg} = \frac{2(v_1 v_2)}{v_1 + v_2}$$

3) There are two possible batches A and B in a class. If we find the average of whole class.

$$\text{no. of students} = a \quad \text{no. of students} = b$$

$$\text{Avg of batch A} = x \quad \text{Avg. of batch B} = y.$$

$$\text{Avg.} = \left(\frac{ax+by}{a+b} \right)$$

$$\text{D Avg of n natural numbers} = \frac{n+1}{2}$$

$$2) \text{Avg of even numbers} = (n+1)$$

$$3) \text{In Arithmetic progression, if no. of terms are,}$$

1. Odd - Average is middle term

2. Even - average of middle two term.

1. When new person is added in the group/family,

case A: If average age increases.

Age of new member = Given previous

$$\text{avg} + \frac{\text{Increase in average after new member added}}{\text{Total members including new member}}$$

case B: If average decrease.

$$\text{Age of new member} = \frac{\text{Given previous avg} + \text{Decrease in average after new member added}}{\text{Total members including new member}}$$

Type 1: average of numbers:

1). Find the average of all numbers b/w 5

and 15 which are divisible by 5.

$$= \frac{5+10+15+20+25+30+35}{7}$$

$$\Rightarrow \frac{145}{7} = 20.71$$

2) The avg of 11 numbers is 30. If the average of first six numbers is 17.5 and that of last six is 42.5, Then what is the sixth number?
Then what is the sixth number?

$$(\text{avg. of eleven}) 11 \times 30 = 330$$

$$6 \times 42.5 = 255$$

$$17.5 \times 6 = 105$$

$$330 - (105 + 255)$$

$$330 - 360 = 30$$

$$\begin{array}{r} 255 \\ 105 \\ \hline 360 \end{array}$$

3. The avg of 15 numbers is 15. Number & 15

If the average of first five numbers is 14 and
that of other 9 numbers is 16 then find
middle number.

$$15 \times 15 = 225$$

$$5 \times 14 = 70$$

$$9 \times 16 = 144$$

$$\begin{array}{r} 90 \\ 54 \\ \hline 144 \end{array}$$

$$225 - 70$$

$$225 - 214$$

$$\textcircled{0} \textcircled{1} \textcircled{1}$$

4. The average of 4 consecutive even numbers is 27. Find the largest of those numbers.

$$\frac{x + x+2 + x+4 + x+6}{4} = 27$$

$$4x + 12 = 27 \times 4$$

$$4x = (27 \times 4) - 12$$

$$= 108 - 12$$

$$x = \frac{108 - 12}{4}$$

$$\therefore \text{Largest number } x = 24 \quad (\text{Largest num} = 24 + 6 = 30)$$

Type 2: Average of weight/age/marks of two or more

5. There are 2 batches A and B of a class.

Batch A consist of 36 students and batch B

consist of 44 students. Find the avg. weight

of whole class. If average weight of batch A is

40kg and that of batch B is 35kg.

$$36 \times 40 = 1440$$

$$44 \times 35 = 1540$$

$$1440 + 1540 = 2980$$

$$36 + 44 = 80$$

$$\textcircled{37.25}$$

$$\frac{2980}{80} =$$

b. In a school, average marks of three batches of 40, 50 and 60 students respectively is 45, 55, 70. Find the average marks of all the students.

$$40 \times 45 = 1800 \quad 50 \times 55 = 2750$$

$$60 \times 70 = 4200 \quad 1800 + 2750 + 4200 = 8750$$

$$\Rightarrow \frac{8750}{40+50+60} = \frac{8750}{150}$$

Type 3: Change in average when one entry is added/replaced.

7. The average age of a class 29 students is 20 years. If the age of teacher is included, then average increased by 3 months. Find the age of teacher.

$$\frac{x}{29} \times 20 = 29 \times 20$$

$$\frac{20}{29} \times 20.25 = 60.750$$

$$\frac{30}{29} \times 20.25 = 60.750$$

$$\frac{20.25 \times 3}{60.750} = 1.50$$

8. 8 years ago, the avg age of a family

of 5 members was 16 years. After a baby is born, the average age of family is same

today. Find the present age of baby.

$$\frac{x}{5} = 16 \Rightarrow 16 \times 5 = 80$$

$$\frac{x+1}{6} = 16 = 86 \quad (86 - 80)$$

⑥

Type 4: Change in average when one entry is entered wrong.

- ④ John's mark were wrongly entered as 83 instead of 63. If the average marks calculated for the class increased by half. Then find the no. of students in class.

no. of students x

$$(x) \times \left(\frac{1}{2}\right) = \left(\frac{x}{2}\right)$$

$$\frac{x}{2} = 83 - 63$$

$$\frac{x}{2} = 20 \Rightarrow 2(20) = 40$$

- ⑤ The mean of 40 observation was 46. Later on it was found that an observation 38 was wrongly taken as 33. Find the correct mean value.

$$40 \times 46 = 1840$$

$$\frac{1840}{40} = 46$$

correct sum = sum of observation +

$$(38 - 33) = 1840 + 5$$

$$\frac{1845}{40} = 46.125 = 1845 \div 10$$

Type 5: Average speed.

11. A person covers a distance of 60km from P to Q at speed of 20km/hr and returns from Q to P at a speed of km/hr. average speed.

$$2(20 \times 20) \Rightarrow \frac{2 \times 400}{30} \Rightarrow 24$$

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12. An express train runs at an average speed of 27 km/hr , including the time of stoppage at stations. Another train runs at an average speed of 41 km/hr . Find how many minutes does train stop in 1 hour.

Train 1 lags train 2 by $(41 - 27)\text{ km} \Rightarrow 14\text{ km}$

Now, we have to find the time, train 2 stops in 1 hour.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\frac{14}{41} = 0.342 \text{ hr}$$

$$0.342 \times 60 \Rightarrow 20.52 \text{ min.}$$

Type 6: Cricket / score inntings:

13. A batsman makes a score of 80 runs in the 16th inntings and increases avg by 3. What is his average 16th inntings?

$$15 \text{ 15th innning} = (x-3)$$

$$(x-3) = 15(x-3)$$

$$15(x-3) + 80 = 16x \quad 16\text{th} = 80$$

$$15x - 45 + 80 = 16x \quad x = 16x$$

$$\boxed{35 = x}$$

In a cricket match, 6 players had an average x of their scores. Average increases by 10 runs, if seventh player makes a score 112. What is the average of first 6 players.

Score of 7 players = $6x + 112$

$$\text{Total average} = x + 10$$

$$\frac{6x + 112}{7} (x + 10)$$

$$x = 48$$

$$6x^2 + 112x + 1120 = 7$$

$$6x^2 + 112x + 1113 = 0$$

(A) Age:-

1) If the current age of person, i.e x then,

$$\text{age after } n \text{ years} = x + n$$

$$\text{age before } n \text{ years} = x - n$$

$$n \text{ times the age} = nx$$

$$A:B \Rightarrow Ax : \text{and } Bx$$

2) If sum of ages of x and y and ratio of their ages is $p:q$ respectively, then we can determine age of y using the formula shown below,

$$y = \frac{\text{Ratio of } y}{\text{Sum of Ratio}} \times \text{Sum of age}$$

$$y = \frac{a}{p+q} (A)$$

Type 1: calculate present age:

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①

What is John's present age, if after 10 years his age will be 5 times his age 5 years back.

$$\begin{aligned}x+10 &= 5(x-5) \\x+10 &= 5x - 25 \\10+25 &= 5x - x \\35 &= 4x \\8.75 &= x\end{aligned}$$

②. Rahul is 15 years older than Rohan. If 5 years ago, Rahul was 3 times as old as Rohan. Find Rahul's present age.

age of Rohan = y .

$$(y+15-5) = 3(y-5)$$

$$y+10 = 3y-15$$

$$10+15 = 3y-y$$

$$25 = 2y$$

$$\frac{25}{2} = y \Rightarrow 12.5$$

age of Rahul

$$\Rightarrow (\text{Rohan}+15) \Rightarrow 12.5 + 15 = 27.5$$

Type 2: Numerical to Determine Ages in ratio form.

3. One year ago, ratio of Henry and Peter age's was 5:6 respectively. After 11 years the ratio became 6:7. How old Peter.

$$\frac{(5x+5)}{(6x+5)} = \frac{6}{7} \Rightarrow (5x+5)7 = (6x+5)6$$

$$35x+35 = 36x+30$$

$$5 = x$$

Current age.

$$x + 10 = 2 \Rightarrow x = 15$$

$$x + 10 \Rightarrow 31$$

4. Age of mother 10 years ago was 3 times the age of her son. After 10 years, mother's age will be twice that of his son. find ratio.

$$\text{at present } x+10 = 3x+10$$

$$\text{10 years earlier } (x+10) - 10 = 3x + 20$$

$$2(x+10) = 3x + 20$$

$$2x + 20 = 3x + 20$$

$$20 = x$$

$$30 : 70$$

Type 3: Numerical Determine Age of a person before x years.

5. Sharad is 60 years old and Santhosh is 80 years old. How many years ago was their ages 11:6?

$$\begin{array}{r} 60 : 80 \\ 50 : 70 \\ \hline 40 : 60 \end{array}$$

$$\frac{(60-x)}{(80-x)} = \frac{11}{6}$$

$$(60-x)6 = (80-x)11$$

$$360 - 6x = 320 - 11x$$

$$40 = 5x$$

$$\frac{40}{5} = 8$$

6. The ratio of Rohan's age 11 years ago and Rahul's age after 11 years is 1:1. If at present, the ratio of their ages is 5:3 then find the ratio b/w Rohan's age 11 years hence and Rahul's age 11 years ago.

$$\text{at present} = 5:3$$

$$\frac{5x-4}{3x+11} = \frac{1}{1} \Rightarrow 5x-4 = 3x+11$$

$$2x = 15$$

$$x = 7.5$$

$$\therefore \text{present} \frac{5x+4}{3x-11} = \frac{27.5+4}{22.5-11} = 3:1.5$$

Type 4: Numericals to Determine Age of Person After x years.

7. 5 years ago, sister's age 5 times the age of brother and the sum of present ages of sister and brother is 34 years. What will be the age of her brother after 6 years?

$$24 - x + 30 - x = x + 6$$

$$(24-x) = 5(30-x)$$

$$(24-x) = 5(x-5)$$

$$24 - x - 5 = 5x - 25$$

$$= 6x$$

$$24 + 25 - 5 = 5x$$

$$5x = 6x$$

$$9 = x. \quad x+6 = 9+6 = 15$$

8. Father is 8 times more than age his daughter. If after 5 years, he would be 3 times of the daughter's age. Then further after 5 years, how many times he would be of his

daughter's age = x years
 father's age = $x + 10$ years

$$x + 3x = 4x \text{ (years)}$$

$$4x + 5 = 3x + 15$$

$$x = 10$$

$$4(10) + 10 = \frac{50}{20} = 2.5$$

$$4x + 5 = (x + 10)$$

$$(10 + 10) = 80$$

HCF and LCM of Numbers:

Co-prime: These numbers do not have common factors b/w given numbers.

HCF [highest Common Factor]. 12, 36, 48

$$12 \Rightarrow 2, 3, 8$$

$$36 \Rightarrow 2, 3, 3, 3 = 2^2 \times 3^2$$

$$48 \Rightarrow 2, 2, 2, 2, 3 = 2^4 \times 3$$

Find HCF and LCM of composite numbers.

1. HCF : 513, 1134, 1215

$$3 | 513, 1134, 1215$$

$$3 | 171, 378, 45$$

$$3 | 57, 186, 15$$

$$3 | 19, 62, 5$$

$$5 | 10, 15, 20$$

$$3 | 10, 3, 4$$

$$2 | 1, 1, 4$$

2. LCM : 10, 15, 20

$$\Rightarrow 60$$

3. Find the largest number of 5-digit divisible by
 18, 15, 18.

9999

$$9999 - 9900 = 99$$

$$18 \Rightarrow 2 \times 3 \times 3 = 2^2 \times 3$$

$$15 \Rightarrow 3 \times 5 = 3 \times 5$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

$$2 \times 15 = 2 \times 3 \times 5 = 2^2 \times 3 \times 5 = 4 \times 9 \times 5 = 20 \times 9 = 180$$

$$\frac{180 \times 5}{900}$$

$$\begin{array}{r} 5 \\ 180 \quad | \quad 9999 \\ 900 \\ \hline 999 \\ 900 \\ \hline 99 \\ 90 \\ \hline 9 \end{array}$$

$$180 \times 5 = 900 \quad 9999 - 99 = 9900$$

Type 2: HCF and LCM of decimal numbers and
 Fractions:-

①.

HCF of 0.63 and 1.05

$$\begin{array}{r} 3 \\ 7 \quad | \quad 63, 105 \\ \hline 21, 35 \\ \downarrow \quad 3, 5 \\ \text{HCF} \end{array}$$

$$3 \times 7 = 21$$

$$0.81$$

5) LCM of 1.05 and 2.1

(2.1)

$$\begin{array}{r} 7 \\ 5 \quad | \quad 105, 910 \\ \hline 15, 30 \\ 3 \quad | \quad 15, 30 \\ \hline 3, 6 \\ 2 \quad | \quad 3, 6 \\ \hline 1, 3 \\ \text{LCM} = 105 \times 2 = 210 \end{array}$$

210

6) HCF of $\frac{8}{3}, \frac{16}{81}, \frac{8}{9}$

$$\text{HCF} = \frac{\text{HCF of numerator}}{\text{LCM of denominator}} = \frac{(2, 16, 8) \text{ HCF}}{3, 81, 9 \text{ (LCM)}}$$

$$= \frac{8}{81}$$

$$\begin{array}{r} 2 \\ 2 \quad | \quad 8, 16, 8 \\ 2 \quad | \quad 1, 8, 4 \\ 2 \quad | \quad 1, 4, 2 \\ \text{HCF} = 2 \end{array}$$

$$1. \text{ LCM } \frac{8}{3}, \frac{8}{9}, \frac{64}{81}, \frac{10}{27} \quad [Q] \text{ Syntex Exam} \\ \text{Algebraic Fraction}$$

LCM = LCM of numerators 280
 HCF of denominators 3

Type 3: Find the numbers/sum of numbers / product
 of numbers if

1) Their ratio and HCF are given

2) Product of HCF and LCM are given.

8. HCF of two numbers is 13. If these two numbers are in the ratio of 15:11 then find the numbers.

HCF = 13

$$\begin{aligned} 15 \times 13 &= 150 + 15 = 195 \\ 11 \times 13 &= 110 + 32 = 142 \end{aligned}$$

9. If the product and HCF of two numbers are 1107 and 27 respectively, Then find the greatest number.

$$1107 = 27 \times 27 \quad \text{HCF} = 27$$

10. Find the sum of two numbers, which are greater than 29 and have HCF and LCM of 89 and 1147 respectively.

$$x \times (x+29) = 29 \times 1147$$

$$29 \times xy = 120263$$

$$xy = \frac{120263}{89}$$

$$xy =$$

$$\begin{array}{r} 8 \\ 29 \times 29 \\ 861 \\ - 58 \\ \hline 841 \end{array}$$

Type 4: Find the least number, which on dividing by another number leaves a remainder

11. find the greatest number, which on dividing 1657 and 2037 leaves remainder 6 and 5.

$$\text{Div} \ 1657 - 6 = 1651 \quad \text{Div} \ 2037 - 5 = 2032$$

If the number does not have any common factor.
then apply Euclidean algorithm

187

$$\begin{array}{r} 1 \\ \hline 1651 \mid 9030 \\ \hline 651 \\ \hline 0381 \end{array}$$

$$\begin{array}{r} 1 \\ \hline 381 \mid 1684 \\ \hline 1584 \\ \hline 102 \end{array}$$

$$\begin{array}{r} 1 \\ \hline 187 \mid 381 \\ \hline 281 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 9 \\ \hline 381 \times 15 \\ \hline 1524 \\ \hline 187 \end{array}$$

$$\begin{array}{r} 1 \\ \hline 187 \times 2 \\ \hline 374 \\ \hline 187 \\ \hline 38 \end{array}$$

12. Find least number which is divided by 5, 6, 7 and 8 leaves 3 but when divided by 9 no remainder. divisible rule.

Sum of digit divided by 9.

1) $1963 = 19$

3) $1683 = 18$

2) $2593 = 18$

4) $1536 = 15$

13. LCM 12, 15, 20, 54 remainder = 8. 18+

$$\begin{array}{r} 3 \mid 18, 15, 20, 54 \\ \hline 2 \mid 6, 5, 10, 18 \\ \hline 5 \mid 3, 5, 5, 9 \\ \hline 9 \end{array}$$

$$3 \times 2 \times 5 \times 2 \times 9$$

$$(\times 60 \times 9)$$

$$540 + 8 \Rightarrow 548$$

Ratio & Proportions:

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1. Duplicate ratio:

It is the ratio of squares of 2 numbers.

$$\frac{x}{y} \Rightarrow \frac{x}{y} = \frac{x^2}{y^2} \text{ (or) } x:y = x^2:y^2$$

2. Sub-Duplicate ratio: It is the ratio b/w square roots of 2 numbers.

$$\frac{x}{y} \Rightarrow \frac{x}{y} = \frac{\sqrt{x}}{\sqrt{y}} \text{ (or) } x:y = \sqrt{x}:\sqrt{y}$$

3. Triplicate Ratio: (cube numbers)

$$\frac{x}{y} \Rightarrow \frac{x}{y} = \frac{x^3}{y^3}$$

4. Sub-Triplicate Ratio: (cube roots)

$$\frac{x}{y} \Rightarrow \frac{x}{y} = \frac{x^{(1/3)}}{y^{(1/3)}}$$

5. Compound ratio: It is the ratio of product of first term in every ratio to that product of second terms in every ratio.

$$\text{Compound Ratio: } (a:x):(b:y),(c:z) = (abc : xyz)$$

6. Inverse Ratio: The ratio formed by interchanging the old places in the ratio to new the inverse ratio of 5:8 is 8:5

Type 1: Proportion:-

Q1. $a:b = 2:3$ and $b:c = 5:7$ and $a:b:c = ?$

$$\begin{array}{ccc} a & b & c \end{array}$$

$$\begin{array}{ccc} 2 & 3 & 3 \end{array}$$

$$\begin{array}{ccc} 5 & 5 & 7 \end{array}$$

$$\underline{\underline{10 : 15 : 21}}$$

Q2. If $A:B:C = 3:4:7$ then what is the ratio of $A/B : B/C : C/A$?

$$\frac{A}{B} = \frac{3x}{4x}, \quad \frac{B}{C} = \frac{4x}{7x}, \quad \frac{C}{A} = \frac{7x}{3x}$$

$$\begin{array}{c} 3, 4, 7 \\ \uparrow \uparrow \uparrow \\ 12 \times 7 = 84 \end{array}$$

$$3 \times 84 = 252/4 = 63$$

$$4 \times 84 = 336/7 = 48$$

$$7 \times 84 = 588/3 = 196$$

Type 2: Division and Distribution of objects in ratio.

(*) If swesh distributes his pens in the ratio of $1/2:1/4:1/5:1/7$ b/w 4 friends A, B, C, D then find the total no. of pens swesh should have?

$$\begin{array}{l} \frac{1}{2} : \frac{1}{4} : \frac{1}{5} : \frac{1}{7} \\ \text{L.C.M.} = 140 \end{array} \quad \begin{array}{r} 140 \\ \times 2 \\ \hline 280 \\ \times 2 \\ \hline 560 \\ \times 5 \\ \hline 2800 \\ \times 7 \\ \hline 19600 \\ \hline 140 \end{array}$$

$$A = (1/2) \times 140 = 70$$

$$B = \frac{140}{4} = 35$$

$$C = \frac{140}{5} = 28$$

$$D = \frac{140}{7} = 20$$

$$\begin{array}{r} 105 \\ \hline 18 \\ 153 \end{array}$$

(4) If RS 1050 is divided by into three parts, proportional to $(\frac{1}{3}) : (\frac{3}{4}) : (\frac{4}{6})$ then what is the first part?

$$\text{LCM for } 3, 4, 6 = 12.$$

$$\frac{4x+9x+8x}{12} = 1050$$

$$21x = 12600$$

$$x = 600$$

$$600 \times \frac{1}{3} = 200.$$

$$\begin{array}{r} 600 \\ 21 \sqrt{12600} \\ \underline{-105} \\ 210 \\ \underline{-105} \\ 60 \\ \underline{-60} \\ 0 \end{array}$$

$$\begin{array}{r} 1050 \times 12 \\ 2100 \\ 1050 \\ \hline 12600 \\ 21 \times 6 \\ \hline 126 \end{array}$$

Type 3: Mixture or content (difference)

(5) In a mixture of 13 litre, the ratio of milk and water is 3:2. If 3 litre of this mixture replaced by 3 litres of milk, then what will be the ratio of milk and water in the newly formed mixture?

If 3 ltr removed then,

$$\text{milk in 10 litres} = 6 \text{ ltr} + 3 = 9 \text{ ltr}$$

$$\text{water in 10 litre} = 4 \text{ ltr} = 4 \text{ ltr}$$

$\boxed{9:4}$

(6). A mixture contains alcohol and water in the ratio of 7:5 if 8 litre water is added to the mixture, then the ratio become 7:9. Find the quantity of alcohol in the given mixture?

$$\frac{7x}{5x+8} = \frac{7}{9} \Rightarrow 7x(9) = 7(5x+8)$$

$$63x = 35x + 56$$

$$62x - 35x = 56$$

$$72x = 56$$

$$x = \frac{56}{72} = \frac{14}{18} = \frac{7}{9}$$

Type 4: Income / Expenditure and Salary:

(1) The annual income of Puja, Hema and Jaya taken together is Rs : 46,000. Puja spends 70% of income. Hema - 80%, Jaya 92% of her income. If their annual savings are 15:11:10. Find the annual saving of Puja?

Puja, saving 0.30%

Hema 0.20%

Jaya 0.08%

$$\frac{0.30A}{15}, \frac{0.20B}{50}, \frac{0.08C}{10} = \frac{A}{50}, \frac{B}{55}, \frac{C}{125}$$

$$(\div) 5 \quad \frac{A}{10} + \frac{B}{55} + \frac{C}{125} = \frac{A+B+C}{46} = \frac{1}{46} = 1000$$

$$A = 1000 \times 10 = 10000$$

$$B = 1000 \times 11 = 11,000$$

$$C = 1000 \times 25 = 25000$$

(2). A man, his wife and daughter worked in garden. The man worked for 3 days, his wife = 2 days, daughter = 4 days. The ratio of daily wage man to woman 5:4 and the ratio man to daughter 5:3. Their total earning is mounted 105. Then daughter daily wage.

days \times wage \uparrow daily wage \uparrow 3(3) = 9

$$(3 \times 5x) + (2 \times 4x) + (4 \times 3x) = 105$$

$$15x + 8x + 12x = 105$$

$$25x = 105$$

$$x = \frac{105}{25}$$

$$x = 3.$$

(9) Amit, Raju and Ram agree to pay their total electricity bill in the prop of 3:4:5. Amit pays 1st day bill = 50, Raju pays 2nd day bill = 55 Ram pays 3rd day bill = 75. How much amount should Amit want to settle the accounts?

$$55 + 50 + 75 = 105 + 75 = \frac{180}{18} = 15$$

$$3(15) = 45$$

$$4(15) = 60$$

$$5(15) = 75$$

(10). Salaries of Ram and Sham are in the ratio of 4:5. If the salary increased by 5000, then new ratio 50:60. present salary of sham?

$$\frac{4x + 5000}{5x + 5000} = \frac{50}{60}$$

$$60(4x + 5000) = 50(5x + 5000)$$

$$240x + 300000 = 250x + 250000$$

$$\frac{50000}{10} = 10x$$

$$5000 = x$$

$$5x + 5000 = 5(5000) + 5000 = 25000 + 5000 = 30000$$

Type 5: Coins and Values:

A bag contains equal no. of 25 p, 50 p and One rupees coins respectively. If the total 105, how many types of each type are present?

~~$$25 : 50 : 100 = 1 : 2 : 4$$~~

$$\frac{100}{10} \times 5$$

$$117 \times 105 \quad (1x7) / 105 = 14$$

$$\& (105) / 7 \quad (3x7) 14 / 105 =$$

$$85 : 50 : 100 \Rightarrow 1 : 2 : 4$$

69 Syntax Error

Abhishek Rathor

$$\frac{1}{x} \times \frac{15}{105} \Rightarrow 15 \times \frac{1}{4} = 60$$

$$\frac{2}{x} \times \frac{15}{105} \Rightarrow 30 \times \frac{1}{2} = 60$$

$$\frac{4}{x} \times \frac{15}{105} \Rightarrow 60 \times \frac{1}{1} = 60$$

(Q). A purse contains 342 coins consist of one rupee, 50 paise, 25 paise. If their values are in the ratio of 11:9:5 then find the 50 paise coins.

$$\frac{11x}{1} = 11x, \quad \frac{9x}{0.5} = 18x, \quad \frac{5x}{0.25} = 20x$$

$$11x + 18x + 20x = 342$$

$$49x = 342$$

$$x = \frac{342}{49}$$

$$x = 7 \text{ (almost)}$$

$$\begin{array}{r} 5 \\ 49 \times 6 \\ \hline 294 \\ 49 \\ \hline 343 \end{array}$$

$$11x = 77$$

$$18x = 126 \Rightarrow 126$$

$$20x = 140$$

Third proportion:

a : and b :: b : c

$$\Rightarrow b^2 : a$$

b

15

Profit & Loss; Discounts:

Cost Price (C.P), Selling Price (S.P), Profit/Gain, Loss.

$$1) \text{Loss \%} = \frac{\text{Loss}}{\text{C.P}} \times 100$$

$$2) \text{Profit \%} = \frac{\text{Profit}}{\text{C.P}} \times 100$$

When shopkeeper earns profit;

$$\text{Cost price} = \frac{100}{(100 + \text{Gain \%})} \times \text{S.P} \quad (\text{Given: Gain \% and S.P of an article})$$

$$\text{Selling Price} = \frac{(100 + \text{Gain \%})}{100} \times \text{C.P} \quad (\text{Given: Gain \% and Cost price of article.})$$

When shopkeeper incurs loss;

$$\text{Cost price} = \frac{100}{(100 - \text{Loss \%})} \times \text{S.P}$$

$$\text{Selling Price} = \frac{(100 - \text{Loss \%})}{100} \times \text{C.P}$$

1) Profit: $S.P > C.P$

2) Loss: $C.P > S.P$

3) If profit earned by selling an article is 25%
then Selling Price = 125% of C.P

4). If loss occurred by selling an article 30%.

$$S.P = 70\% \text{ of C.P}$$

5) A shopkeeper sells two similar items A and B. If A is sold at a gain of $x\%$ and B is sold at loss of $x\%$, then shopkeeper always incurs a loss given by,

$$\text{Loss \%} = \left[\frac{\text{common gain and loss \%}}{10} \right]^2$$

$$= \left[\frac{x}{10} \right]^2.$$

6). A trader sells goods at cost price but uses a weight of x kg instead of y kg and make profit. This profit can be calculated using formula

$$\text{True weight} - \text{False weight} = \text{Error}.$$

$$\text{Gain \%} = \left[\frac{\text{Error}}{(\text{True weight} - \text{Error})} \times 100 \right] \%$$

7) If cost price of x articles is equal to selling price of y articles, then profit can be calculated,

a) C.P of x = S.P of y

b) no. of x articles > no. of y articles

$$\text{Profit \%} = \frac{\text{no. of } x \text{ articles} - \text{no. of } y \text{ articles}}{\text{no. of } y \text{ articles}} \times 100$$

8) If a seller makes $x\%$ above C.P and offers at $y\%$, then profit % or loss % can be calculated by

$$\text{Profit} = (x - y) = \frac{x + y}{100}$$

Syntax Error

Abhishek Pathan

9) Discount :

a) Discount% = $\frac{\text{Discount}}{\text{M.P}} \times 100$

b) If D_1, D_2, D_3 are percentage of successive discounts on M.P, then S.P =

$$\text{M.P} \left(1 - \frac{D_1}{100}\right) \left(1 - \frac{D_2}{100}\right) \left(1 - \frac{D_3}{100}\right)$$

c) If a and b are two successive discounts, then single equivalent discount percentage is given as,

$$a+b - \frac{ab}{100}$$

Type 1: To find profit/loss or selling/cost price.

1. A shopkeeper sells an article for Rs. 200 with a loss of 20%. Find the cost price.

Method 1:

$$\text{S.P} = 200, \text{ loss} = 20\% \Rightarrow \frac{200 \times 100}{100+20} = x$$

$$\frac{2000}{120} = x$$

$$\frac{2000}{8} = x$$

$$250 = x$$

Method 2:

$$\text{Cost price} = \left[\frac{100}{100+\text{loss}\%} \right] \text{S.P}$$

$$= \frac{100}{80} \times 200 \Rightarrow \frac{2000}{8} = 250$$

Q. A trader expects a gain 15%. On his cost price if in a week his sale is of Rs. 580 then what is the profit?

$$\text{Selling Price} = \text{Profit} + \text{C.P}$$

$$= (\text{C.P at } 15\%) + \text{C.P}$$

$$= x + 0.15x$$

$$580 = 1.15x$$

$$\frac{580}{1.15}$$

$$504.34 = x$$

$$\text{Profit} = \text{S.P} - \text{C.P} = 580 - 504.34 = 75.66$$

$$\frac{100 \times 580}{115} = \frac{580 \times 20}{23} = \frac{11600}{23}$$

$$= 580 - 504.34$$

$$= 75.66$$

$$\begin{array}{r} 504.34 \\ 23 \end{array} \overline{)11600} \quad \begin{array}{r} 115 \\ 23 \end{array} \overline{)11600} \quad \begin{array}{r} 100 \\ 98 \end{array} \overline{)98} \quad \begin{array}{r} 69 \\ 69 \end{array} \overline{)0}$$

3. If a boy sells a book for 450 Rupees he gets a loss of 10%. Then find C.P. To gain 10%, what should be the selling price?

$$\begin{array}{r} 450 & 90\% \\ x & 100 \end{array} \Rightarrow \frac{500}{90\%} = x$$

$$\Rightarrow 555$$

$$\text{C.P} = 500$$

$$\text{S.P} = 550$$

4. A merchant sells 30 m of cloth and gains selling price of 10 m. Find the gain percent.

Selling price of item - cloth is profit

$$\text{Profit} = \text{S.P. of 10m} - (\text{S.P. of 20m}) - (\text{C.P. of 30m})$$

S.P. per meter of 20m = cost price of 10m

$$m = 100$$

$$2000 = 3000$$

$$\text{Profit} = \frac{10}{20} \times 100 = 50\%$$

Type 2: calculate profit / Loss if cost price of X articles equals to selling price of Y articles.

$$\text{Profit \%} = \frac{\text{no. of } x - \text{no. of } y}{\text{no. of } y} \times 100$$

(5). S.P. of 10 candles is same as C.P. of 12 candle. profit?

$$\frac{12-10}{10} \times 100 = \frac{2}{10} \times 100 = \frac{1}{5} \times 100 = 20\%$$

(6). S.P. of 110 apples is = to C.P. of 35 apple.

profit (or) loss?

$$= \frac{40-35}{35} \times 100 = \frac{5}{7} \times 100 = \frac{100}{7} = 14.28\%$$

cost price of apple = 40

Selling price of apple = 35

$40 > 35 \Rightarrow \text{Loss}$

Type 3: Find Loss incurred by the shopkeeper.

a) Material A sold at x%. gain & material B sold at y%. loss.

b) Material A sold at x%. gain & material B sold at y%. loss.

(7) A man purchased two plots for Rs. 5,00,000. On one he gains 15% while on other he loses 15%. Find how much does he gain or lose in the transaction.

$$\text{B.P.R}(\%) = \left[\frac{\text{common gain \% Loss}}{10} \right]^2$$

$$= \left[\frac{15}{10} \right]^2 = \frac{225}{100} = \frac{9}{4} = 2.25\%$$

(8). A boy bought camel and carriage for Rs. 5000. He sells the camel at a gain of 20% and the carriage at loss of 10%. If he gains whole 3%. Find the cost of camel.

$$\text{cost of camel} = x \text{ and carriage} = 5000 - x$$

$$20\% \text{ of } x - 10\% \text{ of } (5000 - x) = 3\% \text{ of } 5000$$

$$\left(\frac{20}{100} \times x \right) - \left(\frac{10}{100} \times 5000 - x \right) = \frac{3}{100} \times 5000$$

$$\frac{x}{5} - \frac{(5000 - x)}{10} = 150$$

To cancel the denominator (x 10)

$$\frac{10x}{50} - \frac{(5000 - x) 10}{10} = 150 \times 10$$

$$2x - 5000 + x = 1500$$

$$3x = 6500$$

$$x = \frac{6500}{3}$$

$$x = 2166.67$$

(P) A man sells one camera A for Rs. 7500 at a gain of 20% and another camera B for Rs. 8550 at a loss of 5%. Find loss or gain?

$$\text{Cost Price of } (A) = 6250$$

$$\frac{6250}{7500}$$

$$S.P(A) = 7500 \quad 120\%$$

$$12$$

$$\frac{6250}{7500} \times 120 = \frac{7500}{x} \times 8$$

$$\frac{6250}{7500} \times 120 = \frac{7500}{x} \times 8 \Rightarrow x = \frac{7500 \times 120}{6250 \times 8} = 180$$

$$\text{Cost price of } B = 9000$$

$$9000$$

$$8550 \quad 95\%$$

$$95$$

$$C.P = 6250 + 9000 = 15250$$

$$S.P = \frac{7500 + 8550}{2} = 16050$$

$$\text{Profit \%} = \frac{16050 - 15250}{15250} \times 100 \Rightarrow$$

$$\frac{800}{115250} \times 100 \Rightarrow$$

$$1525 \overline{)800} \begin{matrix} 5.8 \\ -1625 \\ \hline 3750 \\ -3050 \\ \hline 700 \end{matrix}$$

Type 4: Trader sells goods at cost price but uses a weight of x kg instead of y kg (false weights), and makes profit.

$$\text{Gain \%} = \frac{\text{Error}}{\text{False weight}} \times 100\%$$

(10). A shopkeeper sells his goods at cost price but uses a weight of 910 grams for a kg. what his gain percentage?

$$\text{Gain} = \frac{30}{970} \times 100 = \frac{300}{970} = \frac{900}{873} = \frac{97 \times 3}{873}$$

3.09

(11). A dishonest shopkeeper sells his grocery using weight 10% less than true weights and makes a profit of 30%. Find his gain percentage?

Let weight of grocery bag = 1000 gram.

10% less means 90%.

90% of 1000g = 900g

$$\text{calculate S.P} = \frac{100 + \text{Gain}}{100} \times \text{C.P}$$

$$= \frac{130}{100} \times 1000 = 1300$$

$$\text{Gain} = 1300 - 900 = 400$$

$$= \frac{400}{900} \times 100 = \frac{400}{9} = 44.44$$

(12). A dealer sells his good at ~~4/3~~ $\frac{14}{9}$ cost price. If by using false weights he gains $4\frac{8}{23}\%$. Then find the weight he uses for 1kg.

100% gains

$$4\left(\frac{8}{23}\right) = \frac{x}{(1000-x)} \times 100$$

$$\frac{100}{23} = \frac{100x}{1000-x} \Rightarrow 41.66$$

$$43(1000x) = 100(1000-x)$$

$$2300x = 100000 - 100x$$

$$2400x = 100000$$

$$x = \frac{100000}{2400}$$

False weight = $1000 - 41.66 = 958.34$ grams.

Type 5: Numericals on Discount and successive discount | points.

(13). After two successive discounts, a tie with a list price of Rs. 120 is available at Rs. 90. If second discount is 9%, what is first discount?

$$\text{First discount} = x$$

$$91\% \text{ discount at } (100-x)\% \text{ of } 120 = 90$$

$$\frac{91}{100} \times \frac{100-x}{100} \times 120 = 90$$

$$100-x = \frac{90 \times 100 \times 100}{91 \times 120}$$

$$= \frac{900000}{91 \times 120}$$

$$= \frac{900000}{10920}$$

$$= \frac{900000}{1080}$$

$$= \frac{900000}{10920}$$

(14). Find the single equivalent to a series discount of 30%, 20%, and 10%.

$$\frac{90}{100} \times \frac{80}{100} \times \frac{70}{100} = \frac{72 \times 7}{1000} \Rightarrow \frac{504}{1000} \Rightarrow \frac{63}{185}$$

$$\begin{array}{r} 125 \times 5 \\ 625 \\ 625 \\ \hline 500 \end{array}$$

$$\begin{array}{r} 6.50 \times 5 \\ 185 \\ 185 \\ \hline 500 \\ 500 \\ \hline 50 \end{array}$$

$$49.6$$

(15) A dealer marks price of all the goods at 30% above the cost price and assumes that he will make a profit of 15%. If he offers a discount of 15%. Find what will be his actual profit on sale?

$$\text{cost price} = 100\%$$

$$\text{Selling price} = 130\%$$

$$\begin{array}{rcl} 130 & 100\% \\ \times & 15\% \\ \hline 130 \times 15 & 3 \\ \hline 1950 & \\ 30 & \\ \hline 1650 & \\ \frac{1650}{2} & = \frac{39}{2} \\ \hline 19.50 & \end{array}$$

$$\begin{array}{l} \text{Marked price} = 130 - 19.50 \\ = 110.50 \end{array} \Rightarrow 19.50$$

(16). A manufacturer sells a pair of shoes to a wholesale dealer at a profit of 20%. Wholesale sells them to retailer at a profit of 25%. The shoes are again sold to the customer for Rs. 50.50 thereby earning a profit of 30%. Find the cost price of manufacturer.

$$\text{profit earned by manufacturer} = 20\%$$

$$\text{profit earned by wholesaler} = 25\%$$

$$\text{profit earned by retailer} = 30\%$$

$$\text{S.P of shoes} = \text{Rs. } 50.$$

$$\begin{array}{rcl} 126 \\ \frac{120}{100} \times \frac{125}{100} \times \frac{130}{100} = \frac{5050}{100} \\ 105 \quad 4 \quad 20 \end{array}$$

$$\frac{6}{5} \times \frac{5}{4} \times \frac{13}{10} = \frac{5050}{100}$$

$$\begin{array}{r} 30 \times 1 \\ 300 \\ \hline 90 \\ \hline 390 \end{array}$$

$$\frac{390}{250} = \frac{50.50}{100}$$

$$x = \frac{10}{100} \times \frac{10}{200}$$

$$= \frac{100}{100 \times 200}$$

$$= \frac{100}{200}$$

$$= \frac{10}{20}$$

$$= 5$$

$$= 25.89$$

$39 \overline{) 1890}$ $\underline{-15}$ $39 \overline{) 390}$ $\underline{-39}$ 0	39×2 $\underline{78}$ 39×5 $\underline{195}$ $39 \overline{) 380}$ $\underline{-380}$ 0	25.89 $\underline{18}$ 39×8 $\underline{312}$ $39 \overline{) 352}$ $\underline{-352}$ 0
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Simple Interest:-

Simple Interest on principal amount P for T years at the rate of R% is given as,

1) Principle = $\frac{100 \times S.I}{R \times T}$

2) Years = $\frac{100 \times S.I}{P \times R}$

3) Rate of Interest = $\frac{100 \times S.I}{P \times T}$

4) Simple Interest = $\frac{P \times R \times T}{100}$

* The rate of interest is always calculated per year unless specifically noted.

* If any numerical, the time given is specified in months, then converted it into years by simply dividing number of months by 12. If time is given in days, then converted days into years by dividing it with 365.

a) $S.I = \frac{PRM}{1200}$

* If sum of money becomes Z times in T years at simple interest, then rate of interest calculated using,

$$\text{Rate of Interest \%} = \frac{100(Z-1)}{T}$$

If we want to find T

$$\text{Time (T)} = \frac{100(Z-1)}{\text{Rate of Interest \%}}$$

Type I: Find Simple Interest.

- What will be the simple interest on 80,000 at $16\left(\frac{2}{3}\right)\%$ per annum for 9 months?

$$16\left(\frac{2}{3}\right) \Rightarrow \frac{(16 \times 3) + 2}{9 \times 4 \times 3} = \frac{48+2}{3} = \frac{50}{3}$$

$$\text{Time} = \frac{9}{12} = \frac{3}{4}$$

$$= 80,000 \times \frac{3}{4} \times \frac{50}{100}$$

$$= 80,000 \times \frac{50}{4}$$

$$= 20,000 \times \frac{50}{2}$$

$$= 10,000$$

- Find the simple interest on ₹ 5000 at 6% per annum. for the period 5th Feb to 19 April 2015.

$$5\text{th Feb} = 23$$

$$\text{March} = 31$$

$$\text{April} = 19$$

$$\frac{10}{7000} \times 6 \times 72$$

① Syntax Error

$$\frac{36500}{72}$$

$$= ₹ 60.$$

3. Swadesh Borrowers Rs. 10,000 for 2 years at 4% p.a. annum S.I. He lends it to Ramesh 6% p.a. for 2 years. find his gain per year.

$$\frac{10,000 \times 2 \times 4\%}{100} \Rightarrow 200 \times 4 = 800$$

$$\frac{10,000 \times 2 \times 6\%}{100} \Rightarrow 200 \times 6 = 1200$$

$$1200 - 800 = 400 \Rightarrow \text{for 2 years.}$$

$$400/2 = 200 \Rightarrow \text{for 1 year.}$$

Type 2: Find rate of interest!

4. At what rate % p.a will sum of money double in 20 years.

$$R = \frac{100(z-1)}{T} \Rightarrow \frac{100(2-1)}{20} = 5 \quad ⑤$$

5. John took a loan of Rs. 1500 with simple interest for as many years as the rate of interest. If he paid 5110 as interest at the end of loan period, what was the rate of interest.

$$S.I = \frac{P R R}{100} = S.I = \frac{1500 R^2}{100}$$

$$5110 = 15 R^2$$

$$\frac{1836}{540} = R^2$$

$$\frac{18}{18} = R^2$$

$$6 = R$$

6. A sum of Rs. 12,000 amounts to Rs 15000 in 4 years. Find S.I. rate.

$$\text{Rate of Interest} = \frac{\frac{SI - PRT}{T}}{P} = \frac{\frac{3000 \times 100}{18000 \times 4}}{4} = \frac{25}{4} = 6.25\%$$

Type 3 : Principle Amount

(1). Nikil borrowed some money at the rate of interest 5% for 2 years. 8% p.a for next 5 years. 10% p.a for a period beyond 7 years. If he pay total interest of 8000 and at the 10 years. Then find the money nikil borrowed.

$$\begin{aligned} SI &= \frac{PRT}{100} \Rightarrow \frac{P(5 \times 2)}{100} + \frac{P(8 \times 5)}{100} + \frac{(10 \times 3)P}{100} \\ &\Rightarrow \frac{10P + 40P + 30P}{100} = \frac{80P}{100} = 8000 \\ &\Rightarrow \frac{80000}{8} = 10,000 \end{aligned}$$

(2) A sum of was put at S.I. at certain rate for 2 years. Had it been put at 2% higher rate, it would have fetched RS.400 more. Find the sum.

$$\frac{P * (R+2) * 2}{100} - \frac{P * R * 2}{100} = 400$$

$$(PR + 2P) * 2 - PR * 2 = 400 \times 100$$

$$2PR + 4P - 2PR$$

$$4P = 40000$$

$$P = \frac{10000}{4}$$

$$P = 10000$$

(Q) The S.I. on a certain sum of money for 4 years at 15% per annum is Rs 180/- more than S.I. on the same sum for 5 years at 10% per annum. Find sum.

$$S.I. = \frac{PRT}{100} \Rightarrow$$

$$\frac{P(60)}{100} - \frac{P(50)}{100} = 180$$

$$10P = 18000$$

$$P = \frac{18000}{10}$$

$$P = 1800$$

Compound Interest:

A person borrows Rs. 2000 at 10% compound interest. Find the total amount paid by him after 3 years.

If simple means: $2000 + 600 = 2600$

In Compound Interest:

2000	$\xrightarrow{10\%}$	200
1st year	$\xrightarrow{10\%}$	220

Second year $2420 \xrightarrow{10\%} 2112$

Third year $2112 \rightarrow$ amount after 3 years.

Formula:

$$1. \text{ Amount} = P \left[1 + \frac{R}{100} \right]^n \quad (\text{annual})$$

$$2. \text{ Amount} = P \left[1 + \frac{R/2}{100} \right]^{2n} \quad (\text{half-year})$$

$$\text{Amount} = P \left[1 + \frac{R/4}{100} \right]^n \quad (\text{Quarterly})$$

1) Numerical on population.

$\Rightarrow \uparrow P_1 \text{ by } R\%$

$$P_2 = P_1 \left[1 + \frac{R}{100} \right]^n$$

$\Rightarrow \downarrow P_1 \text{ by } R\%$

$$P_2 = P_1 \left[1 - \frac{R}{100} \right]^n$$

2. If interest is compound annually and time is in fraction say $2 \frac{3}{5}$ years.

$$\text{Amount} = P \left[1 + \frac{(3/5)R}{100} \right]^n$$

3. If rates are different for different years.

$$\text{After } n \text{ years} = P \left[1 + \frac{R}{100} \right] \left[1 + \frac{R}{100} \right] \left[1 + \frac{R}{100} \right]$$

4. Present worth of sum of Rs. X due n years, hence $R\%$ p.a compound interest gn.

$$\text{Present worth} = \frac{X}{\left[1 + \frac{R}{100} \right]^n}$$

5. If difference b/w compound interest and simple interest is given find.

a) Two years $\Rightarrow P \left(\frac{R}{100} \right)^2 = \text{SICI} - \text{SI}$.

b) Three years $\Rightarrow P \left[\frac{R^2}{100^2} \right] \times \left[\frac{300+R}{100} \right]$

b) Rule of 72: The rule is used to find,

a) Time required for an amount to double itself, at a given rate of interest,

$$T_{\text{Time}} = \frac{72}{R \times 100}$$

b) Rate at which amount double itself, at a given time.

$$R = \frac{72}{\text{Time} \times 100}$$

$$\text{Future Value} = \text{Present Value} \left[1 + \frac{R}{100} \right]^n$$

But when we have to calculate time required for an account to double itself, at a given rate of interest, the above expression can be written as follows,

$$\text{Future Value (Final amount)} = 2 \text{ Present Value.}$$

$$\frac{\text{Future Value}}{\text{Present Value}} = 2.$$

Type I: Numerical Based on population.

The population of city increased 5% annually but decreased by $\frac{1}{4}$ due to migration. Find the net increase in percent in 3 years.

$$P_2 = P_1 \left[1 + \frac{R}{100} \right]^n \quad 5 - \frac{1}{4} = \frac{80 - 1}{4} = \frac{15}{4}$$

$$P_2 = P_1 \left[1 + \frac{R}{100} \right]^3$$

$$= 100 \left[1 + \frac{19}{100} \right]^3 \Rightarrow 100 \left[\frac{400+19}{400} \right]^3$$

b) Syntax Error
Abhishek Rathi

$$= 100 \left[\frac{119}{100} \right]^3 = 114.93$$

$$= 100 - 114.93 = 14.93\%$$

- 2) The value of a sewing machine depreciates at the rate of 10% after every year. If at the end of 3 years, its value is Rs. 8748, find its purchase price.

$$P_A = P \left[1 - \frac{R}{100} \right]^n$$

$$8748 = P \left[1 - \frac{10}{100} \right]^3$$

$$\frac{8748}{\left(1 - \frac{10}{100} \right)^3} \Rightarrow$$

$$P = 12,000$$

Type 2: Numerical to find compound interest (yearly/half-yearly, quarterly)

- 3) Find the compound interest on \$5000 for

9 months, 6% per annum. If the interest is reckoned quarterly.

$$P = 5000 \left[1 + \frac{6}{4 \times 100} \right]^3$$

$$P = 5000 \left[1 + \frac{3}{500} \right]^3$$

$$= 5228.37$$

$$= 228.37$$

4) Find compound interest on Rs. 20,000 in 8 years at 4% p.a., the interest being compounded half-yearly.

$$\Rightarrow 20,000 \left[1 + \frac{2}{100} \right]^4$$

$$21648.64$$

$$20,000.00$$

$$1648.64$$

5) Find compound interest on Rs. 8500 at 4% per annum for 2 years.

$$8500 \left[1 + \frac{4}{100} \right]^2 = \text{Rs. } 9193.6$$

$$9193.6 - 8500 = 693.6$$

Type 3: Numerical to find rate of interest/years.

b) An amount of Rs. 500 amounts to Rs. 583.20 in two years if compounded yearly. Find the rate of interest per annum.

$$583.20 = 500 \left[1 + \frac{R}{100} \right]^2 \Rightarrow \frac{583.20 \times 10}{500 \times 10} = \left[1 + \frac{R}{100} \right]^2$$

$$\frac{11664}{5000} = \left[1 + \frac{R}{100} \right]^2 \Rightarrow \frac{11664}{10000} = \left[1 + \frac{R}{100} \right]^2$$

$$\left[\frac{108}{100} \right]^2 = \left[1 + \frac{R}{100} \right]^2 \Rightarrow 1 + \frac{R}{100} = \frac{108}{100} \Rightarrow \frac{R}{100} = \frac{108}{100} - 1 \\ \Rightarrow \frac{R}{100} = \frac{108 - 100}{100}$$

$$\frac{R}{100} = \frac{8}{100} \Rightarrow R = \frac{8}{100} \times 100 = 8\%$$

7) A certain sum amounts to Rs. 7000 in 2 years and to Rs. 8000 in 3 years. Find the sum.

D Amount = Rs. 7000 Time 2 years

2) Amount = Rs. 8000 Time 3 years

$$\begin{array}{r} 14.8 \\ - 100 \\ \hline 48 \\ - 30 \\ \hline 18 \\ - 14 \\ \hline 4 \end{array}$$

Simple Interest = $\frac{PRN}{100}$

$$1000 = \frac{7000 \times R \times 1}{100}$$

$$R = 11 \frac{2}{7} \text{ %}$$

$$7000 = P \left[1 + \frac{100}{7 \times 100} \right]^2$$

$$\frac{8}{7} \times \frac{8}{7} \times P = 7000$$

$$P = 5359.37$$

(8). A sum of money double itself at compound interest in 10 years. In how many years will it be eight times?

$$2P = P \left[1 + \frac{R}{100} \right]^n \quad P \left[1 + \frac{R}{100} \right]^n = 8P$$

$$2 = \left[1 + \frac{R}{100} \right]^n$$

$$\left[1 + \frac{R}{100} \right]^n = 8 = 2^3$$

$$\left[1 + \frac{R}{100} \right]^n = \left[\left[1 + \frac{R}{100} \right]^{10} \right]^3$$

$$\left[1 + \frac{R}{100} \right]^n = \left[1 + \frac{R}{100} \right]^{30}$$

\Rightarrow Ans: 30 years

Type 4: Numerical to find sum/ rate of interest, when difference b/w C.I and S.I is specified.

9) The difference between C.I and S.I on a certain sum at 10% per annum for 2 years is Rs. 530. Find the sum.

$$\text{C.I} = P \left[\left(1 + \frac{10}{100}\right)^2 - P \right] \Rightarrow (1.1)^2 = 1.21P \quad (1)$$

$$\text{C.I} = P(1.21) - P = 1.21P - P = 0.21P \\ = \frac{21P}{100}$$

$$\text{S.I} = \frac{PRT}{100} = \frac{P \times 10 \times 2}{100} = \frac{20P}{100} = \frac{P}{5}$$

$$\text{C.I} - \text{S.I} = \frac{21P}{100} - \frac{P}{5} = 530$$

$$= \frac{21P(5) - 100P}{500} = 530$$

$$= \frac{105P - 100P}{500} = 530$$

$$= \frac{5P}{500} = 530 \times 100 = 53000$$

10) The difference b/w C.I and S.I accrued on an amount of Rs. 20,000 in 2 years was Rs 392. Find the rate of interest per annum.

$$\left[P \left[\left(1 + \frac{R}{100}\right)^2 - P \right] - \left[\frac{PRN}{100} \right] \right] = 392$$

$$\left[20000 \left[\left(1 + \frac{R}{100}\right)^2 - 20000 \right] - \left[\frac{20,000 \times R \times 2}{100} \right] \right] = 392$$

$$\left[20000 \left[\frac{(100+R)^2 - 10000}{100} - 20000 \right] - \left[\frac{20,000 \times R \times 2}{100} \right] \right] = 392$$

$$20000 \left[\frac{(100+R)^2 - 10000 - 2000R}{10000} \right] = 392$$

$$2R^2 = 392$$

$$R^2 = 196$$

$$R^2 = 196$$

$$R = 14 \text{ km} \text{ (Radius of Earth)}$$

(II) The compound interest on a certain sum at 50/3 % for 3 years is Rs. 187. Find simple interest on same sum for same period and rate.

$$187 = P \left[\left(1 + \frac{50}{3 \times 100} \right)^3 - P \right]$$

$$187 = P \left(1 + \frac{1}{6} \right)^3 \Rightarrow P \left(\frac{1}{6} \right) \Rightarrow \frac{343}{216} - 1$$

$$\Rightarrow \frac{343 - 216}{216} = \frac{127P}{216}$$

$$127 = \frac{127P}{216}$$

$$216 = P$$

$$S.I = \frac{216 \times 50 \times 3}{200}$$

$$S.I = 108$$

Permutation & Combination:

Permutation: The various ways of arranging a given number of things some (or) all at a time are all called as permutation.

Word formation, number formation, circular formation.

In permutation, objects are to be arranged in particular order.

$${}^n P_r \text{ (or), } P(n, r)$$

Combination:

Each of different groups or selections formed by taking some or all number of objects is called combination.

$${}^n C_r \text{ (or), } C(n, r)$$

Type I: permutation.

1. Find value of ${}^{50} P_8$.

$$\begin{aligned} {}^n P_r &\stackrel{\text{def}}{=} \frac{n!}{(n-r)!} = \frac{50 \times 49 \times 48}{48!} \\ &= 50 \times 49 = 2450 \end{aligned}$$

2. How many words can be formed by using letters of the word (DELHI)

$${}^5 P_5 \Rightarrow 1 \times 2 \times 3 \times 4 \times 5 = 120$$

3. Find the number of ways the letters of the word 'RUBBER' can be arranged?

$$2R \cdot 2B \cdot 7U \cdot 1F$$

The required no. of ways = $N!$

$$\frac{N!}{(R!) \times (B!) \times (U!) \times (F!)} \cdot$$

$$6!$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(R!) \times (B!) \times (U!) \times (F!)} \cdot$$

$$= 720 \times 6 = 4320$$

4. Find in how many ways, the letters of the word 'LEADING' can be arranged in such way that the vowels always come together?

$$\text{No. of Vowels} = EAI \quad (3)$$

$$\text{Remaining consonants} = LDNG \quad (4)$$

$$(5) \Rightarrow 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

$$3! = 6$$

$$120 \times 6 = 720$$

Type 2. combination:

(5). Find value of ${}^{20}C_{17}$.

$$= \frac{n!}{r!(n-r)!} = \frac{20 \times 19 \times 18 \times 17}{17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \cdot$$

$$= \frac{20 \times 19 \times 18 \times 17}{17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \cdot$$

$$= \frac{20 \times 19 \times 18 \times 17}{17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \cdot$$

$$= \frac{20 \times 19 \times 18 \times 17}{17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \cdot$$

6. Out of 5 second consonants and 4 vowels,

how many words of 3 consonants, 2 vowels
can be formed?

$${}^5C_3 \times {}^4C_2 = 10 \times 6 = 60$$

$$\frac{5!}{3!2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{120}{2} = 60$$

$$\frac{4!}{2!2!} \Rightarrow \frac{12 \times 11}{2} = 6$$

$$5! \Rightarrow 1 \times 2 \times 3 \times 4 \times 5 = 120 \\ = 60 \Rightarrow 7200$$

7. A bag contains 2 white marbles, 3 black
marbles and red 4 marbles. Find in how many ways,
3 marbles can be drawn, so that at least one black
marble included in each draw.

$$= {}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + 1$$

$$\left(\frac{3!}{1!2!} \times \frac{6!}{8!1!} \right) + \left(\frac{3!}{2!1!} \times \frac{6!}{11!5!} \right) + 1 \\ \left(\frac{3 \times 2 \times 1}{1 \times 2 \times 2} \times \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{1 \times 2 \times 1 \times 2 \times 3 \times 4} \right) + \left(\frac{1 \times 2 \times 3}{1 \times 2 \times 1} \times \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{1 \times 2 \times 1 \times 3 \times 4 \times 5 \times 1} \right) + 1 \\ (3 \times 15) + (3 \times 6) + 1 \Rightarrow 45 + 19 = \underline{\underline{64}}$$