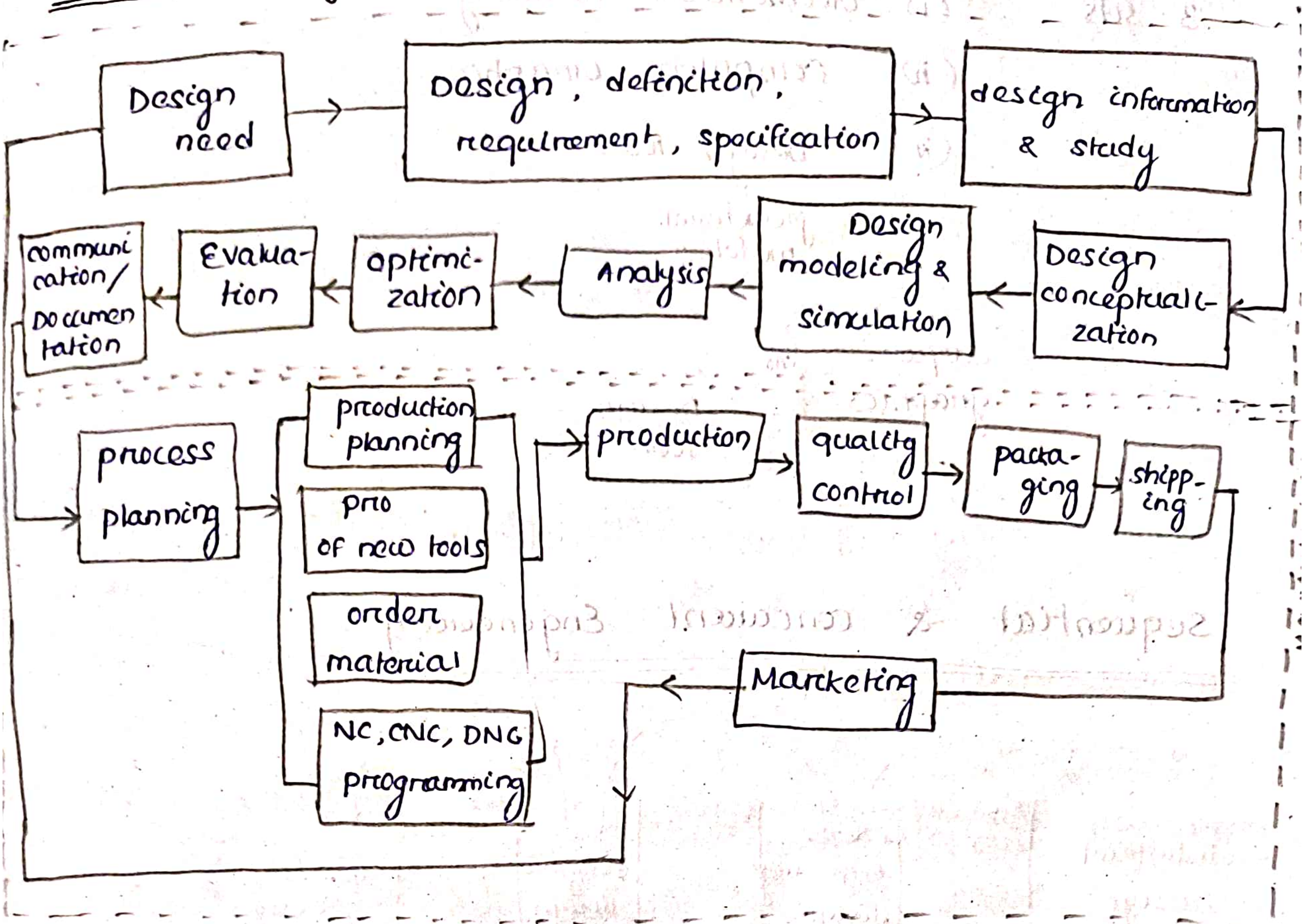
Product Life Cycle :



→ The CAD CAM has been utilised in different ways by different people

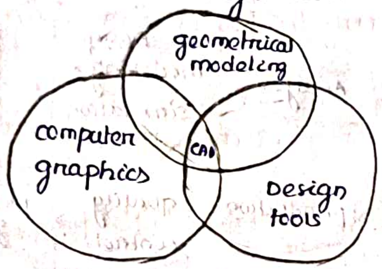
(i) Utilise to produce drawing or documentation of the design

(ii) Generating shadded or 3D images and animated display

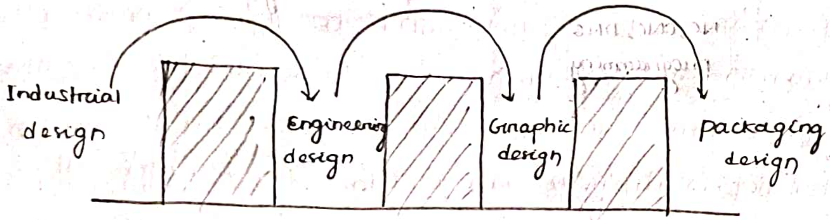
(iii) perform engineering analysis of a geometry model such finite element analysis.

(iv) CAM is also used to perform process planning and generating NC part programming.

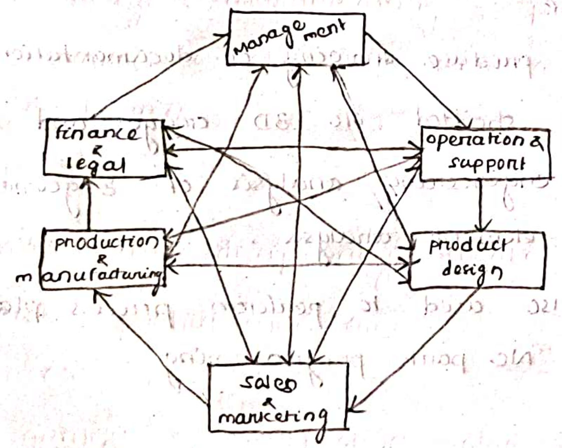
→ The CAD tools can be defined as the intersection of 3 sets :
(i) Geometrical modeling
(ii) Computer Graphics
(iii) Design tools



Sequential & concurent Engineering :



(sequential Engineering)



(concurrent Engineering)

→ The traditional or sequential engineering is the term used to describe linear product developement processes where design and developement steps are carried out one after another focusing on one field of expertise at a time.

→ During the industrial revolution this concept force companies to hire specialised people & department due to product getting more complicated.

→ This in term giving the way to an over the world mentality where each team member or department only focuse on their task and passed the completed work on to the next.

→ This traditional product developement process developes a large vertical structure that only interacts downstreams

Advantages of seq. Engeneering

→ Easy to track progess & understand the requiremen of the project & processes.

→ It is well defined for each team member & department.

→ simple to understant & avoid misinterpretation

→ It is an imposed discipline approach that takes away any misunderstanding.

Disadvantages :

→ Increase product cycle time because of network during the latter stages of product developement.

→ change request from others in latter stage are often very expensive & difficult to manage.

-> without staged feedback & specification with evaluation gets the product might creep out or fall ~~may~~ sort of expectations.

-> Manufacturing & production cost might too difficult & expensive making final unit cost not economical viable.

Concurrent Engineering :

-> Concurrent Engg. is a method of designing and developing Engg. products in which different departments simultaneously work on different stages of engg. product developement.

-> If manged well it helps increase the efficiency of product developement and marketing considerably reducing time and contributing the red^n of overal developement cost while improving final product quality.

-> This streamline approach towards engg. product forces several teams within the organisation such as product design, manufacturing, production, financial service etc to work simultaneously on new product developement.

-> **Advantages** :

-> It increases multi disciplinary collabor.

-> It reduces product cycle time.

-> It reduces cost.

-> Increases quality by supporting entire product cycle.

Increases productivity by stopping mistakes in tracks

-> It gives a competative edge over the competitors.

Disadvantage :

-> complex to manage

-> It relies on everyone working together hence communicatio is critical.

-> Room for mistake small as it impart all the department or discipline involve.

CAD TOOL

-> Based on implimentation in a design environment CAD tools can be defined as the design tools being improved by computer hardware & software throughout its various phases to achieve the design goal efficiently & competatively.

-> The designers will always require tools that provide them with fast and reliable sol^n to design situations that involve iterations and testing of more than one alternative.

-> The CAD tools can vary from geometric tools such as : Manipulation of graphics entities and interference checking on one extreme to customised application programms such as developing, analysis and optimisation routines on the other extreme.

-> In between these two extremes typical tools currently available include tolerance analysis, mass property calculation, space analysis and thermal analysis etc.

Design tools + computer
- Hardware (CPU, I/O, Display)
- Software (FEA, CFD, modelling)

= CAD tool

Reasons for implementing a CAD system :

(i) To increase productivity of designer.

→ The CAD increase the productivity of designer to visualise the product, it's opponents and parts and also reduces the time required in synthesising, analysing & documenting the design.

(ii) To improve the quality of design

→ The CAD system permits a more detail engg. analysis and a large no. of design alternatives can be investigated. The design errors are also reduced because of greater accuracy provided by system

(iii) To improve communication in design

→ The ease of CAD system provide better engg. drawing more standardisation of drawing, better documentation in drawing and less error in drawing.

(iv) To create a database for manufacturing

In the process of creating documentation for product design much of the required database for manufacturing the product can be created.

(v) Improve the efficiency of design

→ It improves the efficiency of design process and the wastage at the design stage can be reduced.

Benefits of CAD

→ Improve engg. productivity

→ Reduce manpower require

→ More efficient operation

→ Customer modifications are easier to make.

→ Low wastage.

→ Improve accuracy of design

→ Better design can be evolved.

→ Saving of materials & machining time by optimisation.

→ Colors can be used to customise the product.

Limitations of CAD

→ The system require large memory and speed.

→ The size of software package is large.

→ It requires highly skilled personnel.

→ It has huge investment.

CAD system Evaluation criteria

The various types of CAD systems are available based on the computer system used such as :

- Main frame based
- Mini computer
- Micro com
- Work station based

The implementation of these types of systems by various venders, software developers, hardware manufactures are available in the market so the selection of a optimised system is require to evaluated through a guideline based on different requirements.

- The selection guideline may vary sharply from one organisation to another but the technical evaluation criteria are largely the same.
- The technical evaluation normally based on certain criteria like :

(i) system consideration
(ii) Geometrical modelling capability
(iii) Design documentation
(iv) Applications

system consideration :

(i) Hardware :

Each work station connected to a central computer called server which has large disk and enough memory to store users files and application program as well as executing these programs.

(ii) Software :

3 major contributing factors are

- The type of operating system in which the software runs.
- The type of user interface.
- Quality of documentation

(iii) Maintainance :

- Repair of hardware components and software updates comprise the ~~ty~~ ~~m~~ of typically maintainance contracts.
- The annual cost of these contracts is substancial about 10-15% and should be consider in deciding on the cost of a system in addition to initial capital investment.

(iv) Support & services :

The vender support typically includes training, field services and technical supports. Most venders provide training courses sometimes on site if necessary.

CAD system selection criteria

(1) System requirement

(2) Geometrical modelling capabilities

(i) Representation technique

→ The geometrical modeling module of a CAD system is its heart. The application module of the system is directly related to and limited by the various representation it supports.

(ii) co-ordinate system & inputs

→ In order to provide the designer with the proper flexibility to generate geometric models various types of co-ordinate systems and co-ordinate inputs to be provided.

→ various types of co-ordinate systems are available such as cartesian co-ordinate system
cylindrical ,,
spherical ,,

(iii) Modeling entities

→ The fact that a system supports a representation scheme is not enough ; It is important to know the specific entities provided by the scheme, the ease to generate, verify and edit these entities should be consider during evaluation.

(iv) Geometric editing & Manipulation

→ It is essential to ensure that these geometric function exist for 3 types of representation. editing fun^n include intersection, trimming, projection etc and manipulation includes translation, rotation, copy, mirror, offset, scaling etc.

(v) Graphic standard support

→ If geometric models data base are to be transfered from one system to another both system must support exchange standard.

(3) Design Documentation

→ After a geometric model is created standard drafting practices are usually applied to it to generate the engg. drawing or the blue print.

→ various views i.e. top view, front view and side view are generated in the proper drawing lay out then dimensions are added and hidden lines are represented by dotted lines, tolerances are specified, general notes and labels are added etc.

(4) Application

(i) Assembling or model merging

→ Generating assemblies and assembly drawing from individual parts is an essential process.

(ii) Design applications

→ There are design packages available to perform applications such as mass property calculation, tolerance analysis, finite element modeling & analysis, injection modeling analysis, mechanism analysis & simulation.

(iii) Manufacturing application

→ The common packages available are tool path generation a verification, NC part programming, post progressing, process planning and CNC programming, etc.

(iv) programming language support

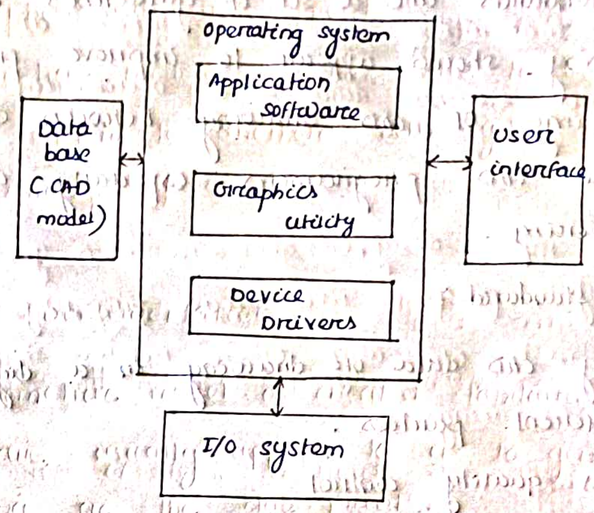
→ It is vital to look into the various levels of programming language a system supports. Attention should be payed to the syntax of graphic program when they are used. If the syntax changes significantly then confusion and panics of user should be expected when syntax error is there.

ASSIGN

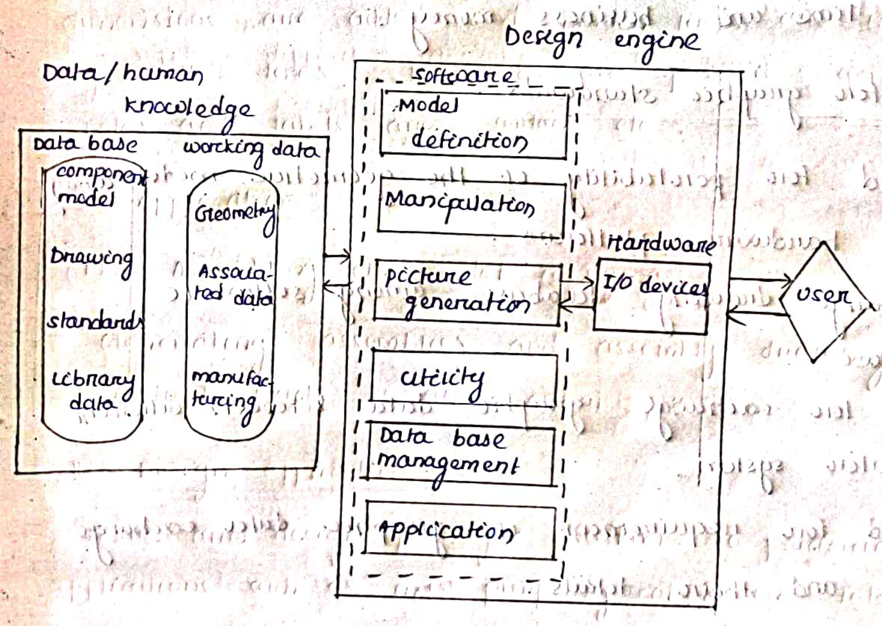
(1) Explain in detail the different components for CAD system architecture

(2) Describe the different devices use for computer graphics generation

CAD System Architecture



Advanced CAD system Architecture



CAD standards

→ CAD standards are a set of guidelines for the way CAD drawing should appear to improve productivity and interchange of CAD drawing between different offices and CAD programmers especially in architecture and engineering.

Why CAD standard :

(i) Sharing CAD data or drawing in a digital format between different parties.

(ii) Simplifies quality control

(iii) Uniformity throughout the world.

(iv) Standardise layers and consistant data appearance.

saves time and business money.

Need for graphic standards

(i) Need for portability of the geometric model among different hardware platforms.

(ii) Exchange drawing database among software database.

(iii) Need for exchange graphic data between different computer system.

(iv) Need for requirement of graphic data exchange format and their details.

Some of the graphic standards are :

GKS (Graphic Kernel system)

PHIGS (programmers Hierarchical interface of graphics system)

IGES (Initial graphics exchange specifications)

DXF (Drawing exchange format)

STEP (standard for the exchange of product data)

DMIS (Dimensional measurement interface specification)

VDI (Vertual device interface)

VDM (" " Metafile)

GKSM (GKS Metafile)

NAPLPS (North American presentation level protocol syntax)

Modelling data used in product description.

Normally 4 types of data are used to describe the product :

(i) shape data

(ii) Non shape data

(iii) Design data

(iv) Manufacturing data

Shape data

Both geometry & tropological interference informations part or form features, font, color etc are considered as the part of shape or geometry information

Non shape data :

→ Graphic data such as shaded images, model global data as measuring units of database and the resolution of storing data base numerical values.

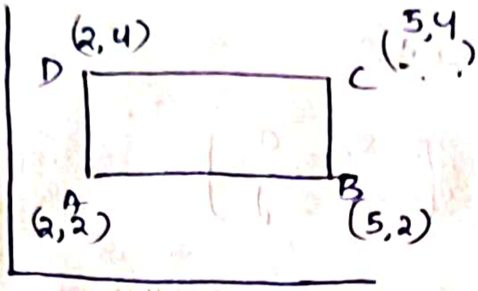
Design data :

Information that designers generate from geometric models for analysis purpose like mass property, finite element mesh or any other related data required for further analysis.

Manufacturing data

Information as tooling equipements, NC tool path, tolerance, process planning, tool design, ~~below~~ bill of material etc.

Translation, Rotation & Scaling



Translate
x → 4 unit
y → 5 unit

A → 2+4 = 6
2+5 = 7

B → 5+4 = 9
2+5 = 7

C → 5+4 = 9
4+5 = 9

D → 2+4 = 6
4+5 = 9

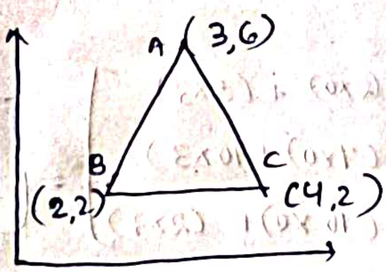
$$\begin{bmatrix} 2 & 2 \\ 5 & 2 \\ 5 & 4 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 4 & 5 \\ 4 & 5 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 9 & 7 \\ 9 & 9 \\ 6 & 9 \end{bmatrix}$$

Scaling

zoom in – scale up Δ Δ
zoom out – scale down Δ Δ
scale in Δ Δ

If
scaling factor > 1 (scale up)
scaling factor < 1 (scale out)
scaling factor = 1 (scale in)

*



Scaling factor : 4

$$\begin{bmatrix} 3 & 6 \\ 2 & 2 \\ 4 & 2 \end{bmatrix} \propto \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \quad \end{bmatrix}$$

A → 3×4 = 12
6×4 = 24

B → 2×4 = 8
2×4 = 8

C → 4×4 = 16

Scaling :

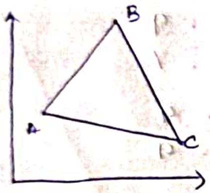
$x' = x \cdot S_x$

$y' = y \cdot S_y$

$$[x' \ y'] = [x \ y] \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$= x \cdot S_x + y \times 0 \qquad x \cdot 0 + y \cdot S_y$$

$$(xS_x \quad yS_y)$$



A (2,5)

B (7,10)

C (10,2)

X – direction → 2 unit

Y – direction → 3 unit

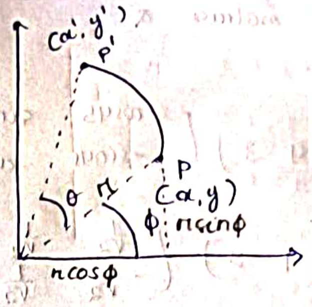
$\boxed{P' = P \cdot S}$

$$P = \begin{array}{c|cc} & X & Y \\ \hline A & 2 & 5 \\ B & 7 & 10 \\ C & 10 & 2 \end{array} \qquad S = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{array}{c} A' \rightarrow \\ B' \rightarrow \\ C' \rightarrow \end{array} \begin{bmatrix} x_1' & y_1' \\ x_2' & y_2' \\ x_3' & y_3' \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 7 & 10 \\ 10 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (2\times2)+(5\times0) & (2\times0)+(5\times3) \\ (7\times2)+(0\times0) & (7\times0)+(10\times3) \\ (10\times2)+(2\times0) & (10\times0)+(2\times3) \end{bmatrix} = \begin{bmatrix} 4 & 15 \\ 14 & 30 \\ 20 & 6 \end{bmatrix}$$

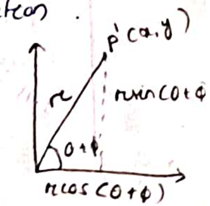
2-D Geometric Rotation :



$[x' \ y'] = [x \ y]$

$\theta = ;$

formulate an expression for the above situation.



$\boxed{x = r\cos\phi}$

$\boxed{y = r\sin\phi}$

$\boxed{x' = r\cos(\theta+\phi)}$

$\boxed{y' = r\sin(\theta+\phi)}$

$x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta$

$y' = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$x' = x\cos\theta - y\sin\theta$

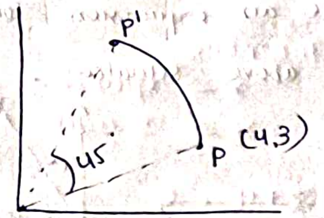
$y' = x\sin\theta + x\cos\theta$

$$[x' \ y'] = [x \ y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \rightarrow \text{Anticlock}$$

$$[x' \ y'] = [x \ y] \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\boxed{[x' \ y'] = [x \ y] \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}} \quad \text{clockwise}$$

Q1) A point (4, 3) is rotated counter clockwise has an angle 45°. Find the rotation matrix & resultant point.



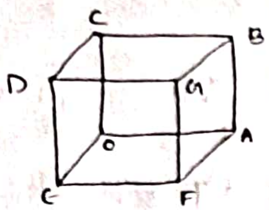
$$[x' \; y'] = [4 \;\; 3] \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix}$$

$$= [4 \;\; 3] \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} (4 \times \frac{1}{\sqrt{2}}) + (3 \times -\frac{1}{\sqrt{2}}) & (4 \times \frac{1}{\sqrt{2}}) + (3 \times \frac{1}{\sqrt{2}}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} \end{bmatrix}$$

Q)



O (0, 0, 0)
A (1, 0, 0)
B (1, 1, 0)
C (0, 1, 0)
D (0, 1, 1)
E (0, 0, 1)
F (1, 0, 1)
G (1, 1, 1)

A unit cube is translated 2 units in x - direction, 3 units in y - dir^n 4 unit in z - direction. Find the translated position. (Row major or Column major)

* Row major

$P' = P \cdot T$

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

column major

$P' = T \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

~~Given question~~ Given $T_x = 2$, $T_y = 3$, $T_z = 4$

column major

$P' = T \cdot P$

$$\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cccccccc} O & A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Row major

$P' = P \cdot T$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

: Scaling Transformation :

Scaling matrix

$$S[x, y, z] = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* perform a scaling matrix transformation of a cube shown below with 2 - units in x - direction, 3 units in Y direction and 2 units in z - direction

Rotation :

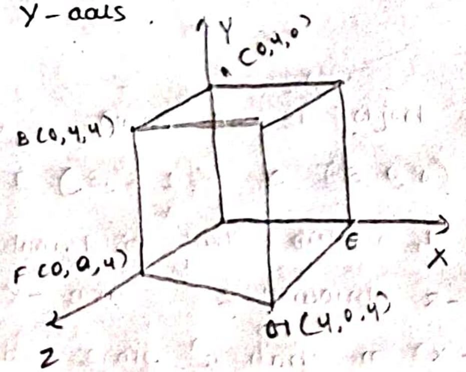
Rotation in x - direction
(counter clockwise)   clockwise $\theta = (-\theta)$

$$x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Y - direction , $Y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$Z = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* perform rotation transformation over a cube 'OABCDEFG' and rotate it through 45° in anticlockwise dirn about Y - axis .



Y - axis

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A homogenous coordinate point P[3,2,1] is translated in x, y, z direction by -2, -2 and -2 unit respectively, followed by successive rotation of 60° about x-axis. Find the final position of homogenous coordinate.

Sol:

Sol: point $P\,[3\ 2\ 1]$

(i) Translation

$$P' = P_T \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

$$P'_T = [3\ 2\ 1\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & -2 & -2 & 1 \end{bmatrix} = [1\ 0\ -1\ 1]$$

(ii) Rotation

$$P'_R = [1\ 0\ -1\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & \sin 60 & 0 \\ 0 & -\sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = [1\ 0.866\ -0.5\ 1]$$

Q2) Given a 3D object with coordinate points A (0,3,1), B (3,3,2), C (3,0,0), D (0,0,0) (a) Apply the translation with distance 1 towards x-axis, 1 towards y-axis, 2 towards z-axis

(b) Scale 2 units in x-direction, 3 units in y-direction and 4 units in z-dirⁿ

Rotate 60° about x-axis

Solⁿ

$$P = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 3 & 1 & 1 \\ 3 & 3 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Translation

$$P' = P \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

$$P'_T = \begin{bmatrix} 0 & 3 & 1 & 1 \\ 3 & 3 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 1 \\ 4 & 1 & 4 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$$

(b) Scaling

$$P'_S = \begin{bmatrix} 1 & 4 & 3 & 1 \\ 4 & 4 & 4 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 12 & 12 & 1 \\ 8 & 12 & 16 & 1 \\ 8 & 3 & 8 & 1 \\ 2 & 3 & 8 & 1 \end{bmatrix}$$

(c) Rotation

$$P_R = \begin{bmatrix} 2 & 12 & 12 & 1 \\ 8 & 12 & 16 & 1 \\ 8 & 3 & 8 & 1 \\ 2 & 3 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & \sin 60 & 0 \\ 0 & -\sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4.3 & 16.33 & 1 \\ 8 & -7.8 & 18.39 & 1 \\ 8 & -5.42 & 6.59 & 1 \\ 2 & -5.42 & 6.59 & 1 \end{bmatrix}$$

Q21 A homogenous coordinate point $P[3,2,1]$ is translated in x, y, z direction by $-2, -2$ and -2 unit respectively, followed by successive rotation of 60° about x-axis. Find the final position of homogenous coordinate.

Sol: point $P\,[3\ 2\ 1]$

(i) Translation

$$P' = P_T \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

$$P'_T = \begin{bmatrix} 3 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 \end{bmatrix}$$

(ii) Rotation

$$P'_R = \begin{bmatrix} 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & \sin 60 & 0 \\ 0 & -\sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.866 & -0.5 & 1 \end{bmatrix}$$

Q2) Given a 3D object with coordinate points A (0,3,1) B (3,3,2), C (3,0,0), D (0,0,0) (a) Apply the translation with distance 1 towards x-axis, 1 towards y-axis, 2 towards z-axis
(b) scale 2 units in x-direction, 3 units in y-direction and 4 units in z-dirⁿ
Rotate 60° about x-axis

Solⁿ

$$P = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0 & 3 & 1 & 1 \\ 3 & 3 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Translation

$$P' = P \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

$$P'_T = \begin{bmatrix} 0 & 3 & 1 & 1 \\ 3 & 3 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 1 \\ 4 & 4 & 4 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$$

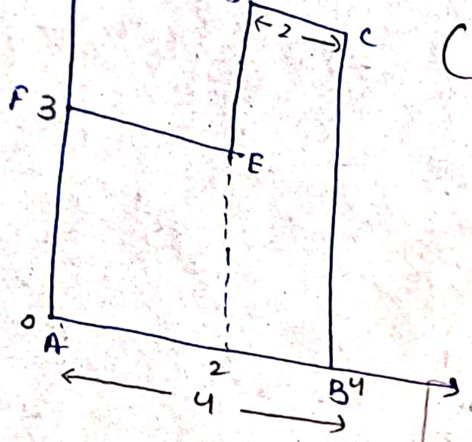
(b) Scaling

$$P'_S = \begin{bmatrix} 1 & 4 & 3 & 1 \\ 4 & 4 & 4 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 12 & 12 & 1 \\ 8 & 12 & 16 & 1 \\ 8 & 3 & 8 & 1 \\ 2 & 3 & 8 & 1 \end{bmatrix}$$

(c) Rotation

$$P_R = \begin{bmatrix} 2 & 12 & 12 & 1 \\ 8 & 12 & 16 & 1 \\ 8 & 3 & 8 & 1 \\ 2 & 3 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 60 & \sin 60 & 0 \\ 0 & -\sin 60 & \cos 60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4.3 & 16.33 & 1 \\ 8 & -7.8 & 18.39 & 1 \\ 8 & -5.42 & 6.59 & 1 \\ 2 & -5.42 & 6.59 & 1 \end{bmatrix}$$

...dinate system



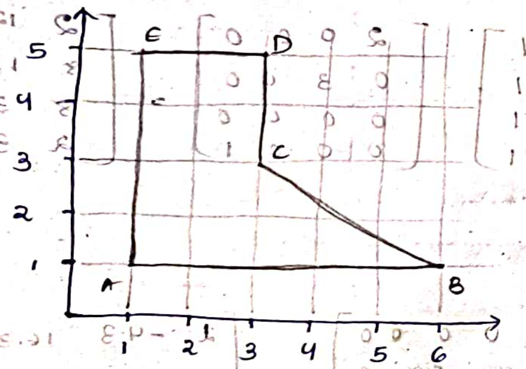
(Absolute co-ordinate system)

Points	A	B	C	D	E	F	A
Coordinates	(0,0)	(4,0)	(4,5)	(2,5)	(2,3)	(0,3)	(0,0)

Command :: Line ↵

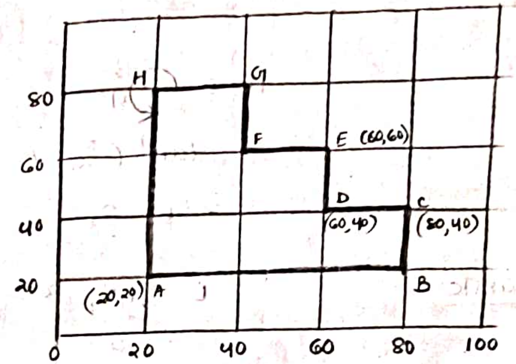
Specify first point : 0, 0 ↵

and point : 4, 0 ↵



(Relative co-ordinate system)

Points	A	B	C	D	E	A
coordinates	(1,1)	@ (5,0)	@ (-3,2)	@ (0,2)	@ (-2,0)	@ (0,-4)



(Degree w.r.t +ve x-axis)

Describe co-ordinates by applying

(a) Absolute co-ordinate system

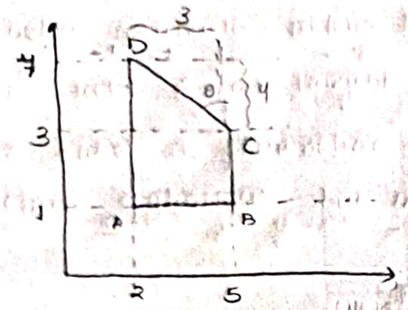
(b) Relative co-ordinate system

(c) polar co-ordinate system

Absolute co-ordinate system

	Absolute	Relative	polar
A	20, 20	@ 20, 20	@ 20, 20
B	80, 20	@ 60, 0	@ 60 <0
C	80, 40	@ 0, 20	@ 20 <90
D	60, 40	@ -20, 0	@ 20 < 180
E	60, 60	@ 0, 20	@ 20 <90
F	40, 60	@ -20, 0	@ 20 <180
G	40, 80	@ 0, 20	@ 20 < 90
H	20, 80	@ -20, 0	@ 20 < 180
A	20, 20	@ 0, -60	@ 20 < 270

* Express in polar co-ordinate system. (w.r.t. +ve x-axis)



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(3/4)$

$= 36.86$

$L = \sqrt{4^2+3^2} = 5$

points	co-ordinates
A	2, 0
B	@ 3 < 0
C	@ 2 < 40
D	@ 5 < 90 + 36.86 = 126.82
A	@ 6 < 270

* Draw the figure from the following polar co-ordinate system.

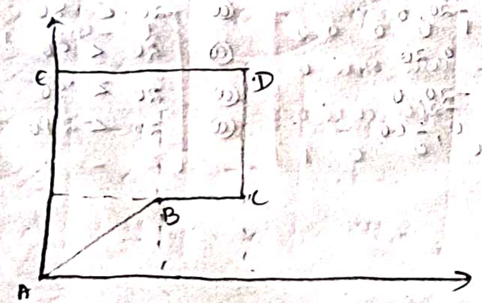
A $(0,0)$

B @ $\sqrt{2^2+2^2}$ $\tan^{-1}(2/2)$ 2.82

C @ 2 < 0   E @ 4 < 180

D @ 4 < 90   A @ 6 < 270

sol^n



Parametric and Geometric Continuity

SPLINES (Curve) → Hermite, Bezier

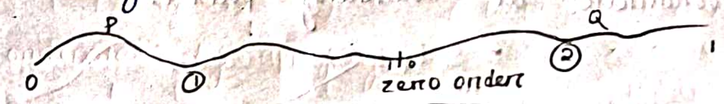
Parametric continuity

(1) zero order parametric continuity

(2) First order " "

(3) second order " "

→ If the ending point of first curve is the starting point of the second curve, it is called zero order parametric continuity.

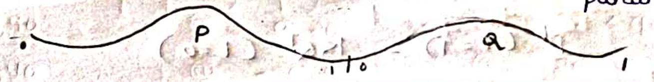


$\boxed{P(1) = Q(0)}$

Q) explain all types of parametric & Geometric continuity with suitable example

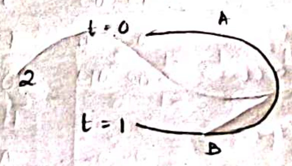
→ The derivative of co-ordinate function for two successive curve sections is equal at joining point is first order parametric continuity

ex : 1



$\boxed{P'(t=1) = Q'(t=0)}$

ex : 2



→ Both the first and second derivative of two curve section are same at intersecting point.

$\boxed{P''(t=1) = Q''(t=0)}$

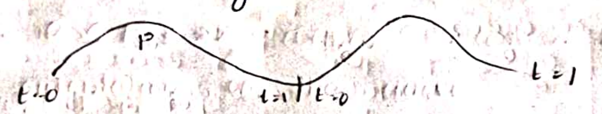
Geometric continuity :

Geometric continuity refers to the way that a curve or surface looks (unit tangent or curvature vector continuity)

If some curve in parametric continuous , then it also geometric continuous but not vise versa .

Zero order Geometric Continuity

It means two curve sections must have same coordinate at boundary point .



$$\boxed{P\,(t=1) = Q(t=0)}$$

First order Geometric continuity

The parametric first derivative is proportional at intersection of two successive sections .

$$P'\,(t=1) \propto Q'\,(t=0)$$

$$P'\,(t=1) = KQ'\,(t=0)$$

Second order Geometric continuity

Both first & second derivative are proportional to their boundary point & tangent vector direction same but magnitude may different .

$$P''\,(t=1) \propto Q''\,(t=0)$$

$$P''\,(t=1) = KQ''\,(t=0)$$

Parametric Equations :

Q3) suppose a curve in the plane is described by parametric equation

$$x = 1 + \sin(t)$$

$$y = -2 + \cos(t)$$

(a) At what point plane corresponds to $\pi/2$

(b) What familiar geometric shape is traced as we allow 't' to increase .

Sol^n :

At $t = \frac{\pi}{2}$ $\quad x = 1 + \sin\left(\frac{\pi}{2}\right)$

$= 1 + 1$

$= 2$

$y = -2 + \cos(t)$

$= -2 + \cos(\pi/2)$

$= -2 + 0 = -2$

(b) $x = 1 + \sin t$ $\quad y = -2 + \cos t$

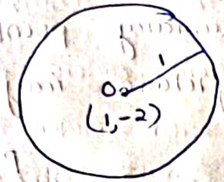
$\Rightarrow \boxed{\sin t = x - 1}$ $\quad \boxed{\cos t = y + 2}$

$\sin^2 t + \cos^2 t = 1$

$\Rightarrow (x-1)^2 + (y+2)^2 = 1$

$\Rightarrow (x-1)^2 + \{y-(-2)\}^2 = 1$

$\Rightarrow (x-h)^2 + (y-k)^2 = r^2$



Centre is $(h, k) = (1, -2)$

and radius $= 1$

Q7// $x = 4\cos t$, $y = 2\sin t$

$0 \le t \le 2\pi$

show orientation of the curve

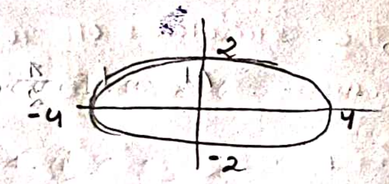
Sol^n : $x = 4\cos t$ $y = 2\sin t$ in

$\cos t = \frac{x}{4}$ $\sin t = \frac{y}{2}$

$\sin^2 t + \cos^2 t = 1$

$\Rightarrow (y/2)^2 + (\frac{x}{4})^2 = 1$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$



ellipse

t	x	y	(x, y)
0	4	0	4, 0
$\pi/2$	0	2	0, 2
π	-4	0	-4, 0
$3\pi/2$	0	-2	0, -2
2π	4	0	4, 0

Bezier curve :

Q8// Explain Bezier curve with a derivation of its expression. (5 mark)

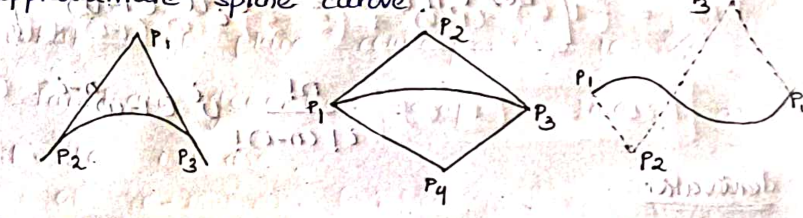
Ans. → A Bezier curve is particularly a line of spline generated from a set of control points ~~in~~ by forming a set of polynomical function.

→ These functions are computed from co-ordinates of control points.

→ A Bezier curve is defined by defining polygon. It has number of properties that makes them highly useful and convenient for curve and surface design.

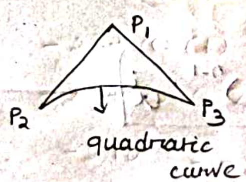
→ It is an approximate spline curve.

Ex :



→ properties of Bezier curve :

(1) Bezier curve is a polynomical of degree one less than number of control points



3 control points : quadratic curve
4 " " : cubic curve

(2) Bezier curve always passes through first and last point. $P(0) = P_0$, $P(1) = P_n$

(3) The slope at the begining of the curve is along the line joining first two control points and slope at the

end of the curve is along the line joining last two control points.

4) Bezier blending function are all positive and sum is equal to 1

$$\boxed{\sum_{i=0}^{n} Bez_{n,i}(u) = 1}$$

5) The curve follows the shape of defining polygon.

(6) The curve lies entirely within the convex hull formed by four control points

$$\boxed{Bez_{i,n}(u) = {}^{n}C_{i}\, u^{i} (1-u)^{n-i}}$$

$$= \frac{n!}{i!\,(n-i)!} u^{i} (1-u)^{n-i}$$

derivation :

The Bezier polynomial is expressed as

$$\boxed{\sum_{k=0}^{n} P_i B_i^n (u)} \quad \text{—— (1)}$$

$$\boxed{B_i^n(t) = \binom{n}{i} (1-u)^{n-i} u^{i}} \quad \text{—— (11)}$$

n is the polynomial degree

i is the index

u, t is the variable.

for n = 3

$$P(u) = P_0 B_0^3(u) + P_1 B_1^3(u) + P_2 B_2^3(u) + P_3 B_3^3(u) \quad \text{—— (3)}$$

$$B_0^3(u) = \binom{3}{0} (1-u)^{3-0} u^0 = 1 (1-u)^3 u^0 \quad \text{—— (4)}$$

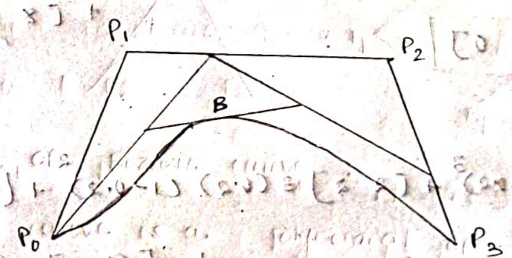
$$B_1^3(u) = \binom{3}{1} (1-u)^{3-1} u^1 = 3(1-u)^2 u^1 \quad \text{—— (5)}$$

$$B_2^3(u) = \binom{3}{2} (1-u)^{3-2} u^2 = 3(1-u)^1 u^2 \quad \text{—— (6)}$$

$$B_3^3(u) = \binom{3}{3} (1-u)^{3-3} u^3 = 1 (1-u)^0 u^3 \quad \text{—— (7)}$$

substituting eqn 4, 5, 6, 7 in eqn (3)

$$\boxed{P(u) = (1-u)^3 P_0 + 3u^1 (1-u)^2 P_1 + 3u^2 (1-u) P_2 + u^3 P_3}$$



Bezier curve

Q7) We are given four control points

$(1,0)$, $(2,2)$, $(6,3)$, $(8,2)$

Determine the five points that can lie on the curve & also draw curve on graph.

$$P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3(1-u)u^2 P_2 + u^3 P_3$$

Assume five different values

u $\{0, 0.2, 0.5, 0.7, 1\}$

$$P(u) = [1\ 0](1-u)^3 + [2\ 2]\,3u(1-u)^2 + [6\ 3]\times 3(1-u)u^2 + [8\ 2]u^3$$

<u>u=0</u>

$$P(0) = [1\ 0](1-0)^3 + \cancel{[2\ 2]}\,3(0)(1-0)^2 + \cancel{[6\ 3]\,3(1-0)}\,0 + [8\ \cancel{2}]\,0^3$$

$$\boxed{P(0) = [1\ 0]}$$

<u>u=0.2</u>

$$P(0.2) = [1\ 0](1-0.2)^3 + [2\ 2]\,3(0.2)(1-0.2)^2 + [6\ 3]\,3(1-0.2)(0.2)^2 + [8\ 2](0.2)^3$$

$$\Rightarrow \boxed{P(0.2) = [1.48424\ \ 0.848]}$$

similarly,

<u>u = 0.5</u>

$$\boxed{P(0.5) = [4.125\ \ 1.75]}$$

<u>u = 0.7</u>

$$\boxed{P(0.7) = [5.412\ \ 2.108]}$$

<u>u = 1</u>

$$\boxed{P(1) = [8\ \ 2]}$$