Q 1. A particle is moving in one dimensional box of infinite height of width 10 A°. calculate the probability of finding the particle within an interval of 1 A° at the centre of the box, when it is in the state of least energy.

⇒ The wave function of the particle is

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

when the particle is in the state of least energy $n = 1$ then

$$\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

The probality of finding the particle in unit interval at the centre of box ($x = L/2$) is

$$p = |\psi_1|^2$$

$$= \left[\sqrt{\frac{2}{L}} \sin \frac{\pi (L/2)}{L} \right]^2$$

$$= \frac{2}{L} \sin^2 \frac{\pi}{2}$$

$$= \frac{2}{L}$$

∴ The prob. of finding the particle within the interval of Δx at the centre of box $= w$

$$w = |\psi_1|^2 \Delta x$$
$$= \frac{2}{L} \Delta x$$

Here $L = 10 A = 10 \times 10^{-10}$ m

$\Delta x = 1 A = 1 \times 10^{-10}$ m

$$\therefore w = \frac{2}{10 \times 10^{-10}} 1 \times 10^{-10}$$

$$\Rightarrow w = 0.2$$

Q 2: Calculate the expectation value of $\langle p_x \rangle$ of the momentum of a particle trapped in one dimensional box.

⇒ The normalized wave function of particle in one dimensional box

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} = \psi^*$$

Now, $\frac{\partial \psi}{\partial x} = \sqrt{\frac{2}{L}}\left(\frac{n\pi}{L}\right)\cos\frac{n\pi x}{L}$

now,

$$\langle p_x \rangle = \int_{-\infty}^{\infty} \psi^* p_x \psi \, dx$$

$$= \int_{-\infty}^{\infty} \psi^* \left(\frac{-ih}{2\pi}\frac{\partial}{\partial x}\right)\psi \, dx$$

$$= \frac{-ih}{2\pi}\int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} \, dx$$

$$= \frac{-ih}{2\pi}\int_{0}^{L} \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \sqrt{\frac{2}{L}}\left(\frac{n\pi}{L}\right)\cos\frac{n\pi x}{L}$$

$$= \frac{-ih}{2\pi}\,\frac{2}{L}\,\frac{n\pi}{L}\int_{0}^{L} \sin\frac{n\pi x}{L}\cos\frac{n\pi x}{L}\,dx$$

$$= 0$$

The expectation value $\langle p_x \rangle$ of the momentum of particle is 0

Q 3. The wave function of electron moving on x-axis like a wave is given by $\psi(x) = 2\sin 2\pi x$. calculat the probability of forming out an

Date:

electron from 0.25 m to 0.5 m length on the axis.

⇒ probability of finding the particle

$$p = \int_{x_1}^{x_2} |\psi|^2 dx$$

$$= \int_{0.25}^{0.5} (2 \sin 2\pi x)^2 dx$$

$$= 4 \int_{0.25}^{0.5} \sin^2 2\pi x \, dx$$

$$= 4 \int_{0.25}^{0.5} \left[\frac{1 - \cos 2 \cdot 2\pi x}{2}\right] dx$$

$$= 4 \cdot \frac{1}{2} \int_{0.25}^{0.5} (1 - \cos 4\pi x) \cdot dx$$

$$= 2 \left[x - \frac{\sin 4\pi x}{4\pi}\right]_{0.25}^{0.5}$$

$$= 2 \left[0.5 - \frac{\sin 2\pi}{4\pi} - 0.25 + \frac{\sin \pi}{4\pi}\right]$$

$$= 2 [0.25 - 0 - 0]$$

$$= 2 \times 0.25$$

$$p = 0.5$$

Q 4. Normalize the one dimensional wave function given by

$$\psi(x) = A \sin\left(\frac{\pi x}{a}\right) \qquad 0 < x < a$$

$$\psi(x) = 0 \qquad \text{outside}$$

⇒ The wave function $\psi(x)$ is said to be normalized if it satisfied the condition

$$\int_{-\infty}^{\infty} \psi \psi^* dx = 1$$

then, Applying this condition,

$$\int_0^a \psi \psi^* dx = 1$$

$$\Rightarrow \int_0^a A \sin\left(\frac{\pi x}{a}\right) A^* \sin\left(\frac{\pi x}{a}\right) . dx = 1$$

$$\Rightarrow AA^* \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx = 1$$

$$\Rightarrow A^2 \int_0^a \left[\frac{1 - \cos 2\pi x/a}{2}\right] dx = 1$$

$$\Rightarrow \frac{A^2}{2} \left[\int_0^a dx - \int_0^a \cos\left(\frac{2\pi x}{a}\right) dx\right] = 1$$

Date:

$$\Rightarrow A^2\left[x - \frac{a}{2\pi}\left(\frac{\sin 2\pi x}{a}\right)\right]_0^a = 2$$

$$\Rightarrow A^2\left[a - 0\right] = 2$$

$$\Rightarrow A^2 a = 2$$

$$\Rightarrow A = \sqrt{\frac{2}{a}}$$

$\therefore$ normalized wave function

$$\psi = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$$

Q5. Find the energy of the neutron in units of electron volt whose De-Broglie wave length is $1\,\mathring{A}$

$\Rightarrow$ given mass of neutron

$= 1.67 \times 10^{-27}$ kg

plank's constant $h = 6.62 \times 10^{-34}$ Joulesec

we have,

De-Broglie wavelength $\lambda = \frac{h}{mv}$ --- (1)

but $K.E = \frac{1}{2}mv^2$

or $E = \frac{1}{2} mv^2$

$\Rightarrow E = \frac{1}{2m} m^2v^2$

$\Rightarrow m^2v^2 = 2mE$

$\Rightarrow mv = \sqrt{2mE}$ --- (2)

From (1) and (2)

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \lambda^2 = \frac{h^2}{2mE}$$

$$\Rightarrow E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (10^{-10})^2}$$

$$E = \frac{13.01 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV}$$

Q6. what would be the wavelength of quantum of radient energy emitted, if an electron transmitted into radiation and converted into one quanta.

⇒ According to plank, the energy associated with the one quanta = $h\nu$ when the energy of an electron is transmitted into radiation then

$$E = mc^2$$

$$\therefore \quad h\nu = mc^2$$

$$\Rightarrow \quad h\frac{c}{\lambda} = mc^2$$

$$\Rightarrow \quad \lambda = \frac{h}{mc}$$

$$= \frac{6.67 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}$$

$$= 0.029 \times 10^{-10} \, m$$

Q.7 calculate the De-Broglie wavelength of an electron moving with velocity $\frac{3}{5}c$.

⇒ Here,

velocity of electron $v = \frac{3}{5}c$

$$\Rightarrow \quad \frac{v}{c} = \frac{3}{5}$$

the De-Broglie wave length

$$\lambda = \frac{h}{mv}$$

since v is comparable with speed of light, m is relativistic mass given by

$$\lambda = \frac{h}{m_0 v}\sqrt{1 - v^2/c^2}$$

$$\left[\because m = \frac{m_0}{\sqrt{1-v^2/c^2}}\right.$$

$$\Rightarrow \lambda = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \frac{3}{5}c}\sqrt{1 - (3/5)^2}$$

$$\Rightarrow \lambda = 0.323 \times 10^{-11} \text{ m}$$

Q8. A fast moving neutron is found to have an associated De-Broglie wave-length of 2×10^{-12} m. Find the K.E, phase velocity and group velocity of De-Broglie waves.

(mass of neutron $= 1.67 \times 10^{-27}$ kg.

$$\Rightarrow \quad k.E. = \frac{1}{2} m v^2 = \frac{1}{2m}(mv)^2$$

$$\therefore \quad k.E. = \frac{1}{2m} p^2 \qquad [mv = p]$$

but $p = \frac{h}{\lambda}$

$$\therefore \quad k.E. = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2$$

$$= \frac{1}{2 \times 1.67 \times 10^{-27}}\left(\frac{6.62 \times 10^{-34}}{2 \times 10^{-12}}\right)^2$$

$$k.E. = 3.280 \times 10^{-17} \, J$$

$$\Rightarrow k.E. = \frac{3.28 \times 10^{-17}}{1.6 \times 10^{-19}} \, eV$$

now, phase velocity $\quad v_p = \nu\lambda$

$$\text{or} \quad v_p = \frac{\omega}{k}$$

$$\Rightarrow v_p = \frac{\hbar\omega}{\hbar k}$$

$$\Rightarrow v_p = E/p$$

$$v_p = \frac{p^2}{2mp} \quad \left[E = \frac{p^2}{2m} \right]$$

$$v_p = \frac{p}{2m}$$

$$v_p = \frac{h}{2m\lambda} \quad \left[p = \frac{h}{\lambda} \right]$$

$$= \frac{6.62 \times 10^{-34}}{2 \times 1.67 \times 10^{-27} \times 2 \times 10^{-12}}$$

$$v_p = 0.988 \times 10^5 \text{ m/s}$$

now, group velocity $v_g = V$ (particle vel)

$$\Rightarrow V = \frac{h}{m\lambda} \quad \left[\because \lambda = \frac{h}{mv} \right]$$

$$\Rightarrow V = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times 2 \times 10^{-12}}$$

$$= 1.976 \times 10^5 \text{ m/s}$$

$$\therefore v_g = 1.976 \times 10^5 \text{ m/s}$$