

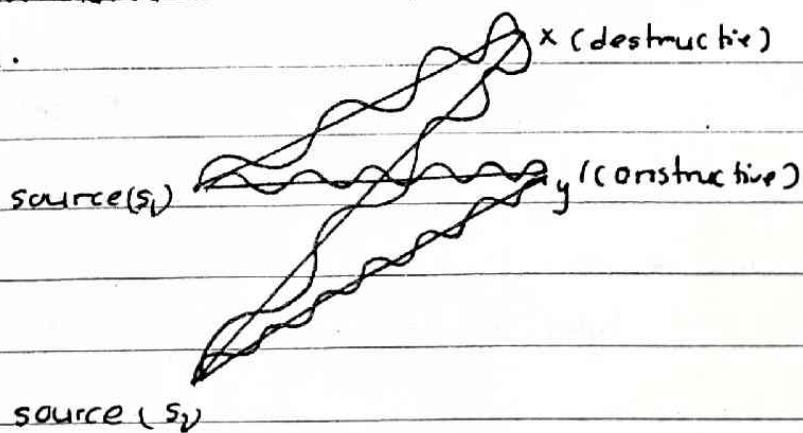
# physical optics

Date . . .



## \* Interference -

The source of light send out its energy uniformly in all direction but If there are two sources of light, the energy distribution is non-uniform in all direction. The phenomenon of non-uniform distribution of light energy due to the superposition of two light waves is called interference of light wave. If the crest of one wave falls on crest of the other or trough of one falls on trough of other the resultant amplitude becomes maximum i.e. intensity of light is maximum at these points. At certain point crest of one falls on trough of other and vice versa, the corresponding point will be of minimum intensity. The former is called constructive interference and later one is called destructive interference.



## \* optical path and geometrical path -

suppose the light travels in an optically denser medium with a velocity  $v$ . If the distance travelled by light in the medium is  $x$ , then time taken by light to travel this distance is given by  $t = \frac{x}{v}$  --- (1)

If the light travels a distance  $L$  in the free space during the same time as it travels in medium then

$$L = ct \quad \dots \dots (1)$$

$$\text{From (1) and (2)} \quad L = \frac{c}{\nu} x \quad \dots \quad (3)$$

If the refractive index of the medium is  $\mu$  then,

$$\mu = c/v \quad \dots \quad (4)$$

$$\text{From (3) and (4)} \quad L = \mu x \quad \dots \quad (*)$$

$\Rightarrow$  optical path = refractive index  $\times$  geometrical path.

optical path is equal to the product of refractive index of a medium and geometrical path.

geometrical path is the distance travelled by light wave in medium and optical path is distance travelled in vacuum during the same time.

### sustained interference -

The interference patterns on the screen are said to be sustained if they remain as permanent patterns. The conditions to sustain interference are as follows -

The two sources of light must be coherent. The two sources are said to be coherent if they emit waves continuously same wavelength or frequency and always have a constant phase difference.

The amplitudes of waves should be equal.

The sources should be monochromatic.

They must be very close to each other.

They should be point sources or very narrow sources.

The screen should be far from sources.

\* condition for maxima and minima -

Let us consider  $s_1$  and  $s_2$  be two coherent sources of light which produce interference fringes.

Let 'P' be the point on the screen.

Suppose  $y_1 = a \sin wt$  be the wave produced by  $s_1$  and  $y_2 = a \sin(wt + \phi)$  be the wave produced by  $s_2$ . Where  $\phi$  is the phase difference between waves from  $s_1$  and  $s_2$ .

Then the resultant displacement of wave is given by

$$y = y_1 + y_2$$

$$\Rightarrow y = a \sin wt + a \sin(wt + \phi)$$

$$\Rightarrow y = a[\sin wt + \sin wt \cos \phi + \cos wt \sin \phi]$$

$$\Rightarrow y = a[\sin wt(1 + \cos \phi) + \cos wt \sin \phi] \quad \dots \text{--- (1)}$$

$$\text{Let } a(1 + \cos \phi) = A \cos \theta \quad \dots \text{--- (2)}$$

$$\text{and } a \sin \phi = A \sin \theta \quad \dots \text{--- (3)}$$

Substituting (2) and (3) in (1)

$$\Rightarrow y = A [\sin wt \cos \theta + \cos wt \sin \theta]$$

$$\Rightarrow y = A \sin(wt + \theta) \quad \dots \text{--- (4)}$$

This is the resultant wave equation with amplitude A.

Now, squaring and adding eqn (1) and (3)

$$\Rightarrow a^2 \cos^2 \theta + A^2 \sin^2 \theta = a^2(1 + \cos \phi)^2 + a^2 \sin^2 \phi$$

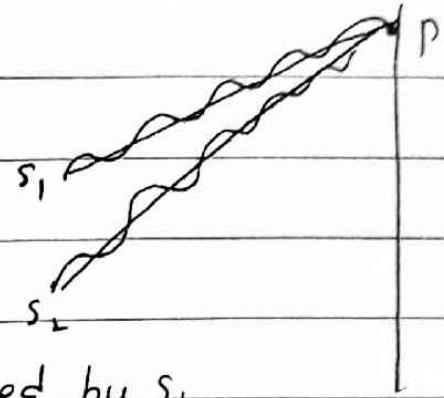
$$\Rightarrow A^2 [\sin^2 \theta + \cos^2 \theta] = a^2 [1 + 2 \cos \phi + \cos^2 \phi + \sin^2 \phi]$$

$$\Rightarrow A^2 = a^2 [2 + 2 \cos \phi]$$

$$\Rightarrow A^2 = 2a^2(1 + \cos \phi)$$

$$\Rightarrow n^2 = 4a^2 \cos^2 \frac{\phi}{2} \quad \dots \text{--- (5)}$$

Since the intensity of wave is directly proportional to the



In case of the amplitude, we can write,

$$\text{Intensity } (I) \propto A^2$$

$$\therefore I \propto q \alpha^2 \cos^2 \frac{\phi}{2} \quad \dots \dots (*)$$

- maximum intensity,  $\cos^2 \frac{\phi}{2} = 1$

$$\Rightarrow \cos \frac{\phi}{2} = \pm 1$$

$$\Rightarrow \cos \frac{\phi}{2} = \cos n\pi, \quad n = 0, 1, 2, 3, \dots$$

$$\Rightarrow \frac{\phi}{2} = n\pi$$

$$\Rightarrow \phi = 2n\pi \quad \dots \dots (6) \quad [\text{phase difference}]$$

have, relation between path difference and phase difference,

$$\text{path diff.} = \frac{\lambda}{2\pi} \times \text{phase diff.}$$

$$= \frac{\lambda}{2\pi} \times 2n\pi$$

$$\text{path diff.} = n\lambda \quad \dots \dots (7)$$

is the condition for maximum intensity i.e bright fringe.

For minimum intensity,  $\cos^2 \frac{\phi}{2} = 0$

$$\Rightarrow \cos \frac{\phi}{2} = 0$$

$$\Rightarrow \cos \frac{\phi}{2} = \cos(2n+1)\pi/2, \quad n = 0, 1, 2, 3, \dots$$

$$\Rightarrow \phi = (2n+1)\pi \quad \dots \dots (8)$$

path diff. =  $\frac{\lambda}{2\pi} \times (2n+1)\pi$

$$\Rightarrow \text{path diff.} = (2n+1) \frac{\lambda}{2} \quad \dots \dots (9)$$

This is the condition for minimum intensity.

i.e dark fringe.

## \* Young's Double slit experiment -

consider a monochromatic

source of light 's'. Let  $s_1$  and

$s_2$  be the two slits which are

equidistant from source s.  $s_1$

and  $s_2$  acts as two coherent

sources of light. Light waves

from  $s_1$  and  $s_2$  overlap each other so as to produce alternative dark and bright fringes.

From fig. the path difference between the waves from  $s_1$  and  $s_2$  reaching at point 'p' on the screen is  $s_2 N_1$  as drawn normal for  $s_1$  to  $s_2 p$ .

since  $s_1$  and  $s_2$  are close to each other and so as to center C;

$s_1 N$  meets CP at  $90^\circ$ . Then  $\angle CS_1 N = 0$  If  $\angle PCO = 0$

∴ considering triangle  $S_1 S_2 N_1$

$$\sin \theta = S_2 N_1 / s_1 s_2$$

$$\Rightarrow \sin \theta = S_2 N_1 / d$$

$$\Rightarrow S_2 N_1 = d \sin \theta \quad \dots (1)$$

where  $d$  = distance bet^n slits

Again, from A PCO,  $\tan \theta = y / D \quad \dots (2)$

where  $y$  = position of fringe from centre of screen and

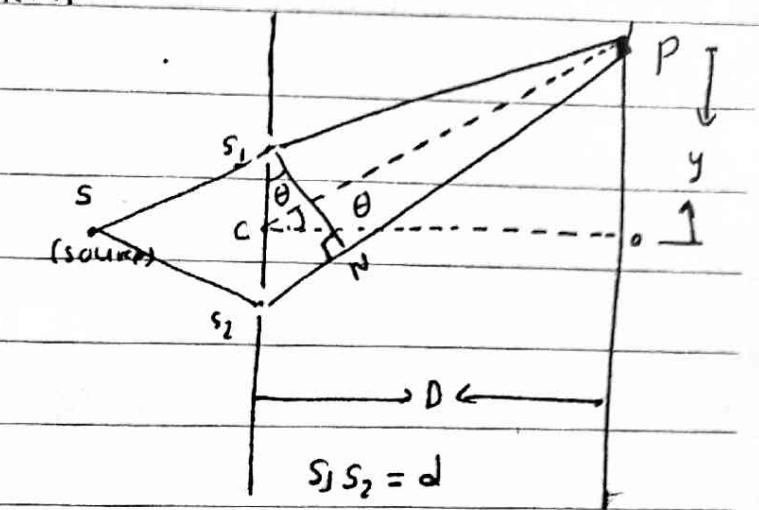
$D$  = distance of screen from slits.

The point 'p' on the screen will correspond to bright fringe if

$$\text{path diff} = n \lambda \quad \dots (3)$$

From (1) and (3)

$$d \sin \theta = n \lambda \quad \dots (4)$$



If  $\theta$  is small then  $\sin \theta \approx \theta$  and  $\tan \theta \approx \theta$

from (2)  $\theta = y/D \text{ --- (5)}$

from (4)  $d\theta = n d \text{ --- (6)}$

from (5) and (6)  $dy = \frac{n d}{D}$

$$\Rightarrow y = \frac{n d D}{d} \text{ --- (7)}$$

when  $n=0$ ,  $y_0 = 0$

when  $n=1$ ,  $y_1 = \frac{\lambda D}{d}$

when  $n=2$ ,  $y_2 = \frac{2 \lambda D}{d}$

distance b/w the two consecutive bright fringes will be,

$$y_1 - y_0 \text{ or } y_2 - y_1, \dots$$

every time we get the diff =  $\frac{\lambda D}{d}$

is called fringe width denoted by  $\beta = \frac{\lambda D}{d} \text{ --- (8)}$

similarly, the point p will correspond to dark fringe if

$$\text{path diff} = (2n+1) \frac{\lambda}{2} \text{ --- (9)}$$

from (1) and (9)

$$ds \sin \theta = (2n+1) \frac{\lambda}{2}$$

$$\Rightarrow d\theta = (2n+1) \frac{\lambda}{2} \quad [ \because \theta = y/D, \text{ from (5)} ]$$

$$\Rightarrow dy = (2n+1) \frac{\lambda}{2}$$

$$\Rightarrow y = \frac{(2n+1) \lambda D}{2d} \text{ --- (10), } n=0, 1, 2, 3, \dots$$

$$\therefore y = \frac{\lambda D}{2d}, \frac{3\lambda D}{2d}, \frac{5\lambda D}{2d}, \dots$$

Then distance b/w two consecutive dark fringes will again,  
 $\lambda D/d$

$$\therefore \beta = \lambda D/d \text{ -- (xx)}$$

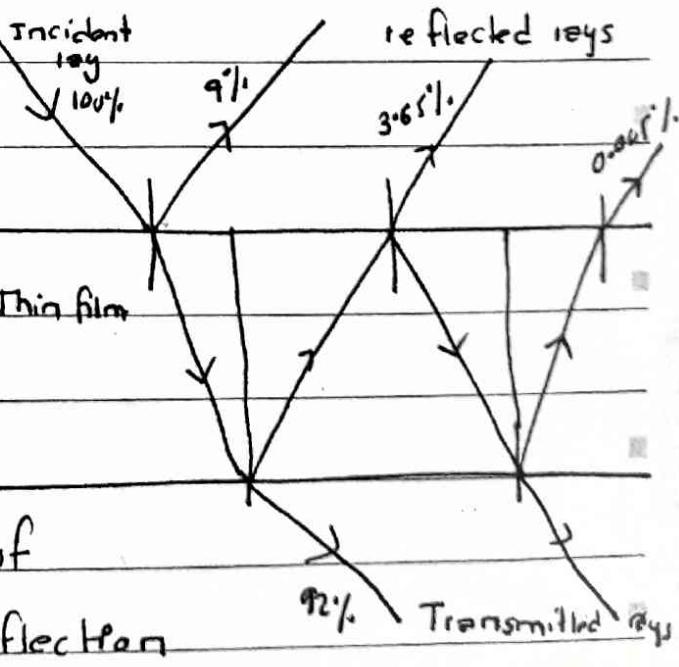
we can conclude that bright and dark fringes are equally spaced.

#### \* Thin film -

An optical medium is called thin film when the thickness is about 1 order of wavelength of light in visible spectrum. Thus, a film of thickness in the range  $0.5 \mu m$  to  $10 \mu m$ , may be considered as a thin film. It may be the thin sheet of transparent medium such as glass, mica or air film enclosed between two transparent plates or a soap bubble.

If the light incident on thin film a small part of it gets reflected from the top surface and major part is transmitted through film.

Again a small part of the transmitted component is reflected back into the film by the bottom surface and rest of it emerges out of the film. Therefore only the first reflection at top surface and first reflection on the bottom surface will be of appreciable strength.



## Interference due to reflection on thin film.

Suppose a ray of light

incident on the screen

surface  $x_1 y_1$  of thin film of

thickness  $t$  and refractive

index  $\mu$ .

part of ray incident at

$B$  such that a part

light is reflected along

and major part is transmitted along

similarly at point 'C' small part is

reflected along  $CD$  and major part is transmitted along  $CF$ .

Also at 'D' major part is transmitted along  $DI$  and small  
portion reflected back along  $DE$ . And at point  $E$ , major part  
transmitted along  $EG$  and remaining part reflected back to  
 $m$ . and so on...

only the rays reflected along  $BH$  and  $CD$  are of appreciable  
strength.

From fig. draw  $\perp DP$  normal on  $BH$ .

Let  $i$  be the angle of incidence and ' $r$ ' be the angle of refraction,  
then, from  $\triangle BPD$ ,

then, path diff. b/w reflected rays from point  $B$  and  $C$  will be

$$\text{path diff} = BC + CD - BP$$

$$-\text{optical path diff} = \mu(BC + CD) - BP \quad \dots (1)$$

in,  $\triangle BGM$

$$\cos r = \frac{MC}{BG} \Rightarrow BG = \frac{MC}{\cos r} \quad \dots (2) \quad [MC = t]$$

As  $\Delta BCM$  and  $\Delta CMD$  are similar, then, also,

$$(D = \frac{t}{\cos r} \quad \dots \dots (3))$$

Again, from a BPD,  $\sin i = BP/BD$

$$\Rightarrow BP = BD \sin i$$

$$\Rightarrow BP = 2BM \sin i \dots \dots (4) \quad [ BM = MD ]$$

Also, from  $\Delta BMC$ ,  $\tan r = \frac{BM}{MC}$

$$\Rightarrow BM = MC \tan r$$

$$\Rightarrow BM = t \tan r \dots \dots (5)$$

From (4) and (5)  $Bp = 2t \tan r \sin i$

$$Bp = 2t \tan r \mu \sin r \quad [\because \mu = \sin i / \sin r]$$

$$\Rightarrow Bp = 2t \mu \frac{\sin^2 r}{\cos r} \dots \dots (6)$$

From (1) and (2), (3), (6) we get,

$$\text{optical path diff.} = \mu \left[ \frac{t}{\cos r} + \frac{t}{\cos r} \right] - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} [ 1 - \sin^2 r ]$$

$$= \frac{2\mu t \cos^2 r}{\cos r}$$

$$\text{path diff.} = 2\mu t \cos r \dots \dots (7)$$

but at point 'B' the ray is reflected from the surface of denser medium so that there is change in phase by  $\pi$  or path diff. is  $\lambda/2$ , i.e. wave losses half of its wavelength on reflection.

from the boundary of rarer to denser medium.

∴ Additional path diff =  $\lambda/2$ .

$$\text{total path diff} = 2\mu t \cos r + \frac{\lambda}{2} \quad \dots (8)$$

when the thickness of the film is negligible, then

$$\text{path diff} = \frac{\lambda}{2}$$

the position of bright fringe,

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu t \cos r = (2n-1) \frac{\lambda}{2} \dots (9)$$

- the position of dark fringe

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$\Rightarrow 2\mu t \cos r = n\lambda \dots (10)$$

If we draw normal EG on CF and consider triangle CGF we can get the condition of interference due to transmitted rays in thin film. Between rays GF and CF,

the path diff = CD + DE - CG

$$\begin{aligned} \text{optical path diff} &= \mu(CD + DE) - CG \\ &= 2\mu t \cos r \end{aligned}$$

There is no additional path difference as at point D, the rays reflected from the surface of rarer medium i.e. there is no phase change.

$$\therefore \text{total path diff} = 2\mu t \cos r \dots (x)$$

transmitted rays overlap to give bright fringe if  $2\mu t \cos r = n\lambda$

give dark fringe if  $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

## \* Newton's Ring -

Newton's rings are the example of fringes of equal thickness.

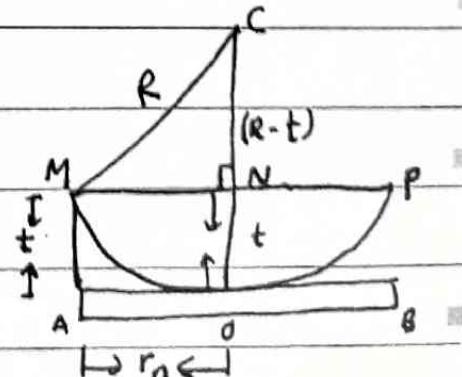
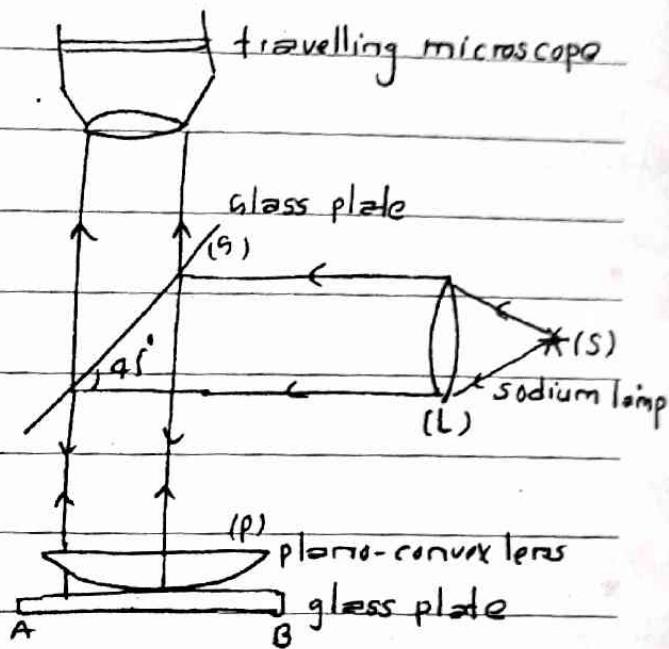
The experimental arrangement for observing Newton's ring is shown in fig.

Monochromatic light from the extended source  $S$  is spread parallel by lens ' $L$ '. These rays are partially reflected by the glass plate  $G$  inclined  $45^\circ$  to horizontal. The reflected rays from  $G$  incident normally to the plane surface of plano-convex lens  $P$  kept on glass plate  $AB$ . The lens ' $P$ ' and glass plate  $AB$  encloses air film of variable thickness.

The rays reflected from upper surface of the film and lower surface of the film overlap so as to produce interference fringes. The fringes are formed between the lens ' $P$ ' and plate  $AB$  as circular fringes are called Newton's ring.

As the thickness of air film at the point contact bet<sup>n</sup> lens ' $P$ ' and plate ' $AB$ ' is zero and gradually increases as we move outward the locus of points where the air film has equal thickness fall on a circle whose centre is the point of contact. Thus the fringes are in circular form.

Let ' $R$ ' be the radius of curvature of plano-convex lens. consider the air film of thickness ' $t$ ' such that  $r_n$  be the radius of  $n^{\text{th}}$  ring whose thickness from center is ' $t$ '.



or the thin film, the path difference for reflected rays

$$= 2ht \cos r + \lambda/2 \quad \dots (1)$$

- air film,  $\mu = 1$

normal incidence,  $r = 0$

$$\therefore (1) \text{ becomes, path diff} = 2t + \lambda/2 \quad \dots (2)$$

or bright fringes, path diff =  $nd \dots (3)$

in (1) and (3)

$$2t + \frac{\lambda}{2} = nd$$

$$\Rightarrow 2t = nd - \lambda/2$$

$$\Rightarrow 2t = (2n-1) \frac{\lambda}{2} \quad \dots (4)$$

in fig.

$$\text{in } \triangle MCN, (MC)^2 = (CM)^2 + (MN)^2$$

$$\Rightarrow R^2 = (R-t)^2 + r_n^2$$

$$\Rightarrow R^2 = R^2 - 2Rt + t^2 + r_n^2$$

$$\Rightarrow r_n^2 = 2Rt - t^2 \quad \dots (5)$$

now, lens is of large aperture,  $R \gg t$

From eqn (5)  $t^2$  can be neglected.

eqn (5) becomes,

$$r_n^2 = 2Rt \quad \dots (6)$$

$$\Rightarrow 2t = \frac{r_n^2}{R} \quad \dots (7)$$

$$\text{in (5) and (7)} \quad \frac{r_n^2}{R} = (2n-1) \frac{\lambda}{2}$$

$$\Rightarrow r_n^2 = (2n-1) \lambda R/2$$

$$\Rightarrow D_n^2 = 2(2n-1) \lambda R \quad \dots (8) \quad [\because r_n = D_n/2]$$

nearly

For  $m^{\text{th}}$  bright ring,  $D_m^2 = 2(2m-1)dR \dots (9)$

Subtracting eqn (8) from (9) we get :

$$D_m^2 - D_n^2 = 2(2m-1)dR - 2(2n-1)dR$$

$$= 2dR [2m-1 - 2n+1]$$

$$= 2dR [2m-2n]$$

$$D_m^2 - D_n^2 = 4(m-n)dR$$

$$\Rightarrow \lambda = \frac{D_m^2 - D_n^2}{4(m-n)dR} \dots (10)$$

This is the wavelength of monochromatic light. Measuring the diameter of  $n^{\text{th}}$  and  $m^{\text{th}}$  ring we can determine the wavelengths. Similar condition can be obtained by considering the condition of dark fringe.

For dark fringe, path diff =  $(2n+1)\lambda/2 \dots (11)$

From (2) and (11)

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2t = n\lambda \dots (12)$$

Now from (7) and (12)

$$\frac{r_n^2}{R} = n\lambda$$

$$\Rightarrow r_n^2 = n\lambda R$$

$$\Rightarrow D_n^2 = 4n\lambda R \dots (13)$$

Similarly for  $m^{\text{th}}$  ring,  $D_m^2 = 4m\lambda R \dots (14)$

From (13) and (14)  $D_m^2 - D_n^2 = 4(m-n)\lambda R$

$$\Rightarrow \lambda = \frac{D_m^2 - D_n^2}{4(m-n)\lambda R} \dots (15)$$

As path diff =  $2t + \lambda/2$

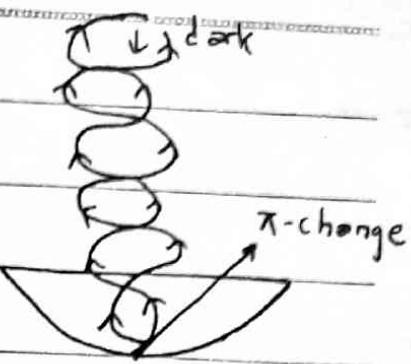
at the point of contact,  $t=0$

$$\therefore \text{total path diff} = \lambda/2$$

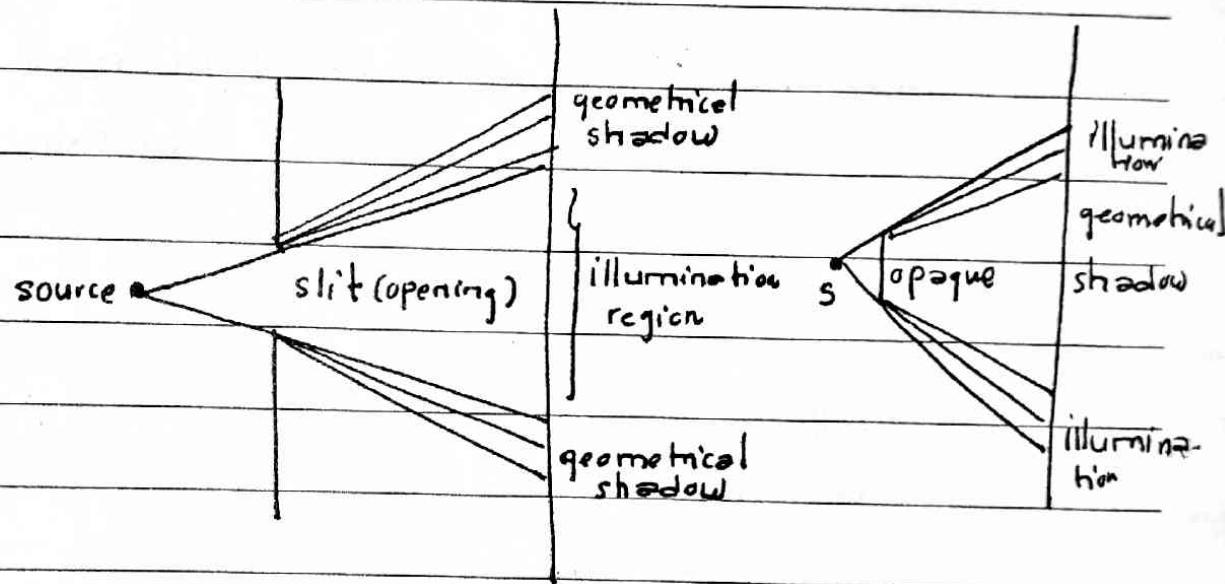
which is the condition of dark fringes.

It means at the point of contact the

phase  $\Delta\phi$  changes by  $\pi$  or path change by  $\lambda/2$  or the wave lost half wave on reflection. Therefore, the center of the Newton's ring always dark.



### Diffraction -



The phenomenon of bending of light from the sharp edge of an obstacle and spreading around the geometrical shadow region called diffraction.

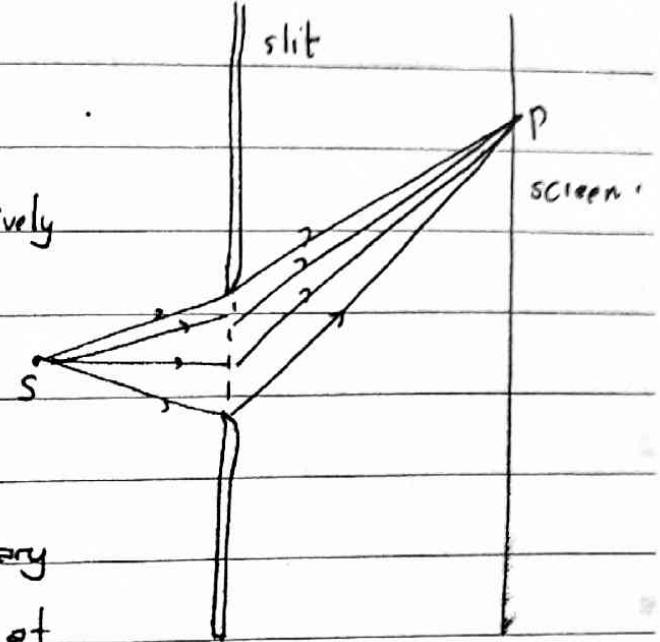
When the opening is large compared to the wavelength of light the wave do not bend round the edges. When the opening is small the bending round the edges is noticeable. When the opening is very small, the wave spread over the entire surface behind the opening. Therefore, diffraction is observable only when size of the obstacle is comparable to wavelength of light.

The diffracted rays from the obstacle overlap each other produce diffraction fringes.

## \* Fresnel diffraction -

In fresnel type of diffraction the source and slit screen are effectively at finite distance from obstacle. It does not require any lenses. The incident wavefront is not planar.

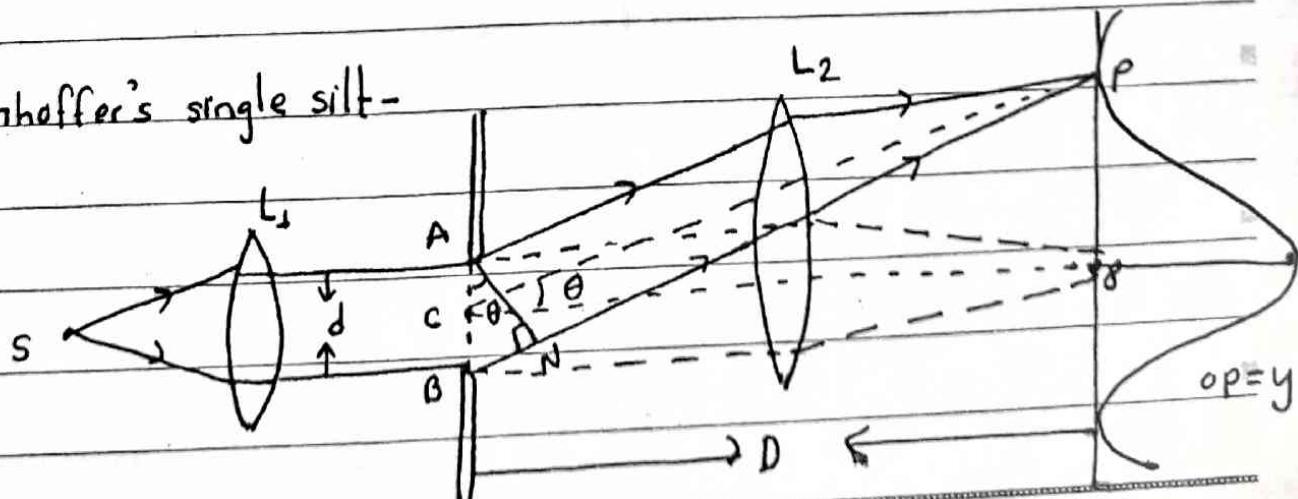
Therefore, the phase of the secondary wavelets are not in same ~~same~~ phase at all the points in the plane of obstacle. The resultant amplitude at any point on the screen is obtained by mutual interference of secondary wavelets from different elements of unblocked portions of wavefront.



## \* Fraunhofer's diffraction -

In this type of diffraction the source and screen are effectively infinite distance from the obstacle. The two converging lens are used. The first one is used to make rays parallel and 2nd one is used to converge the diffracted rays on the screen. The wavelets emerge after unblocked portion of the obstacle are of same phase, which overlap to give diffracted fringes.

## \* Fraunhofer's single slit -



A ray of parallel beam of light (monochromatic) is incident on a slit AB of width 'd'. The diffracted rays are focused on a screen by a convex lens,  $L_2$ , where we get diffraction fringes.

The light wave travelling in the same direction as the incident wave, focus at point 'o' i.e. center of the screen. For the other part of screen all the rays meet at same phase i.e. there is no path difference. Therefore, central fringe is bright called incipie maxima.

The path difference between the rays from A and B reaching at point p = BN

$$\therefore \text{from } A \text{ to } BN, \sin\theta = \frac{BN}{AB}$$

$$\Rightarrow BN = AB \sin\theta$$

$$\Rightarrow BN = d \sin\theta \quad \dots \{1\}$$

path difference is equal to the wavelength of light then point will be the position of minimum intensity

$$\text{i.e. if } d \sin\theta = \lambda$$

in this case the slit is divided into

no equal parts i.e. the whole wavefront

divided into AC and BC and if the

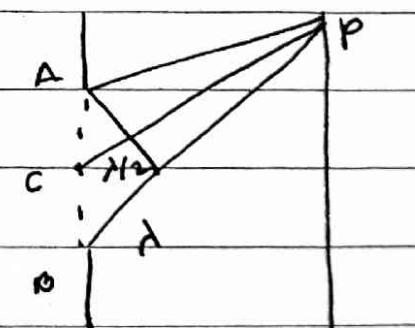
path difference between the waves from A and B is  $\lambda$  then

the path difference between waves from A and C will be  $\lambda/2$  and

the path difference between waves from B and C will be  $\lambda/2$ . As the

point in upper half of AC the corresponding point in lower half BC, they interfere destructively and corresponding point will be dark.

similarly if the path diff =  $2\lambda$ , then we can divide whole wavefront into 4 equal parts so as the wave reaching at



Screen will be minimum intensity.

In general for dark fringe,  $d \sin \theta_n = n\lambda \dots (2)$  [  $n=1, 2, 3, \dots$  ]

If we divide the whole wave front into, 3, 5, 7, ... two of the wave cancel each other and remaining one reinforces the bright fringe and corresponding point will be bright.

In general for bright fringe,

$$d \sin \theta_n = (2n+1) \frac{\lambda}{2} ; (4) \quad n=1, 2, 3, \dots$$

Let  $y_n$  be the position of  $n^{\text{th}}$  minima and 'D' be the distance between slit and screen then,

$$\text{From A CPO, } \tan \theta_n = y_n/D \dots (5)$$

$$\text{For small } \theta, \tan \theta \approx \sin \theta \approx \theta_n$$

$\therefore$  From (3) and (5)

$$d \theta_n = n\lambda \dots (6) \quad [\text{For minima}]$$

$$\text{and } \theta_n = y_n/D \dots (7)$$

From (1) and (7)

$$\frac{dy_n}{D} = n\lambda$$

$$\Rightarrow y_n = \frac{n\lambda D}{d} \dots (8)$$

the difference b/w two consecutive minima,

$$y_{n+1} - y_n = \beta$$

$$\Rightarrow \left( n+1 \right) \frac{\lambda D}{d} - n \frac{\lambda D}{d} = \beta$$

$$\Rightarrow \frac{\lambda D}{d} [n+1-n] = \beta$$

$$\Rightarrow \beta = \frac{\lambda D}{d} \dots (9)$$

similarly for maxima,

$$\text{from eqn (4) and (5)} \quad d \frac{y_n}{D} = (2n+1) \frac{\lambda D}{2}$$

$$\Rightarrow y_n = (2n+1) \frac{\lambda D}{2} \quad \dots (10)$$

∴ difference bet<sup>n</sup> two consecutive maxima,

$$y_{n+1} - y_n = \beta$$

$$\Rightarrow [2(n+1)+1] \frac{\lambda D}{2D} - (2n+1) \frac{\lambda D}{2D} = \beta$$

$$\Rightarrow \frac{\lambda D}{2D} [2n+2+1 - 2n-1] = \beta$$

$$\Rightarrow \beta = \frac{\lambda D}{d} \quad \dots (11)$$

width of central maxima & the distance between the first minima on either side of central maxima.

if  $\alpha$  be the distance of first minima from center of screen then  
width of central maxima =  $2\alpha$

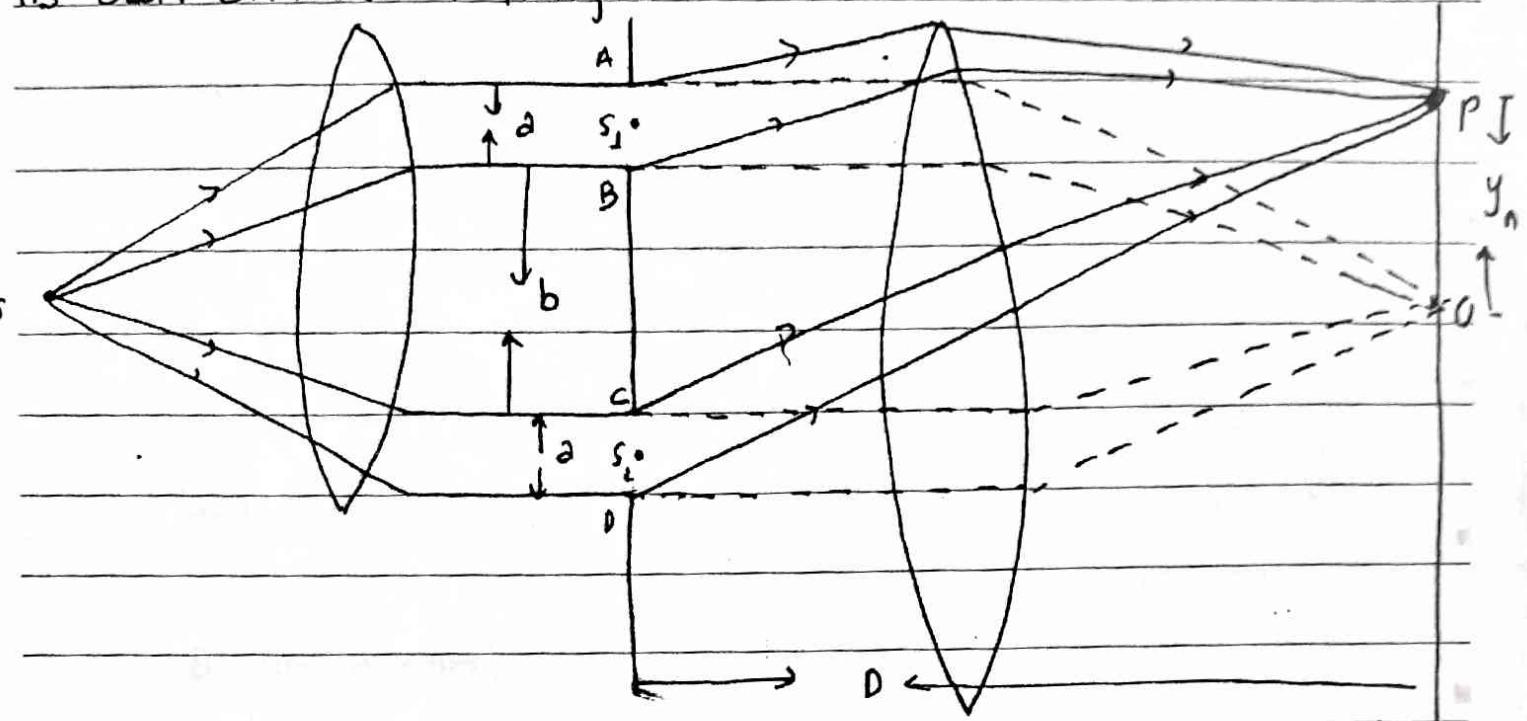
$$\text{but } \alpha = \lambda D / d$$

$$\therefore \text{central width} = 2\lambda D / d$$

Fraunhofer's diffraction through Double slit -

Suppose AB and CD are two identical slits each of width 'a' separated by opaque of width 'b'. When the parallel beam of light incident on the slits, these rays are focused at the center of screen by a convex lens where central maxima is obtained. These two slits together produce light and dark interference fringes. Also each slit produces

Its own diffraction fringes.



### \* Interference maxima and minima -

The mid point of each slit behave as a source. Therefore  $s_1$  and  $s_2$  are two coherent sources and produces interference fringes. The secondary wavelets emerging from  $s_1$  and  $s_2$  at angle  $\theta$  have path difference =  $s_2 N$

From fig,  $\Delta s_1 s_2 N$

$$\sin \theta = \frac{s_2 N}{s_1 s_2}$$

$$\Rightarrow s_2 N = s_1 s_2 \sin \theta$$

$$\Rightarrow s_2 N = (a+b) \sin \theta$$

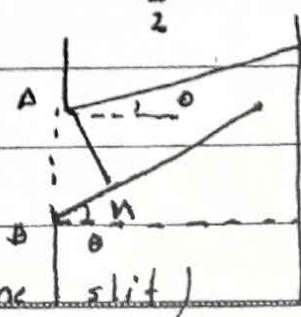
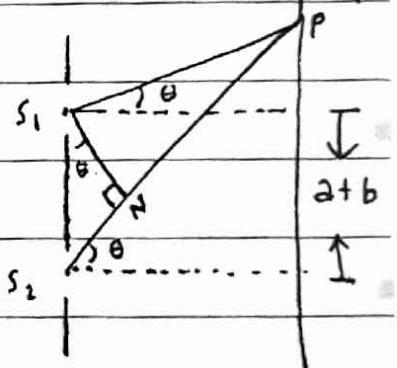
$\therefore$  for interference maxima,  $(a+b) \sin \theta = n\lambda$  [  $n=1, 2, 3, \dots$  ]

For interference minima,  $(a+b) \sin \theta = \frac{(2n+1)\lambda}{2}$

### Diffraction maxima and minima.

The path difference between the rays

diffracted through angle  $\theta$  (considering any one slit)



on the upper and lower surface of slit = BM.

on fig in A BAM,  $\sin\theta = BM/AB$

$$\Rightarrow BM = AB \sin\theta$$

$$\Rightarrow BM = a \sin\theta \dots \text{---(x)}$$

- diffraction minima,  $a \sin\theta = n\lambda$  [  $n = 1, 2, 3, \dots$  ]

- diffraction maxima,  $a \sin\theta = (2n+1)\lambda/2$

Thus we can conclude that the interference depends upon the slits and opaque width but diffraction depends upon only width. The resultant pattern on the screen is the combination interference as well as diffraction fringes. The diffraction fringes intensity is 4 times that of single slit diffraction fringes.

### Diffraction grating -

Diffraction grating is an arrangement of a number of equidistant parallel slits separated by identical opaques. It is made by ruling 1500 to 200 lines on one inch glass plate by diamond point. Each ruled line behaves like opaque and glass between two such line behaves as a slit.

If the rulings are made on transparent sheet of glass then it is called transmission grating and if rulings are made on silvered glass then it is called reflection grating. In reflection grating the ruling line behaves as slit.

The width of slit and opaque is called grating element.

'a' be the slit width and 'b' be the opaque width then  
grating element =  $a+b$

Let N be the number of ruling lines in 1 inch glass plate then

$$N(a+b) = 1 \text{ inch}$$

$$\Rightarrow a+b = \frac{1}{N} \text{ inch}$$

$$\Rightarrow a+b = \frac{2.54}{N} \text{ cm.}$$

If the path difference bet<sup>n</sup> the rays emerges from two consecutive slit is  $nd$  then corresponding point will be bright and if the path difference is  $(2n+1)\frac{\lambda}{2}$ , then corresponding point will be dark.

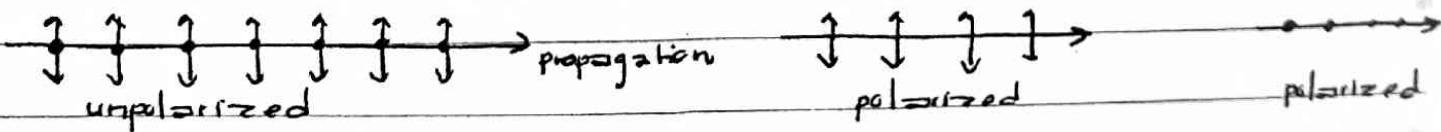
$\therefore$  For maxima, path diff  $\Rightarrow (a+b) \sin\theta = nd$

For minima,  $(a+b) \sin\theta = (2n+1) \frac{\lambda}{2}$

### A polarization.

The light rays in which vibrations are symmetrical and perpendicular to the direction of light wave is called unpolarized light wave. The vibration in horizontal direction is denoted by arrows and vibrations in vertical plane are denoted by dots.

The light wave in which vibrations are confined only in one plane perpendicular to wave propagation is called polarized light and phenomenon is called polarization. Therefore the polarized light wave contains either dot vibrations or arrow vibrations.



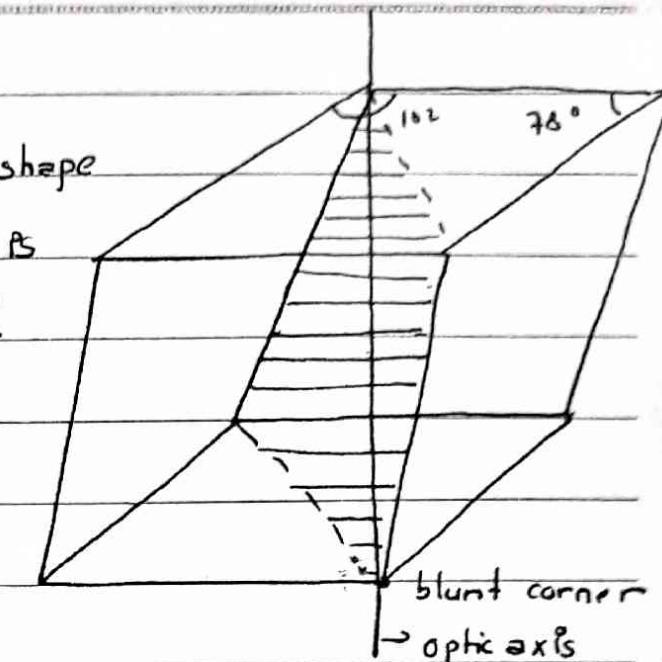


Date . . .

## \* calcite crystal -

It is the crystal of rhombohedral shape bounded by six faces each of which is parallelogram with angle  $102^\circ$  and  $78^\circ$ .

Therefore there are two corners at which all three faces meet at obtuse angle. These corners are called blunt corners. The line joining the blunt corners is called optic axis and any line parallel to this line is also optic axis.

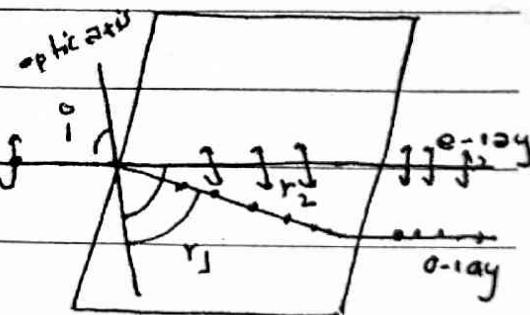


The plane which contains optic axis and perpendicular to the opposite pair of faces is called principle plane. In the fig. shaded area represents a principle plane. There are three such planes.

## \* Double refraction -

When a ray of monochromatic light incident on one face of calcite crystal, it splits into two rays, one containing arrow vibration called extra-ordinary ray which does not follow the snell's law strictly. The velocity of e-ray is different at different surface of the crystal while other ray containing dot vibration called ordinary ray. The velocity of ordinary ray throughout the crystal is same but along the optic axis both rays travel with same speed and we can not separate o-ray and e-ray.

The vibrations of e-ray are parallel to optic axis and vibrations



of o-rays are perpendicular to the axis.

From fig. for ordinary ray,  $\mu_0 = \sin i / \sin r$ ,

for extra-ordinary ray,  $\mu_e = \sin i / \sin r$ ,

but  $r_2 > r_1$

$\therefore \mu_e < \mu_0$

$$\Rightarrow \frac{c}{v_e} < \frac{c}{v_0}$$

[ $v_e$  = velocity of e-ray]

$v_0$  = velocity of o-ray

$$\Rightarrow v_0 < v_e$$

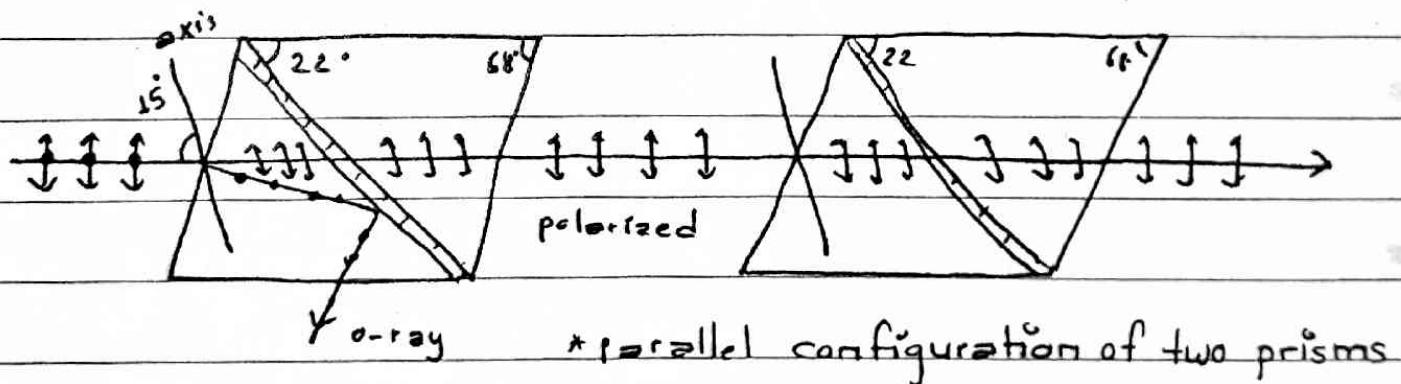
$c$  = speed of light in vacuum

i.e. velocity of extra-ordinary ray is greater than velocity of ordinary ray. Such a crystal is called negative double refracting crystal.

If  $v_0 > v_e$  then such a crystal is called positive double refracting crystal. Calcite is negative crystal and quartz is positive crystal.

The phenomenon in which an unpolarized splits into two rays is called double refraction.

### \* Nicol prism:



A Nicol prism is made from calcite crystal whose length is three times its width. The calcite crystal is ground with until the acute angle of principle plane reaches  $68^\circ$  instead of  $75^\circ$ . The piece is then cut into two parts perpendicular to principle axis and the two new end surfaces again combined together with cement called canada balsam.

The combination is then called Nicol prism. The Canada Balsam behaves like rarer medium for ordinary ray and denser medium for extra-ordinary ray. The refractive index of Canada Balsam lies between the refractive index of o-ray and e-ray.

$$\mu_o = 1.66, \mu_b = 1.55 \text{ and } \mu_e = 1.48$$

When unpolarized light is incident on Nicol prism at an angle about  $15^\circ$  the ray after entering the crystal splits into two rays i.e. ordinary ray and extra-ordinary ray. The refractive index of Canada Balsam is such that the e-ray is transmitted while o-ray gets total internal reflection. Therefore the transmitted light is polarized with vibration parallel to optic axis. Therefore Nicol prism acts as polarizer.

If another Nicol prism is kept parallel to the 1<sup>st</sup> Nicol prism then the transmitted vibration is also transmitted through 2<sup>nd</sup> one if the optic axes of both prisms are parallel otherwise not. Thus Nicol prism acts as both polarizer and analyzer.

#### \* Optical activity -

If two Nicol prisms are held in crossed configuration and if the field of view of polarized light is observed, it will be completely appeared dark. Now if the quartz or calcite crystal cut its face perpendicular to its optic axis is inserted between the prisms such that the light normally incident on the crystal the field of view appears bright indicating that light is not cutoff by the analyzer. In order to cutoff the transmitted light the analyzer has to be rotate through certain angle.

This shows that the plane of polarization is rotated through certain angle when light passes through the crystal.

The ability to rotate the plane of polarization of plane polarized light by certain substance is called optical activity and substance is called optical active substance. For example sugar solution, sodium chloride etc.

#### \* specific rotation -

The specific rotation for a given wavelength of light at given temperature is defined as the rotation produced by one decimeter long column of the solution containing 1 gm of optically active substance per cc of solution.

If 's' be the specific rotation,  $\theta$  be the rotation produced and 'c' be the concentration.

$$\text{Then, } s = \frac{\theta}{lc}$$

$\Rightarrow$  specific rotation =  $\frac{\text{rotation produced in degree}}{\text{length in cm} \times \text{concentration g/cc}}$

$$\Rightarrow s = \frac{10\theta}{lc} \dots (*)$$

[ Here l in decimeter ]

#### \* Quarter wave plate -

A quarter wave plate is the double refracting crystal having optic axis parallel to its refracting face and thickness is adjusted in such a way that it introduced a path difference  $\lambda/4$  or phase.

Difference  $\pi/2$  between o-ray and e-ray, propagating through

If  $\mu_o$  be the refractive index of o-ray,  $\mu_e$  be the refractive index of e-ray and  $t$  be the thickness of plate then,

optical path difference between o-ray and e-ray is

$$\text{For o-ray} = \mu_o t$$

$$\text{For e-ray} = \mu_e t$$

$$\text{total path diff} = \mu_o t - \mu_e t$$

For quarter wave plate, path diff =  $\lambda/4$

$$\therefore \mu_o t - \mu_e t = \lambda/4$$

$$\Rightarrow (\mu_o - \mu_e) t = \frac{\lambda}{4} \quad \dots (*) \quad [\text{for negative crystal}]$$

$$\text{or } (\mu_e - \mu_o) t = \frac{\lambda}{4} \quad \dots (***) \quad [\text{for positive crystal}]$$

\* half wave plate -

A half wave plate is the double refractive crystal having optic axis parallel to its refracting face and thickness is adjusted in such a way that it introduced the path difference  $\lambda/2$  or phase difference  $\pi$  between e-ray and o-ray, emerging from the plate.

$$\text{i.e. } (\mu_e - \mu_o) t = \frac{\lambda}{2} \quad \dots (*)$$

$$\text{or } (\mu_o - \mu_e) t = \frac{\lambda}{2} \quad \dots (***)$$

respectively for positive and negative crystal.