* Mechanism of metalic conduction-

In a metal valence electrons are alomost free and move randomly in all direction. The moving electrons collides with the atom and changes its path contineously. The distance bet^n two successive collision is called free path and the time bet^n two collision is called free time.

Let $s_1, s_2, \dots$ be the free path of electron then mean free path,

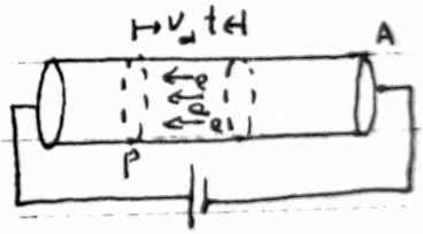
$$\lambda = \frac{s_1 + s_2 + \dots}{n}$$

If $t_1, t_2 \dots$ be the free time then mean free time,

$$\tau = \frac{t_1 + t_2 + \dots}{n}$$

Then average velocity of electron, $\bar{v} = \frac{\lambda}{\tau}$ ----(*)

* current density of a conductor:



consider a conductor of area of cross-section 'A' containing 'n' electrons per unit volume. when an electric field is applied the free electrons move from right to left. If 'q' be the charge crossing the plate 'P' in time 't' then current fllowing through the conductor is $I = \frac{q}{t}$ ....(1)

In time 't' all the electrons cross the plate 'P' which line up to a distance $v_d t$ from it.

where v_d = drift velocity of electron.

The volume of a conductor, $= A v_d t$ ---(2)

∴ The total number of electrons through the plane 'P' in time $t = nAV_d t$ ....(3)

If 'e' be the charge of electron then total charge

$$q = enAV_d t \quad ....(4)$$

From (1) and (4)

$$I = \frac{enAv_d t}{t}$$

$$\Rightarrow I = nev_d A \quad ....(5)$$

$$\Rightarrow \frac{I}{A} = nev_d$$

$$\Rightarrow \vec{J} = ne\vec{v_d} \quad ...(6)$$

where $\vec{J}$ is called current density. i.e current flowing per unit area.

* ohm's law:

It states that the current flowing through the conductor is directly proportional to the potential difference across its two ends. If I be the current and v be the potential then

$$V \propto I$$

$$\Rightarrow V = RI$$

$$\Rightarrow R = V/I \quad ..(7)$$

R is called resistance of the conductor.

* Resistivity and conductivity:

Resistance is the abstruction offered to flow of current by a conductor and depends upon the properties of the object where as resistivity is the obstruction offered to flow the current by unit area of the conductor and depends upon the properties of metal and does not depend upon shape and size of a material. It is a constant quantity where as resistance is a variable.

Resistivity of the conductor is defined as the ratio of electric field to the current density. It is denoted by ρ

$$\therefore \text{Resistivity } (\rho) = \frac{\text{Electric field } (E)}{\text{current density } (J)}$$

$$\Rightarrow \rho = \vec{E}/\vec{J} \quad \cdots (1)$$

If V be the potential difference applied across the length 'l' then, $\vec{E} = \frac{V}{l} \quad \cdots (2)$

If I be the current flowing per unit area 'A' then

$$\vec{J} = \frac{I}{A} \quad \cdots (3)$$

substituting (2) and (3) in (1)

$$\rho = \frac{V/l}{I/A}$$

$$\Rightarrow V = \frac{I\rho l}{A} \quad \cdots (4)$$

comparing eq^n (4) with $V = IR$ we get,

$$R = \frac{\rho l}{A}$$

$$\Rightarrow \rho = RA/l \quad \cdots (5)$$

If $l = 1m$ and $A = 1m^2$ then, $\rho = R$ i.e resistivity is the resistance of the conductor of unit length of unit area. Its unit is ohm-m.

The reciprocal of resistivity is called conductivity. It is given by $\sigma = 1/\rho$ ....(6)

* Relation between current density and conductivity:

we have from ohm's law, $I = V/R$ ...(1)

but $V = El$ ...(2)

$\therefore$ From (1) and (2) $I = \frac{El}{R}$ ...(3)

Again, resistivity, $\rho = \frac{RA}{l}$

$$\Rightarrow R = \rho l/A \quad \text{---(4)}$$

From (3) and (4) $I = \frac{El\,A}{\rho l}$

$$\Rightarrow \frac{I}{A} = \frac{E}{\rho}$$

$$\Rightarrow \vec{J} = \sigma \vec{E} \quad \text{...(5)}$$

* Resistivity interms of mean free path

(Atomic view of resistivity)

Let a metal wire is subjected to an electric field $\vec{E}$. The free electron experiences a force,

$$\vec{F} = e\vec{E} \quad \text{--(1)}$$

The acceleration of electron, $\vec{a} = \frac{\vec{F}}{m}$ ...(2)

From (1) and (2) $\vec{a} = \frac{e\vec{E}}{m}$ ...(3)

where m = mass of electron

e = charge of electron.

Let τ be the mean free time and $\vec{v_d}$ be the drift velocity of electron in time τ then

$$\vec{v_d} = \vec{u} + \vec{a}\tau$$

Initial velocity, $\vec{u} = 0$

$$\therefore v_d = a\tau \quad ---(4)$$

From (3) and (4)

$$\vec{v_d} = \frac{e\vec{E}}{m}\tau \quad ---(5)$$

Again we have resistivity, $\rho = \vec{E}/\vec{J}$

$$\Rightarrow \rho = \frac{\vec{E}}{ne\vec{v_d}} \quad ---(6)$$

From (5) and (6) $\rho = \dfrac{\vec{E}}{ne\, e\vec{E}/m\, \tau}$

$$\Rightarrow \rho = \frac{m}{ne^2\tau} \quad ---(7)$$

Also, average velocity of electron $\bar{v} = \dfrac{\lambda}{\tau} \quad ...(8)$

substituting τ from (8) in ρ

$$\Rightarrow \rho = \frac{m\bar{v}}{ne^2\lambda} \quad ---(9)$$

This is the required resistivity interms of mean free path (λ)

* Mobility:

Drift velocity per unit electric field is called mobility of the electrons. It is denoted by μ.

we have the relation between electrical conductivity and current density with electric field as

$$\vec{J} = \sigma \vec{E} \quad \text{---(1)}$$

Also If 'n' be the number of electrons per unit volume and 'e' be the charge of electron with drift velocity $\vec{v_d}$ then

$$\vec{J} = ne\vec{v_d} \quad \text{...(2)}$$

From (1) and (2)

$$\sigma\vec{E} = ne\vec{v_d}$$

$$\Rightarrow \frac{\sigma}{ne} = \frac{\vec{v_d}}{\vec{E}}$$

$$\Rightarrow \sigma = ne\frac{\vec{v_d}}{\vec{E}}$$

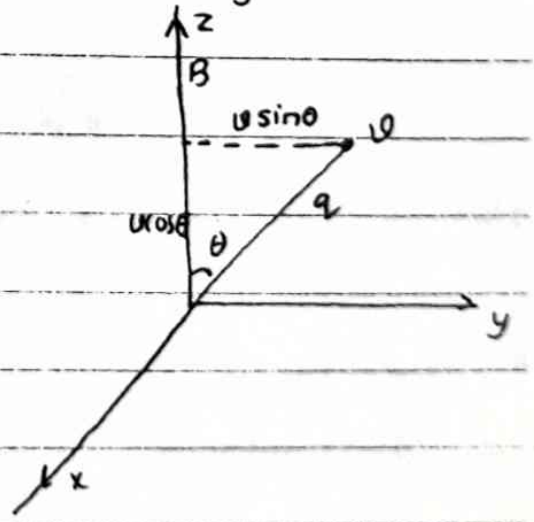
$$\Rightarrow \sigma = ne\mu \quad \text{----(3)}$$

* Magnetic effect of current:

when electric current passes through the conductor magnetic field is produced. For circular current magnetic lines of forces are linear and for linear current magnetic lines of forces are circular.

motion of charge particle in a uniform magnetic field:

consider a charge +q moving with velocity v inside the uniform magnetic field strength 'B'. suppose the magnetic field is along z-axis and charge particle moves along yz plane making angle 'θ' with the direction of magnetic field 'B'.



It is found that charge experience force F_m perpendicular to the plane of v and B such that

$$\vec{F_m} \propto q, \quad --(1)$$

$$\vec{F_m} \propto \vec{B} \quad ---(2)$$

and also proportional to the component of velocity of charge in a direction perpendicular to the direction of B.

i.e $F_m \propto v \sin\theta \quad --(3)$

From (1), (2) and (3)

$$F_m \propto \vec{B} q v \sin\theta$$

$$\Rightarrow \vec{F_m} = k \vec{B} q v \sin\theta$$

For S.I unit $k = 1$.

$$\therefore \vec{F_m} = \vec{B} q v \sin\theta \quad ---(*)$$

$$\Rightarrow \vec{F_m} = q \vec{v} \times \vec{B} \quad ----(4)$$

i.e $\vec{F_m}$ is perpendicular to both plane containing $\vec{v}$ and $\vec{B}$. Also $\vec{F_m}$ is always perpendicular to $\vec{v}$, therefore path of charge is circular in magnetic field

Therefore F_m provides the centripetal force,

$$\therefore F_m = \frac{mv^2}{r} \quad --(5)$$

From (*) and (5)

$$Bqv \sin\theta = \frac{mv^2}{r}$$

If $\theta = 90^\circ$ then, $\quad Bqv = \frac{mv^2}{r}$

$$\Rightarrow Bev = mv^2/r$$

$$\Rightarrow \frac{Be}{m} = \frac{v}{r}$$

$$\Rightarrow \frac{Be}{m} = \omega$$

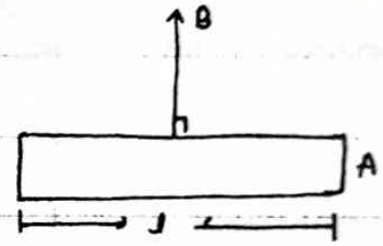
$\Rightarrow \quad \omega = Be/m$

$\Rightarrow \quad 2\pi f = \frac{Be}{m}$

$\Rightarrow \quad f = \frac{Be}{2\pi m} \quad ---- (6)$

This is called cyclotron frequency.

* Magnetic force on a conductor:



consider a conductor of length 'l' and cross-section area 'A' carrying current I which is kept at right angle to the magnetic field $\vec{B}$.

If $\vec{J}$ be the current density, 'n' be the number of electrons per unit volume and 'V_d' be the drift velocity and 'e' be the charge on an electron then,

$\vec{J} = ne\vec{v_d} \quad -- (1)$

$\Rightarrow \quad \frac{I}{A} = ne\vec{v_d}$

$\Rightarrow \quad I = nev_d A \quad --- (2)$

we know that force on each electron $= Bev_d \quad --- (3)$

And the total number of electrons on conductor $= nAl \quad \cdot\cdot (4)$

$\therefore$ The force on a conductor will be

$$\vec{f_m} = Bev_d(nAl)$$

$$\Rightarrow \vec{f_m} = (nev_d A)Bl \quad -- (5)$$

From (2) and (5)

$$\vec{F_m} = BIl \quad -- (6)$$

he fomagnetic force on a conductor.

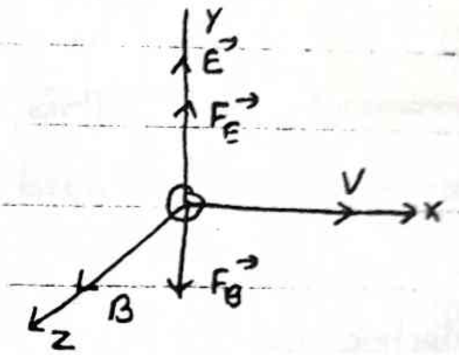
Date

* Lorentz force:

If both an electric and a magnetic field E and B acts on a charge particle, the total force on it can be expressed as

$$\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B} \quad \cdots (1)$$

This force is called lorentz force. The electric part of this force acts only on any charge particle but the magnetic part acts only on moving particles.



one common application of lorentz force occurs when a beam of charged particles passess through the region in which E and B fields are perpendicular to eachother and also perpendicular to the velocity of particles.

If B, V and E are oriented as shown in fig, then electric force $\vec{F_E} = q\vec{E}$ is in opposite direction to the magnetic force $\vec{F_B} = q\,\vec{v}\times\vec{B}$.

we can adjuct the magnetic and electric field until the magnitude of forces are equal in which lorentz force is zero.

ie $F_E = F_B$

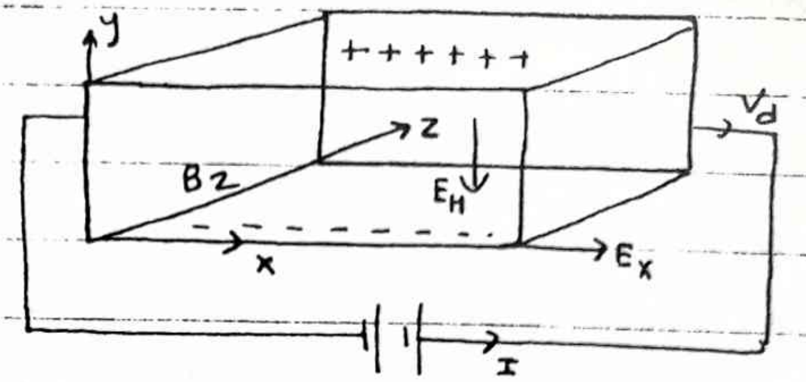
$\Rightarrow qE = qvB$

$\Rightarrow v = \dfrac{E}{B} \quad \cdots\cdots (2)$

$\therefore$ The crossed E and B fields acts as velocity selector. only the particle with speed $v = E/B$ passess through the region unaffected by the magnetic force where as particles with other speeds are deflected. The value of speed is independent of the charge and mass of the particles.

* Hall effect:

when a magnetic field is applied perpendicular to a current carrying conductor, a voltage is developed across the specimen in a direction perpendicular to both current and magnetic field. This phenomena is called Hall effect. The voltage so developed is called Hall voltage.



consider a slab of materal subjected to an external electric field E_x along x-axis and magnetic field B_Z along Z-axis. As a result of electric field, a current density J_x will flow in the direction of E_x. Let the current be carried by electron of charge 'e'. The external magnetic field B_Z will exert a transverse magnetic deflecting force on the electron which causes the electron to drift downward to the lower edge of the specimen. consequently the upper surface collects the excess of positive charge creats the transverse electric field E_H known as Hall field, which opposes the transverse drifting of electrons.

ultimately an equilibrium is reached in which force due to accumulation of electrons becomes equal to the magnetic force and so the flow of electrons stops. Thus net force on electron is zero. i,e lorentz force is zero.

i,e $\quad qE_H = qV_d B_Z$

$\Rightarrow \quad E_H = V_d B_Z \quad ---(1)$

Let 'n' be the free electron density, then current density is

$$J_x = -neV_d \quad --(2)$$

$$\Rightarrow \quad v_d = -\frac{J_x}{ne} \quad ---(3)$$

The negative sign indicates that the current is due to electron.

From (1) and (3) $E_H = -\frac{J_x}{ne} B_z$

$$\Rightarrow E_H = R_H J_x B_z \quad ---(4)$$

where $-1/ne = R_H$ is called Hall coefficient.

Also, we have, mobility $(\mu) = \frac{V_d}{E_x}$

$$\Rightarrow V_d = \mu E_x \quad ---(5)$$

From (1) and (5)

$$E_H = \mu E_x B_z \quad ---(6)$$

From (4) and (6)

$$R_H J_x B_z = \mu E_x B_z$$

$$\Rightarrow R_H = \mu \frac{E_x}{J_x}$$

$$\Rightarrow R_H = \mu/\sigma$$

$$\Rightarrow R_H = \rho\mu \quad ---(7)$$

If 'd' be the width and 't' be the thickness of material then

Hall voltage, $V_H = E_H d \quad --(8)$

From (4) and (8)

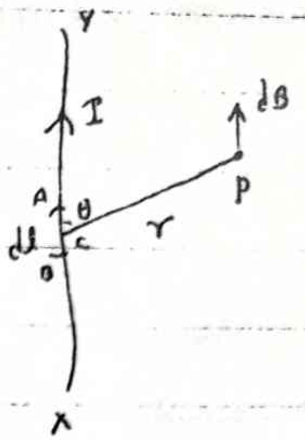
$$V_H = R_H J_x B_z d$$

$$\Rightarrow V_H = R_H \frac{I}{A} B_z d$$

$$\Rightarrow V_H = R_H \frac{I}{dt} B_z d$$

$$\Rightarrow V_H = \frac{R_H B_z I}{t} \quad ---(9)$$

* Biot's and savarts law (Laplace law):



Biot's and savart's law is the law used to find the magnitude of magnetic field strength due to the electric current.

consider a conductor xy carrying current I. as shown in fig. then arround xy the magnetic field is produced. Let 'p' be the point at a distance 'r' from the element length AB. Let θ' be the angle between the element and line joining point 'p' and to its centre 'C'.

According to Biot's and savart's law the strength of magnetic field 'dB' produced at 'p' due to current through element dl of AB is directly proportional to current, element length, sine angle bet^n dl and r. and inversely proportional to square of distance between element length to the point 'p'.

$\therefore \quad dB \propto I$

$dB \propto dl$

$dB \propto \sin\theta$

$dB \propto 1/r^2$

$$\Rightarrow dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

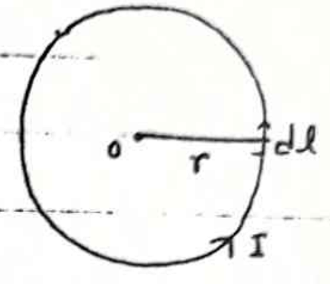
$$\Rightarrow B = \int \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \quad \text{-- (*)}$$

where, $\mu = 4\pi \times 10^{-7}\,\mu m^{-1}$ called permeability of air or free space.

Date

* Application of Biot's and savart's law -

① \# consider a circular coil carrying current I. let r be the radius of the coil consider the element length dl of coil, then according to Biot's and savart's law, field at the centre of coil due to length dl is



$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\sin\theta}{r^2}$$

but $\theta = 90^0$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

Then total field due to whole coil,

$$\int dB = \int \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r^2}\, 2\pi r \qquad \left[\because \int dl = \text{circumference of the coil}\right]$$

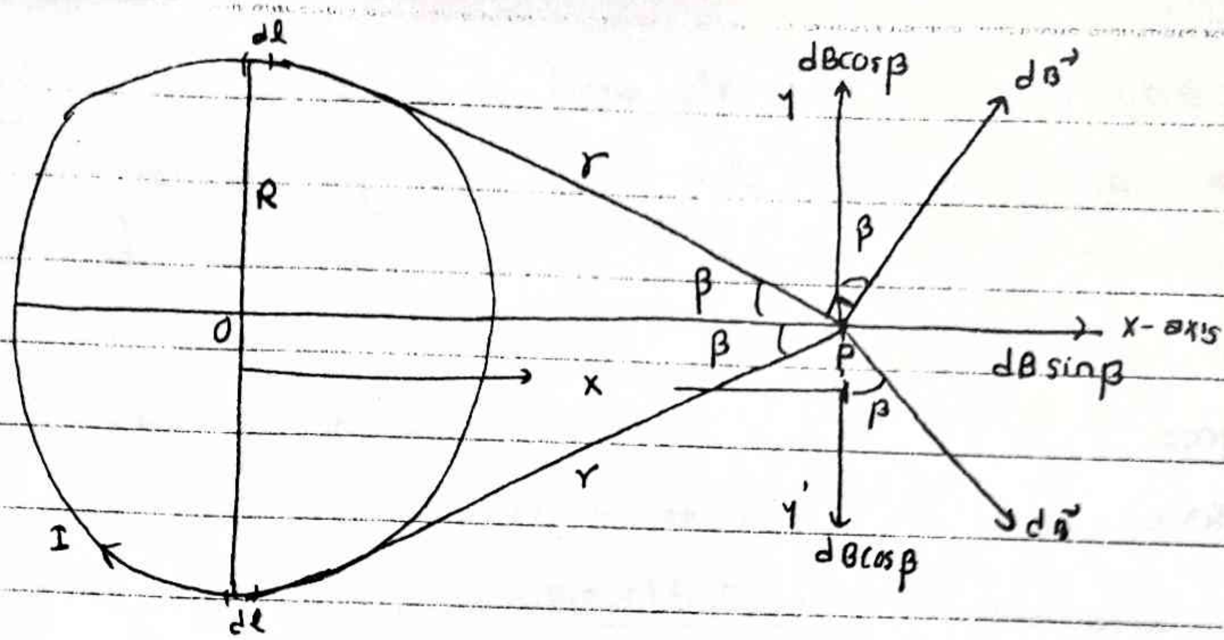
$$\Rightarrow B = \frac{\mu_0 I}{2r} \quad \text{---- } (*)$$

This is the magnetic field strength at the centre of coil.

② \# Magnetic field strength along the axis of coil:

consider a circular coil having radius 'R' carrying current 'I'. we have to find the field at a distance 'x' from the centre of coil, along the axis.

According to Biot's and savart's law, magnetic field



at point 'p' due to element length dl is (for upper element length]

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

but $\theta = 90°$, $\Rightarrow$ $dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$ ----(1)

This field is perpendicular to r.

The component of dB along two perpendicular axis as $dB\cos\beta$ along y-axis and $dB\sin\beta$ along x-axis. where β is the angle made by 'r' with axis.

If we consider a similar element diametrically opposite then the element would produced the same magnetic field dB at a point P. and would have the similar component $dB\cos\beta$ and $dB\sin\beta$. The components due to two current elements along yy' direction will cancel eachother as they are equal in magnitude and opposite in direction.

$\therefore$ magnetic field due to current element along px is $dB\sin\beta$.

$\therefore$ total field at P due to whole coil is

$$B = \int dB\sin\beta \quad ---(2)$$

From (1) and (2) $B = \int \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin\beta$

$$= \frac{\mu_0 I \sin\beta}{4\pi r^2} \int dl$$

$$B = \frac{\mu_0 I}{4\pi r^2} \sin\beta \; 2\pi R \quad \text{--(3)} \quad \left[\because \int dl = \text{circumference of a coil} \right]$$

Also, from fig, $\sin\beta = R/r$ --(4)

From (3) and (4)

$$B = \frac{\mu_0 I}{4\pi r^2} \frac{R}{r} 2\pi R$$

$$\Rightarrow B = \frac{\mu_0 I R^2}{2 r^3} \quad \text{---(5)}$$

Also, From fig, $r^2 = \sqrt{R^2 + x^2}$

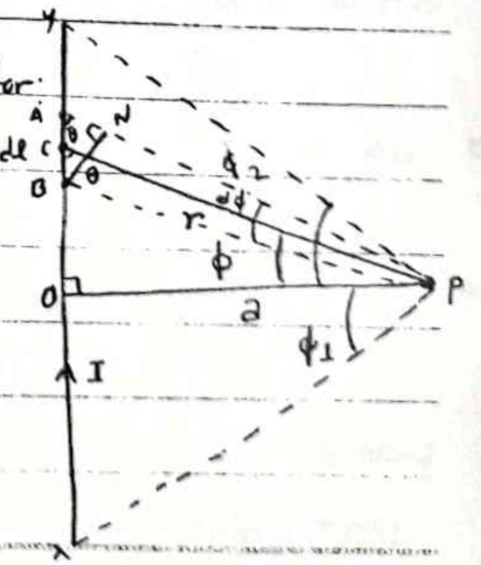
$$\Rightarrow r^3 = (R^2 + x^2)^{3/2} \quad \text{--(6)}$$

From (5) and (6)

$$B = \frac{\mu_0 I R^2}{2[R^2 + x^2]^{3/2}}$$

If the coil contains N-turns then, $B = \frac{\mu_0 N I R^2}{2[R^2+x^2]^{3/2}}$ ---(*)

(2)* magnetic field strength due to a striaght conductor:



consider a infinitely long streight conductor carrying current I. Let 'P' be the point at a perpendicular distance 'a' from the conductor where we have to find the strength of magnetic field.

consider an element length AB

of length dl whose center is 'C'.

Let $PC = r$, $\angle PCO = \theta$, $\angle APB = d\phi$, $\angle OPX = \phi_1$, $\angle OPY = \phi_2$

As the point C and A are very close to each other then,

$\angle BAN$ is also equal to θ

$\therefore \angle BAN = \theta$

Now Draw normal BN on AP then from Δ ABN

$$\sin\theta = \frac{BN}{AB}$$

$$\Rightarrow BN = AB\sin\theta$$

$$\Rightarrow BN = dl\sin\theta \quad ---(1)$$

Also, from Δ BPN,

$$\text{central angle}, (d\phi) = \frac{BN}{r}$$

[$d\phi$ can be taken as the angle substanded by BN of radius r at P]

$$\Rightarrow r\,d\phi = BN \quad --(2)$$

From (1) and (2)

$$dl\sin\theta = r\,d\phi \quad --(3)$$

According to Biot's and savart's law, field at P due to length dl is,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \quad --(4)$$

substituting (3) in (4) we get

$$dB = \frac{\mu_0}{4\pi}\frac{I}{r^2}r\,d\phi$$

$$\Rightarrow dB = \frac{\mu_0 I\,d\phi}{4\pi r} \quad --(5)$$

Also, from Δ CPO, $\cos\phi = a/r$

$$\Rightarrow r = \frac{a}{\cos\phi} \quad ---(6)$$

From (5) and (6)

$$dB = \frac{\mu_0 I}{4\pi a} \cos\phi \, d\phi$$

∴ The total field at 'p' due to whole conductor.

$$\int dB = \int_{-\phi_1}^{\phi_2} \frac{\mu_0 I}{4\pi a} \cos\phi \, d\phi$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi \right]_{-\phi_1}^{\phi_2}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a} \left[\sin\phi_1 + \sin\phi_2 \right]$$

If the conductor is infinitely long then $\phi_1 = \phi_2 = \pi/2$

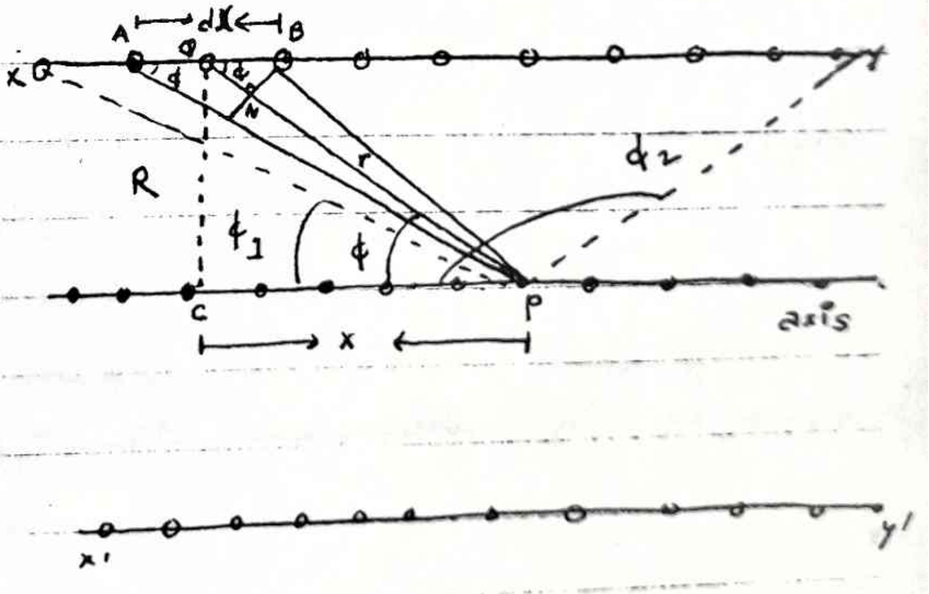
$$\therefore B = \frac{\mu_0 I}{4\pi a} [1 + 1]$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi a} \quad ---(*)$$

---

④ Magnetic field due to solenoid:

Let us consider a long solenoid of radius R carrying current I. Let the number of turns per unit length of the solenoid be 'n'.



suppose we have to find the magnetic field strength B at any point 'p' on the axis of the solenoid at a distance 'r' from the centre of element length of AB of length dx and

The magnetic field B at point 'p' can be regarded as the resultant of field B due to number of coils such as AB of length dx, in which the solenoid may be imagined to be divided. Each of such coil has $n dx$ turns. If the point 'p' lies at a distance x from the center of the coil AB (perpendicular distance) then according to Biot's and savart's law magnetic field at a point on the axis of circular coil,

$$dB = \frac{\mu_0 I R^2}{2r^3} \times \text{number of turns}$$

$$\Rightarrow dB = \frac{\mu_0 I R^2 n dx}{2r^3} \quad \text{- - - -(1)}$$

From fig.: $QP = r$, $\angle BQP = \phi = \angle QPC$, $QC \perp$ on axis.

$BN \perp AP$,

Join point p to two end points of solenoid.

Let $\angle XPC = \phi_1$, $\angle YPC = \phi_2$ and $\angle APB = d\phi$, $\angle BPA = \phi$

From Δ ABN,

$$\sin\phi = \frac{BN}{AB}$$

$$\Rightarrow BN = AB \sin\phi$$

$$\Rightarrow BN = dx \sin\phi \quad \text{- - -(2)}$$

Also, from fig.

$$d\phi = \frac{BN}{r}$$

$$\Rightarrow BN = r\, d\phi \quad \text{- - (3)}$$

from (2) and (3) $\quad dx \sin\phi = r\, d\phi$

$$\Rightarrow \quad dx = \frac{r\,d\phi}{\sin\phi} \quad \text{--- (4)}$$

substituting (4) in (1)

$$dB = \frac{\mu_0 I R^2 n}{2r^3} \frac{r\,d\phi}{\sin\phi} \quad \text{--- (5)}$$

But $\sin\phi = \frac{R}{r}$ --- (6) [From Δ QPC]

From (5) and (6) $dB = \frac{\mu_0 n I R^2 r}{2r^3} \frac{d\phi}{R} r$

$$\Rightarrow dB = \frac{\mu_0 n I}{2} \frac{R}{r} d\phi$$

$$\Rightarrow dB = \frac{\mu_0 n I}{2} \sin\phi\, d\phi \quad \text{--- (7)}$$

Then total field due to solenoid.

$$\int dB = \int_{\phi_1}^{\phi_2} \frac{\mu_0 n I}{2} \sin\phi\, d\phi$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[-\cos\phi\right]_{\phi_1}^{\phi_2}$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[\cos\phi_1 - \cos\phi_2\right]$$

For infinitely long solenoid, $\phi_1 = 0$, $\phi_2 = \pi$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[1 - (-1)\right]$$

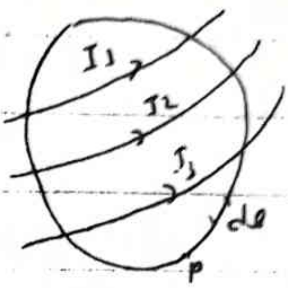
$$\Rightarrow B = \mu_0 n I \quad \text{--- (8)}$$

If point 'p' lies at one end of solenoid then $\phi_1 = 0$, $\phi_2 = \pi/2$

then $B = \frac{\mu_0 n I}{2}\left[1 - 0\right] \Rightarrow B = \frac{\mu_0 n I}{2}$ --- (9)

* Ampere's law:

It states that the line integral of magnetic field round a closed path is equal to the μ_0 times the current enclosed by that path.



As shown in the fig the closed path 'p' encloses current $I_1, I_2 \ldots\ldots$ let B the magnetic field any point of path. Now consider an element length dl of the path at p. Then line integral of magnetic field $= \int \vec{B} \cdot \vec{dl}$

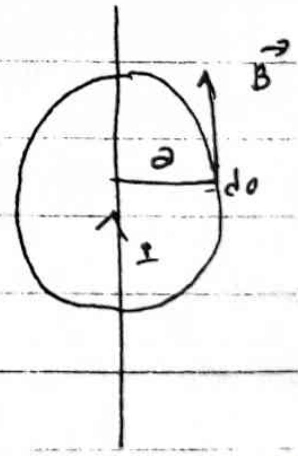
From Ampere's law,

$$\int \vec{B} \cdot \vec{dl} = \mu_0 (I_1 + I_2 + \ldots\ldots)$$

$$\Rightarrow \int \vec{B} \cdot \vec{dl} = \mu_0 I \quad \text{-- (x)}$$

Application of Ampere's law:

① Magnetic field due to a straight conductor.



Let us consider a straight conductor carrying current I. As we know that for linear current the magnetic lines of forces are circular and tangent to the lines of forces give the direction of magnetic field. To find the magnetic field at any point at a perpendicular distance 'a' from the conductor, draw a circle of radius 'a'.

Let 'B' be the magnetic field at a distance 'a' from the conductor, then according to Ampere's law

$$\int \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\Rightarrow \int B\,dl\cos\theta = \mu_0 I$$

since B has only tangential component, $\theta = 0$, i,e angle between B and dl.

$$\therefore \int B\,dl = \mu_0 I$$

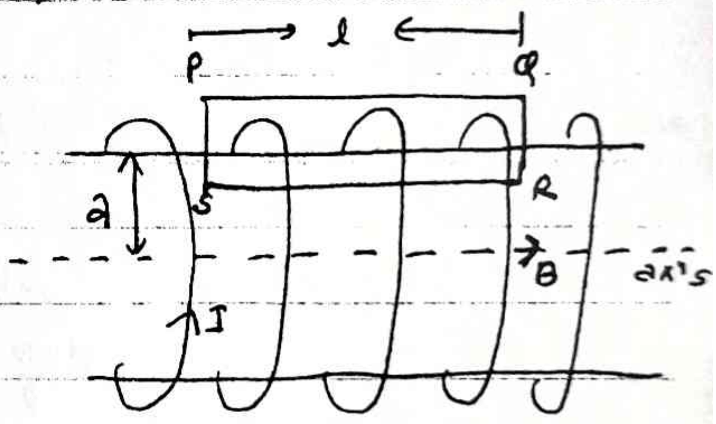
$$\Rightarrow B\int dl = \mu_0 I \quad ---(1)$$

But $\int dl = 2\pi a$, circumference of closed path.

$\therefore$ From (1) $B\,2\pi a = \mu_0 I$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi a} \quad ---(2)$$

(2) Magnetic field due to a solenoid:



The magnetic field inside the solenoid is parallel to the axis and magnetic field outside the solenoid is zero.

Let us consider a rectangular path PQRS as shown in fig.

Let N be the number of turns of solenoid enclosed by path PQRS and l be the length of the path.

$$\text{i,e } PQ = RS = l$$

Then total line integral of magnetic field of path PQRS is

$$\oint_{PQRS} \vec{B}\cdot d\vec{l} = \int_P^Q \vec{B}\cdot d\vec{l} + \int_Q^R \vec{B}\cdot d\vec{l} + \int_R^S \vec{B}\cdot d\vec{l} + \int_S^P \vec{B}\cdot d\vec{l}$$

But the line integral from P to Q, ~~the~~ lies outside the solenoid, $B = 0$

The line integral from Q to R, $\vec{B}$ and $d\vec{l}$ are perpendicular to each other, i,e $\theta = 90^\circ$

line integral from R to S, angle between $\vec{B}$ and dl is 0°

and line integral from S to P again $\theta = 90^\circ$

$$\therefore \oint_{PQRS} \vec{B}\cdot\vec{dl} = \int_P^Q \underset{0}{B}\, dl \cos 0^\circ + \int_Q^R B\,dl \underset{0}{\cos 90^\circ} + \int_R^S B\,dl \cos 0^\circ + \int_S^P B\,dl \cos 90^\circ$$

$$= 0 + 0 + \int_R^S B\,dl \cos 0^\circ + 0$$

$$\Rightarrow \int_{PQRS} \vec{B}\cdot\vec{dl} = \int_R^S B\,dl$$

$$\Rightarrow \int_{PQRS} \vec{B}\cdot\vec{dl} = B\int_R^S dl$$

$$\Rightarrow \int_{PQRS} \vec{B}\cdot\vec{dl} = Bl \quad ---- (1)$$

Now from ampere's law, $\displaystyle\int_{PQRS} \vec{B}\cdot\vec{dl} = \mu_0 N I$ --(2) [For N turns]

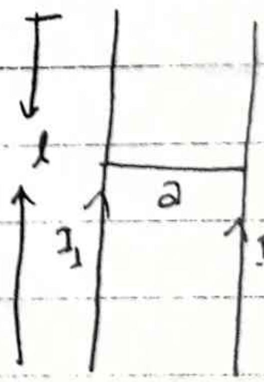
From (1) and (2) $\mu_0 N I = Bl$

$$\Rightarrow B = \frac{\mu_0 N I}{l}$$

$$\Rightarrow B = \mu_0 n I \quad --(3) \qquad [n = N/l]$$

* Force between two parallel conductors: (magnetic force)

consider two straight conductor carrying current I_1 and I_2 resp. Let 'a' be the perpendicular distance between the conductors. Then the wire carrying current I_1 produces magnetic field B_1 whose magnitude at the location of wire carrying current I_2 is



$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad ---(1)$$

Thus the wire which carries current I_2 can be considered to be in an external magnetic field B_1.

$\therefore$ The force experienced by conductor 2nd will be

$$F_{21} = B_1 I_2 l \quad --(2)$$

From (1) and (2)

$$F_{21} = \frac{\mu_0 I I_2 l}{2\pi a}$$

$$\Rightarrow \frac{F_{21}}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad ---(3)$$

similarly the wire carrying current I_2 produces magnetic field B_2 whose magnitude at the location of wire carrying current I_1 is $B_2 = \frac{\mu_0 I_2}{2\pi a} \quad ---(4)$

Thus the wire which carries current I_1 can be considered to be in an external magnetic field B_2

$\therefore$ Force experienced by conductor 1st will be

$$F_{12} = B_2 I_1 l \quad --(5)$$

From (4) and (5)

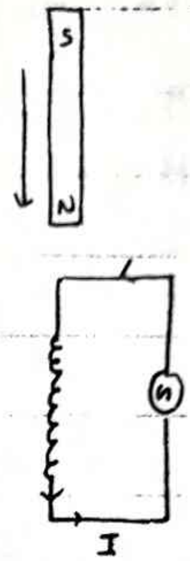
$$F_{12} = \frac{\mu_0 I_2 I_1 l}{2\pi a}$$

$$\Rightarrow \frac{F_{12}}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad ---(6)$$

From (3) and (6)

$$\vec{F_{21}} = \vec{F_{12}}$$

* Faradays law of electromagnetic induction



The phenomenon by which an electric current is produced in a closed coil due to the relative motion between it and magnetic field is called electromagnetic induction. The current so produced is called induced current and the emf under which the current flows is called induced emf.

The magnetic lines of forces per unit cross-section area represents the magnetic induction B, when the lines of forces are perpendicular to the area.

i,e $B = \dfrac{\text{magnetic lines of forces}}{\text{Area}}$ (perpendicular to area)

$\Rightarrow B = \dfrac{\text{magnetic flux}(\phi_m)}{\text{Area}(A)}$

$\Rightarrow \phi_m = BA$

If the coil contains N turns then,

$$\phi_m = NBA \text{ ---- } (*)$$

whenever there is change in flux linked with the coil, induced emf is produced.

The induced emf last as the change in flux continuous.

The magnitude of induced emf is directly proportional to the rate of change of flux.

i.e $E \propto d\phi_m/dt$

$\Rightarrow E = -k\, d\phi_m/dt$

In S.I. unit k=1. $\therefore$ $E = -\dfrac{d\phi_m}{dt}$ ---(**)

- sign indicates that the induced emf oppose the change in flux.

* self induction:

When current is flowing in closed loop, it produces magnetic field and hence magnetic field has flux through the area bounded by the loop. If the current changes with time, the flux also changes and hence emf is induced and is called self induction.

∴ The flux linked with a coil (loop) is directly proportional to the current flowing through it.

i.e $\phi_m \propto I$

$\Rightarrow \phi_m = LI$ - - (1)

L is called coefficient of self induction.

If $I = 1$ A and then $\phi_m = L$,

the self inductance of coil is numerically equal to the flux linked with coil when unit current flowing through it.

Also, From Faraday's law,

$$E = -\frac{d\phi_m}{dt} \quad ---(2)$$

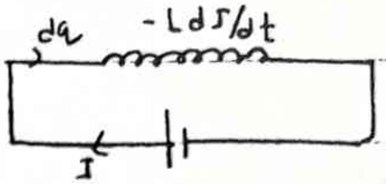
From (1) and (2) $\quad E = -L\frac{dI}{dt} \quad --(3)$

when $\frac{dI}{dt} = 1\,Ap^{-1}$ then,

$$E = -L$$

The self inductance is also numerically equal to the back emf in the coil when the rate of change of current through the coil is 1 Amp sec^{-1}.

* workdone against back emf:



The current in the circuit due to charge dq in time dt as

$$I = \frac{dq}{dt}$$

$$\Rightarrow dq = I\,dt \cdots (1)$$

The workdone against back emf = emf x charge

$$\Rightarrow dw = E\,dq \cdots (2)$$

From (1) and (2) $\quad dw = E I dt \cdots (3)$

Also, back emf $E = -L \frac{dI}{dt} \cdots (4)$

From (3) and (4) $\quad dw = L \frac{dI}{dt} I\,dt \cdots (5)$ [Magnitude only]

Total workdone against back emf is

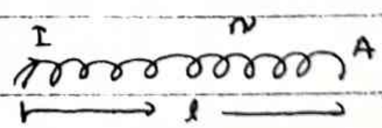
$$\int dw = \int L \frac{dI}{dt} I\,dt$$

$$\Rightarrow \int dw = L \int I\,dI$$

$$\Rightarrow w = \frac{1}{2} L I^2$$

The workdone is change into magnetic energy of a coil.

* self inductance of solenoid:



Let N be the number of turns l be the length and 'A' be the cross-section area of the coil, through which current I is flowing.

If 'n' be the number of turns per unit length then magnetic flux density along the axis of solenoid is

$$B = \mu_0 n I \quad \text{---(1)}$$

Then flux through the solenoid,

$$\phi_m = NBA \quad \text{---(2)}$$

From (1) and (2)

$$\phi_m = N\mu_0 n I A \quad \cdots(3)$$

Also, flux $(\phi_m) = IL \quad \cdots(4)$

From (3) and (4)

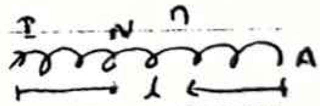
$$N\mu_0 n I A = IL$$

$$\Rightarrow L = \frac{\mu_0 N n I A}{I}$$

$$\Rightarrow L = \frac{\mu_0 N^2 A}{l} \quad \text{----(5)} \qquad [\, n = N/l \,]$$

* Energy density of solenoid:

The magnetic energy per unit volume of the inductor (solenoid) is called energy density of the solenoid. It is denoted by U_m and given by



$$U_m = \frac{\text{~~Electr~~magnetic energy}}{\text{volume}}$$

$$U_m = \frac{\frac{1}{2} L I^2}{Al} \quad \text{---(1)}$$

but $L = \mu_0 N^2 A / l \quad \cdots(2)$

From (1) and (2)

$$U_m = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \frac{I^2}{Al}$$

$$\Rightarrow U_m = \frac{1}{2}\mu_0\left(\frac{N}{l}\right)^2 I^2$$

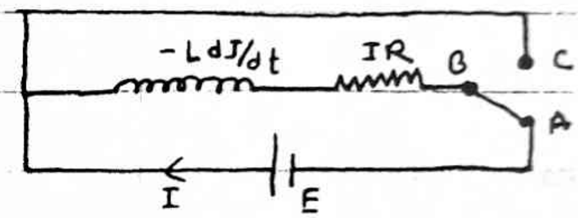
$$= \frac{1}{2\mu_0}\left(\mu_0 \frac{N}{l} I\right)^2$$

$$= \frac{1}{2\mu_0}(\mu_0 n I)^2$$

$$U_m = \frac{1}{2\mu_0} B^2 \quad \text{----(3)}$$

* Growth and decay of current through L-R circuit:

The inductor of inductance 'L' and resistor of resistance 'R' is connected in series with the source of emf 'E' as shown in fig.



when the terminal A and B are connected, the current in the circuit starts growing. Let I be the current in the circuit at any time 't'.

Then according to krichoff's 2nd law,

$$E + \left(-L\frac{dI}{dt}\right) = IR$$

$$\Rightarrow \frac{E}{L} - \frac{dI}{dt} = \frac{IR}{L}$$

$$\Rightarrow \frac{dI}{dt} = \frac{E}{L} - \frac{IR}{L}$$

$$\Rightarrow \frac{dI}{dt} = \frac{R}{L}\left[\frac{E}{R} - I\right]$$

$$\Rightarrow \frac{dI}{[E/R - I]} = \frac{R}{L}\,dt \quad \text{-----}(1)$$

Let $E/R - I = X \quad \cdot\cdot(2)$

then differentiating eq^n (2) w.r.to x

$$-dI = dx \quad \text{--}(3)$$

substituting (2) and (3) in (1)

$$\frac{-dx}{X} = \frac{R}{L}\,dt$$

$$\Rightarrow \int \frac{dx}{X} = \int -\frac{R}{L}\,dt$$

Integratin $\Rightarrow$ $\log_e X = -\frac{R}{L}\,dt + k$ (constant) ---(4)

Now replace X by $E/R - I$ then eq^n (4) becomes

$$\log_e\left(\frac{E}{R} - I\right) = -\frac{R}{L}\,dt + k \quad \cdot\cdot(5)$$

when $t = 0$, $I = 0$ then (5) becomes

$$\log_e\left(\frac{E}{R}\right) = k \quad \text{--}(6)$$

From (5) and (6)

$$\log_e\left[\frac{E}{R} - I\right] = -\frac{R}{L}\,dt + \log_e\left(\frac{E}{R}\right)$$

$$\Rightarrow \log_e\left[\frac{E/R - I}{E/R}\right] = -\frac{R}{L}\,t$$

$$\Rightarrow 1 - \frac{I}{(E/R)} = e^{-R/L\,t}$$

$$\Rightarrow I = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right] \quad \text{---}(7)$$

when $t = 0$, $I = 0$

when $t = \infty$, $I = E/R$ i.e maximum current $= I_0$

$\therefore$ From eqn (7)

$$I = I_0 \left[1 - e^{-\frac{R}{L}t}\right] \quad \text{--- (8)}$$

For one time constant, $\frac{R}{L} t = 1$

$$\therefore I = I_0 \left[1 - e^{-1}\right]$$

$$= I_0 \left[1 - \frac{1}{e}\right]$$

$$= I_0 \left[1 - \frac{1}{2.7}\right]$$

$$= 0.63 I_0$$

$$\Rightarrow I = 63\% \text{ of } I_0 \quad \text{--- (9)}$$

In one time constant the current in the circuit reaches 63% of maximum current.

Now, for decay:

when the current in the circuit reaches maximum then terminal A and B is disconnected and terminal A and C are connected, the current starts decaying.

The circuit eqn becomes

$$-L \frac{dI}{dt} = IR$$

$$\Rightarrow \frac{dI}{I} = -\frac{R}{L} dt \quad \text{--- (10)}$$

Now, integrating $\int \frac{dI}{I} = \int -\frac{R}{L} dt$

Date

$$\Rightarrow \log_e I = -\frac{R}{L} t + k \text{ (constant)} \quad \text{--- (11)}$$

when $t = 0$, $I = I_0$ then eq^n (11) becomes

$$\log_e I_0 = k \quad \text{--- (12)}$$

From (11) and (12)

$$\log_e I = -\frac{R}{L} t + \log_e I_0$$

$$\Rightarrow \log_e \left(\frac{I}{I_0}\right) = -\frac{R}{L} t$$

$$\Rightarrow \frac{I}{I_0} = e^{-\frac{R}{L} t}$$

$$\Rightarrow I = I_0 e^{-\frac{R}{L} t} \quad \text{---- (13)}$$

At $t = 0$, $I = I_0$

At $t = \infty$, $I = 0$

The eq^n (13) shows how the current in circuit decays.