Electrostatics
(chapter-6)

Date . .

* Electric field:

The electric field of the charge is defined as the region around the charge in which its influence can be experienced.

* Electric field intensity:

The force experienced by a unit positive charge placed at a point in an electric field of an another charge is called electric field intensity.

Let 'q' be the charge surrounded by its own electric field and q_0 be a test charge placed at a distance 'r' from charge 'q'. Then according to coulombs law, force between the charges q and q_0 is

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2} \quad ----(1)$$

If $\vec{E}$ be the electric field strength then,

$$\vec{E} = \frac{\vec{F}}{q_0} \quad ----(2)$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad ----(3) \quad [\text{From (1) and (2)}]$$

where $1/4\pi\epsilon_0$ is constant

and ϵ_0 = permitivity in free space

Numerically $1/4\pi\epsilon_0 = 9\times10^9 \; Nm^2C^{-2}$

or $\epsilon_0 = 8.85\times10^{-12} \; N^{-1}m^{-2}C^2$

The electric field intensity is also defined as the number of electric lines of force passing normally through unit area.

i.e $\vec{E} = \frac{\text{electric lines of force (normally crossing)}}{\text{Area}}$

Date

$\Rightarrow \vec{E} = \frac{\text{flux}(\phi)}{\text{Area}(A)}$

$\Rightarrow \phi = \vec{E} A$ ---- (1)

* Gauss law of electrostatics:

It states that the total number of electric lines of forces crossing normally through certain area i,e electric flux is equal to $1/\epsilon_0$ times toal charge enclosed by that surface.

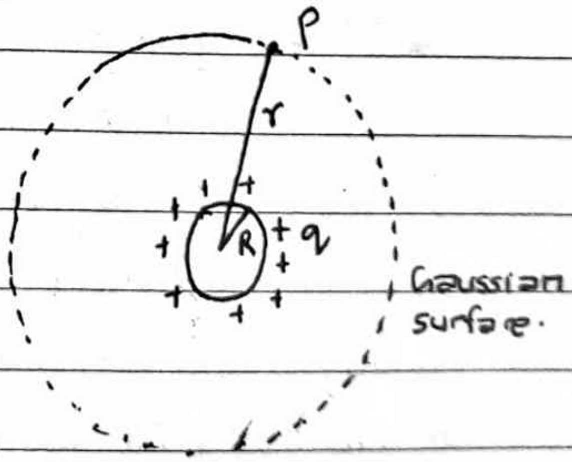
i,e electric flux $(\phi) = \frac{1}{\epsilon_0} \times$ charge enclosed (q)

$$\Rightarrow \phi = \frac{1}{\epsilon_0} \times q \quad \text{----} (*)$$

The surface enclosed is celled Gaussian surface.

* Application of Gauss law

① Electric field intensity due to charged sphere:



Let us consider a sphere of radius R having total charge 'q'. Let the point 'p' lies at a distance 'r' from the center of sphere where we have to find the intensity $\vec{E}$.

Now draw a gaussian surface through point 'p' in the form of sphere of radius 'r'.

case I

when point 'p' lies outside the sphere $r > R$,

Now, from Gauss law, total flux $(\phi) = \frac{1}{\epsilon_0} \times$ charge enclosed (q)

$\Rightarrow \phi = q/\epsilon_0$

$\Rightarrow \vec{E}A = \dfrac{q}{\epsilon_0}$

$\Rightarrow \vec{E} = q/A\epsilon_0$ ----- (1)

where A = Area of Gaussian surface

$A = 4\pi r^2$ --- (2)

$\therefore$ From (1) and (2) $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$ -- (3)

Case II

when point 'p' lies at the surface of the sphere.

In this case $r = R$

$\therefore$ Field intensity at surface, $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R^2}$ ---- (4)

when point 'p' lies inside the sphere (III case):

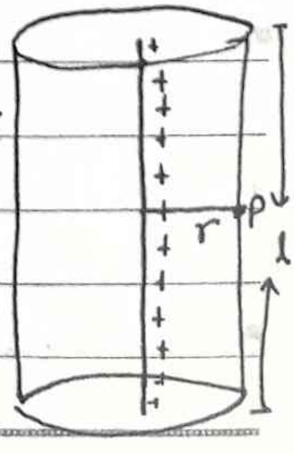
In this case, $r < R$, and no charge enclosed by the Gaussian surface.

$\therefore \phi = \dfrac{1}{\epsilon_0} \times 0$

$\Rightarrow \vec{E}A = 0$

$\Rightarrow A \neq 0$

$\therefore \vec{E} = 0$ .... (5)



(2) Electric field intensity due to linear charged conductor:

Let us consider an infinitely long conductor with linear charge density λ.

i.e $\lambda = \dfrac{\text{charge}(q)}{\text{length}(l)}$

Let 'p' be the point at a perpendicular distance 'r' from the conductor. To determine the electric field intensity draw the Gaussian fsurface in the form of cylinder of length l and radius 'r'.

From the Gauss law,

$$\text{electric flux } (\phi) = \frac{1}{\epsilon_0} \times \text{charge enclosed } (q)$$

$$\Rightarrow \phi = 1/\epsilon_0 \, q$$

$$\Rightarrow \vec{E}A = \frac{1}{\epsilon_0} \lambda l \quad \text{---} (*)$$

$$\Rightarrow \vec{E} = \frac{\lambda l}{A\epsilon_0}$$

but $A = 2\pi r l$ [Area of Gaussian surfae]

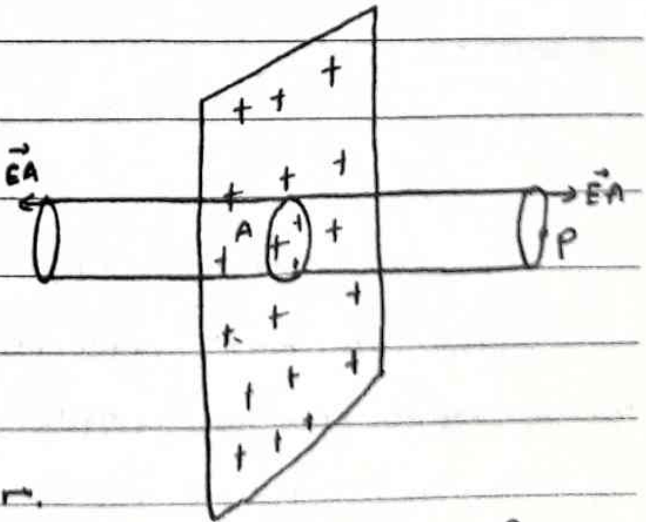
$$\therefore \vec{E} = \frac{\lambda l}{2\pi r l \epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi \epsilon_0 r} \quad \text{---} (**)$$

(3) Electric field intensity due to infinite plane charged conductor:

consider a plane charged conductor having surface charge density σ i.e

$$\sigma = \frac{\text{Charge } (q)}{\text{Area } (A)} \quad \text{---} (*)$$



Let us calculate the field intensity at a point 'p' close to the conductor.

A convenient Gaussian surface is a closed cylinder of cross-

Date . .

section area 'A' to pierce the plane. since the flux from the charged plane conductor is normal to the plane, no flux cross through the curved surface of the cylinder. From symmetry the field has the same magnitude at the ends caps.

The flux through each end cap $= \vec{E}A$

$\therefore$ From Gauss law,

$$\text{Total flux } (\phi) = \frac{1}{\epsilon_0} \text{ times total charge enclosed } (q)$$

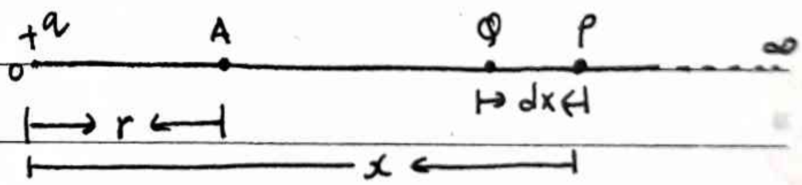
$$\Rightarrow \vec{E}A + \vec{E}A = 1/\epsilon_0 \, \sigma A$$

$$\Rightarrow 2\vec{E}A = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\sigma}{2\epsilon_0} \quad \text{- - - (**)}$$

* Electric potential at a point:

The potential at a point in an electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point against electric forces.



consider a point 'A' in an electric field at a distance 'r' from an isolated point charge +q placed at 'O'.

Let $W_{\infty A}$ be the amount of workdone in moving unit positive charge from ∞ to point 'A' then potential at point 'A' will be

$$V_A = W_{\infty A} \quad \text{- - - (1)}$$

consider at any instant of time the positive charge is at point 'p' at a distance 'X' from 'O' then force experienced by unit charge is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2} \quad --(2)$$

If a test charge moves an infinitesimal displacement $pq = dx$, then small amount of workdone from p to q is

$$dW = \vec{E}(-dx) \quad --(3)$$

From (2) and (3) $\quad dW = -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\,dx \quad --(4)$

The negative sign indicates that the work is done againsts the electrostatic force.

$\therefore$ Amount of workdone from ∞ to A

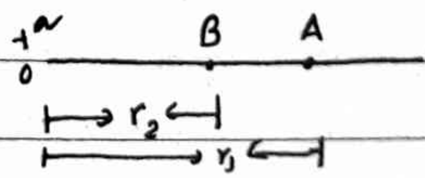
$$\int_{\infty}^{A} dW = -\frac{q}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{x^2}\,dx$$

$$\Rightarrow W_{\infty A} = -\frac{q}{4\pi\epsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow V_A = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$$

$\therefore$ potential at 'A', $V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad ----(5)$

If charge moved from distance r_1 to r_2 then there will be potential difference between them. If V_A be the potential at A and V_B be the potential at B then,



$$V_B - V_A = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$\Rightarrow$ potential difference $= \dfrac{\text{change in work done}}{\text{charge}}$

⇒ change in workdone = potential x charge

➡ The much amount of workdone is converted into potential energy. Therefore,

Potential energy = potential x charge.

* Relation betⁿ electric field and potential:

suppose the electric field at a point $\vec{r}$ due to charge distribution is $\vec{E}$ and electric potential at the same point is 'V'. suppose a point charge 'q' is displaced slightly from the point $\vec{r}$ to $\vec{r} + d\vec{r}$. Then force on the charge is

$$\vec{F} = q\vec{E} \quad \text{----(1)}$$

Then workdone by this force during the displacement $d\vec{r}$ is

$$dw = \vec{F}\cdot d\vec{r}$$

$$\Rightarrow dw = q\vec{E}\cdot d\vec{r}$$

The change in workdone, $dw = -q\vec{E}\cdot d\vec{r}$ --(2)

we have, Change in potential = $\frac{\text{change in workdone}}{\text{charge}}$

$$\Rightarrow dv = \frac{-q\vec{E}\cdot d\vec{r}}{q}$$

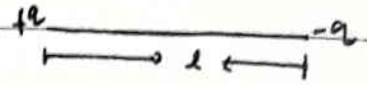
$$\Rightarrow dv = -\vec{E}\cdot d\vec{r}$$

$$\Rightarrow \vec{E} = -\frac{dv}{dr} \quad \text{----(3)}$$

This is the relation between $\vec{E}$ and V.

∴ It concludes that electric field strength is the negative gradient of potential.

* Electric dipole:

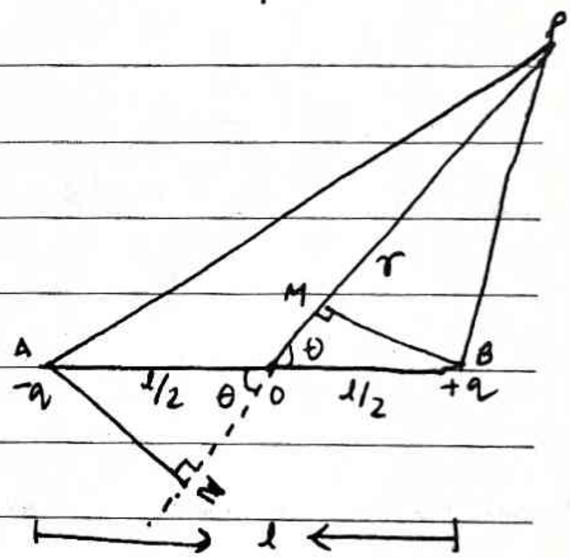


Two equal and opposite charges separated by a finite distance constitutes an electric dipole. The product of length of separation and magnitude of charge gives the dipole moment. It is denoted by $\vec{p}$ and

$$\vec{p} = \text{charge } (q) \times \text{dipole length } (l)$$

$$\Rightarrow \vec{p} = q\vec{l}$$

* Electric potential and field due to electric dipole:



Suppose AB is an electric dipole of length l consitutes by two equal and opposite charges -q and +q placed at A and B resp. We have to find the electric potential and field at point 'p' at a distance 'r' from the center of dipole 'o'.

Let $\angle BOP = \theta$, then $\angle AON = \theta$

since $AB = l$, then $AO = BO = l/2$

Now, Draw the normal BM on op ($BM \perp op$) and normal AN on producing op. ($AN \perp$ to producing op)

Now, from Δ OBM,

$$\cos\theta = \frac{OM}{OB}$$

$$\Rightarrow OM = OB\cos\theta$$

$$\Rightarrow OM = \frac{l}{2}\cos\theta \quad --- (1)$$

Date . .

Similarly from Δ OAN, $ON = \frac{l}{2}\cos\theta$ ---- (2)

Now $PM \approx PB$

$\therefore PB = OP - OM$ $[\because PM = OP - OM]$

$\Rightarrow PB = r - \frac{l}{2}\cos\theta$ ---- (3)

Again, $PN = ON + OP$

$PN = r + \frac{l}{2}\cos\theta$

but $PN \approx AP$

$\therefore AP = r + \frac{l}{2}\cos\theta$ ---- (4)

Then potential at P, due to $-q$ charge is

$$V_1 = -\frac{1}{4\pi\epsilon_0}\frac{q}{AP} \quad ---(5)$$

and potential at 'p' due to $+q$ charge at B is

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{BP} \quad ---(6)$$

Then total potential at 'p' due to dipole,

$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{AP} + \frac{1}{BP}\right] \quad ---(7)$$

Now, substituting AP and BP from (3) and (4) in (7) we get

$$V = \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r + l/_2\cos\theta} + \frac{1}{r - l/_2\cos\theta}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{-r + l/_2\cos\theta + r + l/_2\cos\theta}{r^2 - l^2/_4\cos^2\theta}\right]$$

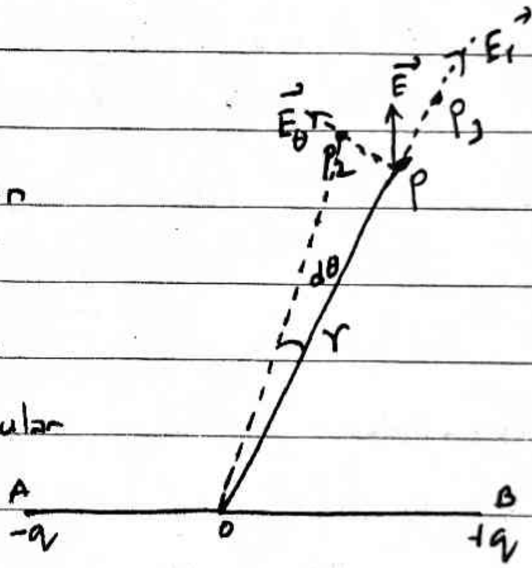
Date . .

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0} \frac{l\cos\theta}{r^2 - l^2/4 \cos^2\theta}$$

For small dipole, $r^2 >> \frac{l^2}{4}\cos^2\theta$

$$\therefore V = \frac{ql\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \quad \text{-----} (8) \qquad [\because ql = p]$$



The electric field at point 'p' can be obtained by resolving the field into two perpendicular components $\vec{E_r}$ along the op and $\vec{E_\theta}$ perpendicular to it.

In going from P to P_1 the angle does not change and distance changes by small amount 'dr'

$$\therefore PP_1 = dr \text{ ---} (9)$$

But in going from P to P_2 the angle changes by small amount '$d\theta$' while distance remains almost same from centre of dipole.

$$\therefore PP_2 = r\,d\theta \text{ ----} (10)$$

Now, electric field at P in PP_1 direction,

$$\vec{E_r} = -\frac{dV}{dP_1}$$

$$\Rightarrow \vec{E_r} = -\frac{dV}{dr}$$

$$= -\frac{d}{dr}\left[\frac{p\cos\theta}{4\pi\epsilon_0 r^2}\right]$$

$$\Rightarrow \vec{E_r} = -\frac{p\cos\theta}{4\pi\epsilon_0}\left[-\frac{2}{r^3}\right].$$

$$\Rightarrow \vec{E_r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3} \quad ----(11)$$

Again, electric field at p in PP_2 direction,

$$\vec{E_\theta} = -\frac{dv}{PP_2}$$

$$\Rightarrow \vec{E_\theta} = -\frac{d}{rd\theta}\left[\frac{p\cos\theta}{4\pi\epsilon_0 r^2}\right]$$

$$= -\frac{p}{4\pi\epsilon_0 r^3}\left[-\sin\theta\right]$$

$$\therefore \vec{E_\theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3} \quad ----(12)$$

$\therefore$ The resultant field at p,

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \sqrt{\left(\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\epsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + 1 - \cos^2\theta}$$

$$E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

when $\theta = 0$, $V = \dfrac{p}{4\pi\epsilon_0 r^2}$

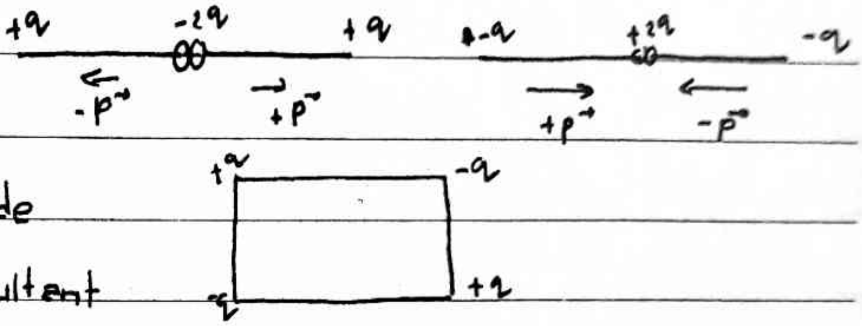
and $E = \dfrac{2p}{4\pi\epsilon_0 r^3}$

$\Big\}$ along the axis (position of end on) if $\theta = 90^{\circ}$ then,

$$V = 0$$ (perpendicular bisector)

and $$E = \frac{p}{4\pi\epsilon_0 r^3}$$ (Broad side on position)

* Quadrupole:

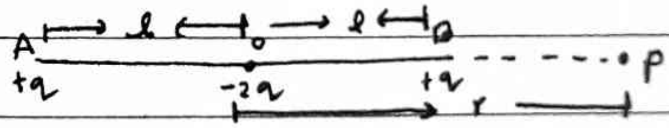


An electric quardrupole is two equal and opposite dipoles that doesnot coincide in space so that their resultant effects at distant points do not quite cancel. The various configurations of quadrupole system is shown in fig.

The quadrupole moment, $Q = 2ql^2$ ---(*)

where l is the distance between two charges in each dipole.

when point 'p' lies along the quadruple:



Let OA and OB be two dipoles each of length 'l'.

It may be noted that though the total charge on the system as a whole is zero, the potential and intensity is not zero.

Let us consider a point 'p' at a distance 'r' from the centre of quadrupole along the axis.

Let V_1, V_2 and V_3 be the potential at 'p' due to charges at A, O, and B resp. Then

$$V = V_1 + V_2 + V_3 \quad \text{---(1)}$$

Date . .

where, $V_1 = \frac{q}{4\pi\epsilon_0}\frac{1}{r+l}$ ---(1)

$V_2 = \frac{1}{4\pi\epsilon_0}\frac{(-2q)}{r}$ ---(3)

and $V_3 = \frac{1}{4\pi\epsilon_0}\frac{q}{r-l}$ ---(4)

$\therefore$ From (1), (2), (3) and (4)

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r+l} - \frac{2}{r} + \frac{1}{r-l}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2 - rl - 2r^2 + 2l^2 + r^2 + rl}{r(r^2-l^2)}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{2l^2}{r^3(1 - l^2/r^2)}$$

$$V = \frac{Q}{4\pi\epsilon_0 r^3} \quad \text{-----(5)} \qquad \left[\frac{l^2}{r^2} \text{ is neglected}\right]$$

And field,

$$E = -\frac{dV}{dr}$$

$$\Rightarrow E = -\frac{d}{dr}\left[\frac{Q}{4\pi\epsilon_0 r^3}\right]$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0}\frac{3}{r^4}$$

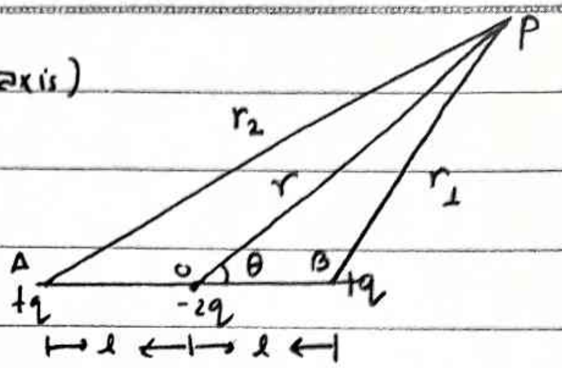
$$\Rightarrow E = \frac{3Q}{4\pi\epsilon_0 r^3} \quad \text{-----(6)}$$

Date . .

Case II: (when point p doesnot lies along axis)

The electric potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_1} - \frac{2q}{r} + \frac{q}{r_2}\right].$$



$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{2}{r} + \frac{1}{r_2}\right] \text{----}(7)$$

Let $\angle POB = \theta$ then

From Δ OPB, $r_1^2 = r^2 + l^2 - 2rl\cos\theta$

$$\Rightarrow r_1^2 = r^2\left[1 + \frac{l^2}{r^2} - \frac{2l\cos\theta}{r}\right].$$

$$\Rightarrow r_1 = r\left[1 + \frac{l^2}{r^2} - \frac{2l\cos\theta}{r}\right]^{1/2}$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right]^{-1/2}$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{1}{2}\left(-\frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right) + \frac{3}{8}\left(-\frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right)^2 + \cdots\cdots\right.$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{l\cos\theta}{r} - \frac{l^2}{2r^2} + \frac{3}{8}\times\frac{4l^2\cos^2\theta}{r^2} - \frac{3}{8}\times 4\frac{l\cos\theta}{r}\frac{l^2}{r^2} + \frac{l^4}{r^4} + \cdots\right.$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r} + \frac{l\cos\theta}{r^2} - \frac{l^2}{2r^3} + \frac{3l^2\cos^2\theta}{2r^3} - \frac{3}{2}\frac{l^3\cos\theta}{r^4} + \frac{l^4}{r^5} + \cdots\cdots$$

Retaining the terms upto r^3 and neglecting higher order terms

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r} + \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1) \text{ -----}(8)$$

similarly,

from Δ OPA, $r_2^2 = r^2 + l^2 + 2rl\cos\theta$ --(9)

and $$\frac{1}{r_2} = \frac{1}{r} - \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1) \text{ ---}(9)$$

From (7), (8) and (9)

$$v = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1) - \frac{2}{r} + \frac{1}{r} - \frac{l\cos\theta}{r^2} + \frac{l^2}{2r^3}(3\cos^2\theta - 1)\right]$$

$$\Rightarrow v = \frac{q}{4\pi\epsilon_0}\left[\frac{2l^2}{2r^3}(3\cos^2\theta - 1)\right]$$

$$\Rightarrow v = \frac{2ql^2}{4\pi\epsilon_0 r^3}\frac{(3\cos^2\theta - 1)}{2}$$

$$\Rightarrow V = \frac{Q}{4\pi\epsilon_0 r^3}\frac{(3\cos^2\theta - 1)}{2} \quad \cdots (10)$$

* capacitance:

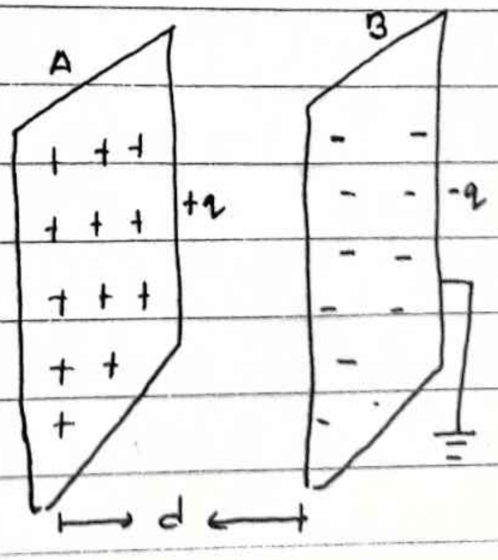
when a conductor given some charge, It is raised to some potential. If more and more charge is given, Its potential Increases accordingly. If 'q' be the charge given and 'V' be the potential then

$$q \propto V$$

$$\Rightarrow q = CV \quad \cdots (1)$$

'C' is constant called capacitance of the conductor.

* parallel plate capacitor:



The parallel plate capacitor consists of two conducting plates placed parallel to each other. Let 'A' be the area of each plate and 'd' be the distance between them. Let $\vec{E_0}$ be the field intensity.

If charge +q is given to plate 'A' the charge -q is dinduced on the left of plate B and +q on the right side of plate B. when plate B is earthed the +q charges of plate B is flows to earth. Hence plate 'A' is completely positive charged and plate B is completely negative charged. Hence electric field ($\vec{E_0}$) is developed between the plates.

If 'σ' be the surface charge density then total charge enclosed,

$q = \sigma A$ ---(1)

According to Gauss law

$$\text{flux}(\phi) = \frac{1}{\epsilon_0} \times \text{charge enclosed}(q)$$

$$\Rightarrow \vec{E_0} A = 1/\epsilon_0 \times \sigma A$$

$$\Rightarrow \vec{E_0} = \frac{1}{\epsilon_0}\sigma \quad \text{----(2)}$$

Also, the potential difference bet^n plates

$$V = \vec{E_0} d \quad \text{---(3)}$$

From (2) and (3)

$$V = \frac{\sigma}{\epsilon_0} d \quad \text{---(4)}$$

If 'c' be the capacitance of parallel plates then,

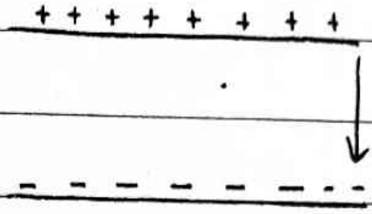
$$C = \frac{q}{V} \quad \text{----(5)}$$

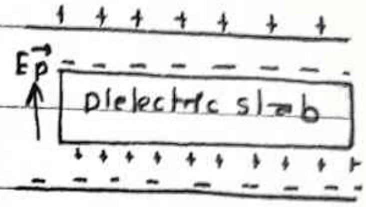
From (4) and (5) $\quad C = \frac{q\,\epsilon_0}{\sigma d}$

$$\Rightarrow C = \frac{q}{\frac{q}{A} d}\epsilon_0$$

$$\Rightarrow C = \frac{\epsilon_0 A}{d} \quad \text{---(*)}$$

* Dielectric constant:





consider a charged parallel plate capacitor as shown in fig. Let $\vec{E_0}$ be the electric field strength.

Now introduced a dielectric slab of non-polar molecules between the plates then due to induction, at the top of the slab negative charges -q and at the bottom surface positive charges +q appears. Due to these induced charges the electric field $\vec{E_p}$ is setup inside the slab which is opposite to $\vec{E_0}$.

Therefore resultant field, $\vec{E} = \vec{E_0} - \vec{E_p}$ .... (*)

The dielectric constant is defined as the ratio of electric field in absence of dielectric to the field in presence of dielectric and denoted by k.

Therefore, $k = \dfrac{\vec{E_0}}{\vec{E_0} - E_p}$

$\Rightarrow \quad k = \dfrac{\vec{E_0}}{\vec{E}}$ .... (* *)

i.e $k > 1$

The dielectric constant is also defined interms of capacitance. If V be the potential in absence of slab and V' be the potential in presence of slab and 'd' be the distance betn plates then,

$V = \vec{E_0}\, d$ ....(* * v)

and $V' = \vec{E}\, d$ ...(* * * v)

From (* * v) and (* * * v) $\quad \dfrac{V}{V'} = \dfrac{\vec{E_0}}{\vec{E}}$ .... (#)

From (* *) and (#) $\Rightarrow \quad V/V' = k$ .... (# #)

If 'c' be the capacitance in absence of dielectric and 'c'' be the capacitance in presence of slab then,

$$V = q/C \quad \cdots\cdots (\cdot)$$

and $$V' = q/C' \quad \cdots\cdots (\cdot\cdot)$$

substituting $(\cdot)$ and $(\cdot\cdot)$ in (# #) we get

$$k = \frac{q/C}{q/C'}$$

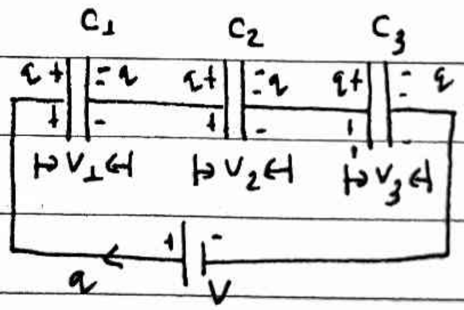
$$\Rightarrow k = \frac{C'}{C}$$

$$\Rightarrow C' = kC \quad \cdots\cdots (\cdot\cdot\cdot)$$

As $k > 1 \Rightarrow C' > C$

Therefore the significance of introducing the slab is to increase the capacitance of the capacitor.

* capacitor in series:



Let C_1, C_2 and C_3 be the capacitance of three capacitors connected in series with source of voltage 'v'.

If q be the charge suppled by the source then each capacitor has same amount of charge (q).

If V_1, V_2 and V_3 be the potential across the three capacitor resp

Then, $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$ and $V_3 = \frac{q}{C_3}$

but, $V = V_1 + V_2 + V_3$

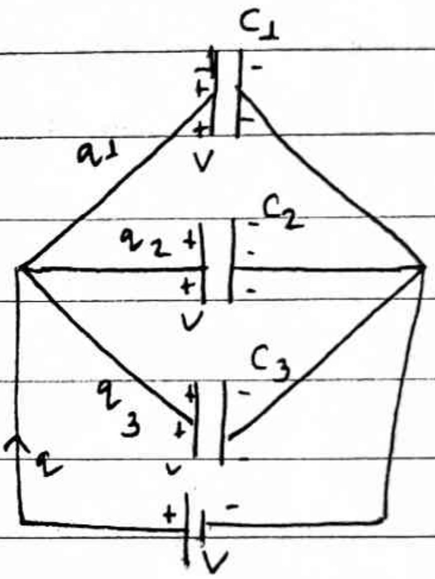
$$\Rightarrow V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\Rightarrow \frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

If C be the total capacitance of series combination then,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

* capacitors in parallel:



Let C_1, C_2 and C_3 be the capacitance of three capacitors connected in parallel with source of voltage 'V'.

As each capacitor is directly connected to the source each capacitor gains same voltage as of source (V) but charges on capacitor is different.

If q_1, q_2 and q_3 be the charges on the three capacitors and 'q' be the charge supplied by source

Then, $q = q_1 + q_2 + q_3$

but,

$$q_1 = VC_1 \quad , \quad q_2 = VC_2 \quad \text{and} \quad q_3 = VC_3$$

$$\therefore q = VC_1 + VC_2 + VC_3$$

$$\Rightarrow \frac{q}{V} = C_1 + C_2 + C_3$$

If C be the capacitance of parallel combination then

$$q/V = C$$

$$\therefore \quad C = C_1 + C_2 + C_3$$

* Energy stored in a capacitor:

Let us consider a capacitor having capacitance 'C'. Let V be the potential of the capacitor being connected to a battery. If 'q' be the charge on plate of capacitor then,

$$q = VC \quad \cdots (1)$$

suppose a battery supplies a charge dq to the capacitor at constant potential V. Then according to the definition of potential difference the small amount of workdone

$$dW = V\,dq$$

$$\Rightarrow dW = \frac{q}{C}\,dq \qquad [\text{From }(1)]$$

Total workdone to add charge 'q' on the capacitor is

$$\int dW = \int \frac{q}{C}\,dq$$

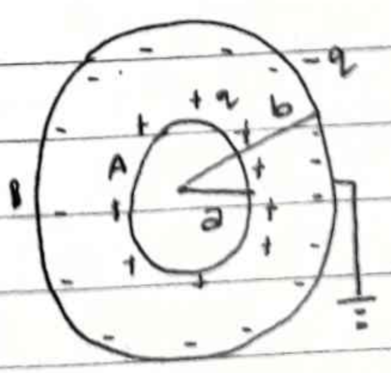
$$\Rightarrow W = \frac{1}{2}\frac{q^2}{C} \quad \cdots (*)$$

since the workdone is stored inside the capacitor in the form of electric potential energy

$$\therefore \text{Energy stored}, = \frac{1}{2}\frac{q^2}{C} \quad \cdots (*)(*)$$

* spherical capacitor:

spherical capacitor consists of two spherical shell (co-centric) of radius 'a' and 'b' resp. as shown in fig. when a positive charge +q is given to the inner spherical shell, it induces negative charge



Date . .

$-q$ on inner surface of outer shell and positive charge $+q$ on outer surface. If outer spherical shell is earthed then inner shell is completely positive and outer shell is completely negative.

The potential at any point on the surface of inner spherical shell, $V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{a}$ ---(1)

similarly, the potential at any point on the surface of outer shell, $V_B = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{b}$ ---(2)

Then total potential of capacitor,

$$V = V_A + V_B$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\Rightarrow \frac{q}{V} = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

If 'C' be the capacitance of the spherical shells then, $q/V = C$

$$\therefore \quad C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right) \quad ---(3)$$

If C' be the capacitance of inner spherical shell then,

From eq^n (1) $\frac{q}{V_A} = 4\pi\epsilon_0 a$

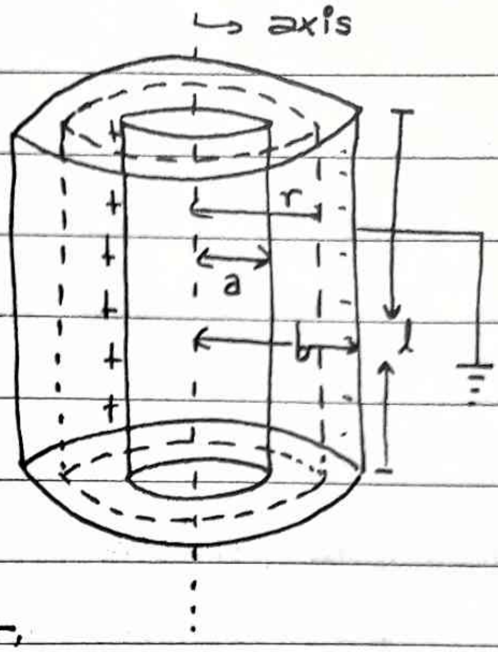
$$\Rightarrow C' = 4\pi\epsilon_0 a \quad ---(4)$$

comparing (3) and (4), $C > C'$

This shows that two spherical ~~capacitors~~ shells leads to the increase in capacitance.

* cylindrical capacitor-

It consists of two co-axial cylinders of same length. The inner cylinder of radius 'a' is given some positive charge and outer cylinder of radius 'b' is connected to earth.



Let 'l' be the length of the cylinder, and $\vec{E}$ be the electric field strength at a distance 'r' from the axis. Then draw a Gaussian surface in the form of cylinder of radius 'r'.

Then according to Gauss law,

$$\phi = \frac{1}{\epsilon_0} \times \text{charge enclosed } (q)$$

$$\Rightarrow \phi = q/\epsilon_0$$

$$\Rightarrow \vec{E}A = q/\epsilon_0$$

$$\Rightarrow \vec{E} = \frac{q}{A\epsilon_0} \quad ----(1)$$

Now the potential difference betn the cylinders

$$dV = -\vec{E}\cdot d\vec{r}$$

Integrating $\int dV = \int -E\,dr$

$$\Rightarrow V = \int -\frac{q}{A\epsilon_0}\,dr \quad \text{[From (1)]}$$

$$\Rightarrow V = \int_b^a -\frac{q}{2\pi r l \epsilon_0}\,dr \quad \text{[A = Area of Gaussian surface]}$$

$$\Rightarrow V = \frac{-q}{2\pi l \epsilon_0} \int_b^a \frac{1}{r}\,dr$$

$$\Rightarrow V = \frac{-q}{2\pi \epsilon_0 l} \Big[\log_e r\Big]_b^a$$

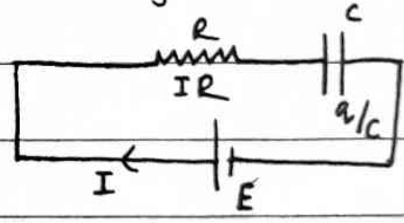
$$\Rightarrow V = \frac{-q}{2\pi \epsilon_0 l} \Big[\log_e a - \log_e b\Big]$$

$$\Rightarrow V = \frac{q}{2\pi \epsilon_0 l} \Big[\log_e\left(\frac{b}{a}\right)\Big]$$

$$\Rightarrow \frac{q}{V} = \frac{2\pi \epsilon_0 l}{\log_e(b/a)}$$

$$\Rightarrow C = \frac{2\pi \epsilon_0 l}{\log_e(b/a)} \quad ----(*)$$

* charging and discharging of capacitor through resistor:
(R-C circuit)



The resistor 'R' is connected to the capacitor 'C' and to the source of emf 'E' in series as shown in fig.

suppose the charge on the capacitor and current in the circuit are 'q' and 'I' resp. at time 't'.

Then potential drop on the capacitor $= q/C$ --(1)

and on the resistor $= IR$ --(1)

Also the charge deposited on the positive plate of capacitor in time dt is, $dq = I\,dt$ --(3)

$$\Rightarrow I = \frac{dq}{dt} \quad ---(4)$$

But from the circuit above, $E = IR + \frac{q}{C}$ ----(5)

Date . .

$\Rightarrow E - q/C = IR$

$\Rightarrow EC - q = RC\, I$

$\Rightarrow EC - q = RC \dfrac{dq}{dt}$

$\Rightarrow \dfrac{dq}{EC-q} = \dfrac{1}{RC}\, dt$ ----(6)

Let $EC - q = x$ --(7)

then differentiating (7) w.r.to x

$-dq = dx$ --(8)

substituting (7) and (8) in (6)

$\Rightarrow -\dfrac{dx}{x} = \dfrac{1}{RC}\, dt$

Integrating on both sides $\int \dfrac{dx}{x} = \int \dfrac{-1}{RC}\, dt$

$\Rightarrow \log_e x = -\dfrac{1}{RC}\, t + k$ (constant) ---(9)

From (7) and (9) $\log_e (EC - q) = \dfrac{-t}{RC} + k$ --(10)

when $t = 0$, $q = 0$

$\therefore$ From (10) $\log_e (EC) = k$ --(11)

substituting (11) in (10)

$\log_e (EC - q) = \dfrac{-t}{RC} + \log_e (EC)$

$\Rightarrow \log (EC - q) - \log (EC) = -t/RC$

$\Rightarrow \log \left(\dfrac{EC - q}{EC}\right) = -\dfrac{t}{RC}$

$\Rightarrow 1 - q/EC = e^{-t/RC}$

Date

$\Rightarrow q = EC(1 - e^{-t/RC})$ ---- (12)

If $t=0$, $q=0$

If $t=\infty$, $q = EC$ i.e maximum charge on capacitor,

$\therefore$ max. charge, $q_0 = EC$ --(13)

$\therefore$ From (12) and (13)

$q = q_0(1 - e^{-t/RC})$ ---(14)

This gives the relation of charging of capacitor.

The constant 'RC' has dimension of time and is called time constant of circuit. In one time constant, $t/RC = 1$

$\therefore q = q_0[1 - e^{-1}]$

$\Rightarrow q = 63\%$ of q_0 --- (15)

i.e At one time constant the charge on capacitor is 63% of maximum charge.

Discharging:

The resistor and capacitor are disconnected with the source after charging the capacitor fully. i.e If the plates of a charged capacitor are connected through a conducting wire, the capacitor gets discharged. Again there is a flow of charge through the wire and hence there is a current.

suppose the capacitor of capacitance 'C' has a charge q_0, at time $t=0$, the plates are connected to the resister R.

Let the charge on the capacitor be 'q' and current be I_0 at time 't' then, from circuit,

$$\frac{q}{C} + IR = 0$$

$$\Rightarrow q/C = -IR$$

$$\Rightarrow \frac{q}{C} = -R\frac{dq}{dt}$$

Date . .

$$\frac{dq}{q} = -\frac{1}{RC}\, dt$$

Integrating $\int \frac{dq}{q} = \int -\frac{1}{RC}\, dt$

$$\Rightarrow \log_e q = -\frac{t}{RC} + k \text{ (constant)} \quad \text{----(16)}$$

when $t=0$, $q = q_0$

$\therefore \log_e(q_0) = k$ ---(17)

substituting (17) in (16)

$$\log_e q = -\frac{t}{RC} + \log_e q_0$$

$$\Rightarrow \log_e(q/q_0) = -t/RC$$

$$\Rightarrow q = q_0 e^{-t/RC} \quad \text{----} (18)$$

In principle, discharging is completed only at $t = \infty$

The constant RC is time constant.

At one time constant $t/RC = 1$

then $q = q_0 e^{-1}$

$$\Rightarrow q = \frac{q_0}{e}$$

$$\Rightarrow q = \frac{q_0}{2.178}$$

$$\Rightarrow q = 0.37\, q_0$$

$$\Rightarrow q = 37\%\ \text{of}\ q_0$$

i.e 37% of discharging is completed in one time constant