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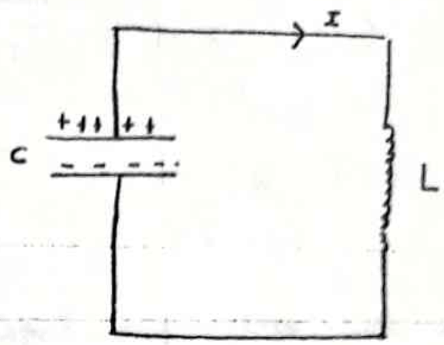
L-C oscillation: (undamped)



when a charged capacitor is connected across the coil, the current set up in the circuit buid up magnetic field i,e the electrical energy is changed into magnetic energy. when capacitor is fully discharged Its whole electrical energy converted into magnetic energy.

After then, the magnetic energy starts to decey which charges the capacitor in opposite direction by sending an opposite current i,e magnetic energy is changed into electrical energy. In this way, an oscillation is set up in between L and C and is called L-C oscillation.

In the absence of resistence the total energy remains constant. The frequency of oscillation i,e called resonant frequency and the oscillation is undamped.

Total energy, U = Electrical energy + Magnetic energy

$$\Rightarrow U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2} L I^2$$

$$\Rightarrow \frac{dU}{dt} = \frac{1}{2C} 2q \frac{dq}{dt} + \frac{1}{2} L\, 2I \frac{dI}{dt}$$

$$\Rightarrow 0 = \frac{q}{C}\frac{dq}{dt} + LI\frac{dI}{dt} \quad \text{---(1)}$$

But, $I = \dfrac{dq}{dt}$ --(2)

and $\dfrac{dI}{dt} = \dfrac{d^2q}{dt^2}$ --(3)

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From (1), (2) and (3)

$$\Rightarrow \frac{q}{C} I + L I \frac{d^2 q}{dt^2} = 0$$

$$\Rightarrow \frac{q}{C} + L \frac{d^2 q}{dt^2} = 0$$

$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{1}{LC} q = 0 \quad ---(4)$$

This equation is analogous to the eq^n of simple harmonic motion.

i,e $\frac{d^2 x}{dt^2} + \omega^2 x = 0 \quad --(5)$

comparing eq^n (4) and (5)

$$\omega^2 = \frac{1}{LC}$$

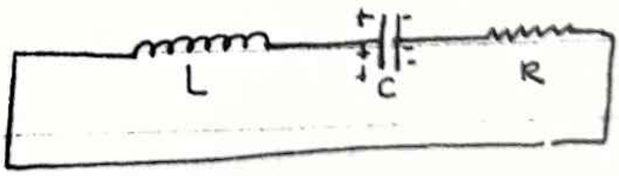
$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} \quad ---(6)$$

This is called resonant or undamped frequency.

* L-C oscillation (Damped)



when the charged capacitor is connected with the resistor and inductor as shown in fig. The energy is dissipated at the rate of I^2R and then the frequency of oscillation is continuously decreased or damped.

we have, total energy, U = Electrical energy + Magnetic energy

$$\Rightarrow U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$$

$$\Rightarrow \frac{dU}{dt} = \frac{1}{2}\frac{2q}{C}\frac{dq}{dt} + \frac{1}{2}L\,2I\frac{dI}{dt}$$

$$\Rightarrow -I^2R = \frac{q}{C}\frac{dq}{dt} + LI\frac{dI}{dt}$$

$$\Rightarrow -I^2R = \frac{q}{C}I + LI\frac{d^2q}{dt^2}$$

$$\Rightarrow -IR = \frac{q}{C} + L\frac{d^2q}{dt^2}$$

$$\Rightarrow -\frac{dq}{dt}R = \frac{q}{C} + L\frac{d^2q}{dt^2}$$

$$\Rightarrow -\frac{dq}{dt}\frac{R}{L} = \frac{q}{LC} + \frac{d^2q}{dt^2}$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0 \quad \text{----} (*)$$

It is in the form of eqn of damped simple harmonic motion

i.e. $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$ ---(**) $\left[\because \frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega^2x = 0\right]$

comparing (*) and (**)

$$\frac{b}{m} = \frac{R}{L}$$

$$\frac{k}{m} = \frac{1}{LC}$$

then frequency of damped harmonic oscillation is given by

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$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right) - \left(\frac{b}{2m}\right)^2} \qquad \left[\because f = \frac{1}{2\pi}\sqrt{\omega^2 - \left(\frac{r}{2}\right)^2}\right]$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \text{---}(\text{xxx})$$

This is the damped frequency.

If $R=0$ then, $f = f_0$

* Displacement current:

In the case of charging and discharging of the capacitor, the electric field betn the plates of capacitor changes continuously. Due to this change in electric field, the flux also changes which causes the current between the plates. This current due to the change in electric field is called displacement current and is denoted by I_d.

Therefore, the current in the circuit is always the sum of current due to a flow of charge and due to the change in electric field.

we have from Gauss law of electrostatics,

$$\text{flux}(\phi) = \frac{1}{\epsilon_0} \times \text{charge enclosed}(q)$$

$$\Rightarrow EA = \frac{q}{\epsilon_0}$$

$\therefore$ Field between the plates of capacitor,

$$E = \frac{q}{A\epsilon_0} \quad \text{----}(1)$$

$$\Rightarrow q = A\epsilon_0 E.$$

$$\Rightarrow dq/dt = A\epsilon_0\, dE/dt$$

$$\Rightarrow I_d = \epsilon_0 A \frac{dE}{dt} \quad ---(2)$$

$$\Rightarrow I_d = \epsilon_0 \frac{d}{dt}(EA)$$

$$\Rightarrow I_d = \epsilon_0 \frac{d\phi}{dt} \quad ----(3)$$

Eqn (2) can also be written as $\frac{I_d}{A} = \epsilon_0 \frac{dE}{dt}$

$$\Rightarrow J_d = \epsilon_0 \frac{dE}{dt} \quad ---(4)$$

This eqn represents the displacement current density.

Representation of integral:

① Line integral of any vector $\vec{A}$ is $\oint \vec{A} \cdot \vec{dl} \quad ---(1)$

② surface integral of a vector $\vec{A}$ is

$$\int_s \vec{A} \cdot \vec{da} \text{ or } \iint \vec{A} \cdot \vec{da} \quad ---(2)$$

③ volume integral of a vector $\vec{A}$ is

$$\int_v \vec{A} \cdot \vec{dv} \text{ or } \iiint \vec{A} \cdot \vec{dv} \quad --(3)$$

The surface integral of any vector gives its flux.
For example if $\vec{A}$ vector is the electric field vector then, from eqn (2) replace $\vec{A}$ by $\vec{E}$ then

$$\int_s \vec{E} \cdot \vec{da} = \vec{E} \int_s \vec{da} = EA$$

$$= \phi \quad \text{(i.e electric flux)}$$

If $\vec{A}$ be the magnetic field vector ($\vec{B}$) then,

$$\int_S \vec{B} \cdot d\vec{a}$$

$$= B^{\circ} \int_S da$$

$$= BA$$

$$= \phi \quad \text{(magnetic flux)}.$$

Gauss divergence theorem -

It states that the surface integral of a vector is equal to volume integral of divergence of that vector.

If $\vec{A}$ be the vector then the statement can be written as

$$\int_S \vec{A} \cdot d\vec{a} = \int_V (\operatorname{div} \vec{A})\, dV$$

where $\operatorname{div} \vec{A} = \vec{\nabla} \cdot \vec{A}$

and $\vec{\nabla}$ is the three dimensional operator.

$$\vec{\nabla} = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$$

* Stoke's theorem -

It states that the line integral of a vector is equal to the surface integral of curl of that vector.

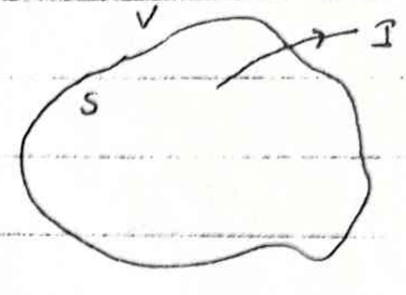
If $\vec{A}$ be the vector then stoke's theorem can be written as

$$\int_l \vec{A} \cdot d\vec{l} = \int_S (\operatorname{curl} \vec{A})\, da$$

where $\operatorname{curl} \vec{A} = \vec{\nabla} \times \vec{A}$

$\vec{\nabla}$ is the three dimensional operator.

* Equation of continuity -



Let us consider a surface 's' enclosing volume 'v'. Let 'ds' be the small element of this surface. Further let $\vec{J}$ be the current density at a point on the surface element Then the current leaving the volume 'V' bounded by the surface 'ds' is given by

$$I = \int_S \vec{J} \cdot d\vec{s} \quad ---(1)$$

As some charge is leaving the volume, correspondingly the same amount of charge diminishes with in that volume. we can express this fact as

$$I = -\frac{dq}{dt} \quad ----(2)$$

If ρ be the volume charge density (i,e charge per unit volume) then,

$$q_\rho = \int_V \rho \, dv \quad ---(3)$$

From (2) and (3) $I = -\frac{\partial}{\partial t} \int_V \rho \, dv \quad ---(4)$

eqn (4) becomes in form of partial derivative as ρ is the function of both position and time, and volume is fixed the time derivative operates only on the function ρ.

Now, from (1) and (4)

$$\int_S \vec{J} \cdot d\vec{s} = -\frac{\partial}{\partial t} \int_V \rho \, dv \quad --(5)$$

using Gauss divergence theorem,

$$\int_S \vec{J} \cdot d\vec{s} = \int_V (\mathrm{div}\,\vec{J}) \, dV \quad --(6)$$

From (5) and (6)

$$\int_V (\text{div}\, \vec{J})\, dV = -\frac{\partial}{\partial t}\int_V \rho\, dV$$

$$\Rightarrow \text{div}\, \vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{--- } (*)$$

$$\Rightarrow \vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

This is called equation of continuity.

From eqⁿ (*) we can conclude that charge can not be flow away from the given volume without diminishing the amount of charge existing within the volume.

* Maxwell's electromagnetic wave equations:

(1) 1st equation:

From Gauss law of electrostatics,

we have, $\phi = \frac{1}{\epsilon_0} \times$ charged enclosed.

$$\Rightarrow \vec{E}A = \frac{1}{\epsilon_0} q$$

$$\Rightarrow \int_S \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0}\int_V \rho\, dV \quad \text{--- } (1)$$

Now, using Gauss divergence theorem,

$$\int_S \vec{E}\cdot d\vec{a} = \int_V (\text{div}\, \vec{E})\, dV \quad \text{--- } (2)$$

From (1) and (2)

$$\int_V (\text{div}\, \vec{E})\, dV = \frac{1}{\epsilon_0}\int_V \rho\, dV$$

$\Rightarrow \text{div}\,\vec{E} = \rho/\epsilon_0$

$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ ----(3) [Differential form]

(2) 2nd Equation:

The magnetic flux through a closed surface is given by the surface integral of magnetic field strength i.e

$$\text{magnetic flux} = \int_S \vec{B}\cdot d\vec{a} \quad \text{---(1)}$$

but magnetic flux through a closed path is always zero. This means monopole of magnetic does not exists.

$$\therefore \int_S \vec{B}\cdot d\vec{a} = 0 \quad \text{----(2)}$$

using Gauss divergence theorem, $\int_S \vec{B}\cdot d\vec{a} = \int_V (\text{div}\,\vec{B})\,dV$ (3)

From (2) and (3) $\int_V (\text{div}\,\vec{B})\,dV = 0$

$\Rightarrow \text{div}\,\vec{B} = 0$

$\Rightarrow \vec{\nabla}\cdot\vec{B} = 0$ ---- (4) [Differential form]

(3) 3rd ~~4th~~ equation -

~~From the faraday's law of electromagnetic induction, time integral of magnetic field round a closed~~

From the faraday's law of electromagnetic induction, the induced emf is equal to the rate of change of magnetic flux.

i.e $e = -\frac{d\phi_m}{dt}$ ---(1)

where ϕ_m = magnetic flux

As the surface integral gives the flux,

$$\phi_m = \int_S \vec{B} \cdot d\vec{a} \quad \text{---} (1)$$

also, emf is the amount of workdone to move 1 coulomb charge round a closed path.

$$\therefore \quad e = \text{force} \times \text{displacement}$$

$$e = \int_l E \, dl \quad \text{----} (3)$$

ie line integral of electric field gives emf.

From (1), (2) and (3) [substituting (2) and (3) in (1)]

$$\int_l \vec{E} \, dl = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a} \quad \text{---} (4)$$

$\Rightarrow$ using ~~Gauss divergence~~ stoke's theorem,

$$\int_l E \, dl = \int_S (\text{curl } \vec{E}) \, da \quad \text{--} (5)$$

From (4) and (5)

$$\int_S (\text{curl } \vec{E}) \, da = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{a}$$

$$\Rightarrow \quad \text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{----} (6)$$

(4) 4th equation:-

we have from Ampere's law, line integral of magnetic field round a closed path is equal to μ_0 times current enclosed:

$$\int_l \vec{B}\cdot d\vec{l} = \mu_0(I + I_d) \quad ----(1)$$

But, current due to flow of charge and current due to change in field can be expressed as

$$I = \int_S \vec{J}\cdot d\vec{a} \quad --(2)$$

$$I_d = \int_S \vec{J_d}\cdot d\vec{a} \quad ---(3)$$

substituting (2) and (3) in (1)

$$\int_l \vec{B}\cdot d\vec{l} = \mu_0 \int_S (\vec{J} + \vec{J_d})\cdot d\vec{a} \quad --(4)$$

Now, using stoke's law,

$$\int_l \vec{B}\cdot d\vec{l} = \int_S (\text{curl}\,\vec{B})\, d\vec{a} \quad --(5)$$

From (4) and (5)

$$\int_S (\text{curl}\,\vec{B})\, da = \mu_0 \int_S (\vec{J} + \vec{J_d})\cdot d\vec{a}$$

$$\Rightarrow \text{curl}\,\vec{B} = \mu_0[\vec{J} + \vec{J_d}]$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\vec{J_d} \quad ---(6)$$

Also, $\vec{J_d} = \epsilon_0 \frac{d\vec{E}}{dt}$ --(7)

From (6) and (7)

$$\vec{\nabla}\wedge\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{d\vec{E}}{dt} \quad ---(8)$$

∴ Maxwell's eqns are

(1) $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

(2) $\vec{\nabla}\cdot\vec{B} = 0$

(3) $\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{\partial t}$

In free space, there is no current density and charge density then the eqns becomes

(1) $\vec{\nabla} \cdot \vec{E} = 0$

(2) $\vec{\nabla} \cdot \vec{B} = 0$

(3) $\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

* speed of electromagnetic wave in free space:

we have from maxwell's third eqn

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---(1)}$$

Now taking curl on both sides we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) \quad \text{--(2)}$$

using vector triple product rule,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = (\vec{\nabla} \cdot \vec{E}) \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \vec{E} \quad \text{--(3)}$$

and from Maxwell's Fouith equation.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{\partial t} \quad \text{--(4)}$$

substituting (3) and (4) in (2) we get

$$(\vec{\nabla} \cdot \vec{E}) \vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla}) \vec{E} = -\frac{\partial}{\partial t} \left[\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$$

but for free space $\vec{\nabla} \cdot \vec{E} = 0$, and $\vec{J} = 0$

$$-(\vec{\nabla}\cdot\vec{\nabla})E = -\frac{\partial}{\partial t}\left[\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right]$$

$$\Rightarrow \nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad ---(5)$$

This represents the electromagnetic wave equation of electric vectors similarly the wave eqn in magnetic vector form.

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad ---(6)$$

eqn (5) and (6) is analogous to the equation of wave equation,

$$\nabla^2 x = \frac{1}{c^2} \frac{\partial^2 x}{\partial t^2} \quad ---(7)$$

Comparing eqn (5) and (7) or (6) and (7)

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\Rightarrow c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$\Rightarrow c = 3 \times 10^8 \ m/s$$

Therefore electromagnetic wave travels with speed of light.

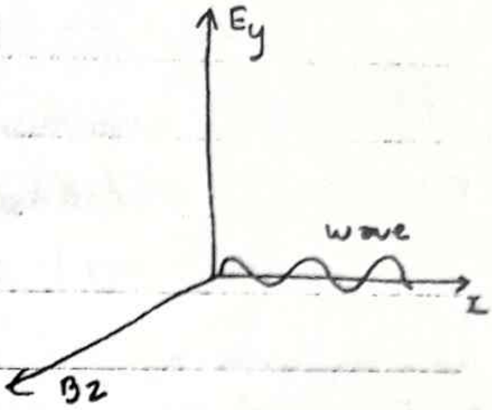
* To show $E_0/B_0 = C$

we have the electromagnetic wave equation in ~~in~~ electric vector is

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad ---(1)$$

and electromagnetic wave eqn in magnetic vector form is

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad \text{---(2)}$$



Let the wave travel in x- direction, electric field is along y direction and magnetic field is along z-direction. Then eqn (1) and (2) can be expressed in one dimensional case as

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{---(3)}$$

and
$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2} \quad \text{---(4)}$$

The solution of eqn (3) and (4) is

$$E_y = E_0 \sin(\omega t - kx) \quad \text{--(5)}$$
$$B_z = B_0 \sin(\omega t - kx) \quad \text{---(6)}$$

Now,
From Maxwell's third eqn.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times (\vec{i}E_x + \vec{j}E_y + \vec{k}E_z) = -\frac{\partial}{\partial t}\left[\vec{i}B_x + \vec{j}B_y + \vec{k}B_z\right]$$

but $E_x = E_z = B_x = B_y = 0$

$$\therefore \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \vec{j}E_y = -\frac{\partial}{\partial t}\left[\vec{k}B_z\right]$$

$$\Rightarrow \vec{k}\frac{\partial E_y}{\partial x} + 0 - \vec{i}\frac{\partial E_y}{\partial z} = -\vec{k}\frac{\partial B_z}{\partial t}$$

Now, comparing the coeffici of $\vec{k}$

we got.

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad ----(7)$$

substituting (5) and (6) in (7)

$$\frac{\partial}{\partial x}\left[E_0 \sin(\omega t - kx)\right] = \frac{-\partial}{\partial t}\left[B_0 \sin(\omega t - kx)\right]$$

$$\Rightarrow E_0 \cos(\omega t - kx)(-k) = -B_0 \cos(\omega t - kx)(\omega)$$

$$\Rightarrow -E_0 k \cos(\omega t - kx) = -B_0 \omega \cos(\cancel{kx}\ \omega t - kx)$$

$$\Rightarrow E_0 k = B_0 \omega$$

$$\Rightarrow \frac{E_0}{B_0} = \frac{\omega}{k}$$

$$\Rightarrow \frac{E_0}{B_0} = \frac{2\pi f}{2\pi/\lambda}$$

$$\Rightarrow \frac{E_0}{B_0} = f\lambda$$

$$\Rightarrow \frac{E_0}{B_0} = c \quad ----(8)$$

* poynting vector:

we can describe the energy transfer interms of the rate of energy flow per unit area or power per unit area, by a vector 's' is called poynting vector.

Let us calculate the energy 'du' passing during time 'dt' through a unit area perpendicular to the direction of propagation of the wave. In a time dt, wave front moves a distance

$d = c\,dt \quad ----(1)$

If 'u' be the electrical energy density then,

$$du = u\,c\,dt \quad ---(2)$$

the value of $u = \frac{1}{2}\epsilon_0 E^2$ ---(3)

For electromagnetic wave electrical energy density is equal to magnetic energy density —

$$\therefore u = \frac{1}{2\mu_0} B^2 \text{ ---(4)}$$

From (3) and (4) $\frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2$

$$\Rightarrow \frac{B}{\sqrt{\mu_0}} = \sqrt{\epsilon_0}\, E \text{ ---(5)} \Rightarrow B = \sqrt{\mu_0\epsilon_0}\, E \text{ ---(6)}$$

From eq (2) and (3)

$$dU = \frac{1}{2}\epsilon_0 E^2 c\, dt \text{ ---(6)}$$

From (6) and (6) $dU = \frac{1}{2}\epsilon_0 \frac{B}{\sqrt{\mu_0\epsilon_0}} E\, c\, dt$ $\left[\because E = \frac{B}{\sqrt{\mu_0\epsilon_0}}\right]$

$$\Rightarrow dU = \frac{1}{2}\epsilon_0 c^2 EB\, dt \text{ ---(7)} \quad \left[\frac{1}{\sqrt{\mu_0\epsilon_0}} = c\right]$$

Or From (2) and (4)

$$dU = \frac{1}{2\mu_0} B^2 c\, dt \text{ ---(8)}$$

$$= \frac{1}{2\mu_0} B\, B\, c\, dt$$

$$= \frac{1}{2\mu_0}\sqrt{\mu_0\epsilon_0}\, E\, B\, c\, dt$$

$$dU = \frac{1}{2\mu_0} EB\, dt \text{ ---(8)} \quad \left[\because \sqrt{\mu_0\epsilon_0} = \frac{1}{c}\right]$$

From (2) and (8) we say that, the term EB represents the magnitude of energy flux density vector. This corresponding vector 'S' is then,

$$\vec{S} = \frac{1}{2\mu_0}(\vec{E} \times \vec{B}) \text{ ----(9)} \quad [\text{From } (8)] \qquad \left(\frac{1}{A}\frac{dU}{dt} = s\right)$$

$$\text{or, } \quad \vec{S} = \frac{\epsilon_0 c^2}{2}(\vec{E} \times \vec{B}) \text{ ---(10)} \quad [\text{From } (7)]$$

'S' is the poynting vector has dimension of ~~per~~ energy perunit time perunit area.

Intensity is the average power per unit area.

$$I = \frac{P_{avg}}{A}$$

$$\Rightarrow I = \frac{1}{2\mu_0 c} E_0^2$$

$$I \propto E_0^2$$

* Radiation pressure and momentum-

Electromagnetic waves transport energy. Therefore, they carry momentum and exhibit a force in the direction of propagation. The momentum is a property of the field alone and it is not associated with any moving mass. The momentum density, that is momentum dp per volume dv is given by

$$\frac{dp}{dv} = \frac{1}{2\mu_0 c^2} EB$$

$$\Rightarrow \frac{dp}{dv} = \frac{S}{c^2} \text{ --- (11)}$$

The volume dv occupied by an electronmagnetic wave that passes through an area 'A' in time dt is

$$\Rightarrow \quad dv = A c\, dt \quad \text{---(2)}$$

Therefore the momentum flow rate per unit area is

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{C} \quad \text{--(3)} \qquad [\because \text{substituting (2) in (1)}]$$

This is the momentum transfered per unit surface area per unit time. The average rate of momentum transfer per unit area is given by,

$$\frac{1}{A}\frac{dp}{dt} = \frac{S_{avg}}{C}$$

$$\frac{1}{A}\frac{dp}{dt} = \frac{I}{C} \quad \text{---(4)}$$

This momentum is responsible for the radiation pressure.

* Electrical Energy density and magnetic energy density.

we have, electrical energy density, $U_E = \frac{1}{2}\epsilon_0 E_0^2$ ---(1)

magnetic energy density, $U_B = \frac{1}{2\mu_0} B_0^2$ .. (2)

Also, from electromagnetism, $\frac{E_0}{B_0} = C$

$$\Rightarrow E_0 = B_0 C \quad \text{---(3)}$$

And, $\frac{1}{\sqrt{\mu_0 \epsilon_0}} = C$

$$\Rightarrow c^2 = 1/\mu_0\epsilon_0 \quad \text{....(4)}$$

From (1) and (3)

$$U_E = \frac{1}{2}\epsilon_0 B_0^2 C^2 \quad \text{---(5)}$$

From (4) and (5)

$$U_E = \frac{1}{2} \epsilon_0 B_0^2 \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow U_E = \frac{1}{2\mu_0} B^2$$

$$\Rightarrow U_E = U_B$$

i,e electrical energy density is equal to magnetic energy density.