Q1. A wave of frequency 500 cycles per sec has velocity (phase) of 350 m/s. How apart are two points 60° out of phase?

⇒ we have frequency (f) = 500 cycles per

phase velocity (u) = 350 m/s

phase difference (ϕ) = 60°

$= \frac{\pi}{180} \times 60$

$= \frac{\pi}{3}$ radian

we have phase velocity = freq. x wavelength

$\Rightarrow u = f\lambda$

$\Rightarrow \lambda = \frac{u}{f}$

$\Rightarrow \lambda = \frac{350}{500}$

$\lambda = 0.7\, m$

Now, phase diff $(\phi) = \frac{2\pi}{\lambda} \times$ path diff (x)

$\Rightarrow \phi = \frac{2\pi}{\lambda} x$

$$\Rightarrow x = \frac{\phi}{2\pi} \lambda$$

$$= \frac{\pi}{3 \times 2\pi} \; 0.7$$

$$x = 0.12\,m$$

Q2. A wave of frequency 500 cycle per sec has phase velocity 350 m/s. what is the phase difference between two displacements of a certain point at time 0.001 sec apart?

$\Rightarrow$ Frequency $(f) = 500$ cycles per sec

phase velocity $(u) = 350$ m/s.

$\phi = ?$

time, $t = 0.001$ sec.

Now,

phase velocity = freq. x wavelength

$$u = f\lambda$$

$$\Rightarrow \lambda = \frac{u}{f}$$

$$\Rightarrow \lambda = \frac{350}{500}$$

$\Rightarrow \lambda = 0.7\ m$

Now, phase diff (ϕ) = $\frac{2\pi}{\lambda} \times$ path diff (x)

$$\Rightarrow \phi = \frac{2\pi}{0.7} \times x$$

but distance covered at time 0.001 sec

is x = velocity $\times$ time ~~time~~

$$x = 350\ \cancel{sec}\ \times 0.001$$

$$x = \cancel{0.001} = 0.35\ m$$

$$\therefore \phi = \frac{2\pi}{0.7} \times \frac{0.350}{\cancel{0.001}}$$

$$\Rightarrow \phi = \pi \text{ radian.}$$

Q.3. The equation of a transverse wave travelling in a rope is given by $y = -10 \sin [0.01x - 2t]$. Find the amplitude, frequency, velocity and wavelength

$\Rightarrow$ The given equation is

$$y = -10 \sin [0.01x - 2t]$$

$$\Rightarrow y = 10 \sin [2t - 0.01x]$$

comparing eqn with

$y = A \sin[\omega t - kx]$ we get

Amplitude $(A) = 10$

Angular frequency $\omega = 2$ rad s^{-1}

$\Rightarrow 2\pi f = 2$

$\Rightarrow f = \frac{1}{\pi}$ cycle per sec.

Again, wave number $k = 0.01$

$\Rightarrow \frac{2\pi}{\lambda} = 0.01$

$\Rightarrow \lambda = \frac{2\pi}{0.01}$

$\Rightarrow \lambda = 628.57$ m

And velocity of wave = frequency x wavelength

$\Rightarrow u = f\lambda$

$\Rightarrow u = \frac{1}{\pi} \times \frac{2\pi}{0.01}$

$\Rightarrow u = 200$ m/s

Q 4. calculate the frequency and maximum particle velocity due to wave represented by $y(x,t) = 0.03 \sin(60\pi t - 0.03\pi x)$

The values of x and y are in cm.

⇒ we have given,

$$y(x,t) = 0.03 \sin(60\pi t - 0.03\pi x)$$

comparing with eq^n

$$y = A \sin(\omega t - kx) \text{ we get}$$

$A = 0.03$ cm

$\omega = 60\pi$

and $k = 0.03\pi$

but maximum particle velocity is $A\omega$

$$\therefore V_{max} = A\omega$$

$$= 0.03 \times 60\pi$$

$$V_{max} = 5.6 \text{ cm/sec}$$

Again,

$$k = \frac{2\pi}{\lambda}$$

$$\Rightarrow 0.03\pi = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 2/0.03$$

$$\Rightarrow \lambda = 66.66 \text{ cm}$$

Then, again, $\omega = 2\pi f$

$$\Rightarrow 60\pi = 2\pi f$$

$\Rightarrow f = 30$ cycle per sec

wave velocity $(u) = f\lambda$

$= 30 \times 66.66$

$u = 1999.8$ cm/sec

Q5. A stretched string has a linear mass density of 50 g/cm and a tension of 10 N. A wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz is travelling in -ve x direction. write the wave equation with appropriate units.

$\Rightarrow$ linear mass density $(\mu) = 5$ g/cm

$\Rightarrow \mu = \frac{5 \times 10^{-3}}{10^{-2}}$ kg/m

$\Rightarrow \mu = 0.5$ kg/m

Tension $(T) = 10$ N

amplitude $(A) = 0.12$ mm

$A = 0.12 \times 10^{-3}$ m

$A = 0.00012$ m --- (1)

frequency $= 100$ Hz

but the velocity of traverse wave along stretched string

$$u = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow u = \sqrt{\frac{10}{0.5}}\ m/sec$$

$$\Rightarrow u = \cancel{5\sqrt{2}}\ \sqrt{20}\ m/s$$

Also, u = frequency x wavelength

$$\Rightarrow u = 100 \times \lambda$$

$$\Rightarrow \lambda = \frac{\sqrt{20}}{100}$$

$$\Rightarrow \lambda = \frac{2\sqrt{5}}{100}$$

$$\Rightarrow \lambda = \frac{\sqrt{5}}{50}\ m$$

Now, $k = \frac{2\pi}{\lambda}$

$$\Rightarrow k = \frac{2\pi \times 50}{\sqrt{5}}$$

$$\Rightarrow k = \frac{100\pi}{\sqrt{5}} \quad \text{---} (2)$$

and $\omega = 2\pi f$

$$\Rightarrow \omega = 2\pi \times 100$$

$$\Rightarrow \omega = 200\pi \quad \text{---} (3)$$

If wave travelling in -ve direction then

$y = A \sin(\omega t + kx)$ --- (4)

Now, substituting (1) (2) and (3) in (4)

$$y = 0.12 \times 10^{-3} \sin\left[200\pi t + \frac{100}{\sqrt{5}}\pi x\right]$$

Q.6. A string of mass 2 g for unit length carries progressive wave of amplitude 1.5 cm, frequency $60\ s^{-1}$ and speed 20 m/s. calculate (a) energy per metre length of the wire? (b) The rate of energy propagation in the wave.

⇒ mass of string (m) = 2g = 2×10^{-3} kg

amplitude of wave (A) = 1.5 cm

$= 1.5 \times 10^{-2}$ m

frequency (f) = 60 /sec.

velocity (u) = 20 m/s.

Now, the energy per unit length of wire

$$= \frac{1}{2} m\omega^2 A^2$$

$$= \frac{1}{2} m (2\pi f)^2 A^2$$

$= \frac{1}{2} \times 2 \times 10^{-3} \times (2 \times \pi \times 60)^2 \times (1.5 \times 10^{-3})^2$

$= 0.031$ J

Now, the rate of transmission of energy in the wave for given length

$= \frac{1}{2} m \omega^2 A^2 u$

$= 0.031 \times 20$ J/sec

$= 0.63$ J/sec

Q 3. Two waves are simultaneously passing through a string. The equations of the waves are given by

$y_1 = A_1 \sin k (x - vt)$

$y_2 = A_2 \sin k (x - vt + x_0)$

where the waves number $k = 6.28\ cm^{-1}$ and $x_0 = 1.50$ cm. The amplitudes are $A_1 = 5$ mm and $A_2 = 4$ mm. Find the phase difference between the waves and amplitude of resultant wave.

⇒ The phase of the first wave $= k(x - vt)$

and phase of 2nd wave, $= k(x - vt + x_0)$

The phase difference is

$$\delta = k(x - vt + x_0) - k(x - vt)$$

$$\Rightarrow \delta = kx_0$$

$$\Rightarrow \delta = 6.28 \times 1.5$$

$$= 2\pi \times 1.5$$

$$\delta = 3\pi \qquad [(2n+1)\pi]$$

The waves satisfy the condition of destructive interference. The amplitude of the resultant wave is then given by

$$|A_1 - A_2| = |5 - 4|$$

$$= 1 \text{ mm}$$

Q8. Two travelling waves of equal amplitudes and equal frequencies moves in opposite directions along a string. They interfere to produce a standing wave having the equation $y = A \cos kx \sin \omega t$ in which $A = 1$ mm, $k = 1.57$ cm^{-1} and $\omega = 78.5$ s^{-1}.

(a) Find the velocity of the travelling wave

(b) node closest to the origin, $x > 0$.

(c) antinode closest to the origin, $x > 0$

(i) Find the amplitude of the particle at $x = 2.33$ cm.

$\Rightarrow$ The velocity of either of the wave is $u = \frac{\omega}{k}$ $\left[\because \omega = 2\pi f, \ k = 2\pi/\lambda \right]$

$\Rightarrow u = \frac{78.5}{1.57}$

$\Rightarrow u = 50 \text{ cms}^{-1}$

For node $\cos kx = 0$

For smallest positive x,

$\cos kx = \cos \frac{\pi}{2}$

$\Rightarrow kx = \pi/2$

$\Rightarrow x = \pi/2k = \frac{\pi}{2.2\pi}\lambda = \frac{\lambda}{4}$

$\Rightarrow x = \frac{3.14}{2 \times 1.57}$

$\Rightarrow x = 1$ cm

Also, for anti node, $|\cos kx| = 1$

$kx = |\cos \pi|$

$\Rightarrow kx = \pi$

$\Rightarrow x = \frac{\pi}{k} = \frac{\pi}{2\pi}\lambda = \frac{\lambda}{2}$ $\Rightarrow x = 2$ cm now, amplitude of particle at $x = 2.23\,1s$

$A \cos kx$

$= 1 \times \cos(1.57 \times 2.33)$

$= 0.86$ mm

Q 9. The average power transmitted through a given point on a string supporting a sine wave is 0.20 W. when the amplitude of the wave is 2mm. what power will be transmitted through this point if the amplitude is increased to 3mm.

$\Rightarrow$ power transmitted when amplitude is 2mm, $P_1 = 0.20$ W

let P_2 be the power transmitted when amplitude becomes 3 mm then,

$$P_1 = \frac{1}{2} \mu A_1^2 \omega^2 u \quad \text{--- (1)}$$

$$P_2 = \frac{1}{2} \mu A_2^2 \omega^2 u \quad \text{--- (2)}$$

From (1) and (2) $\frac{P_1}{P_2} = \frac{A_1^2}{A_2^2}$

$$\Rightarrow P_2 = P_1 \left(\frac{A_2}{A_1}\right)^2$$

$$\Rightarrow P_2 = 0.20 \left(\frac{2}{3}\right)^2$$

$$\Rightarrow P_2 = 0.45 W$$

Q10. show that particle acceleration is equal to the product of square of wave velocity and curvature of displacement

⇒ we have,

particle displacement

$$y = A \sin(\omega t - kx) \quad \cdots (1)$$

Now, differentiating eqn (1) w.r.t t

$$\frac{dy}{dt} = A\omega \cos(\omega t - kx)$$

Again, differentiating w.r.to t we get particle acceleration

$$\frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t - kx) \quad \cdots (2)$$

Again, diff eqn (1) w.r.t. x

$$\frac{dy}{dx} = -Ak \cos(\omega t - kx)$$

Again differentiate with respect to x we get curvature of displacement

$$\frac{d^2y}{dx^2} = -Ak^2 \sin(\omega t - kx) \quad \text{---} (3)$$

From (2) and (3)

$$\frac{d^2y/dt^2}{d^2y/dx^2} = \frac{-A\omega^2 \sin(\omega t - kx)}{-Ak^2 \sin(\omega t - kx)}$$

$$\Rightarrow \frac{d^2y}{dt^2} = \frac{\omega^2}{k^2} \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dt^2} = \left(\frac{\omega}{k}\right)^2 \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dt^2} = (f\lambda)^2 \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dt^2} = u^2 \frac{d^2y}{dx^2}.$$

Hence proved