$$= \frac{q}{4\pi\epsilon_o r}\left[\frac{-d^2}{2r^2} - \frac{d^2}{2r^2} - \frac{d\cos\theta}{r} + \frac{d\cos\theta}{r} + \frac{6}{8}\left(\frac{2d\cos\theta}{r}\right)^2\right]$$

$$= \frac{q}{4\pi\epsilon_o r}\left[\frac{-d^2}{r^2} + \frac{6}{8}\cdot\frac{4d^2\cos^2\theta}{r^2}\right]$$

$$= \frac{q}{4\pi\epsilon_o r}\left[\frac{3d^2\cos^2\theta}{r^2} - \frac{d^2}{r^2}\right]$$

$$= \frac{q}{4\pi\epsilon_o r}\cdot\frac{d^2}{r^2}(3\cos^2\theta - 1)$$

$$V = \frac{q.d^2}{4\pi\epsilon_o r^3}(3\cos^2\theta - 1)$$

This is the electric potential at any point 'P' at an angle θ at a distance r from the centre of the quadrupole.

1. **For axial line, θ = 0**

$$\therefore V = \frac{q.d^2.2}{4\pi\epsilon_o r^3} = \frac{Q}{4\pi\epsilon_o r^3} \quad [\because q.2d^2 = Q]$$

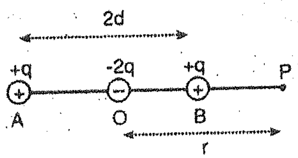
2. **For equatorial line, θ = 90°.**

$$\therefore V = \frac{q.d^2}{4\pi\epsilon_o r^3}.(1) = \frac{Q}{8\pi\epsilon_o r^3}$$

Potential along the axial line of Quadrupole

(Alternation approach)

Consider an electric quadrupole with separation 2d. P is any point at a distance 'r' from centre of quadrupole at which electric potential is to be determined. From figure, total electric potential at P is



$$V = V_A + V_B + V_o$$

$$= \frac{q}{4\pi\epsilon_o(r+d)} + \frac{q}{4\pi\epsilon_o(r-d)} - \frac{2q}{4\pi\epsilon_o r}$$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{1}{r+d} + \frac{1}{r-d} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{r(r-d) + r(r+d) - 2(r+d)(r-d)}{r(r^2-d^2)}\right]$$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{r^2 - rd + r^2 + rd - 2r^2 + 2d^2}{r(r^2-d^2)}\right]$$

$$= \frac{2qd^2}{4\pi\epsilon_o r(r^2-d^2)} = \frac{Q}{4\pi\epsilon_o r(r^2-d^2)}$$

In case of very short quadrupole we have, r > > d

$$\therefore V = \frac{Q}{4\pi\epsilon_o r^3}$$

Solved Examples

1. **What is the magnitude of point charge choosen so that the electric field 50 cm away has magnitude of 2 N/C.**

Solution:

Here, E = 2N /C, r = 50 cm = 0.5 m, q = ?

We have, $E = \frac{q}{4\pi\epsilon_o r^2} = \frac{9\times10^9\times q}{(0.5)^2}$

$q = 5.5\times10^{-11}$ C

2. **Two metal spheres are 3 cm in radius and carry charges of + 1 × 10^{-8} C and - 3 × 10^{-8} C respectively, assumed to be at centre of the sphere. If their centres are 2 meter apart calculate i) the potential of the point half way between their centers, and ii) the potential of each sphere.**

Solution:

Here, $q_1 = 1\times10^{-8}$ C, $q_2 = -3\times10^{-8}$ C, radius of each sphere, R = 3 cm = 0.03 m, distance between two charges r = 2 m.

i. Here, $r_1 = r_2 = 1$ m

$$V = \frac{q_1}{4\pi\epsilon_o r_1} + \frac{q_2}{4\pi\epsilon_o r_2} = \frac{1}{4\pi\epsilon_o}(q_1 + q_2)$$

$$= 9\times10^9(1\times10^{-8} - 3\times10^{-8}) = -180 \text{ Volts.}$$

ii. The potential at the surface of first sphere is due to its own charge q_1 plus potential due to second sphere.

$$V_1 = \frac{q_1}{4\pi\epsilon_0 R} + \frac{q_2}{4\pi\epsilon_0 r} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

$$= 9 \times 10^9 \left(\frac{1 \times 10^{-8}}{0.03} - \frac{3 \times 10^8}{2}\right) = 2865 \text{ volts}$$

The potential at the surface of second sphere is due to its own charge q_2 plus potential due to first sphere.

$$V_2 = \frac{q_2}{4\pi\epsilon_0 R} + \frac{q_1}{4\pi\epsilon_0 r} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_2}{R} + \frac{q_1}{r}\right)$$

$$= 9 \times 10^9 \left(\frac{-3 \times 10^8}{0.03} + \frac{1 \times 10^{-8}}{2}\right) = -8995 \text{ volts}$$

3. **Three charges + 1 × 10^{-7} C, - 4 × 10^{-7} C and + 2 ×10^{-7} C are placed at the three vertices of an equilateral triangle of side 0.1 m. Find the minimum amount of work required to dismantle this structure.**

OR

Three charges +q, +2q and - 4q are placed at the three vertices of an equilateral triangle of side 10 cm. What is the mutual potential energy of the system of the charges?

Solution:

i. Here, $q_1 = +q$, $q_2 = +2q$ and $q_3 = -4q$, $a = 0.1$ m

The total energy of configuration is given by,

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{a} + \frac{q_2q_3}{a} + \frac{q_3q_1}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(+q)(+2q)}{a} + \frac{(+2q)(-4q)}{a} + \frac{(-4q)(+q)}{a}\right]$$

$$U = -\frac{10q^2}{4\pi\epsilon_0 a}$$

ii) Here $q = 1 \times 10^{-7}$ C

$$\therefore U = -\frac{10 \times 9 \times 10^9 \times (1 \times 10^{-7})^2}{0.1} = -9 \times 10^{-3} \text{ J}$$

4. **Twenty seven identical drops of mercury are charged simultaneously to the same potential of 10 Volt. What will be the potential if all the drops are made to combine to form one large drop? Assume the drops to be spherical.**

Solution:

Let, r be the radius of each small drop, R be the radius of large drop and q be the charge on each drop.

then, $\frac{4\pi}{3}R^3 = 27 \times \frac{4\pi}{3}r^3$

$\Rightarrow R = 3r$

The electrical potential of each small drop, $V_s = \frac{q}{4\pi\epsilon_0 r}$ and that of large drop, $V_L = \frac{27q}{4\pi\epsilon_0 R} = \frac{27q}{4\pi\epsilon_0 (3r)} = \frac{9q}{4\pi\epsilon_0 r}$

$\therefore V_L = 9\,V_S = 9 \times 10 = 90$volts.

5. **What is the potential gradient in volts/ meter at a distance of 10^{-12} m from the centre of the gold nucleus? What is the gradient at the nuclear surface of radius R = 5 × 10^{-15} m? [Atomic number of gold = 79 and e = 1.6 × 10^{-19} C]**

Solution:

i. Since the potential gradient is also known as electric field.

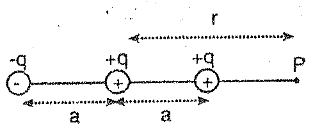
$$E = \frac{dV}{dr} = \frac{q}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\frac{79 \times 1.6 \times 10^{-9}}{(10^{-12})^2} = 1.14 \times 10^{17} \text{ V/m}$$

ii. The potential gradient at the surface of gold nucleus of radius $R = 5 \times 10^{-15}$ m is given by,

$$E = \frac{dV}{dR} = \frac{q}{4\pi\epsilon_0 R^2} = \frac{1}{4\pi\epsilon_0}\frac{79 \times 1.6 \times 10^{-19}}{(5 \times 10^{-15})^2} = 4.55 \times 10^{21} \text{ V/m}$$

6. **For the charge configuration of the figure, show that V (r) at a point 'P' on the line (assuming r > >a) is given by**

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right).$$



Solution:

From figure, the total potential at P is given by

$$V = \frac{(-q)}{4\pi\epsilon_0 (r+a)} + \frac{q}{4\pi\epsilon_0 r} + \frac{q}{4\pi\epsilon_0 (r-a)}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{r+a-r+a}{r^2-a^2}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{2a}{r^2-a^2}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{2a}{r^2}\right] (\because r >> a)$$

$$\therefore V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r} + \frac{2qa}{r^2}\right]$$

7. A spherical drop of water carrying a charge of 30 PC has potential of 500 V at its surface (with V = 0 at infinity) a) what is the radius of the drop? b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

Solution:

Here, $q = 30\text{ PC} = 30 \times 10^{-12}$ C, V = 500 Volts.

a. $V = \frac{q}{4\pi\epsilon_0 r} \Rightarrow r = \frac{q}{4\pi\epsilon_0 V} = \frac{9\times10^9 \times 30\times10^{-12}}{500} = 5.4 \times 10^{-4}$ m

b. After the drops are combined, the total volume is double of the volume of an original drop.

$$\Rightarrow \frac{4\pi}{3}R^3 = 2 \times \frac{4\pi}{3}r^3 \Rightarrow R^3 = 2r^3 \Rightarrow R = 2^{1/3} r = 6.8\times10^{-4}\text{m}$$

$$\therefore V = \frac{2q}{4\pi\epsilon_0 R} = \frac{2\times 9\times10^9 \times 30\times10^{-12}}{6.8\times10^{-4}} = 794.12 \text{ volts}$$

8. A small conducting sphere of mass m having charge +q is suspended by a thread of length l. The sphere is placed in uniform electric field of strength E directed vertically up ward. Show that this sphere oscillate with period. $T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$, if the electrostatic force acting on the sphere is less than gravitational force.

Solution:

The force acting on the sphere are:

i. Tension T along the string.

ii. Weight mg acting vertically downwards.

iii. Electrical force qE vertically upwards.

The resultant force acting vertically down wards is (mg - qE) i.e resultant force, F = mg - qE

T = mg - qE

mg' = mg - qE

$$g' = g - \frac{qE}{m}$$

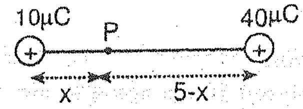
Effective acceleration, $g' = g - \frac{qE}{m}$

$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$$

9. Two small spheres of charge 10 μC and 40 μC are placed 5 cm apart. Find the location of a point between them where the field strength is zero.

Solution:

Here, $q_1 = 10\ \mu C = 10 \times 10^{-6}$ C, $q_2 = 40\ \mu C = 40 \times 10^{-6}$ C, r = 5 cm = 0.05 m



Let, the electric field be zero at the distance x from charge 10 μ c.

⇒ The net electric field is given by, $E = E_1 - E_2 = 0$

⇒ $E_1 = E_2$

$$\frac{1}{4\pi\epsilon_0}\frac{10\times10^{-6}}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{40\times10^{-6}}{(5-x)^2}$$

$4x^2 = (5-x)^2$

$2x = 5 - x$

$3x = 5$

$x = 5/3 = 1.67$ cm

10. Assume that earth has surface charge density of electron per meter square. Calculate the earth's electric field and potential on the earth surface. Given that radius of the earth is 6400 km.

Solution:

Here, $\sigma = 1.6 \times 10^{-19}$ C/m^2, R = 6400 km = 6.4×10^6m

If q is charge on earth, the electric field on it's surface is

$$E = \frac{q}{4\pi\epsilon_o R^2} = \frac{1}{\epsilon_o}\frac{q}{4\pi R^2} = \frac{1}{\epsilon_o}\frac{q}{A} = \frac{\sigma}{\epsilon_o} = \frac{1.6 \times 10^{-19}}{8.85 \times 10^{-12}} = 180.79 \text{ N/C}$$

And, the potential on the surface of earth is given by,

$$V = \frac{q}{4\pi\epsilon_o R} = \frac{R}{\epsilon_o} \cdot \frac{q}{4\pi R^2}$$

$$= \frac{\sigma R}{\epsilon_o} = \frac{1.6 \times 10^{-19} \times 6.4 \times 10^6}{8.85 \times 10^{-12}} = 1.157 \times 10^9 \text{ Volts}$$

11. The electric potential V (in volts) varies with x (in metre) according to the relation $V = 5 + 4x^2$. Calculate the force experienced by a negative charge of 2×10^{-6} C located at x =0.5 m.

Solution:

Here, $V = 5 + 4x^2$, $q = -2 \times 10^{-6}$ C and x = 0.5 m

$$\Rightarrow E = -\frac{dV}{dx} = -8x$$

$$F = qE = -2 \times 10^{-6} \times (-8x)$$

At x = 0.5 m, force, $F = 8 \times 2 \times 0.5 \times 10^{-6} = 8 \times 10^{-6}$ N

12. Two positive point charges of 12 and 8 micro coulombs respectively are placed 10 cm apart in air. Calculate the amount of work done to bring them 4 cm closer.

Solution:

Here, $q_1 = 12 \times 10^{-6}$ C, $q_2 = 8 \times 10^{-6}$ C.

i. Electrostatic potential energy when the charges are 10 cm = 0.1 apart is

$$W_1 = \frac{q_1 q_2}{4\pi\epsilon_o r} = \frac{12 \times 10^{-6} \times 8 \times 10^{-6}}{4\pi\epsilon_o \times 0.1} = \frac{9.6 \times 10^{-10}}{4\pi\epsilon_o}$$

ii. Potential energy when the charges are brought 4 cm closer i.e. when they are (10 - 4) cm = 6 cm = 0.06 m apart is,

$$W_2 = \frac{12 \times 10^{-6} \times 8 \times 10^{-6}}{4\pi\epsilon_o \times 0.06} = \frac{16 \times 10^{-10}}{4\pi\epsilon_o}$$

$\therefore$ Work done $= W_2 - W_1 = (16 - 9.6) \times 10^{-10} \times 9 \times 10^9 = 5.8$ J

13. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is the same for both spheres. Show that, the electric potential at the common centre is $v = \frac{\sigma}{\epsilon_0}(r_1 + r_2)$.

Solution:

The electric potential at the common centre is

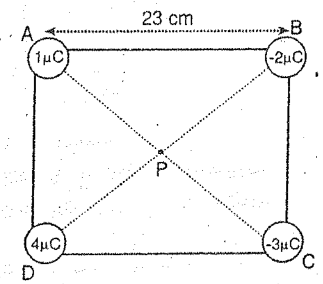
$$V = \frac{q_1}{4\pi\epsilon_o r_1} + \frac{q_2}{4\pi\epsilon_o r_2}$$

$$= \frac{q_1}{4\pi r_1^2} \cdot \frac{r_1}{\epsilon_o} + \frac{q_2}{4\pi r_2^2} \cdot \frac{r_2}{\epsilon_o} = \frac{\sigma r_1}{\epsilon_o} + \frac{\sigma r_2}{\epsilon_o}$$

$$= \frac{\sigma}{\epsilon_o}(r_1 + r_2)$$

14. What will be the potential at the centre of the square of side 23 cm, if the charges 1 μc, - 2μc, - 3μc and 4 μc are placed at the corners.

Solution:



From figure

$AC = \sqrt{23^2 + 23^2} = 32.5269$

$AP = \frac{AC}{2}$

AP = 16.26 cm = 0.163 m

AP = BP = CP = DP = 0.163 m

The total potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{1\times10^{-6}}{AP} + \frac{(-2\times10^{-6})}{BP} + \frac{(-3\times10^{-6})}{CP} + \frac{4\times10^{-6}}{DP}\right]$$

$$= \frac{1}{4\pi\epsilon_0}[1-2-3+4]\times\frac{10^{-6}}{0.163}$$

$$= 0 \text{ volt}$$

15. A metallic sphere A of radius 'a' carries a charge Q. It is brought in contact with an uncharged sphere B of radius b. Calculate the amount of charge left on the sphere A interms of a, b and Q.

Solution:

Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$V_1 = V_2 \Rightarrow \frac{Q-q}{4\pi\epsilon_0 a} = \frac{q}{4\pi\epsilon_0 b}$$

$$\Rightarrow Q - q = \frac{a}{b}.q \Rightarrow Q = \left(1+\frac{a}{b}\right)q = \left(\frac{a+b}{b}\right).q$$

$$\Rightarrow q = \frac{bQ}{(a+b)}$$

Charge left on A $= Q - q = Q - \frac{bQ}{(a+b)} = \left(\frac{a+b-b}{a+b}\right).Q = \frac{aQ}{(a+b)}$

16. Charge of uniform volume density 3.2 μc/m³ fill a non conducting solid sphere of radius 5 cm. What is the magnitude of electric field at a) 3.5 cm b) 8 cm from the centre of the sphere.

Solution:

Here, $\rho = 3.2\times10^{-6}$ C/m³, R = 5 cm = 0.05 m

a. Here, r = 3.5 cm = 0.035 m

$$E = \frac{q'}{4\pi\epsilon_0 r^2}$$

$$q' = \frac{4\pi}{3}r^3.\rho \Rightarrow E = \frac{4\pi}{3}r^3.\rho.\frac{1}{4\pi\epsilon_0 r^2} = \frac{\rho r}{3\epsilon_0}$$

$$\Rightarrow E = \frac{3.2\times10^{-6}\times0.035}{3\times8.85\times10^{-12}} = 4.22\times10^3 \text{ N/C}$$

b. Here, r = 8 cm = 0.08 m

$$E = \frac{q}{4\pi\epsilon_0 r^2}, \quad q = \frac{4\pi}{3}R^3.\rho$$

$$\Rightarrow E = \frac{4\pi}{3}R^3\rho.\frac{1}{4\pi\epsilon_0 r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

$$= \frac{3.2\times10^{-6}\times(0.05)^3}{3\times8.85\times10^{-12}\times(0.08)^2} = 2.35\times10^3 \text{ N/C}$$

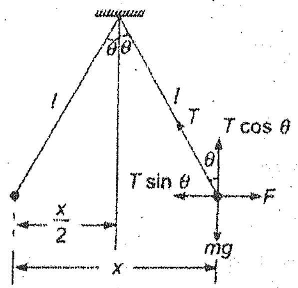
17. Two identical tiny spheres, each of mass m are hung from insulating string of equal length *l*. when an equal charge q is given to each sphere, they repel each other. Show that the separation x between them is given by $x = \left(\frac{q^2 l}{2\pi\epsilon_0 mg}\right)^{1/3}$

Solution:

From figure, at equilibrium condition,

$T\sin\theta = F_E$ .....(1)

And $T\cos\theta = mg$ .....(2)



Where, T is tension on string, $F_E = \frac{q^2}{4\pi\epsilon_0 x^2}$ is the electric force between two charges.

Dividing (1) by (2) $\tan\theta = \frac{F_E}{mg} = \frac{q^2}{4\pi\epsilon_0 x^2 mg}$

for small θ, $\tan\theta \approx \theta \Rightarrow \theta = \frac{q^2}{4\pi\epsilon_0 x^2 mg}$ ....(3)

Again from figure, $\sin\theta = \frac{x}{2l}$

for small θ, $\sin\theta \approx \theta \Rightarrow \theta = \frac{x}{2l}$ ...(4)

from (3) and (4) $\frac{x}{2l} = \frac{q^2}{4\pi\epsilon_o x^2 mg} \Rightarrow x^3 = \frac{q^2 l}{2\pi\epsilon_o mg}$

$$x = \left[\frac{q^2 l}{2\pi\epsilon_o mg}\right]^{1/3}$$

18. A charge of 5×10^{-5} C is distributed between two spheres. It is found that they repel each other with a force of 1N when their centres are 2m apart. Find the charge on each sphere.

Solution:

Here, $q_1 + q_2 = 5 \times 10^{-5}$ c.........(1)

$F = \frac{q_1 q_2}{4\pi\epsilon_o r^2} \Rightarrow 1 = \frac{q_1 q_2}{4\pi\epsilon_o 2^2} \Rightarrow q_1 q_2 \times 9 \times 10^9 = 4$

$\Rightarrow q_1 q_2 = 4.44 \times 10^{-10}$ .....(2)

We have, $(q_1 - q_2) = [(q_1 + q_2)^2 - 4 q_1 q_2]^{1/2}$

$= [(5 \times 10^{-5})^2 - 4 \times 4.44 \times 10^{-10}]^{1/2}$

$q_1 - q_2 = 2.69 \times 10^{-5}$ .....(3)

Adding equation (1) and (3)

$2q_1 = 7.69 \times 10^{-5}$

$q_1 = 3.84 \times 10^{-5}$ C

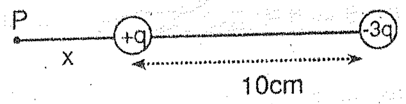
from (3), $q_2 = q_1 - 2.69 \times 10^{-5}$

$= 1.15 \times 10^{-5}$ C.

19. Two point charges + q and - 3q are separated by a distance d = 10 cm. Locate the points on the line passing through the two charges where a) electric potential is zero and b) electric field is zero.

Solution:

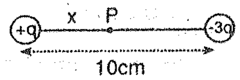
At a point P at a distance x to the left of charge + q, the electric potential is,



$$V = \frac{1}{4\pi\epsilon_o}\left(\frac{q}{x} - \frac{3q}{d+x}\right)$$

If V = 0, $\frac{q}{x} = \frac{3q}{d+x} \Rightarrow 3x = d + x \Rightarrow x = \frac{d}{2} = \frac{10}{2} = 5$ cm

At a point 'P' between two charges, at a distance x to the right of charge +q,



$$V = \frac{1}{4\pi\epsilon_o}\left(\frac{q}{x} - \frac{3q}{d-x}\right)$$

If V = 0, $\frac{q}{x} = \frac{3q}{d-x} \Rightarrow 3x = d - x \Rightarrow x = \frac{d}{4} = \frac{10}{4} = 2.5$cm

At a point P at a distance x to the left of +q, the electric field is

$$E = \frac{q}{4\pi\epsilon_o x^2} - \frac{3q}{4\pi\epsilon_o (x+d)^2}$$

If E = 0,

$$\frac{q}{4\pi\epsilon_o x^2} = \frac{3q}{4\pi\epsilon_o (x+d)^2} \Rightarrow \frac{1}{x^2} = \frac{3}{(x+d)^2}$$

$x^2 + 2xd + d^2 = 3x^2$

$2x^2 - 2xd - d^2 = 0$

$$x = \frac{2d \pm \sqrt{4d^2 + 8d^2}}{4} = \frac{20 \pm 34.64}{4}$$

The positive root is, x = 13.66 cm.

20. Two charges 4q and - q are placed at a distance r apart. A charge Q is placed exactly mid way between them, what will be the value of Q so that charge - q experiences no net force?

Solution:

Here, net force on charge -q is

$$F = \frac{1}{4\pi\epsilon_o}\frac{(-q) \times 4q}{r^2} + \frac{1}{4\pi\epsilon_o}\frac{(-q).Q}{(r/2)^2}$$

If, F = 0,

$$\frac{-4q^2}{r^2} - \frac{4Qq}{r^2} = 0$$

$-q - Q = O$

$Q = - q$

Exercises

1. Define electric field intensity, electronic polarization, and electric displacement. Find the relation between them. Explain the physical significance of the three vectors.
2. Find the relation between electric field intensity and potential.
3. Define electric dipole and dipole moment. Obtain an expression for electric field due to dipole.
4. State Gauss law. Apply Gauss law to calculate the intensity due to charged sphere at the points outside, inside and at the surface of the sphere.
5. What is electric dipole? Find out potential and field due to electric dipole at a distance r from its centre.
6. Define Quadrupole. Find field and potential due to electric quadrupole.
7. prove that electric field due to short dipole at axial point is twice that at equatorial line.
8. For a given short electric dipole, show that the electric potential at any point at a distance r is $V = \frac{P\cos\theta}{r^2}$, where θ is the angle made by r to the dipole axis and P is its dipole moment. Using above relation find an expression for resultant electric field intensity at that point.
9. Find the potential at any point at an angle θ at a distance r from the centre of the short dipole. What result do you obtain if the point is along axial line.
10. Derive an expression for the electric field at any point on the axis of the short linear quadrupole.
11. What is electric quadrupole? Finding an expression for electric potential at any point on axial line at a distance r from centre of short quadrupole, show that electric field at that point is inversely proportional to r^3.

12. Define electric dipole. Find an expression for electric potential at any point in space due to dipole of length 2a. Could you extend this relation to calculate electric field intensity? If so how?
13. What is dipole? Derive an expression for electric field due to dipole at the points (i) axis of the dipole and ii) perpendicular bisector of dipole.
14. Explain Gauss law in free space. How the law is modified for the presence of dielectric material?
15. Two point charges +4q and +q are 30 cm apart. At what point on the line joining them is the electric field zero?
16. Electrostatic force between two charges placed in vaccum is F. If the charges are placed at the same separation, in a medium of dielectric constant (relative permittivity) K, find the force between the charges in medium.
17. Two tiny spheres carrying charges of 1 μC and 3 μC are placed 8 cm apart in air. What is the potential at a point 3 cm from the mid point in a plane normal to the line passing through the mid point.
18. Four point charges +q, +q, - q and -q are placed respectively at the corners of a square of side a. Find the electric potential at the centre of the square.
19. A cube of side 'b' has a charge q at each of its vertices. what is the potential at the centre of the cube.
20. Find the electric potential at the surface of an atomic nucleus (z = 50) of radius 9.0×10^{-3} cm.
21. A point charge $q_1 = +1.0 \times 10^{-8}$ C placed at a distance of 10 cm from another point charge $q_2 = +2.0 \times 10^{-8}$ C. At what point on the line joining the two charges is the electric field zero?
22. Two identical oppositely charged spheres, with their centres 0.5 m apart, attract each other with a force of 0.108 N. The spheres are connected by a conducting wire. When the wire is removed, they repel each other with a force of 0.036 N. Find the initial charges on the sphere.
23. Find the work done on assembling four charges +q, -q, +q and - q on the corners of a square of side a.

24. The electric potential V due to a charge in the surrounding space at any point x - meters from the charge is given by the relation, $V = 8x + 3x^2$ volts. Find the electric field intensity at a point 1.5 m from the charge. consider the medium is air.

25. Two equal and opposite charge of magnitude 2×10^{-7} C are 15 cm apart i) What are the magnitude and direction of $\vec{E}$ at a point mid way between the charges? b) what force (magnitude and direction) would act on an electron placed there?

CAPACITOR

A capacitor is a device that stores electric potential energy or electric charge.

Two conducting plates enclosing an insulating (dielectric) material form a capacitor.

To store charge it is connected in series with power supply. To transfer charge from one conductor to the other some work must be done, This work done is stored as electric potential energy.

Capacitance:

For a capacitor, the ratio of the charge on each conducting plate to the potential difference is a constant quantity, called the capacitance.

$$\frac{\text{Charge}}{\text{potential difference}} = \frac{q}{V} = C \text{ (capacitance)}$$

$$\text{unit} = \frac{\text{coulomb}}{\text{volt}} = \frac{C}{V} = \text{Farad}$$

It is also defined as the 'ability of a capacitor to store electric potential energy'.

The capacitance depends on the size and shapes of the conductors and on the insulating material between them. Due to the presence of dielectric capacitance increase. This happen because redistribution of charge called polarisation takes place with in the insulating material.

Types of Capacitors:

Before studying about types of capacitor we need to know the method for calculation of capacitance.

Calculating the capacitance:

1. Assume a charge q on the plate under consideration.
2. Calculate the electric field $\vec{E}$ between the plates in terms of this charge using Gauss's law.

When $t = RC$, from equation (5) $q = q_o e^{-1} = \frac{q_o}{e} = 0.37\, q_o$

$\therefore q = 37\%$ of q_o

Hence, time constant of a discharging circuit is the time at which the charge stored in capacitor fall to 37% of its initial value.

Solved Examples

1. **A 100 μF capacitor is charged to a potential difference of 50 volts. The charging battery is then disconnected, the capacitor is then connected to second capacitor in parallel. If the measured potential drop to 35 volts. What is the capacitance of second capacitor.**

Solution:

before connection, $V = 50$ volts, $C = 100\,\mu F$, $Q = CV$.

After connection let V_o be the common potential, Q_1 be the charge on first capacitor and Q_2 be the charge on second capacitor.

$\therefore \quad Q_1 = C_1 V_1 = CV_o$

$Q_2 = C_2 V_2 = C_2 V_o$

Also, $Q = Q_1 + Q_2$

$CV = CV_o + C_2 V_o$

$$\Rightarrow C_2 = \left(\frac{V - V_o}{V_o}\right) . C = \frac{(50 - 35) \times 100}{35} = 42.86\,\mu F$$

2. **A parallel plate capacitor with air as dielectric is charged to a potential V. It is then connected to an uncharged parallel plate capacitor filled with wax of dielectric constant K. Calculate common potential of both capacitor.**

Solution:

before connection let, Q and C are the charge and capacitance of capacitor.

Here $K = 1$ and $Q = CV$

After connection, let V_o be the common potential

$\therefore$ For first capacitor, $Q_1 = C_1 V_1 = CV_o$ for second capacitor, $Q_2 = C_2 V_2 = C_2 V_o = KCV_o$ $[\because \frac{C_2}{C_1} = k]$

Also, $Q = Q_1 + Q_2$

$CV = CV_o + KCV_o$

$V = V_o (1 + K)$

$$V_o = \frac{V}{1 + K}$$

3. **An air - filled parallel plate capacitor has a capacitance of 10^{-12} F. The separation between the plates is doubled and wax is inserted between them. Which increases the capacitance to 2×10^{-12} F. Calculate the dielectric constant of wax.**

Solution:

Here, $C_1 = 10^{-12}$ F, $d_1 = d$, $A_1 = A$ and $\epsilon_1 = \epsilon_o$

$$C_1 = \frac{\epsilon_1 A_1}{d_1} = \frac{\epsilon_o A}{d} \quad ......(1)$$

When d is increased to 2d and wax is introduced

$C_2 = 2 \times 10^{-12}$F, $d_2 = 2d$, $A_2 = A$ and $\epsilon_2 = \epsilon$ (say)

$$\therefore C_2 = \frac{\epsilon_2 A_2}{d_2} = \frac{\epsilon A}{2d} = \frac{K\epsilon_o A}{2d} \quad .....(2)$$

$$\Rightarrow \frac{C_2}{C_1} = \frac{K}{2}$$

$$\Rightarrow K = \frac{2 \times 2 \times 10^{-12}}{10^{-12}} = 4$$

4. **If the charge on a capacitor is increased by 2 coulomb, the energy stored in it increased by 21%. Find the original charge on the capacitor.**

Solution:

Since, $U_1 = \frac{q_1^2}{2c}$ and $U_2 = \frac{q_2^2}{2c}$

Therefore, $\frac{U_2 - U_1}{U_1} = \frac{q_2^2 - q_1^2}{q_1^2}$

Given, $\frac{U_2 - U_1}{U_1} = 21\% = 0.21$

$$\Rightarrow \frac{q_2^2 - q_1^2}{q_1^2} = 0.21 \Rightarrow q_2^2 - q_1^2 = 0.21\, q_1^2$$

$q_2^2 = 1.21\ q_1^2$

$q_2 = 1.1\ q_1$ ......(1)

Also from question, $q_2 = q_1 + 2C$

$\Rightarrow 1.1q_1 = q_1 + 2$

$q_1 = \frac{2}{0.1} = 20\ C$

Therefore, original charge on capacitor, $q_1 = 20$ C.

5. **The plates of a parallel plate capacitor are 10 cm apart and have area equal to $2m^2$. If the charge on each plate is 8.85×10^{-10}C. Find the electric field at a point between the plates and the capacitance of the capacitor.**

Solution:

Here, $d = 10\ cm = 10^{-2}m$, $A = 2m^2$, $q = 8.85 \times 10^{-10}$ C.

We have, $E = \frac{q}{\epsilon_0 A} = \frac{8.85 \times 10^{-10}}{8.85 \times 10^{-12} \times 2} = 50\ N/C$

and, $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2}{10^{-2}} = 1.77 \times 10^{-9}\ F$

6. **A leaky parallel plate capacitor is completely filled with a material having dielectric constant K and conductivity σ. Show that the time constant of capacitor is $\frac{K\epsilon_0}{\sigma}$.**

Solution:

Let A be the plate area and 'd' the plate separation. The resistivity of material is given by, $\rho = \frac{RA}{d}$ where R is the resistance of the material. The conductivity σ is, $\sigma = \frac{1}{\rho} = \frac{d}{RA}$ or $R = \frac{d}{\sigma A}$.

The capacitance of capacitor, $C = \frac{\epsilon A}{d} = \frac{K\epsilon_0 A}{d}$

$\therefore$ time constant, $\tau = RC = \frac{d}{\sigma A} \cdot \frac{K\epsilon_0 A}{d} = \frac{K\epsilon_0}{\sigma}$.

7. **Show that capacitance due to a charged sphere of radius 'r' is $4\pi\epsilon_0 r$. The total charge on the sphere is supposed to be concentrated at the centre.**

Solution:

we have, $V = \frac{q}{4\pi\epsilon_0 r}$

Since, $C = \frac{q}{V}$

$\Rightarrow C = 4\pi\epsilon_0 r$

8. **If n drops, each of capacitance C, coalesce to form a single big drop. Find the capacitance of big drop.**

Solution:

The mass of small drop, $M_1 = \frac{4\pi}{3} r^3 \times \rho$

The mass of big drop, $M_2 = \frac{4\pi}{3} R^3 \times \rho$

Since $M_2 = nM_1$

$\Rightarrow \frac{4\pi}{3} R^3 = n.\frac{4\pi}{3} r^3 \Rightarrow R^3 = nr^3$

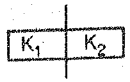
$\therefore R = n^{1/3}$ r.

$\therefore$ capacitance of big drop $= 4\pi\epsilon_0 R = 4\pi\epsilon_0 .\ n^{1/3} .r$

$= 4\pi\epsilon_0 r\ n^{1/3}$

$= n^{1/3}\ C$

9. **A parallel plate capacitor is filled with two dielectrics as shown in figure. Show that the capacitance is given by,**



$C = \frac{\epsilon_0 A}{d} \frac{(K_1 + K_2)}{2}$

Solution:

We have,

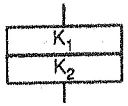
$C_1 = \frac{\epsilon_1 (A/2)}{d} = \frac{K_1\epsilon_0 A}{2d}$ $[\therefore K_1 = \epsilon_r = \frac{\epsilon}{\epsilon_0}]$

and $C_2 = \frac{\epsilon_2 (A/2)}{d} = \frac{K_2\epsilon_0 A}{2d}$

The given parallel plate capacitor is equivalent to a combination of two capacitors in parallel combination.

$$\therefore \quad C = C_1 + C_2 = \frac{K_1 \epsilon_0 A}{2d} + \frac{K_2 \epsilon_0 A}{2d} = \frac{\epsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

10. A parallel plate capacitor is filled with two dielectrics as shown in figures. Show that the capacitance is given by $C = \frac{2\epsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$



Solution: We have, $C_1 = \frac{K_1 \epsilon_0 A}{(d/2)} = \frac{2K_1 \epsilon_0 A}{d}$

and $C_2 = \frac{K_2 \epsilon_0 A}{(d/2)} = \frac{2 K_2 \epsilon_0 A}{d}$

The given capacitor is equivalent to a combination of two capacitors in series.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \epsilon_0 A} + \frac{d}{2K_2 \epsilon_0 A} = \frac{d}{2\epsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$= \frac{d}{2\epsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$\therefore \quad C = \frac{2\epsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

11. A dielectric slab of thickness 'b' is inserted between the plates of a parallel plate capacitor of plate separation 'd'. (b < d). Show that the capacitance is given by $C = \frac{K \epsilon_0 A}{Kd - b(K-1)}$.

Solution:

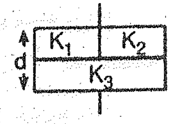
The capacitor can be considered as an arrangement of two capacitors in series, one consisting of a dielectric K with thickness b and area A and another with air gap of thickness (d - b) and area A.

$$\therefore \quad C_1 = \frac{K\epsilon_0 A}{b} \text{ and } C_2 = \frac{\epsilon_0 A}{d - b}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{b}{K\epsilon_0 A} + \frac{d - b}{\epsilon_0 A} = \frac{b + dK - bK}{K \epsilon_0 A}$$

$$C = \frac{K\epsilon_0 A}{b + dK - bK} = \frac{K \epsilon_0 A}{Kd - b(K-1)}$$

12. A parallel plate capacitor of area A. plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown in figure. Find C.



Solution:

We have,

$$C_1 = \frac{K_1 \epsilon_0 (A/2)}{(d/2)}$$

$$= \frac{K_1 \epsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \epsilon_0 (A/2)}{d/2} = \frac{K_2 \epsilon_0 A}{d}$$

$$\text{and, } C_3 = \frac{K_3 \epsilon_0 A}{(d/2)} = \frac{2 K_3 \epsilon_0 A}{d}$$

The capacitors, C_1 and C_2 are in parallel and their equivalent capacitance is

$$\therefore C' = C_1 + C_2 = \frac{(K_1 + K_2) \epsilon_0 A}{d}$$

This combination is in series with C_3

$$\therefore \quad \frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} = \frac{d}{(K_1 + K_2) \epsilon_0 A} + \frac{d}{2K_3 \epsilon_0 A}$$

$$= \left(\frac{1}{K_1 + K_2} + \frac{1}{2K_3}\right)\frac{d}{\epsilon_0 A} = \frac{(K_1 + K_2 + 2K_3)}{2K_3(K_1 + K_2)}\frac{d}{\epsilon_0 A}$$

$$\therefore \quad C = \frac{\epsilon_0 A}{d}\frac{2K_3 (K_1 + K_2)}{(K_1 + K_2 + 2K_3)} = \frac{2\epsilon_0 A K_3 (K_1 + K_2)}{d (K_1 + K_2 + 2K_3)}$$

13. Two capacitors 2μF and 4 μF are connected in parallel across 300 volts pd. calculate the total energy in the system.

Solution:

Here, $C_1 = 2\mu F = 2 \times 10^{-6}$ F, $C_2 = 4\ \mu F = 4 \times 10^{-6}$ F, V = 300 volts.

Since C_1 and C_2 are connected in parallel, so their equivalent capacitance is given by.

$C = C_1 + C_2 = 6 \times 10^{-6}$ F

$$\therefore \quad U = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 300^2 = 0.27 \text{ Joule}$$

14. A capacitor of capacitance C is discharged through a resistor of resistance R. After how many time constant is the stored energy becomes one fourth of initial value.

Solution:

We have, $U = \frac{q^2}{2C} = \frac{q_o^2}{2C} e^{-2t/RC} = U_o e^{-2t/RC}$

According to question, $U = \frac{U_o}{4}$

$\frac{U_o}{4} = U_o e^{-2t/RC}$

$e^{2t/RC} = 4$

$t = \frac{\ln 4}{2} RC = 0.693\ RC$

$\Rightarrow t = 0.693\ \tau$

15. A capacitor of capacitance C is charged through a resistor R. Calculate the time at which the potential across the resistor is equal to the potential across the capacitor.

Solution:

Here, $V_R = V_C \Rightarrow IR = \frac{q}{C}$

$RI_o e^{-t/RC} = \frac{q_o}{C}(1 - e^{-t/RC})$

$\Rightarrow e^{-t/RC} = 1 - e^{-t/RC}$

$2e^{-t/RC} = 1$

$e^{t/RC} = 2$

$t = \ln(2).RC$

$t = 0.693\ RC$

16. A parallel plate capacitor has a capacitance of 100 μμF, a plate area of 100 cm² and mica as dielectric. At 50 volts potential difference calculate a) E in mica, b) the free charge on the plates and (c) the induced surface charge (Given K = 5.4 for mica).

Solution:

Here, $K = 5.4$, $C = 100\ \mu\mu F = 100\times10^{-12} F$, $A = 100 cm^2 = 10^{-2} m^2$

$V = 50$ volts.

b. We have, $q = CV = 100 \times 10^{-12} \times 50 = 5\times10^{-9} C$

c. The induced surface charge is $q' = q\left(1 - \frac{1}{k}\right) = 5\times10^{-9}\left(1 - \frac{1}{5.4}\right)$

$= 4.1\times10^{-9}$ C

a. Electric field in mica is,

$$E = \frac{q}{k \epsilon_0 A} = \frac{5\times10^{-9}}{5.4\times8.85\times10^{-12}\times10^{-2}} = 1.05\times10^4 V/m$$

Exercise

1. Define capacitance. Obtain an expression for capacitance of parallel plate capacitor. Explain the effect of dielectric on the capacitance of capacitor.
2. What is capacitor? Write down the expression of capacitance of cylindrical capacitor which has inner and outer radius a and b respectively. Explain why induced charge is always less in magnitude than freecharge.
3. State Gauss law in free space. Prove that the relation $q' = q\left(1 - \frac{1}{k}\right)$ where q is free charge, q' is polarized charge and k is dielectric constant.
4. Show that energy per unit volume in electric field is proportional to square of electric field.
5. Prove that capacitance per unit length of cylindrical capacitor varies inversely with logarithm of ratio of external and internal radii.
6. State and explain Gauss law in dielectrics. Explain the meaning of three electric vectors $\vec{P}$, $\vec{E}$ and $\vec{D}$. And show that, $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$.
7. A capacitor is made of two concentric spherical plates of radii a and 'b' of inner and outer spheres respectively. If the outer plate is charged positively and inner plate is earthed, prove that the capacitance of such capacitor is given as $C = 4\pi\epsilon_o\left(\frac{b^2}{b-a}\right)$
8. A spherical capacitor consists of two concentric spherical shell of radii a and b (with b > a). Show that the capacitance $C = 4\pi\epsilon_0\frac{ab}{b-a}$

9. Differentiate between polar and non polar dielectrics. Using Gauss law in dielectrics establish relation of electric field with displacement vector and polarization vector.

10. Define capacitance. Give a general method to calculate capacitance of a capacitor. Apply it to find capacitance of spherical capacitor.

11. State Gauss law in free space. How the law is modified if the dielectric materials are present? Then prove the relation $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$. Where symbols carry their usual meaning.

12. Write the circuit equation for a charging RC circuit. Solve it to find charge and current. Explain the meaning of time constant. Provide qualitative sketch for charge and current varying with time.

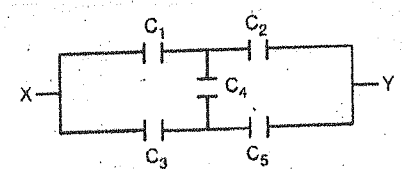
13. Show that capacitance of any capacitor depends on the geometry of the capacitor.

14. Define electric flux. Calculate the capacitance of cylindrical capacitor.

15. Obtain expressions for growth and decay of charges in the RC circuits. Explain how you will measure experimentally the capacitance of given capacitor.

16. A 100 μF capacitor is charged to a potential difference of 50 volts. The charging battery is then disconnected, the capacitor is then connected to second capacitor in parallel. If the measured potential drop to 35 volts, what is the capacitance of the second capacitor.

17. What is equivalent capacitance between the point x and y in the circuit given below ($C_1 = C_2 = C_3 = C_4 = 5$ μF).



18. What distance should the plates each of area 0.1m×0.2m of an air capacitor be placed in order to have same capacitance as a spherical conductor of radius 0.5m.

19. A parallel plate capacitor having plate area 100 cm^2 and separation 1.0 mm holds a charge of 0.12 μC when connected to a 120 V battery. Find the dielectric constant of material filling the gap.

20. A parallel plate capacitor has a circular plate of 8 cm radius and 1mm separation. What charge appear on the plates, if a p.d. of 100 volt is applied?

21. An air filled parallel plate capacitor has a capacitance of 1.1 PF. The separation of the plate is halved and wax is inserted between them. The new capacitance is 8.8 PF. Find the dielectric constant of wax.

22. A long cylindrical conductor has length 1m and is surrounded by a co-axial cylindrical shell with inner radius double that of long cylindrical conductor. Calculate the capacitance assuming that there is vacuum in space between cylinders.

23. Calculate the capacitance of the earth, viewed as spherical conductor of radius 6400 km.

24. A parallel plate capacitor has circular plates of 8.2 cm radius and 1.3 mm separation in air. (i) Calculate the capacitance (ii) What charge will appear on the plates if potential difference of 120 V is applied.

25. In an RC circuit ε = 12 V, R = 1.40 MΩ and C = 1.80 μF (i) Calculate the time constant (ii) maximum charge that will appear on the capacitor during charging. (iii) how long does it take for the charge to build up to 16 μC?

26. Derive an expression of capacitance of a co-axial cable used in transmission line having inner radius 'a', outer radius 'b' and length 'l'. If a = 0.1mm and b = 0.6mm. Calculate capacitance per metre of the cable. Assume that the space between the conductors is filled with polystyrene of dielectric constant 2.6.

27. Two dielectric slabs are inserted between the plates of parallel plate capacitor. The thickness of the slab with dielectric constant k_1 is t_1 and of other slab having dielectric constant k_2 is t_2. The plates of capacitors are $t = t_1 + t_2$ apart, with area of each plate being A. Find the capacitance of new arrangement.

28. A capacitor of capacitance 400 μF is initially charged by a source of 50 V. After fully charging, it is disconnected from the source and is

joined to another capacitor of 200 μF capacitance. What is the loss of energy due to redistribution of charge?

29. What is capacitance of a spherical capacitor made of two concentric spheres of radii 6 and 6.01 cm? The space between them is filled with a medium with relative permittivity equal to 8.

30. A capacitor C is discharged through a resistor, R
    i. After how many time constant does its charge fall to half of its initial value.
    ii. After how many time constant does the stored energy drop to half of its initial value.

31. A circuit contains 40 kΩ resistor and fully charged 800 μF capacitor. What will be the charge remained on the capacitor after 1 ms?

32. What is the capacitance of cylindrical capacitor made of two co-axial cylinders of length 2 cm and of radii 2mm and 2.1 mm. The space between the cylinders is filled with a medium of relative permittivity 7.8.

❑❑❑

Chapter 7

Electricity and Magnetism

Current:

The time rate of flow of electric charge is called current.

i.e. $I = \frac{q}{t}$

All moving charges can not constitute an electric current. If there is to be an electric current through a given surface there must be net flow of charge through that surface.

Unit, S.I. Coulomb per second = Ampere.

current is a scalar quantity because both charge and time are scalars. The arrows in electric circuit is to indicate that charge is moving.

Direction of current:

A current arrow is drawn in the direction in which positive charge would move, even if the actual charge carries are negative and move in the opposite direction. This convention is made because in most situations, the motion of positive charge carries have the same effect as the actual motion of negative charge carriers in opposite direction.

Current density:

The current density is equal to the current per unit area. If $\vec{dA}$ is the small area element, then the amount of current though the element is $\vec{J} \cdot \vec{dA}$. Here $\vec{dA}$ is perpendicular to the direction of current.

∴ Total current through the surface, $I = \int \vec{J} \cdot \vec{dA}$

If $\vec{J}$ is uniform and parallel to $\vec{dA}$ then

$$I = \int J\,dA = J\int dA = JA.$$

$$\text{or, } J = \frac{I}{A}$$