DM Assignment 1

Narayan Somaiya

June 2nd 2024

Problem 1

Sol. To prove: There does not exist any positive number such that $n^2+n^3 = 100$
Method 1: $n^3 = 100 - n^2$
$n^3 = (10-n)(10+n) > 0$ because $n > 0$. Now $n > 0$ and $n < 10$ since if $n > 10$ then

$$(10 - n) < 0$$

which we can't accept as n will become negative then. Now $n^2 = 100 - n^3$, so, $n = \sqrt{100 - n^3}$. Now $\sqrt{100 - n^3} > 0$ so all $n > 4$ is trashed. Now for n = 1, 2, 3, 4 $\sqrt{100 - n^3}$ will not be a perfect square so n will not satisfy this equation as a number belonging to positive integers.

Method 2:- Manual trial and error.

Problem 2

Sol. To prove:

$$n^2 + 1 \geq 2^n$$

when n is a positive integer with $1 \geq n \geq 4$.
For $n = 1$, $n^2 + 1 = 1 + 1 = 2$ and $2^n = 2^1 = 2 \therefore \mathrm{n}^2 + 1 \geq 2^n$ when $n = 1$.
For $n = 2$, $n^2 + 1 = 4 + 1 = 5$ and $2^n = 2^2 = 4 \therefore \mathrm{n}^2 + 1 \geq 2^n$ when $n = 2$.
For $n = 3$, $n^2 + 1 = 9 + 1 = 10$ and $2^n = 2^3 = 8 \therefore \mathrm{n}^2 + 1 \geq 2^n$ when $n = 3$.
For $n = 4$, $n^2 + 1 = 16 + 1 = 17$ and $2^n = 2^4 = 16 \therefore \mathrm{n}^2 + 1 \geq 2^n$ when $n = 4$.

Problem 3

Sol. Some compound proposition of p, q, r and s is true when exactly 3 of them is true and otherwise it is false.
Compound proposition of four propositional variables is difficult to think but we are comfortable with two, right, so as per our conditions let us take $A = p \land q \land r$ so that A is True only when p, q, and r true simultaneously otherwise A is False. Also we know $A \land \neg s$ truth table.

A	s	$\neg s$	$A \land \neg s$
T	F	T	T
T	T	F	F
F	F	T	F
F	T	F	F

Hence for s = F when A = T satisfies our condition as

s = T is forbidden when A = T because we need exactly 3 T's when A = F it means at least one of p, q or r is F. So $p \wedge q \wedge r \wedge \neg s$ satisfies our required condition but it restricts us to fixed truth values for our variables. So in order to remove this restriction we add similar propositions as it won't change the nature of our proposition as if "this or this or this..." i.e.,
$(p \wedge q \wedge r \wedge \neg s) \vee (p \wedge q \wedge \neg r \wedge s) \vee (p \wedge \neg q \wedge r \wedge s) \vee (\neg p \wedge q \wedge r \wedge s)$
So required answer is as above.

Problem 4
Sol. Can't understand question clearly. Googled it also but didn't get it from there as well.

Problem 5
Sol. Let $a \in A$ and hence by definition of power set that it is a set of all subsets of A, $a \in P(A)$.
Given that $P(A) \subseteq P(B) \implies a \in P(B)$. Hence all elements of $A \in P(B)$, hence we can say that all elements of set A also belongs to set B. Hence $A \subseteq B$. Hence proved.

Problem 6
Sol. Given: $A \cap B = A$.
To prove: $A \subseteq B$ iff $A \cap B = A$
Let $a \in A$, so as per given information $a \in A \cap B$ also. Hence $a \in B$ follows that. Hence $A \subseteq B$. Hence proved