Project 3

CS325 — Spring 2015

by Group 2
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1 Transshipment Model

Part A

1. min(
   15*P1W1R1 + 16*P1W1R2 + 17*P1W1R3 + 20*P1W1R4 + 27*P1W2R3 + 23*P1W2R4 + 25*P1W2R5 + 29*P1W2R6 + 16*P2W1R1 + 17*P2W1R2 + 18*P2W1R3 + 21*P2W1R4 + 20*P2W2R3 + 16*P2W2R4 + 18*P2W2R5 + 22*P2W2R6 + 18*P3W1R1 + 19*P3W1R2 + 20*P3W1R3 + 23*P3W1R4 + 20*P3W2R3 + 16*P3W2R4 + 18*P3W2R5 + 22*P3W2R6 + 23*P3W3R4 + 21*P3W3R5 + 21*P3W3R6 + 15*P3W3R7 + 26*P4W2R3 + 22*P4W2R4 + 24*P4W2R5 + 28*P4W2R6 + 22*P4W3R4 + 20*P4W3R5 + 20*P4W3R6 + 14*P4W3R7
)

   (Supply constraints)
   P1W1R1 + P1W1R2 + P1W1R3 + P1W1R4 + P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 ≤ 150,
   P2W1R1 + P2W1R2 + P2W1R3 + P2W1R4 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 ≤ 450,
   P3W1R1 + P3W1R2 + P3W1R3 + P3W1R4 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P3W3R4 + P3W3R5 + P3W3R6 + P3W3R7 ¡= 250,
   P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 + P4W3R4 + P4W3R5 + P4W3R6 + P4W3R7 ≤ 150,
   (Demand constraints)
   P1W1R1 + P2W1R1 + P3W1R1 ≥ 100,
   P1W1R2 + P2W1R2 + P3W1R2 ≥ 150,
   P1W1R3 + P1W2R3 + P2W1R3 + P2W2R3 + P3W1R3 + P3W2R3 + P4W2R3 ≥ 100,
   P1W1R4 + P1W2R4 + P2W1R4 + P2W2R4 + P3W1R4 + P3W2R4 + P3W3R4 + P4W2R4 + P4W3R4 ≥ 200,
   P1W2R5 + P2W2R5 + P3W2R5 + P3W3R5 + P4W2R5 + P4W3R5 ≥ 200,
   P1W2R6 + P2W2R6 + P3W2R6 + P3W3R6 + P4W2R6 + P4W3R6 ≥ 150,
   P3W3R7 + P4W3R7 ≥ 100

2. The input was the following:

```
MIN 15 P1W1R1 + 16 P1W1R2 + 17 P1W1R3 + 20 P1W1R4 + 27 P1W2R3 + 23 P1W2R4 +
25 P1W2R5 + 29 P1W2R6 + 16 P2W1R1 + 17 P2W1R2 + 18 P2W1R3 + 21 P2W1R4 +
20 P2W2R3 + 16 P2W2R4 + 18 P2W2R5 + 22 P2W2R6 + 18 P3W1R1 + 19 P3W1R2 +
20 P3W1R3 + 23 P3W1R4 + 20 P3W2R3 + 16 P3W2R4 + 18 P3W2R5 + 22 P3W2R6 +
```

```
23 P3W3R4 + 21 P3W3R5 + 21 P3W3R6 + 15 P3W3R7 + 26 P4W2R3 + 22 P4W2R4 +
24 P4W2R5 + 28 P4W2R6 + 22 P4W3R4 + 20 P4W3R5 + 20 P4W3R6 + 14 P4W3R7
ST
! Supply Constraints
P1W1R1 + P1W1R2 + P1W1R3 + P1W1R4 + P1W2R3 + P1W2R4 + P1W2R5 + P1W2R6 < 150
P2W1R1 + P2W1R2 + P2W1R3 + P2W1R4 + P2W2R3 + P2W2R4 + P2W2R5 + P2W2R6 < 450
P3W1R1 + P3W1R2 + P3W1R3 + P3W1R4 + P3W2R3 + P3W2R4 + P3W2R5 + P3W2R6 + P3W3R4 +
   P3W3R5 + P3W3R6 + P3W3R7 < 250
P4W2R3 + P4W2R4 + P4W2R5 + P4W2R6 + P4W3R4 + P4W3R5 + P4W3R6 + P4W3R7 < 150

! Demand Constraints
P1W1R1 + P2W1R1 + P3W1R1 > 100
P1W1R2 + P2W1R2 + P3W1R2 > 150
P1W1R3 + P1W2R3 + P2W1R3 + P2W2R3 + P3W1R3 + P3W2R3 + P4W2R3 > 100
P1W1R4 + P1W2R4 + P2W1R4 + P2W2R4 + P3W1R4 + P3W2R4 + P3W3R4 + P4W2R4 + P4W3R4 > 200
P1W2R5 + P2W2R5 + P3W2R5 + P3W3R5 + P4W2R5 + P4W3R5 > 200
P1W2R6 + P2W2R6 + P3W2R6 + P3W3R6 + P4W2R6 + P4W3R6 > 150
P3W3R7 + P4W3R7 > 100
END
```

The following was the result:

```
 LP OPTIMUM FOUND AT STEP      6

 OBJECTIVE FUNCTION VALUE

 1)      17100.00

 VARIABLE        VALUE          REDUCED COST
 P1W1R1         0.000000          0.000000
 P1W1R2        50.000000          0.000000
 P1W1R3       100.000000          0.000000
 P1W1R4         0.000000          5.000000
 P1W2R3         0.000000         10.000000
 P1W2R4         0.000000          8.000000
 P1W2R5         0.000000          8.000000
 P1W2R6         0.000000          9.000000
 P2W1R1       100.000000          0.000000
```

P2W1R2	100.000000	0.000000
P2W1R3	0.000000	0.000000
P2W1R4	0.000000	5.000000
P2W2R3	0.000000	2.000000
P2W2R4	50.000000	0.000000
P2W2R5	200.000000	0.000000
P2W2R6	0.000000	1.000000
P3W1R1	0.000000	2.000000
P3W1R2	0.000000	2.000000
P3W1R3	0.000000	2.000000
P3W1R4	0.000000	7.000000
P3W2R3	0.000000	2.000000
P3W2R4	150.000000	0.000000
P3W2R5	0.000000	0.000000
P3W2R6	0.000000	1.000000
P3W3R4	0.000000	7.000000
P3W3R5	0.000000	3.000000
P3W3R6	100.000000	0.000000
P3W3R7	0.000000	0.000000
P4W2R3	0.000000	9.000000
P4W2R4	0.000000	7.000000
P4W2R5	0.000000	7.000000
P4W2R6	0.000000	8.000000
P4W3R4	0.000000	7.000000
P4W3R5	0.000000	3.000000
P4W3R6	50.000000	0.000000
P4W3R7	100.000000	0.000000

3. Ship 50 from P1 to R2 through W1
   Ship 100 from P1 to R3 through W1
   Ship 100 from P2 to R1 through W1
   Ship 100 from P2 to R2 through W1
   Ship 50 from P2 to R4 through W2
   Ship 200 from P2 to R5 through W2
   Ship 150 from P3 to R4 through W2
   Ship 100 from P3 to R6 through W3
   Ship 50 from P4 to R6 through W3
   Ship 100 from P4 to R7 through W3

Cost: \$17100.00

Part B

Infeasible because it is not possible to meet the demand for R5, R6, and R7.
Without W2, P1 and P2 can only reach R-4, which means P3 and P4 are the only two that can reach R5-7.
We can use P1 and P2 supplies (600) to meet the demand for R1-4 (550). This leaves the most supplies possible for P3 and P4 to meet the demand for R5-7.
However, the supplies for P3 and P4 (400) are not sufficient to meet the demand for R5-7 (450). Since the remaining 50 refrigerators needed for R5-7 cannot come from P1 or P2 since W2 has closed, the problem is infeasible.

Part C

Ship 50 from P1 to R1 through W1
Ship 100 from P1 to R3 through W1
Ship 50 from P2 to R1 through W1
Ship 150 from P2 to R2 through W1
Ship 150 from P2 to R4 through W1
Ship 50 from P2 to R4 through W2
Ship 50 from P2 to R5 through W2
Ship 100 from P3 to R5 through W3
Ship 150 from P3 to R6 through W3
Ship 50 from P4 to R5 through W3
Ship 100 from P4 to R7 through W3
Cost: \$18300.00

Part D

P = Number of plants
W = Number of warehouses
R = Number of retailers

$$min\left\{\sum_{i=1}^{P}\left(\sum_{j=1}^{W}\left(\sum_{k=1}^{R}(costofx(i,j,k))\right)\right)\right\}$$

s.t.

```
for i from 1 to P{
```

```
Sum(j = 1 to W Sum(k = 1 to R (x(i,j,k)) ) < supply of i)
}
for k from 1 to R{
Sum( i = 1 to P Sum( j = 1 to W (x(i,j,k)) ) < demand of k)
}
```

2 Modified from DPV 7.16

Part A

1. The following is the objective function for this part of the problem. We know this because the question requires us to find the solution with the least number of calories, and the following equation has the necessary values to be able to figure that out.
   Note: The variables ($v_1 ... v_8$) correspond to the ingredients mentioned in the problem in order, starting from Tomato to Oil.

$$Minimize(21v_1 + 16v_2 + 40v_3 + 41v_4 + 585v_5 + 120v_6 + 164v_7 + 884v_8)$$

The following are all the constraints that the problem mentioned in the beginning.
Protein:

$$(0.85v_1 + 1.62v_2 + 2.86v_3 + 0.93v_4 + 23.4v_5 + 16v_6 + 9v_7) \geq 15$$

Fat:

$$8 \geq (0.33v_1 + 0.2v_2 + 0.39v_3 + 0.24v_4 + 48.7v_5 + 5v_6 + 2.6v_7 + 100v_8) \geq 2$$

Carbohydrates:

$$(4.64v_1 + 2.37v_2 + 3.63v_3 + 9.58v_4 + 15v_5 + 3v_6 + 27v_7) \geq 4$$

Sodium:

$$(9v_1 + 28v_2 + 65v_3 + 69v_4 + 3.8v_5 + 120v_6 + 78v_7) \leq 200$$

Leafy Greens:

$$\frac{v_2 + v_3}{v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8} \geq \frac{4}{10}$$

$$v_2 + v_3 \geq \frac{4}{10} \cdot (v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8)$$

$$v_2 + v_3 \geq 0.4(v_1) + 0.4(v_2) + 0.4(v_3) + 0.4(v_4) + 0.4(v_5) + 0.4(v_6) + 0.4(v_7) + 0.4(v_8)$$

$$v_2 + v_3 - 0.4(v_1) - 0.4(v_2) - 0.4(v_3) - 0.4(v_4) - 0.4(v_5) - 0.4(v_6) - 0.4(v_7) - 0.4(v_8) \geq 0$$

In the above equation, counting from the top, I know that v_2 and v_3 are Lettuce and Spinach. Also, the last equation is a simplified version of the first.

2. LINDO was used for this problem and the following is what we got.

```
MIN 21V1 + 16V2 + 40V3 + 41V4 + 585V5 + 120V6 + 164V7 + 884V8
ST
0.85V1 + 1.62V2 + 2.86V3 + 0.93V4 + 23.4V5 + 16V6 + 9V7 >= 15
0.33V1 + 0.2V2 + 0.39V3 + 0.24V4 + 48.7V5 + 5V6 + 2.6V7 + 100V8 >= 2
0.33V1 + 0.2V2 + 0.39V3 + 0.24V4 + 48.7V5 + 5V6 + 2.6V7 + 100V8 <= 8
4.64V1 + 2.37V2 + 3.63V3 + 9.58V4 + 15V5 + 3V6 + 27V7 >= 4
9V1 + 28V2 + 65V3 + 69V4 + 3.8V5 + 120V6 + 78V7 <= 200
V2 + V3 - 0.4V1 - .4V2 - .4V3 - .4V4 - .4V5 - .4V6 - .4V7 - .4V8 > 0
V1 >= 0
V2 >= 0
V3 >= 0
V4 >= 0
V5 >= 0
V6 >= 0
V7 >= 0
V8 >= 0
END
```

The result we got back is the following:

```
LP OPTIMUM FOUND AT STEP     12
OBJECTIVE FUNCTION VALUE
1)      114.7541
VARIABLE        VALUE          REDUCED COST
V1         0.000000         16.901640
V2         0.585480          0.000000
V3         0.000000         14.513662
V4         0.000000         36.289616
V5         0.000000        408.387970
V6         0.878220          0.000000
V7         0.000000         97.551910
V8         0.000000        886.404358
```

3. We take the values we got in the problem and multiply them by the corresponding costs as mentioned in the question. The following is the answer:

$$0.58548(0.75) + 0.87822(2.15) = \$2.33$$

Part B

1. The objective function for this equation is the following:

$$Minimize(1.00v_1 + 0.75v_2 + 0.50v_3 + 0.50v_4 + 0.45v_5 + 2.15v_6 + 0.95v_7 + 2.00v_8)$$

   The constraint equations are mostly the same from the previous problem. The only addition is the objective function from Part A. We know that the least calorie is going to be 114.75, so we decided to add a constraint so that the calorie count will be greater than or equal to that value.

$$21V1 + 16V2 + 40V3 + 41V4 + 585V5 + 120V6 + 164V7 + 884V8 >= 114.75$$

2. Once again LINDO was used for this problem.

```
MIN 1.00V1 + 0.75V2 + 0.50V3 + 0.50V4 + 0.45V5 + 2.15V6 + 0.95V7 + 2V8
ST
0.85V1 + 1.62V2 + 2.86V3 + 0.93V4 + 23.4V5 + 16V6 + 9v7 >= 15
0.33V1 + 0.2V2 + 0.39V3 + 0.24V4 + 48.7V5 + 5V6 + 2.6V7 + 100v8 >= 2
0.33V1 + 0.2V2 + 0.39V3 + 0.24V4 + 48.7V5 + 5V6 + 2.6V7 + 100V8 <= 8
4.64V1 + 2.37V2 + 3.63V3 + 9.58V4 + 15V5 + 3V6 + 27V7 >= 4
9V1 + 28V2 + 65V3 + 69V4 + 3.8V5 + 120V6 + 78V7 <= 200
V2 + V3 - .4V1 - .4V2 - .4V3 - .4V4 - .4V5 - .4V6 - .4V7 - .4V8 > 0
21V1 + 16V2 + 40V3 + 41V4 + 585V5 + 120V6 + 164V7 + 884V8 >= 114.75
V1 >= 0
V2 >= 0
V3 >= 0
V4 >= 0
V5 >= 0
V6 >= 0
V7 >= 0
V8 >= 0
END
```

   The following is the result.

```
LP OPTIMUM FOUND AT STEP      0
OBJECTIVE FUNCTION VALUE
1)      1.554133
VARIABLE        VALUE          REDUCED COST
```

V1	0.000000	1.002081
V2	0.000000	0.402912
V3	0.832298	0.000000
V4	0.000000	0.486914
V5	0.096083	0.000000
V6	0.000000	0.405609
V7	1.152364	0.000000
V8	0.000000	7.281258

3. Similar to that of Part A, we use the values we got above and apply them to the Energy column in the table that was given to us.

$$40(0.832298) + 585(0.096083) + 164(1.152364) = 278.49 calories$$

Part C

1. It's important to start off by saying that there are always sacrifices to be made. It comes down to priority, and which requirement is more important. This being said, there are a variety of things that Veronica could do to reach her goal. One of the first things to take notice is how both sunflower seeds and oil are really high in calorie. If she wanted to advertise that her salads were less than 250 calories, perhaps looking for an alternative to sunflower seeds and oil may help. If there were ingredients with lesser calories, then the chances of increasing the combinations for the salad are high.
   On the other hand, Veronica could buy cheaper ingredients to be able to meet her cost requirement. This also possibly means forgoing her calorie requirement. Another cost saving technique might be trying to use the least number of ingredients. Part A actually happened to do this. With just lettuce and smoked tofu, while the salad seemed slightly boring, it met all the requirements that were set in the problem. On the same idea, perhaps looking for newer ingredients that could do achieve the same effect would be useful too. The possible solution maybe involves widening the original requirements that were set. For example, decreasing the protein requirement to 13g, increasing the fat range to 2-10g, 200-220mg of sodium, and 35% leafy greens by mass. With the widening of ranges, Veronica would get more opportunities to either save money, or decrease the calorie count.

2. From Part A, I would suggest using calorie effective method. The ingredients involved here are lettuce and smoked tofu. The cost is \$2.33, but the calorie count will be 114.75.

3. If she sold these, Veronica would make about \$2.67. This definitely does not meet the expectation she set, but its important to note that the calorie count is far below 250, and

she will be selling more of them. What she loses in profit per salad, she will make it up in the amount of salads she sells.

Note: Customer enjoyment is not taken into account in this problem, but if that was taken into account, then it will definitely matter! We do believe that it will make a difference.

3 Regression Solution via Linear Programming

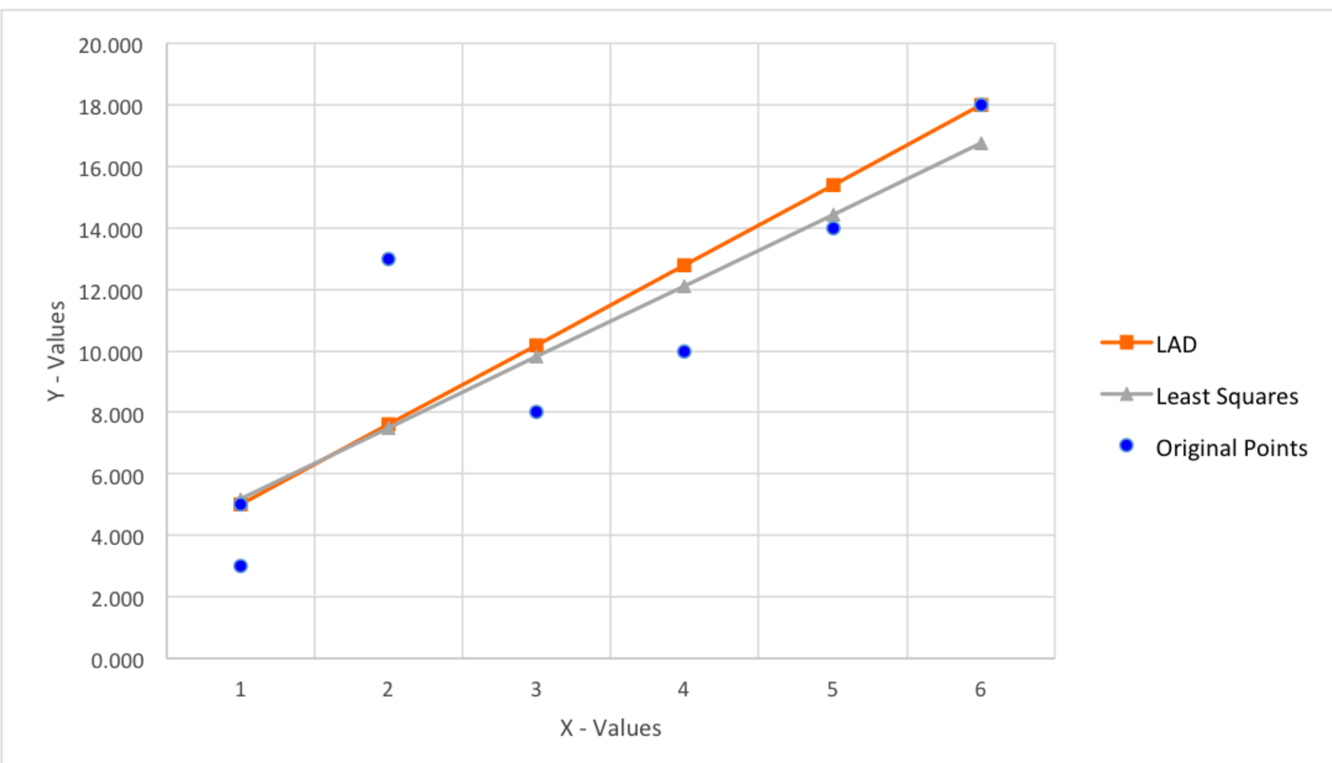
Part A

Objective: $\min \sum_{i=1}^{n} |y_i - (a_1 x_i + a_0)|$

Constraint Equations

$a_0 + a_1 + z_1 \geq 3$	$a_0 + a_1 - z_1 \leq 3$
$a_0 + a_1 + z_2 \geq 5$	$a_0 + a_1 - z_2 \leq 5$
$a_0 + 2a_1 + z_3 \geq 13$	$a_0 + 2a_1 - z_3 \leq 13$
$a_0 + 3a_1 + z_4 \geq 8$	$a_0 + 3a_1 - z_4 \leq 8$
$a_0 + 4a_1 + z_5 \geq 10$	$a_0 + 4a_1 - z_5 \leq 10$
$a_0 + 5a_1 + z_6 \geq 14$	$a_0 + 5a_1 - z_6 \leq 14$
$a_0 + 6a_1 + z_7 \geq 18$	$a_0 + 6a_1 - z_7 \leq 18$

Sum of absolute deviations: 13.8



This was calculated in Lindo using the constraint equations listed above to obtain the LAD equation $y = 2.6x + 2.4$. In this case, the LAD is close to the original Least Squares solution, especially for x-values close to 0. It appears that as the value of x grows, the gap between the 2 lines will widen.

Lindo code:

```
MIN X + Y + Z
ST
    A0+A1+Z1>=3
    A0+A1-Z1<=3
    A0+A1+Z2>=5
    A0+A1-Z2<=5
    A0+2A1+Z3>=13
    A0+2A1-Z3<=13
    A0+3A1+Z4>=8
    A0+3A1-Z4<=8
    A0+4A1+Z5>=10
    A0+4A1-Z5<=10
    A0+5A1+Z6>=14
    A0+5A1-Z6<=14
    A0+6A1+Z7>=18
    A0+6A1-Z7<=18
A0>0
A1>0
END
```

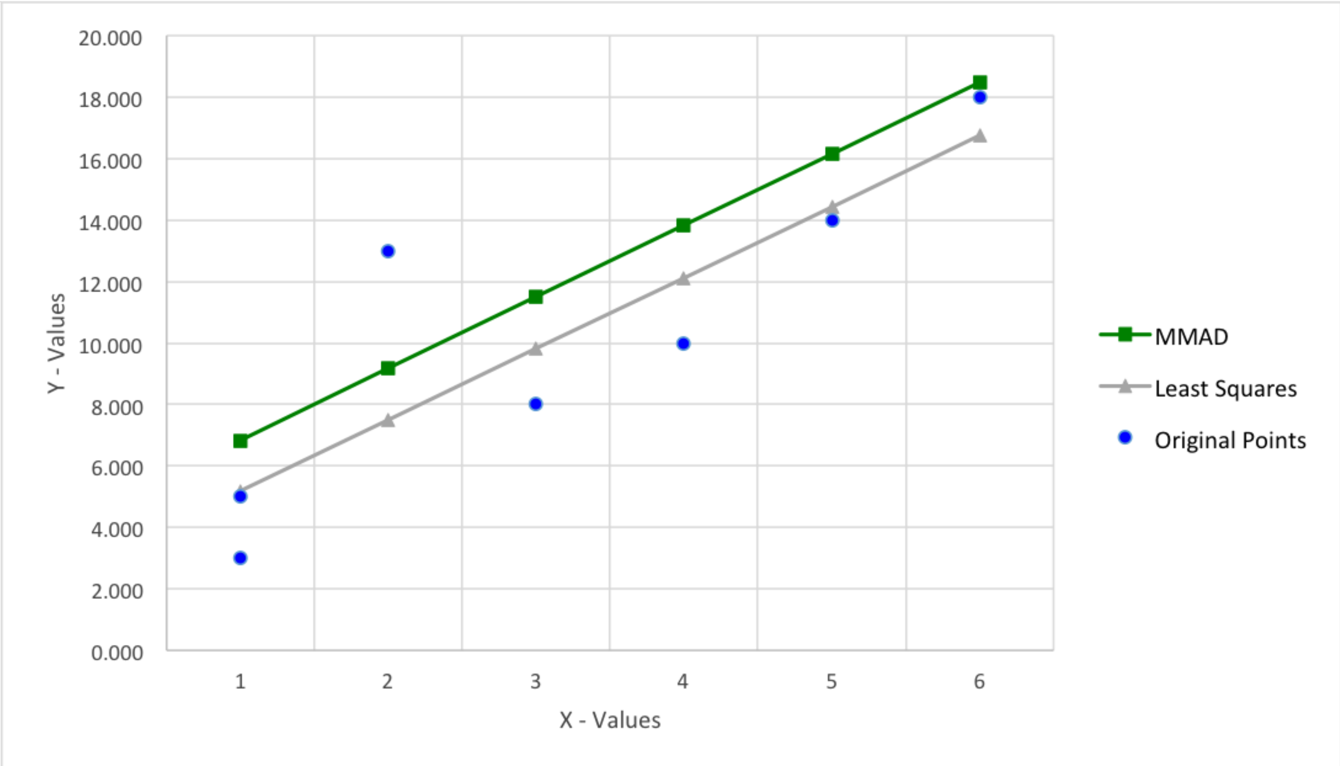
Part B

Objective: min $max|y_i - (a_1 x_i + a_0)|$

Constraint Equations

$a_0 + a_1 + z \geq 3$	$a_0 + a_1 - z \leq 3$
$a_0 + a_1 + z \geq 5$	$a_0 + a_1 - z \leq 5$
$a_0 + 2a_1 + z \geq 13$	$a_0 + 2a_1 - z \leq 13$
$a_0 + 3a_1 + z \geq 8$	$a_0 + 3a_1 - z \leq 8$
$a_0 + 4a_1 + z \geq 10$	$a_0 + 4a_1 - z \leq 10$
$a_0 + 5a_1 + z \geq 14$	$a_0 + 5a_1 - z \leq 14$
$a_0 + 6a_1 + z \geq 18$	$a_0 + 6a_1 - z \leq 18$

min of the max absolute deviations: 3.8333



This was calculated again in Lindo using the constraint equations above to obtain the MMAD equation $y = 2.3x + 4.5$. By contrast, the MMAD parallels the Least Squares by almost a full y-value above the Least Squares line. This makes sense since we are merely optimizing or taking the minimum of the maximum possible deviation.

Lindo code:

```
MIN X + Y + Z
ST
    A0+A1+Z>=3
    A0+A1-Z<=3
    A0+A1+Z>=5
    A0+A1-Z<=5
    A0+2A1+Z>=13
    A0+2A1-Z<=13
    A0+3A1+Z>=8
    A0+3A1-Z<=8
    A0+4A1+Z>=10
    A0+4A1-Z<=10
```

```
    A0+5A1+Z>=14
    A0+5A1-Z<=14
    A0+6A1+Z>=18
    A0+6A1-Z<=18
A0>0
A1>0
END
```

Intuitively, it is not immediately apparent that a non-linear dataset could exist that would produce the same line using all 3 methods, but after some thought and discussion a solution presented itself. Take the following dataset:

(x,y) = { (1,2), (1, 3), (1, 4), (7, 5), (7,6), (7, 7) }

Testing all 3 methods in Lindo produced the same regression equation: $y = 0.5x + 2.5$. Therefore, a dataset does exist that will yield the same line via all 3 methods.

Part C

Objective: min $\sum_{i=1}^{n} |y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)|$ as an LP.

Constraint Equations

$a_2 + a_1 + a_0 + z_1 \geq 5$	$a_2 + a_1 + a_0 - z_1 \leq 5$
$2a_2 + a_1 + a_0 + z_2 \geq 9$	$2a_2 + a_1 + a_0 - z_2 \leq 9$
$2a_2 + 2a_1 + a_0 + z_3 \geq 12$	$2a_2 + 2a_1 + a_0 - z_3 \leq 12$
$a_2 + 0a_1 + a_0 + z_4 \geq 3$	$a_2 + 0a_1 + a_0 - z_4 \leq 3$
$0a_2 + 0a_1 + a_0 + z_5 \geq 0$	$0a_2 + 0a_1 + a_0 - z_5 \leq 0$
$3a_2 + a_1 + a_0 + z_6 \geq 11$	$3a_2 + a_1 + a_0 - z_6 \leq 11$

This was again calculated in Lindo using the constraint equations above to obtain the LAD equation $y = 3x_2 + 3x_1$.

```
MIN W + X + Y + Z
ST
    A0+A1+A2+Z1>=5
    A0+A1+A2-Z1<=5
    A0+A1+2A2+Z2>=9
    A0+A1+2A2-Z2<=9
    A0+2A1+2A2+Z3>=12
    A0+2A1+2A2-Z3<=12
    A0+0A1+A2+Z4>=3
```

```
    A0+0A1+A2-Z4<=3
    A0+0A1+0A2+Z5>=0
    A0+0A1+0A2-Z5<=0
    A0+A1+3A2+Z6>=11
    A0+A1+3A2-Z6<=11
A0>0
A1>0
A2>0
END
```