Project 3

CS325 — Spring 2015

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1 Transshipment Model

Part A

Part B

Part C

Part D

2 Modified from DPV 7.16

Part A

Part B

Part C

3 Regression Solution via Linear Programming

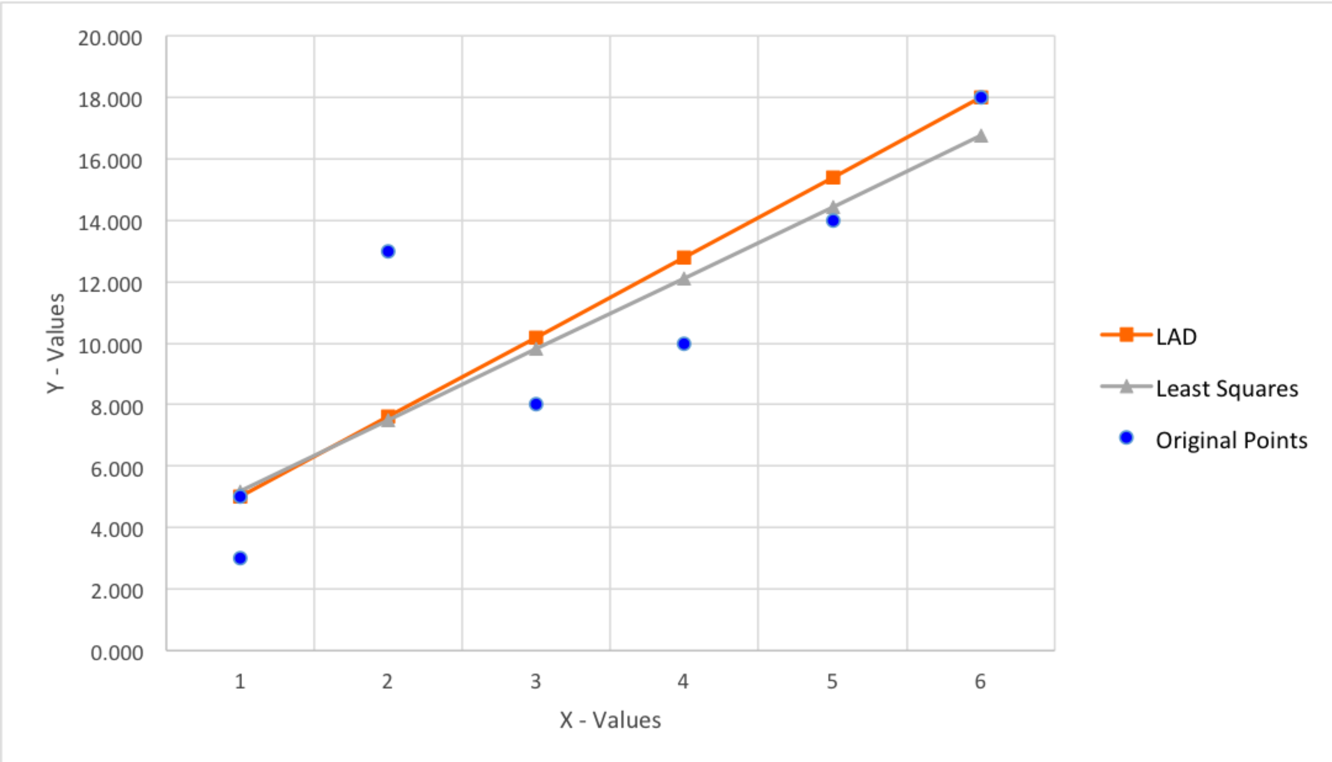
Part A

Objective: $\min \sum_{i=1}^{n} |y_i - (a_1 x_i + a_0)|$ as an LP.

Constraint Equations

$a_0 + a_1 + z_1 \geq 3$	$a_0 + a_1 - z_1 \leq 3$
$a_0 + a_1 + z_2 \geq 5$	$a_0 + a_1 - z_2 \leq 5$
$a_0 + 2a_1 + z_3 \geq 13$	$a_0 + 2a_1 - z_3 \leq 13$
$a_0 + 3a_1 + z_4 \geq 8$	$a_0 + 3a_1 - z_4 \leq 8$
$a_0 + 4a_1 + z_5 \geq 10$	$a_0 + 4a_1 - z_5 \leq 10$
$a_0 + 5a_1 + z_6 \geq 14$	$a_0 + 5a_1 - z_6 \leq 14$
$a_0 + 6a_1 + z_7 \geq 18$	$a_0 + 6a_1 - z_7 \leq 18$

Sum of absolute deviations: 13.8



This was calculated in Lindo using the constraint equations listed above to obtain the LAD equation $y = 2.6x + 2.4$. In this case, the LAD is close to the original Least Squares solution, especially for x-values close to 0. It appears that as the value of x grows, the gap between the 2 lines will widen.

Lindo code:

```
MIN X + Y + Z
ST
    A0+A1+Z1>=3
    A0+A1-Z1<=3
    A0+A1+Z2>=5
    A0+A1-Z2<=5
    A0+2A1+Z3>=13
```

```
    A0+2A1-Z3<=13
    A0+3A1+Z4>=8
    A0+3A1-Z4<=8
    A0+4A1+Z5>=10
    A0+4A1-Z5<=10
    A0+5A1+Z6>=14
    A0+5A1-Z6<=14
    A0+6A1+Z7>=18
    A0+6A1-Z7<=18
A0>0
A1>0
END
```

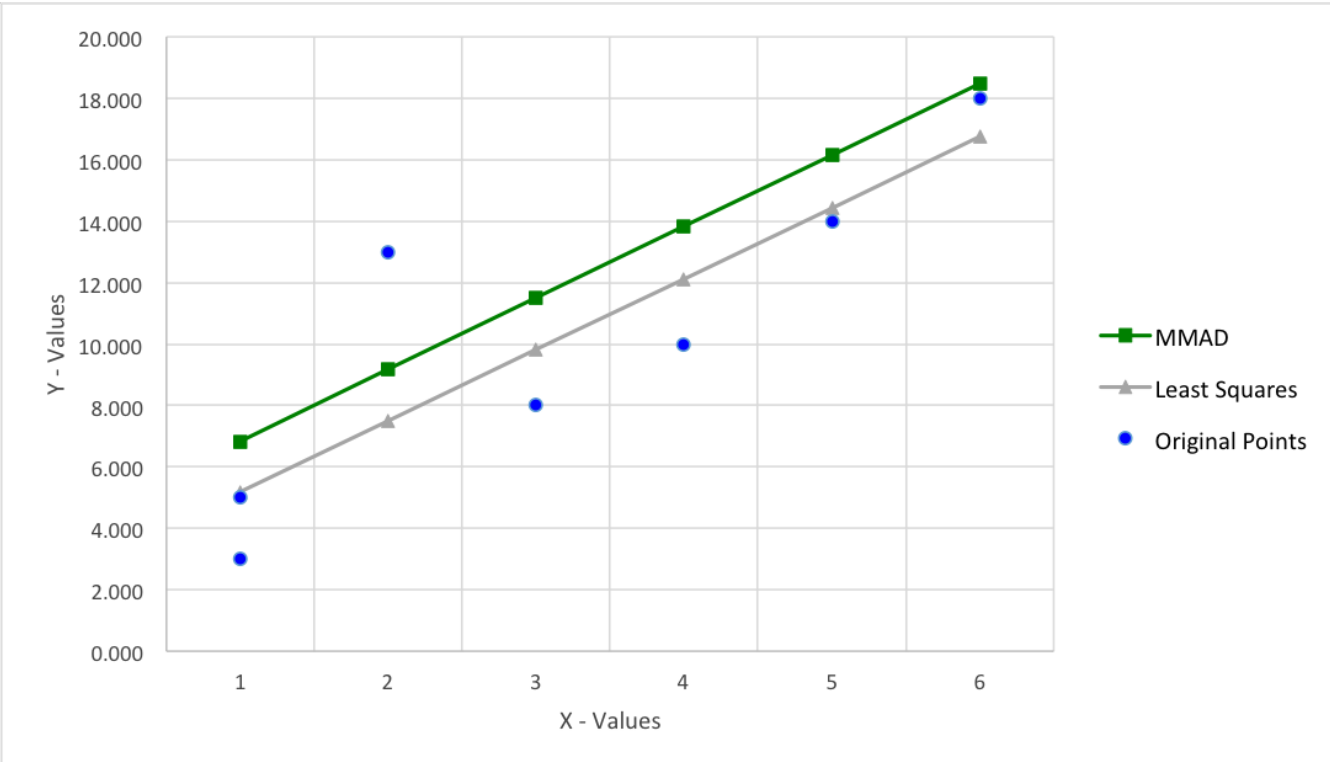
Part B

Objective: min $max|y_i - (a_1x_i + a_0)|$ as an LP.

Constraint Equations

$a_0 + a_1 + z \geq 3$	$a_0 + a_1 - z \leq 3$
$a_0 + a_1 + z \geq 5$	$a_0 + a_1 - z \leq 5$
$a_0 + 2a_1 + z \geq 13$	$a_0 + 2a_1 - z \leq 13$
$a_0 + 3a_1 + z \geq 8$	$a_0 + 3a_1 - z \leq 8$
$a_0 + 4a_1 + z \geq 10$	$a_0 + 4a_1 - z \leq 10$
$a_0 + 5a_1 + z \geq 14$	$a_0 + 5a_1 - z \leq 14$
$a_0 + 6a_1 + z \geq 18$	$a_0 + 6a_1 - z \leq 18$

min of the max absolute deviations: 3.8333



This was calculated again in Lindo using the constraint equations above to obtain the MMAD equation $y = 2.3x + 4.5$. By contrast, the MMAD parallels the Least Squares by almost a full y-value above the Least Squares line. This makes sense since we are merely optimizing or taking the minimum of the maximum possible deviation.

Lindo code:

```
MIN X + Y + Z
ST
    A0+A1+Z>=3
    A0+A1-Z<=3
    A0+A1+Z>=5
    A0+A1-Z<=5
    A0+2A1+Z>=13
    A0+2A1-Z<=13
```

```
    A0+3A1+Z>=8
    A0+3A1-Z<=8
    A0+4A1+Z>=10
    A0+4A1-Z<=10
    A0+5A1+Z>=14
    A0+5A1-Z<=14
    A0+6A1+Z>=18
    A0+6A1-Z<=18
A0>0
A1>0
END
```

Intuitively, it is not immediately apparent that a non-linear dataset could exist that would produce the same line using all 3 methods, but after some thought and discussion a solution presented itself. Take the following dataset:

(x,y) = (1,2), (1, 3), (1, 4), (7, 5), (7,6), (7, 7)

Testing all 3 methods in Lindo produced the same regression equation: $y = 0.5x + 2.5$. Therefore, a dataset does exist that will yield the same line via all 3 methods.

Part C

Objective: $\min \sum_{i=1}^{n} |y_i - (a_2 x_{2i} + a_1 x_{1i} + a_0)|$ as an LP.

Constraint Equations

$$
\begin{array}{ll}
a_2 + a_1 + a_0 + z_1 \geq 5 & a_2 + a_1 + a_0 - z_1 \leq 5 \\
2a_2 + a_1 + a_0 + z_2 \geq 9 & 2a_2 + a_1 + a_0 - z_2 \leq 9 \\
2a_2 + 2a_1 + a_0 + z_3 \geq 12 & 2a_2 + 2a_1 + a_0 - z_3 \leq 12 \\
a_2 + 0a_1 + a_0 + z_4 \geq 3 & a_2 + 0a_1 + a_0 - z_4 \leq 3 \\
0a_2 + 0a_1 + a_0 + z_5 \geq 0 & 0a_2 + 0a_1 + a_0 - z_5 \leq 0 \\
3a_2 + a_1 + a_0 + z_6 \geq 11 & 3a_2 + a_1 + a_0 - z_6 \leq 11
\end{array}
$$

This was again calculated in Lindo using the constraint equations above to obtain the LAD equation $y = 3x_2 + 3x_1$.

```
MIN W + X + Y + Z
ST
    A0+A1+A2+Z1>=5
    A0+A1+A2-Z1<=5
    A0+A1+2A2+Z2>=9
    A0+A1+2A2-Z2<=9
    A0+2A1+2A2+Z3>=12
    A0+2A1+2A2-Z3<=12
    A0+0A1+A2+Z4>=3
    A0+0A1+A2-Z4<=3
    A0+0A1+0A2+Z5>=0
    A0+0A1+0A2-Z5<=0
    A0+A1+3A2+Z6>=11
    A0+A1+3A2-Z6<=11
A0>0
A1>0
A2>0
END
```