SPHINCS Interim Report

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Introduction

Over the past 10 weeks, we have been focusing on SPHINCS for our project. Most of the work leading up to the past couple weeks have been reading documents that explain the tools being used in SPHINCS. The single biggest challenge for us was primarily getting acquainted with the material. To properly, and throughly, understand SPHINCS we needed to get caught up with a lot of reading. There were multiple papers that required time and dedication to fully understand. Understanding the tools and technologies is crucial if we want to be successful. On top of this, we had the added challenge of figuring out how to piece together the technologies, and how SPHINCS uses them.
While we had a lot of catching up to do, we also did get a chance to work on something new. The actual focus is transcribed in this paper. With help from Professor Yavuz, our goal was to come up with a simpler signature scheme that withheld the security that can be incorporated into SPHINCS. The scheme we propose is called Lamport+.

Preliminaries

This section is utilized to briefly talk about existing signature schemes. The sole reason for this is so future references to the specific schemes are not ambiguous.

Lamport Signature Scheme

The Lamport Signature Scheme was created by Leslie Lamport in 1989, and it is the simplest signature scheme that exists. It is also the first One-Time signature that was invented. This scheme makes use of a cryptographic hash function that has already been predetermined.
To put it simply, the idea behind the scheme can be split into two separate pieces. Signer first generates the secret keys, public keys, hashes messages, and gets a signature that is passed off to the verifier. Now, the verifier's job more or less is to follow similar steps so they end up with a similar result. Thus, it can be argued that the message and signature could only come from the signer and no one else. The detailed version of the scheme is mentioned in the Lamport+ section.

WOTS

The primary idea behind the scheme is to break the messages into little blocks, that get processed together, and having an input run through a hash function several times. The

number of iterations entirely depends on the message that needs to be signed.
WOTS was built on top of the Lamport signature scheme, and the expectation is for it to be intuitive in its logic, but it's not the case. The complexity of the scheme is heavily influences by figuring out the number of iterations necessary for a value to go through the hash function.

WOTS+

WOTS+ is very similar to WOTS, expect for the addition of XORing random elements every time a value is iterated over hash function. In the key generation phase, WOTS+ generates a set of random numbers that will serve for XORing. Just like the keys are split into chunks, so are the random elements. They get incorporated in the following recursive chaining function

$$c_k^i(x, \mathbf{r}) = f_k(c_k^{i-1}(x, \mathbf{r}) \oplus r_i) \tag{1}$$

Th equation is strictly $i > 0$, but in the case of $i = 0$, $c_0^k(x, \mathbf{r}) = x$. The equation is clever in that it makes sure to XOR different values for every iteration.

Lamport+ Signature Scheme

Lamport+ Signature Scheme is the new scheme we are proposing. It not only brings the simplicity of the original Lamport scheme, but also pulls in elements of the WOTS+ scheme. Our hope is that the original scheme's security is withheld, if not enhanced. Please note that the security of the proposed scheme has not been proven, but it can very well be inferred from the previous. Similar to how WOTS+ introduces XORing of randomized elements to WOTS, the same principle is introduced to Lamport. The following is meant to give an idea of how Lamport+ would work as a One Time Signature, before we extrapolate it to hash chains. Take note of the following variables:

- $i \in \{0, 1\}$, denotes bit pair
- $n \in \mathbb{N}$, the security parameter
- $s \xleftarrow{\$} \{0, 1\}^m$, seed for the PRNG
- PRNG $g : \{0, 1\}^m \to \{0, 1\}^n, m < n$
- Cryographic hash function $f_k : \{0, 1\}^n \to \{0, 1\}^n, k \in \mathcal{K}$

Chaining Function

$$\begin{aligned} c^i(x) &= f_k(x_{1,n} \oplus g(x_{n+1,n+m})), i > 0 \\ c^0(x) &= x_{1,n} \end{aligned} \tag{2}$$

The idea for the chaining function, $c : \{0, 1\}^{n+m} \to \{0, 1\}^n$, is borrowed from WOTS+. Intuitively, the function just splits the seed from the passed in argument, and uses it

to XOR a PRN. The subscript, $x_{a,b}$ defines the specific bits used, i.e. $x_a, ..., x_b$. $a < b \leq (n+m)$ is assumed to be true.

Key Generation
Input: Security parameter n
Output: Secret key sk, Public key pk

$$\begin{aligned} sk &= ((x_1^0, x_1^1), ..., (x_n^0, x_n^1)) \\ &= ((c_1^0(x)||s_0, c_1^0(x)||s_1), ...(c_n^0(x)||s_0, c_n^0(x)||s_1)), x \in \{0,1\}^{n+m} \\ pk &= ((y_1^0, y_1^1), ..., (y_n^0, y_n^1)) \\ &= ((c_1^1(x_1^0)||s_0, c_1^1(x_1^1)||s_1), ...(c_n^1(x_n^0)||s_0, c_n^1(x_n^0)||s_1)) \end{aligned} \tag{3}$$

Signature Generation
Input: Secret key sk, hashed message M
Output: Signature σ

$$\sigma = (\sigma_1, ..., \sigma_n) = (x_1^{M_1}, ..., x_n^{M_n}), M_i \in \{0,1\} \tag{4}$$

Signature Verification
Input: Public key pk, hashed message M, and signature σ
Output: Verification pass or fail

$$pk = (y_1^i, ..., y_n^i) \stackrel{?}{=} (c_1^1(\sigma_1)||\sigma_{n+1,n+m}, ..., c_1^1(\sigma_n)||\sigma_{n+1,n+m}) \tag{5}$$

Lamport+ Hash Chain

Hash chains aid One-Time signatures to be used multiple times with a single key. This can be applied to the Lamport+ signature scheme. The underlying idea here is that after the public key is generated in the scheme, another set of keys are generated based on the previous public key getting hashed. As you can guess, expect the last public key of the chain, all keys before act as a private key.
There are two important things to make note of here. First off, we introduce a new parameter, which we call $l \in \mathbb{N}, l > 0$. This parameter helps keep track of how long the chain is, or how many messages can be signed. Secondly, the public keys will hold new PRNG seeds. These seeds will help generate XOR values to help produce the following set of public keys. The concept is better understood once all the steps are explained.

Key Generation
Input: Security parameter n
Output: Secret key sk, Public key pk

$$\begin{aligned} sk_k &= ((x_1^{0,k}, x_1^{1,k}), ..., (x_j^{0,k}, x_j^{1,k})) \\ &= ((c_1^k(x_1^{0,k-1})||s_0, c_1^k(x^{1,k-1})||s_1), ..., (c_j^k(x_j^{0,k-1})||s_0, c_j^k(x_j^{1,k-1})||s_1)), \\ &\quad \text{where } s \stackrel{\$}{\leftarrow} \{0,1\}^m, 0 \leq k \leq l, 1 \leq j \leq n \end{aligned} \tag{6}$$

Like you could guess, there will no separate public key that will be generated. If the hash chain is properly generated, this is how it should look:

$$
\begin{aligned}
sk_0 &= ((x_1^{0,0}, x_1^{1,0}), ..., (x_n^{0,0}, x_n^{1,0})) = ((c_1^0(x)||s_0, c_1^0(x)||s_1), ...(c_n^0(x)||s_0, c_n^0(x)||s_1)) \\
sk_1 &= ((x_1^{0,1}, x_1^{1,1}), ..., (x_n^{0,1}, x_n^{1,1})) = ((c_1^1(x_1^{0,0})||s_0, c_1^1(x_1^{1,0})||s_1), ...(c_n^1(x_n^{0,0})||s_0, c_n^1(x_n^{1,0})||s_1)) \\
&\vdots \\
sk_l &= ((x_1^{0,l}, x_1^{1,l}), ..., (x_n^{0,l}, x_n^{1,l})) = ((c_1^1(x_1^{0,l-1})||s_0, c_1^1(x_1^{1,l-1})||s_1), ...(c_n^1(x_n^{0,l-1})||s_0, c_n^1(x_n^{1,l-1})||s_1))
\end{aligned}
\tag{7}
$$

Signature Generation
Input: Secret key sk, hashed message M, Lamport parameter l
Output: Signature σ, Updated Lamport parameter l

$$
\begin{aligned}
\sigma &= (\sigma_1, ..., \sigma_n) = (x_1^{M_1,l-1}, ..., x_n^{M_n,l-1}), M_i \in \{0,1\} \\
l &= l - 1
\end{aligned}
\tag{8}
$$

Signature Verification
Input: Public key pk, hashed message M, and signature σ
Output: Verification pass or fail

$$
pk = (y_1^l, ..., y_n^l) \stackrel{?}{=} (c_1^1(\sigma_1)||\sigma_{n+1,n+m}, ..., c_1^1(\sigma_n)||\sigma_{n+1,n+m}) \tag{9}
$$

Lamport+ Hash Tree

The Lamport+ Hash Tree incorporates both the Lamport+ hash chains, and Merkle Hash Trees. The scheme shown here is loosely based on XMSS trees from [6]. The hash tree allows us to sign One of the most important things to note is that once a signature has been produced from all hash chains, the hash tree needs to be reconstructed. The caveat here is the need to know which leaves have already produced signatures already. An index variable i keep track of the next leaf index to be used, and the signer is required to generate signatures from left to right. Once $i = 2^H - 1$, we can determine that signatures for all chains have been used once, and the tree needs to be rehashed using the next set of keys from each of the hash chains.

- H : Height of tree
- i : Index of node at a level from left to right
- $f_k : \{0,1\}^{2n} \rightarrow \{0,1\}^n$
- Node : $\{0,1\}^{n+m}$

Key Generation
Begin by choosing an $H \geq 1$ variable first. This will allow us to have 2^H number of Lamport+ hash chains. The hash chains now need to be created. The following

equation shows to construct inner nodes. The leaf nodes will be the hashes of the public key of the hash chains, which is the last set of keys in the chain.
Input: 2^H hashed public keys of Lamport+ hash chains
Output: Lamport+ Hash Tree

$$\begin{aligned} Node_{i,h} &= f_k((Node_{i,h-1} \oplus g(s_0))||(Node_{i+1,h-1} \oplus g(s_1)))||s \\ &\text{where } 0 \leq i < 2^H, 0 \leq h < H \end{aligned} \tag{10}$$

Signature Generation
Once a signature is formed, the tree needs to be recalculated, as one of the leaves is going to store a new public key for one of the chains. All nodes on authentication path will be recalculated basically. New seeds stored for parent nodes? Seeds generated every time path needs to be reconstructed.
Before producing signature from a particular chain, make sure $l > 0$, because l dictates how many signatures can be produced from that chain.
Input: Hashed message M
Output: Authentication path A, signature σ, index i

$$a_h = \begin{cases} 1, & \text{if} \lfloor \frac{i}{2^h} \rfloor \equiv 0 \bmod 2 \\ 0, & \text{if} \lfloor \frac{i}{2^h} \rfloor \equiv 1 \bmod 2 \end{cases} \tag{11}$$

Signature Verification
There are two main steps to look out for here. First, using the secret key to verify the hashed message M. Once the OTS is verified, we have the leaf node for the hash tree. Secondly, traversing the hash tree helps us check the authenticity for the signature.
Input: Authentication path A, signature σ, hashed message M, Global public key pk when message was signed
Output: Verification pass or fail

$$0 = 0 \tag{12}$$

Security Considerations

Briefly talk about where and the idea for Lamport+ came about. Why are random values XORed?

Hash tree, new random values generated every time so adversary can't just use previous random values.

Conclusion

While an implementation for Lamport+ Hash Tree was created, it definitely can be made better. Given enough time, the algorithm could be optimized to not only be

faster, but cut down on costs, and even make a smaller memory footprint. This isn't entirely a promise, but a potential future work that would be worth investing time in.

In this report, we presented the Lamport+ Signature Scheme, which is much simpler than WOTS and WOTS+. WOTS+ is currently being used in SPHINCS. SPHINCS is indeed stateless, but if Lamport+ were to be integrated instead, the system would become stateful. We went through an example or two to make sure that the scheme worked and it was possible, but have not spent more time on it. We want to go over it again, and see how it fairs against edge cases and such. The plan is also to try to implement it, and integrate it with SPHINCS to see the results.

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