

$$\text{pos: } m_c: \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$$

$$m_p: \begin{bmatrix} -l \sin(x_2) + x_1 \\ l \cos(x_2) \end{bmatrix}$$

$$\text{vel: } m_c: \begin{bmatrix} \dot{x}_1 \\ 0 \end{bmatrix}$$

$$m_p: \begin{bmatrix} -l \cos(x_2) \dot{x}_2 + \dot{x}_1 \\ -l \sin(x_2) \dot{x}_2 \end{bmatrix}$$

$$L = K - P$$

$$K = \frac{1}{2} m_c \dot{x}_1^2 + \frac{1}{2} m_p [(-l \cos(x_2) \dot{x}_2 + \dot{x}_1)^2 + (-l \sin(x_2) \dot{x}_2)^2]$$

$$P = m_p g l \cos(x_2)$$

$$\begin{aligned}
 L &= \frac{1}{2} m_c \dot{x}_1^2 + \frac{1}{2} m_p [(-l \cos(x_2) \dot{x}_2 + \dot{x}_1)^2 \\
 &+ (-l \sin(x_2) \dot{x}_2)^2] + \frac{1}{2} I \dot{x}_2^2 - m_p g l \cos(x_2) \\
 &= \frac{1}{2} m_c \dot{x}_1^2 + \frac{1}{2} m_p [l^2 \dot{x}_2^2 - l \cos(x_2) \dot{x}_1 + \dot{x}_1^2] \\
 &\quad - m_p g l \cos(x_2)
 \end{aligned}$$

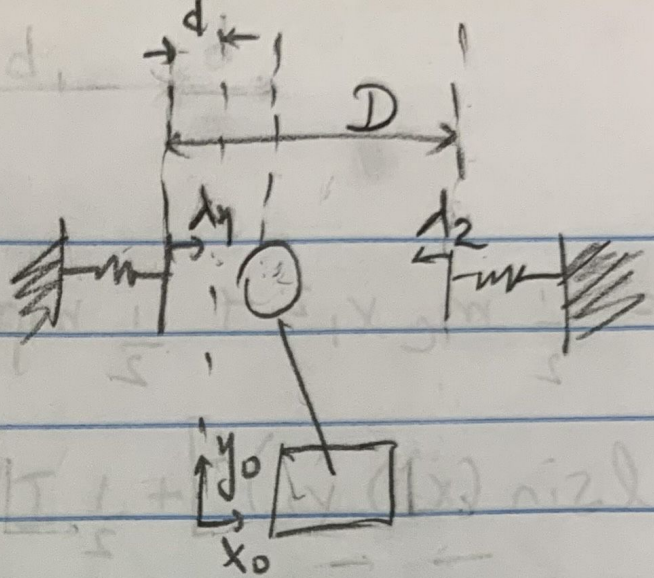
$$\begin{aligned}
 \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_1} - \frac{\partial L}{\partial x_1} &= \frac{d}{dt} \left[m_c \dot{x}_1 - \frac{m_p l \cos(x_2)}{2} + m_p \dot{x}_1 \right] \\
 &= \left[m_c \ddot{x}_1 + \frac{m_p l \sin(x_2)}{2} \dot{x}_2 + m_p \ddot{x}_1 \right]
 \end{aligned}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_2} - \frac{\partial L}{\partial x_2} = \frac{d}{dt} \left[m_p l^2 \dot{x}_2 + I \dot{x}_2 \right] -$$

$$+ \frac{1}{2} m_p \sin(x_2) \dot{x}_1 + m_p g l \sin(x_2)$$

$$= (m_p l^2 + I) \ddot{x}_2 - \frac{m_p \sin(x_2)}{2} \dot{x}_1 + m_p g l \sin(x_2)$$

$$\Rightarrow \begin{bmatrix} m_c + m_p & 0 \\ m_p & m_p l^2 + I \end{bmatrix} \ddot{x} + \begin{bmatrix} 0 & m_p l \sin(x_2)/2 \\ -m_p \sin(x_2)/2 & 0 \end{bmatrix} \dot{x} \\
 + \begin{bmatrix} 0 \\ m_p g l \sin(x_2) \end{bmatrix} = \begin{bmatrix} U \\ 0 \end{bmatrix} + W^T d$$



$$g_1 = (x_1 - l \sin(x_2)) - d$$

$$g_2 = D - g_1$$

$$\sigma = \begin{bmatrix} \dot{x}_1 - l \cos(x_2) \dot{x}_2 \\ -\dot{x}_1 + l \cos(x_2) \dot{x}_2 \end{bmatrix}$$

$$\sigma = W^T \dot{x} = \begin{bmatrix} 1 & -l \cos(x_2) \\ -1 & l \cos(x_2) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$