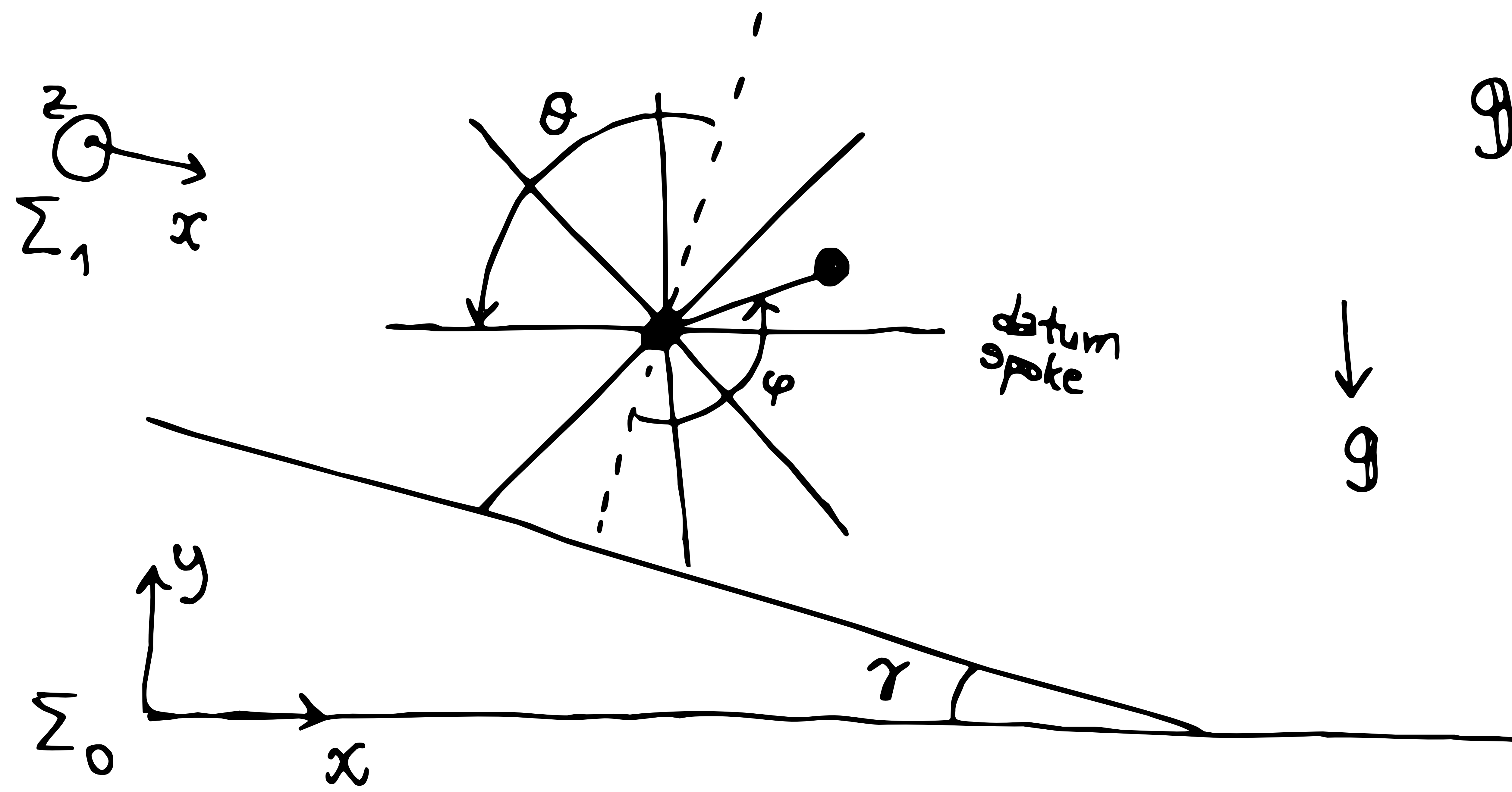


$$g^1 = R_{01}^T g^0 = R_{z,\gamma} g^0$$
$$= g\begin{bmatrix} -s_\gamma \\ c_\gamma \end{bmatrix}.$$

Kinetic Energy (written in Σ_1)

$$K_1 = \frac{1}{2} m_1 (\dot{x} + \dot{y})^2 + \frac{1}{2} I_1 \dot{\theta}^2$$

$$K_2 = \frac{1}{2} m_2 \left[(\dot{x} + l_2 c_\varphi \dot{\varphi})^2 + (\dot{y} + l_2 s_\varphi \dot{\varphi})^2 \right] + \frac{1}{2} I_2 \dot{\varphi}^2$$

$$\boxed{K = K_1 + K_2 = \frac{1}{2} m_t (\dot{x} + \dot{y})^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2}(I_2 + m_2 l_2^2)\dot{\varphi}^2 + m_2 l_2 (c_\varphi \dot{x} + s_\varphi \dot{y})\dot{\varphi}}$$

Position vector to the end of the torso

$$p_2 = p_1 + p_t$$
$$= \begin{pmatrix} x \\ y \end{pmatrix} + l_2 \begin{pmatrix} s_\varphi \\ -c_\varphi \end{pmatrix}$$

Velocity:

$$\dot{p}_2 = \begin{pmatrix} \dot{x} + l_2 c_\varphi \dot{\varphi} \\ \dot{y} + l_2 s_\varphi \dot{\varphi} \end{pmatrix}$$

Potential Energy

$$P_1 = m_1 (p_1 \cdot -g) = m_1 g(-x s_\gamma + y c_\gamma)$$

$$P_2 = m_2 (p_2 \cdot -g) = m_2 g\left(-s_\gamma (x + l_2 s_\varphi) + c_\gamma (y - l_2 c_\varphi)\right)$$
$$= m_2 g(-x s_\gamma + y c_\gamma) - m_2 g l_2 c_{\varphi\gamma}; \quad * \; c_{ab} = \cos(a-b).$$

$$\boxed{P = P_1 + P_2 = m_t g(-x s_\gamma + y c_\gamma) - m_2 g l_2 c_{\varphi\gamma}.}$$

$$p_1^0 = R_{01}^T p_1^1 + c$$
$$= \begin{bmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + c$$
$$= \begin{bmatrix} x c_\gamma + y s_\gamma \\ \boxed{-x s_\gamma + y c_\gamma} \end{bmatrix} + c$$

$$p_1^0 \cdot e_2 = \underline{-x s_\gamma + y c_\gamma}$$

The Lagrangian

$$\mathcal{K} = K_1 + K_2 = \frac{1}{2} m_t (\dot{x}+\dot{y})^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2}(I_2 + m_2 \ell_2^2)\dot{\varphi}^2 + m_2 \ell_2 (c_\varphi \dot{x} + s_\varphi \dot{y})\dot{\varphi}$$

$$\mathcal{P} = P_1 + P_2 = m_t g(-x s_\gamma + y c_\gamma) - m_2 g \ell_2 c_{\varphi\gamma} .$$

$$\boxed{\begin{aligned}\mathcal{L} = \mathcal{K} - \mathcal{P} &= \frac{1}{2} m_t (\dot{x}+\dot{y})^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2}(I_2 + m_2 \ell_2^2)^2 \dot{\varphi}^2 + m_2 \ell_2 (c_\varphi \dot{x} + s_\varphi \dot{y})\dot{\varphi} \\ &\quad + m_t g(x s_\gamma - y c_\gamma) + m_2 g \ell_2 c_{\varphi\gamma} .\end{aligned}}$$

Equations of Motion

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = m_t \ddot{x} + m_2 \ell_2 c_\varphi \ddot{\varphi} - m_2 \ell_2 s_\varphi \dot{\varphi}^2 - m_t g s_\gamma = 0.$$

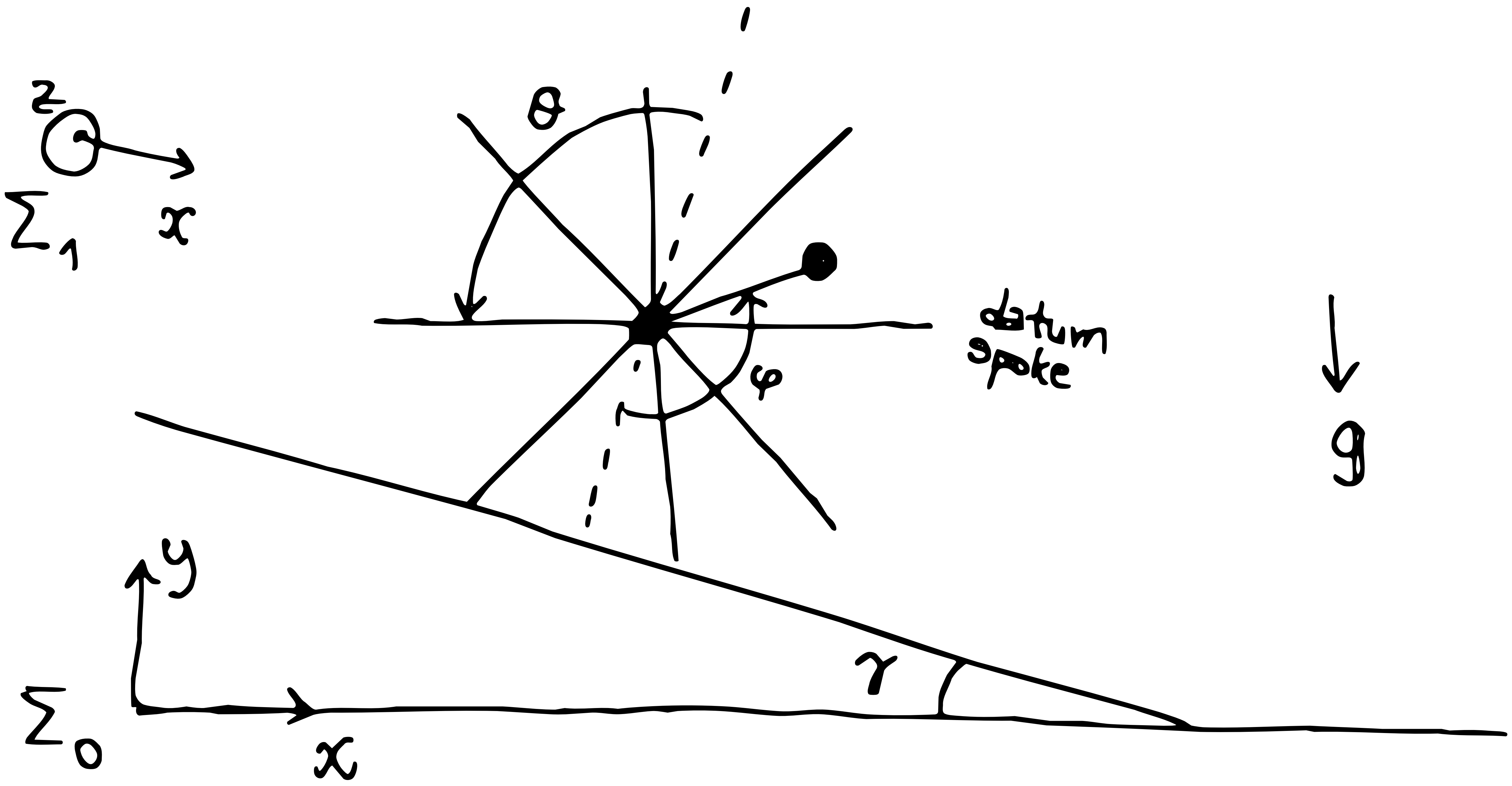
$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} - \frac{\partial \mathcal{L}}{\partial y} = m_t \ddot{y} + m_2 \ell_2 s_\varphi \ddot{\varphi} + m_2 \ell_2 c_\varphi \dot{\varphi}^2 - m_t g c_\gamma = 0.$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \frac{\partial \mathcal{L}}{\partial \varphi} = (I_2 + m_2 \ell_2^2)\ddot{\varphi} + m_2 \ell_2 (c_\varphi \ddot{x} + s_\varphi \ddot{y}) + m_2 \ell_2 (-s_\varphi \dot{x}\dot{\varphi} + c_\varphi \dot{y}\dot{\varphi})$$

$$- m_2 g \ell_2 s_{\varphi\gamma} = u$$

$\boxed{* \; s_{ab} = \sin(a-b)}$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = I_1 \ddot{\theta} = -u .$$



What is the gap function?

At any instant one of the spokes had better be touching.

The location of the k^{th} spoke is $(k = 0, \dots, n-1)$

$$p_{s,k} = \begin{bmatrix} x \\ y \end{bmatrix} + \ell_1 R_{z,2\alpha} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x - \ell_1 \sin(2k\alpha) \\ y + \ell_1 \cos(2k\alpha) \end{bmatrix}$$

Hence the gap functions are

$$\boxed{{}^{*} \quad q = (x, y, \varphi, \theta)}$$

$$g_k(y) = y + \ell_1 \cos(2k\alpha).$$

In the figure above, we have $g_1(y) = 0$ and $g_2(y) = \varepsilon > 0$, but ε small. For all other $k \in \{0, 3, 4, 5, 6, 7\}$, $g_k(y) = 0$.

Gradients are

$$\nabla g_k(q) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \quad \text{for all } k = 0, 1, \dots, n-1.$$

For inelastic collision, we must set $\varepsilon_n \equiv 0$.