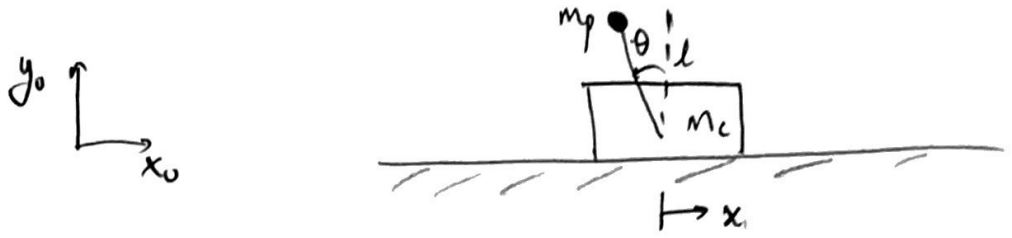

pose : m_c : $\begin{bmatrix} x \\ 0 \end{bmatrix}$

$m_p = \begin{bmatrix} -l\sin(\theta)+x \\ l\cos(\theta) \end{bmatrix}$

velocities : $m_c : \begin{bmatrix} \dot{x} \\ 0 \end{bmatrix}$, $m_p : \begin{bmatrix} -l\cos(\theta)\dot{\theta} + \dot{x} \\ -l\sin(\theta)\dot{\theta} \end{bmatrix}$

$$K = \frac{1}{2} m_c \dot{x}^2 + \frac{1}{2} m_p \left[(-l\cos(\theta)\dot{\theta}+\dot{x})^2 + (-l\sin(\theta)\dot{\theta})^2 \right] + \frac{1}{2} I \dot{\theta}^2$$

$$P = m_p g l \cos(\theta)$$

$$\mathcal{L} = K - P = \frac{1}{2} m_c \dot{x}^2 + \frac{1}{2} m_p \left[(l^2\dot{\theta}^2\cos^2(\theta) - 2l\dot{x}\dot{\theta}\cos(\theta) + \dot{x}^2) + (l^2\sin^2(\theta)\dot{\theta}^2) \right]$$

$$+ \frac{1}{2} I \dot{\theta}^2 - m_p g l \cos(\theta)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\left[m_c\dot{x} - m_p l \dot{\theta}\cos(\theta) + m_p \dot{x} \right]$$

$$= m_c \ddot{x} - m_p l \ddot{\theta}\cos(\theta) + m_p l \dot{\theta}^2 \sin(\theta) + m_p \ddot{x}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\left[m_p l^2 \dot{\theta}\cos^2(\theta) - m_p l \dot{x}\cos(\theta) + m_p l^2 \sin^2(\theta)\dot{\theta} + I\dot{\theta} \right]$$

$$- \left[-m_p l^2 \dot{\theta}^2 \cancel{\cos(\theta)\sin(\theta)} + m_p l \dot{x}\dot{\theta}\sin(\theta) + \cancel{m_p l^2 \dot{\theta}^2 \sin(\theta)\cos(\theta)} \right.$$

$$\left. + m_p g l \sin(\theta) \right]$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = m_p l^2 \cos^2(\theta)\,\ddot{\theta} - 2m_p l^2 \dot{\theta}^2 \cos(\theta)\sin(\theta) - m_p l \ddot{x}\cos(\theta)$$

$$+ m_p l \dot{x} \sin(\theta)\dot{\theta} + 2m_p l^2 \sin(\theta)\cos(\theta)\dot{\theta}^2 + m_p l^2 \sin^2(\theta)\ddot{\theta}$$

$$+ I\ddot{\theta} - \left[m_p l \dot{x}\dot{\theta}\sin(\theta) + m_p g l \sin(\theta) \right]$$

$$= m_p l^2 \ddot{\theta} + I\ddot{\theta} - m_p l \cos(\theta)\ddot{x} - m_p g l \sin(\theta)$$

$\Rightarrow$

friction

$$\begin{bmatrix} m_c + m_p & -m_p l\cos(\theta) \\ -m_p l\cos(\theta) & m_p l^2 + I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} b & m_p l \dot{\theta}\sin(\theta) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -m_p g l \sin(\theta) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + W \cdot \lambda$$