Notation:

- $V = \begin{pmatrix} V_{1x} & V_{1y} \\ V_{2x} & V_{2y} \\ \dots & \\ V_{nx} & V_{ny} \end{pmatrix}$, $V \in \mathbb{R}^{n\times 2}$

n - is the number of vertices

- $\delta(a \in A) = \begin{cases} 0, & \text{if } a \notin A \\ 1, & \text{if } a \in A \end{cases}$

2.5.3. Derive an expression for $\frac{\partial x_f}{\partial V}$

$$\frac{x_f}{\partial V} = \frac{\partial\left(\frac{1}{3}\sum_{v_i \in f} v_{ix}\right)}{\partial V} = \frac{1}{3}\sum_{v_i \in f} \frac{\partial V_{ix}}{\partial V} = \frac{1}{3} W$$

$$W = \begin{cases} W_{:,1} = 0 \\ W_{i,0} = \delta(v_i \in f), \end{cases} \quad W \in \mathbb{R}^{n\times 2}$$

2.5.4. Derive an expression for $\frac{\partial E_{sh}}{\partial V}$

$$\frac{\partial E_{sh}}{\partial V} = \frac{\partial\left(\frac{1}{2}\sum_{e \in E}(l_e - L_e)^2\right)}{\partial V} = \frac{1}{2}\sum_{e \in E}\frac{\partial\left((l_e - L_e)^2\right)}{\partial V} =$$

$$= \frac{1}{2}\sum_{e \in E}\left(2(l_e - L_e)\frac{\partial(l_e - L_e)}{\partial V}\right) = \sum_{e \in E}(l_e - L_e)\underbrace{\frac{\partial(l_e - L_e)}{\partial V}}_{} \ominus$$

$$\frac{\partial(l_e - L_e)}{\partial V} = -\frac{\partial L_e}{\partial V} = \frac{\partial\left(\sqrt{(V_{2x} - V_{1x})^2 + (V_{2y} - V_{1y})^2}\right)}{\partial V} = \frac{1}{2}G$$

P_e

$$G_{ij} = \delta(v_i \in e) \cdot \frac{\partial P_e}{\partial V_{ij}}, \quad G \in \mathbb{R}^{n\times 2}$$

$$\ominus \sum_{e \in E}(l_e - L_e) \cdot G_e$$