1.3.1. from Assignment 1

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1 Area centroid of a quadrilateral

We know that for a shape Ω, which is divided into N non degraded triangle meshes, the following applies:

$$x_{cm} = \frac{1}{A_\Omega} \sum_{i}^{N} A_{f_i} x_{f_i} \tag{1}$$

where x_{cm} is the centroid of Ω, A_Ω is it's total area, A_{f_i} is the area of the i-th face, and x_{f_i} is the centroid of the according face. $x_{f_i} \in \mathbb{R}^2$

As can be seen, our example on Figure 1 is divided into two triangular meshes, which means that in our particular case, Equation 1 takes the following form:

$$x_{cm} = \frac{1}{A_{f_1} + A_{f_2}} (A_{f_1} x_{f_1} + A_{f_2} x_{f_2}) \tag{2}$$

We can now compute every value. Since f_1 and f_2 are triangles:

$$x_{f_1} = \frac{1}{3}[2+3+3, 3+2+1] = [\frac{8}{3}, 2] \tag{3}$$

$$x_{f_2} = \frac{1}{3}[3+3+10, 3+2+1] = [\frac{16}{3}, 2] \tag{4}$$

We compute the areas of triangles using $A_{f_i} = \frac{1}{2} h_i * r_i$, where h_i is the height of the triangle, and r_i is the length of the base

$$A_{f_1} = 1 \tag{5}$$

$$A_{f_2} = 7 \tag{6}$$

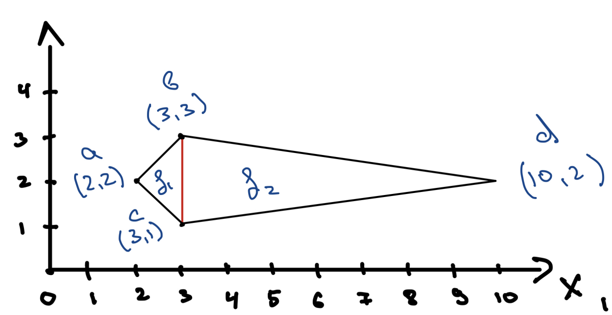

Figure 1: Example of a quadrilateral on. Red line denotes suggested line for dividing the initial figure into two triangles.

This way we get

$$x_{cm} = \frac{1}{8}(1 * [\frac{8}{3}, 2] + 7 * [\frac{16}{3}, 2]) = [5, 2] \tag{7}$$

Alternatively, if we compute the mean of all vertex coordinates, we will get:

$$x_{mean} = \frac{1}{4}([3 + 3 + 2 + 10, 1 + 2 + 2 + 3]) = [4.5, 2] \tag{8}$$

We can see that the result in Equation 8 differs from Equation 7.

This way the general answer to "Is the area centroid of a quadrilateral equal to the average of its vertices?" is No! ■