The "ka-Ta-pa-ya" scheme
and its application to
mELAkarta raagas of Carnatic music

The ka-Ta-pa-ya scheme:
The "ka-Ta-pa-ya" rule used by ancient Indian mathematicians and grammarians is a tool to map names to numbers. Writing the consonants of the Sanskrit alphabet as four groups with "ka, Ta, pa, ya" as the beginning letters of the groups we get

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1   2   3  4   5   6   7   8  9    0
ka kha ga gha ~ma cha Cha ja jha ~na
Ta Tha Da Dha  Na ta  tha da dha  na
pa pha ba bha  ma
ya ra  la va   Sa sha sa  ha
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Now, each letter of the group is numbered from 1 through 9 and 0 for the tenth letter. Thus, ka is 1, sa is 7, ma is 5, na is 0 and so on. So to indicate the number 356 for example one would try and come up with a word involving the third, fifth and sixth letters of the groups like "gaNitam" or "lESaca". However, in the Indian tradition, the digits of a number are written left to right in the increasing order of their place value - exactly opposite the way we are used to writing in the western way. Therefore 356 would be indicated using letters in the 6th, 5th, and 3rd positions of the group e.g. "triSUlaM".

There apparently were upto 4 flavors of this scheme in use in ancient India. These differ in how to interpret the conjoint consonant. The popular scheme was to use only the last consonant. And any consonant not attached to a vowel is to be disregarded. These rules should be used while decoding a phrase in "katapayadi" scheme.

The following phrase found in "sadratnamAla" a treatise on astronomy,
bhadram budhi siddha janma gaNita Sraddha@h mayadbhUpagi@h
when decoded yields
4 2 3 9 7 8 5 3 5 6 2 9 5 1 4 1 3
which when reversed gives
3 1 4 1 5 9 2 6 5 3 5 8 7 9 3 2 4
which is readily recognised as the digits in "pi" (except that the 17th digit is wrong - it should be 3) :-)!

(source: The article "The Katapayadi Formula and Modern Hashing Technique" by Anand V Raman, appearing in "Computing Science in Ancient India", edited by T.R.N.Rao and Subhash Kak, published by the Center for Advanced Computer Studies, University of South Western Louisiana, Louisiana LA 70504)

mELakarta raagas:
The raagas of Carnatic music are said to be derived from a definite set of 72 ragas known as mELakarta or janaka or sampoorNa raagas.

Before we go any further let me remind you that in an octave there are 12 tones each separated by a half note. These 12 tones are named as the 7 notes and variations on some of those 7 notes. These names and their western music equivalents are as follows:

1. shaDjamam	S	Doh	C	
2. Suddha rishabham	R1		C# or Db (read as C-sharp or D-flat)	
3. chatuSruti rishabham	R2			
Suddha gAndhAram	G1	Re	D	
4. shaTSruti rishabham	R3			
sAdhAraNa gAndhAram	G2		D# or Eb	
5. antara gAndhAram	G3	Me	E	
6. Suddha madhyamam	M1	Fa	F	
7. prati madhyamam	M2		F# or Gb	
8. pancamam	P	Sol	G	
9. Suddha dhaivatam	D1		G# or Ab	

10. chatuSruti dhaivatam	D2		
Suddha nishaadam	N1	La	A
11. shaTSruti dhaivatam	D3		
kaiSika nishaadam	N2		A# or Bb
12. kAkali nishaadam	N3	Ti	B

The properties of the janaka raagas are :
a) they contain all the 7 notes of an octave (hence the name saMpoorNa) exactly once in the scale.
b) the tones of the notes must all be in ascending order in the aarOhaNa. i.e You cannot pick S, R3, G1 ... because the tone of R3 is higher than the tone of G1. Also the ArOhaNa/avarOhaNa cannot have jumps back and forth like S, G3, R1, ... or S, N2, P, D1 ... etc..
c) the avarOhaNa should contain the same notes as ArOhaNa in the reverse order.

Given these properties/rules, we can easily surmise that there cannot be more than 72 sampoorNa raagas. Because of the need for ascending order of tones the permissible combinations of R,G and D,N are limited to 6 each, viz. R1G1, R1G2, R1G3, R2G2, R2G3, R3G3 and D1N1, D1N2, D1N3, D2N2, D2N3, D3N3. There are two varieties of M viz. M1, M2. So the number of possible different sampoorNa raagas is 6 x 6 x 2 = 72. So if these 72 are arranged in a regular order, we can figure out the scale of a janaka raaga if its number in the list is given. Now, from our "kaTapayaadi" scheme if we can name the raaga in such a way that the name yields the number, then we have further reduced the memorising!

Application of "kaTapayaadi" to mELakarta raagas

That is exactly what venkaTamakhi of the 18th century is purported to have done. He applied the "kaTapaya" scheme to name the janaka raagas to fit their place in the mELakarta list. Some of these already had suitable names and some had unsuitable names that were in common use. He changed those names a little to fit the naming scheme. Thus "kalyANi" becomes "mEcha kalyANi","SankarAbharaNam" becomes "dheera SankarAbharaNam" etc.

Under this naming scheme, the number of a janaka raaga is obtained by decoding the first two letters using the "kaTapaya" scheme. For the naming scheme used for the mELakarta raagas, apart from the decoding rules mentioned above for, conjoint consonants, in case one of the consonants is from the 'ya' group, the first consonant is to be considered instead of the last. And finally, to get back to our familiar western notation, reverse the decoded digits.

For example:
"kharaharapriya" : kha = 2 and ra = 2 i.e 22 reversing the digits : 22
"shaNmukhapriya" : sha = 6 and mu = 5 i.e 65 reversing the digits : 56
"naThabhairavi" : na = 0 and Tha = 2 i.e 02 reversing the digits : 20
"divyamaNi" : di = 8 and va = 4 i.e 84 reversing the digits : 48

Once you get the number, figuring out the notes is easy. The 72 raagas are arranged such that the first 36 raagas contain M1 and the next 36 contain M2. In each half, the various possible combinations of R,G and D,N occur cyclically with the R,Gs varying slower than the D,Ns.
i.e: for the first six raagas
R1G1 occurs with each of D1N1, D1N2, D1N3, D2N2, D2N3, D3N3
for the next six raagas
R1G2 occurs with each of D1N1, D1N2, D1N3, D2N2, D2N3, D3N3
and so on.

So given a janaka raaga number you perform the following calculation:
1. if NUM is from 1-36, raaga has M1, from 37-72 raaga has M2.
2. if NUM is greater than 36 subtract 36 from it.
3. divide NUM by 6;
   a. if remainder=0
      i. the sixth D,N combination occurs.
      ii. the quotient gives which of the R,G combinations occurs.
   b. if remainder is not zero
      i. the remainder gives which of the D,N combinations occurs.
      ii. the quotient+1 gives which of the R,G combinations occurs.

Taking the example of "shaNmukhapriya":
From the "kaTapaya" rule its number is 56.
56 is greater than 36. So M2 occurs.
56-36=20.
20 divided by 6 : quotient=3, remainder=2
so 3+1=4th RG combination : R2G2 occurs.
and 2nd DN combination : D1N2 occurs.
So shaNmukhapriya has the notes:
S R2 G2 M2 P D1 N2 S

Another example : "varuNapriya"
From the "kaTapaya" rule its number is 24.
24 is less than 36. So M1 occurs.
24 divided by 6 : quotient=4, remainder=0

so 4th R,G combination: R2G2 occurs.
and 6th D,N combination: D3N3 occurs.
thus varuNapriya has the notes:
S R2 G2 M1 P D3 N3 S

65
56 → 36
↳ M_2

Shanmuka 4 prize.

3, R_2, G_2, M_2, á

→ D_1, N_2.

b) 20
3 + 1
18
2

1.