Definitions

A compound statement is called a ***tautology(T_0)*** if it is true for all truth value assignments for its component statements. If a compound statement is false for all such assignments, then it is called a ***contradiction(F_0)***.

E.g.

Example

Consider the following:

Arguments

Consider an argument:

If any one of is false, then no matter what truth value has, the implication is true.

Logical Equivalence

Consider the following:

p	q			

Logical Equivalence

$p \rightarrow q$

$p \leftrightarrow q$

$p \oplus r$

Logical Equivalence

If s_1 and s_2 are statements and $s_1 \leftrightarrow s_2$ is a tautology, then s_1, s_2 must have the same corresponding truth values and $s_1 \Leftrightarrow s_2$.

Similarly, when s_1 and s_2 are logically equivalent statements (I.e. $s_1 \Leftrightarrow s_2$) then the compound statement $s_1 \leftrightarrow s_2$ is a tautology.

The Laws of Logic

Law of Double Negation	
DeMorgan's Laws	
Commutative Laws	
Associative Laws	
Distributive Laws	

The Laws of Logic - cont'd…

Idempotent Laws	
Identity Laws	
Inverse Laws	
Domination Laws	
Absorption Laws	

Duals

All the laws, aside from the Law of Double Negation, **fall naturally into pairs.**

Definition: Let s be a statement. If s contains no logical connectives other than $\wedge$ and $\vee$, then the **dual** of s, denoted s^d, is the statement obtained from s by replacing each occurrence of $\wedge$ and $\vee$ by $\vee$ and $\wedge$, respectively, and each occurrence of T_0 and F_0 by F_0 and T_0, respectively.

Duals

Theorem 2.1 (*The Principle of Duality*): Let s and t be statements that contain no logical connectives other than and . If $s \Leftrightarrow t$, then $s^d \Leftrightarrow t^d$

First Substitution Rule

Suppose that the compound statement P is a tautology. If p is a *primitive* statement that appears in P and we replace *each* occurrence of p by the *same* statement q, then the resulting compound statement P_1 is also a tautology.

E.g.

Second Substitution Rule

Let P be a compound statement where p is an arbitrary statement that appears in P, and let q be a statement such that $q \Leftrightarrow p$. Suppose that in P we replace one or more occurrences of p by q. Then this replacement yields the compound statement P_1. Under these circumstances $P_1 \Leftrightarrow P$.

E.g.

Example

Negate and simplify the compound statement

$(p \wedge q) \to \ r$

Example

p: Joan goes to Lake George.

q: Mary pays for Joan’s shopping spree.

$p \to q$: If Joan goes to Lake George, then Mary will pay for Joan’s shopping spree.

What is the negation of $p \to q$?

Implications

p	q	$\neg p$	$\neg q$	$p \to q$			

Example

p: Jeff is concerned about his cholesterol level.

q: Jeff walks at least 5 km’s a week.

$p \to q$: If Jeff is concerned about his cholesterol level, then he will walk at least 5 km’s a week.

Contrapositive:

Converse:

Inverse:

Simplification

Simplify $(p \lor q) \land \neg(\neg p \land q)$

Simplification

Simplify $\neg[\neg [(p \lor q) \land r] \lor \neg q]$

Logical Implication: Rules of Inference

Recall that an argument:

$(p_1 \land p_2 \land p_3 \land \ldots \land p_n) \to q$

Is a valid argument if and only if

$(p_1 \land p_2 \land p_3 \land \ldots \land p_n) \to q$

is a tautology.

Example

Consider the **statements**: *p*: Roger studies. *q*: Roger plays tennis. *r*: Roger passes discrete mathematics.

Consider the **premises**:
p_1: If Roger studies, then he will pass discrete math.
p_2: If Roger doesn't play tennis, then he'll study.
p_3: Roger failed discrete mathematics.

Is the argument $(p_1 \land p_2 \land p_3) \to q$ valid?