

CS7.301 Machine, Data, and Learning

Assignment 05A: Partially Observable Markov Decision Process

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Parameters Table

x	0.79
y	2

POMDP Formulation Table

Number of States	Five
Possible States	S1, S2, S3, S4, S5
Number of Actions	Two
Possible Actions	Left (L), Right (R)
Number of Observations	Two
Possible Observations	Green, Red
Discount Factor	One (1.0)
Action Probability	0.79

Observation State Probability Table

P (Observation = Red State = Red)	0.85
P (Observation = Green State = Red)	0.15
P (Observation = Green State = Green)	0.90
P (Observation = Red State = Green)	0.10

States Table

States	S1	S2	S3	S4	S5
Color	Red	Red	Green	Green	Red

Probability Transition Matrix

	S1, L	S1, R	S2, L	S2, R	S3, L	S3, R	S4, L	S4, R	S5, L	S5, R
S1	0.79	0.21	0.79	0.21	0	0	0	0	0	0
S2	0.21	0.79	0	0	0.79	0.21	0	0	0	0
S3	0	0	0.21	0.79	0	0	0.79	0.21	0	0
S4	0	0	0	0	0.21	0.79	0	0	0.79	0.21
S5	0	0	0	0	0	0	0.21	0.79	0.21	0.79

Initial State: One of the red states i.e. S1 S2, or S5

States	S1	S2	S3	S4	S5
Belief State	0.33	0.33	0	0	0.33

Action 01: Agent took the action Right.

Observation 01: Agent observed Red.

s	S1	S2	S3	S4	S5
b(s)	0.33	0.33	0	0	0.33
$p = P(S1 s, R)$	0.21	0.21	0	0	0
$p * b(s)$	0.0693	0.0693	0	0	0

$$b(S1) := P(\text{Observation} = \text{Red} | \text{State} = \text{Red}) * \sum (p * b(s)) = 0.85 * 0.1386 = 0.11781$$

s	S1	S2	S3	S4	S5
b(s)	0.33	0.33	0	0	0.33
$p = P(S2 s, R)$	0.79	0	0.21	0	0
$p * b(s)$	0.2607	0	0	0	0

$$b(S2) := P(\text{Observation} = \text{Red} | \text{State} = \text{Red}) * \sum (p * b(s)) = 0.85 * 0.2607 = 0.221595$$

s	S1	S2	S3	S4	S5
b(s)	0.33	0.33	0	0	0.33
$p = P(S3 s, R)$	0	0.79	0	0.21	0
$p * b(s)$	0	0.2607	0	0	0

$$b(S3) := P(\text{Observation} = \text{Red} | \text{State} = \text{Green}) * \sum (p * b(s)) = 0.10 * 0.2607 = 0.02607$$

s	S1	S2	S3	S4	S5
b(s)	0.33	0.33	0	0	0.33
$p = P(S4 s, R)$	0	0	0.79	0	0.21
$p * b(s)$	0	0	0	0	0.0693

$$b(S4) := P(\text{Observation} = \text{Red} | \text{State} = \text{Green}) * \sum (p * b(s)) = 0.10 * 0.0693 = 0.00693$$

s	S1	S2	S3	S4	S5
b(s)	0.33	0.33	0	0	0.33
p = P (S5 s, R)	0	0	0	0.79	0.79
p * b(s)	0	0	0	0	0.2607

$$b(S5) := P (\text{Observation} = \text{Red} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.85 * 0.2607 = 0.221595$$

Current Belief State: After Action 01

States	S1	S2	S3	S4	S5
b Value Before Normalization	0.11781	0.221595	0.02607	0.00693	0.221595
Belief State	0.198333333	0.373055556	0.043888889	0.011666667	0.373055556

Action 02: Agent took the action Left.

Observation 02: Agent observed Green.

s	S1	S2	S3	S4	S5
b(s)	0.198333333	0.373055556	0.043888889	0.011666667	0.373055556
p = P (S1 s, L)	0.79	0.79	0	0	0
p * b(s)	0.156683333	0.294713889	0	0	0

$$b(S1) := P (\text{Observation} = \text{Green} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.15 * 0.451397222 = 0.067709583$$

s	S1	S2	S3	S4	S5
b(s)	0.198333333	0.373055556	0.043888889	0.011666667	0.373055556
p = P (S2 s, L)	0.21	0	0.79	0	0
p * b(s)	0.04165	0	0.034672222	0	0

$$b(S2) := P (\text{Observation} = \text{Green} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.15 * 0.076322222 = 0.011448333$$

s	S1	S2	S3	S4	S5
b(s)	0.198333333	0.373055556	0.043888889	0.011666667	0.373055556
p = P (S3 s, L)	0	0.21	0	0.79	0
p * b(s)	0	0.078341667	0	0.009216667	0

$$b(S3) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Green}) * \sum (p * b(s)) = 0.90 * 0.087558334 = 0.078802501$$

s	S1	S2	S3	S4	S5
b(s)	0.198333333	0.373055556	0.043888889	0.011666667	0.373055556
p = P (S4 s, L)	0	0	0.21	0	0.79
p * b(s)	0	0	0.009216667	0	0.294713889

$$b(S4) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Green}) * \sum (p * b(s)) = 0.90 * 0.303930556 = 0.2735375$$

s	S1	S2	S3	S4	S5
b(s)	0.198333333	0.373055556	0.043888889	0.011666667	0.373055556
p = P (S5 s, L)	0	0	0	0.21	0.21
p * b(s)	0	0	0	0.00245	0.078341667

$$b(S5) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.15 * 0.080791667 = 0.01211875$$

Current Belief State: After Action 02

States	S1	S2	S3	S4	S5
b Value Before Normalization	0.067709583	0.011448333	0.078802501	0.2735375	0.01211875
Belief State	0.152630837	0.025806815	0.177636474	0.616607807	0.027318067

Action 03: Agent took the action Left.

Observation 03: Agent observed Green.

s	S1	S2	S3	S4	S5
b(s)	0.152630837	0.025806815	0.177636474	0.616607807	0.027318067
p = P (S1 s, L)	0.79	0.79	0	0	0
p * b(s)	0.120578361	0.020387384	0	0	0

$$b(S1) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.15 * 0.140965745 = 0.021144862$$

s	S1	S2	S3	S4	S5
b(s)	0.152630837	0.025806815	0.177636474	0.616607807	0.027318067
p = P (S2 s, L)	0.21	0	0.79	0	0
p * b(s)	0.032052476	0	0.140332814	0	0

$$b(S2) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.15 * 0.17238529 = 0.021144862$$

s	S1	S2	S3	S4	S5
b(s)	0.152630837	0.025806815	0.177636474	0.616607807	0.027318067
p = P (S3 s, L)	0	0.21	0	0.79	0
p * b(s)	0	0.005419431	0	0.487120167	0

$$b(S3) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Green}) * \sum (p * b(s)) = 0.90 * 0.492539598 = 0.443285638$$

s	S1	S2	S3	S4	S5
b(s)	0.152630837	0.025806815	0.177636474	0.616607807	0.027318067
p = P (S4 s, L)	0	0	0.21	0	0.79
p * b(s)	0	0	0.03730366	0	0.021581273

$$b(S4) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Green}) * \sum (p * b(s)) = 0.90 * 0.058884933 = 0.05299644$$

s	S1	S2	S3	S4	S5
b(s)	0.152630837	0.025806815	0.177636474	0.616607807	0.027318067
p = P (S5 s, L)	0	0	0	0.21	0.21
p * b(s)	0	0	0	0.129487639	0.005736794

$$b(S5) := P(\text{Observation} = \text{Green} \mid \text{State} = \text{Red}) * \sum (p * b(s)) = 0.15 * 0.135224433 = 0.020283665$$

Current Belief State: After Action 03

States	S1	S2	S3	S4	S5
b Value Before Normalization	0.021144862	0.021144862	0.443285638	0.05299644	0.020283665
Belief State	0.037836011	0.037836011	0.793202652	0.094830315	0.03629501

Reference(s)

CPS 570 Partially Observable Markov Decision Processes by Prof. Ron Parr; Duke University

<http://db.cs.duke.edu/courses/compsci590.2/fall17/POMDPs.pdf>