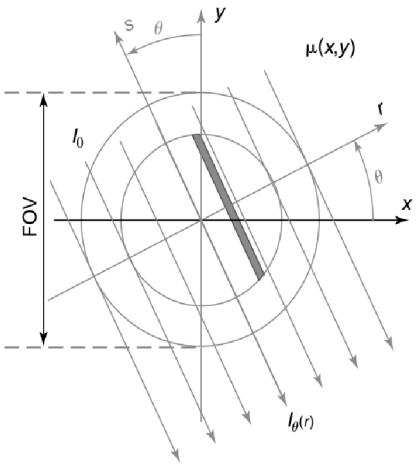
Question 01: Why is the Radon Transform of an image known as Sinogram?

If we apply the radon transform on a Dirac Delta function - an object function representing a single dot in the coordinate system, then we end up with a sine way in the projection coordinate system.

Radon transform is a linear operator. Given that each possible object function can be expressed as a linear combination of an endless Dirac Delta function, we can say that the projection of any object is the sum of a number of sine waves.



Here, we consider parallel beam geometry. The X-ray beams make an angle θ with the y-axis and are at distance r from the origin. The attenuated intensity of the X-ray beam is I_0. The new coordinate system (r, s) is defined by rotating (x, y) over an angle θ.

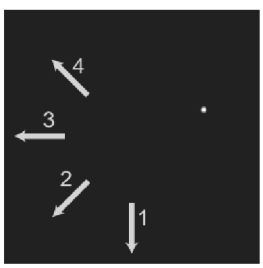
$$I_\theta(r) = I_0 \cdot e^{-\int_{L_{r,\theta}} \mu(x,y)\,ds}$$
$$= I_0 \cdot e^{-\int_{L_{r,\theta}} \mu(r\cdot\cos\theta - s\cdot\sin\theta,\, r\cdot\sin\theta + s\cdot\cos\theta)\,ds}$$

For a fixed angle θ, the measured intensity profile as a function of r is given above. $L_{r,\theta}$ is the line that makes an angle θ with the y-axis at distance r from the origin.

$$p_\theta(r) = -\ln\frac{I_\theta(r)}{I_0}$$
$$= \int_{L_{r,\theta}} \mu(r\cdot\cos\theta - s\cdot\sin\theta,\, r\cdot\sin\theta + s\cdot\cos\theta)\,ds,$$

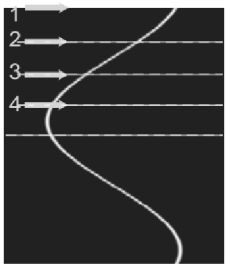
Each intensity profile is transformed into an attenuated profile. $p_\theta(r)$ is the projection of the function μ(x, y) along the angle θ. Stacking all these projections $p_\theta(r)$ results in a 2D dataset p(r, θ) called a sinogram.

If we assume a distribution μ(x, y) containing a single dot, the corresponding projection function p(r, θ) has a sinusoidal shape. In (a), and (b), we find the image and surface plot of distribution μ(x, y) containing one single dot. The four arrows indicate the views that correspond to four arbitrary projection directions. In ©, we find the 360^0 sinogram obtained by projecting μ(x, y).



(a)


(b)



(c)

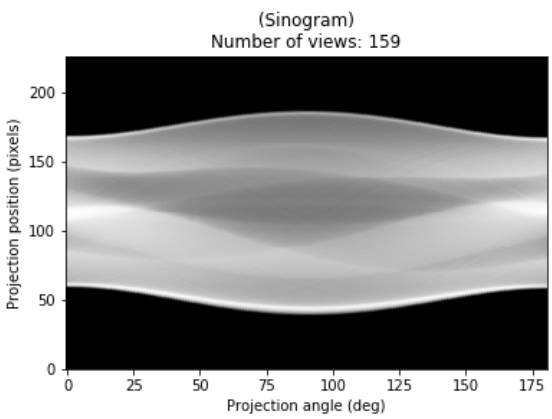
Question 02: Implement Radon Transform

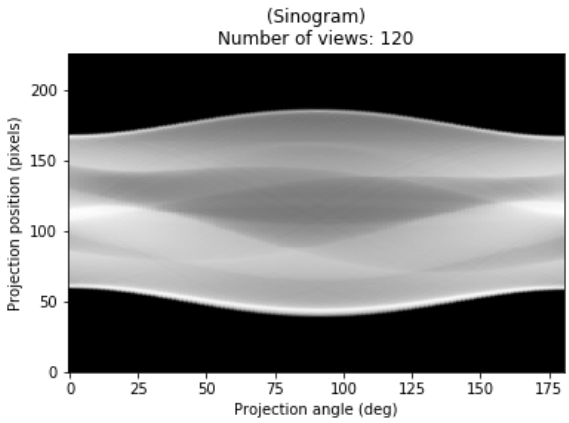
The Python implementation of Radon Transform takes in an image input, and considers the center of the image to be along the rotation axis. The input image is zero-padded. The projection angles are also passed as an input, and the function returns the sinogram of the image given specified projection angles. We use the Shepp-Logan phantom provided by skimage library for simulation purposes.

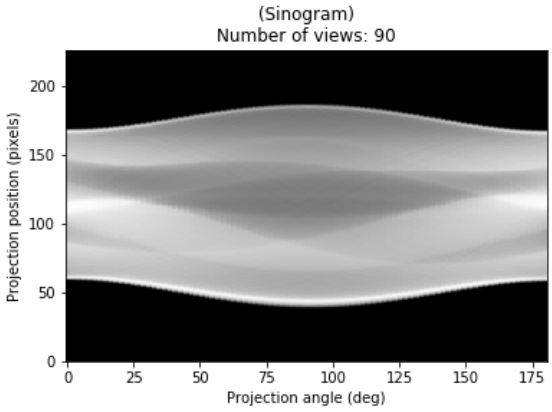
Note: The results generated using all the possible variations of hyperparameters can be found inside the Code/results folder. Only selected samples are shown here. All files are appropriately named.

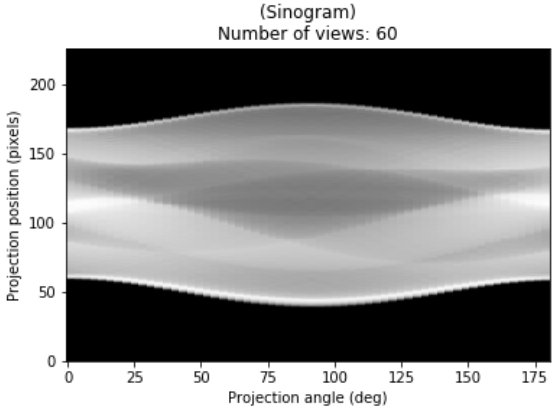
Observations:
Low number of projections reduces the image reconstruction quality because of the sparse sampling of image data. As the number of angles decreases, we find that we consider only a limited number of projections about the object image, and this doesn't completely represent the object.

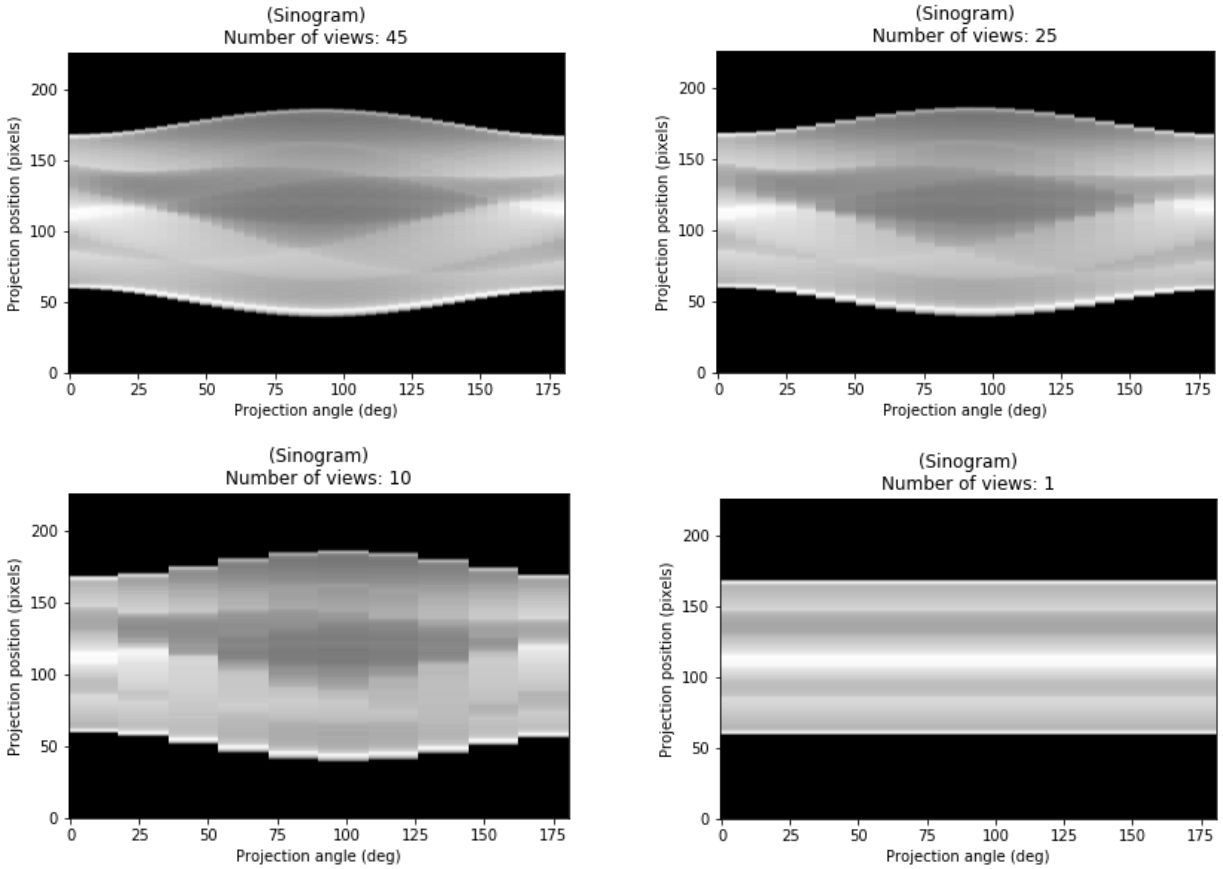
As we reduce the number of detectors (the range of r), it results in limiting spatial resolution in the resulting image. The record of intensity of the beams passing through the tissue is minimized. As the number of detectors near the number of pixels in each dimension, we observe better reconstruction results.











Question 03: Analytical Reconstruction Algorithms

General Note: Results corresponding to all the possible values of hyperparameters are generated by the code. The file is over a few GBs, and hence, have attached the drive link of the folder. Samples are shown here.

I. Backprojection

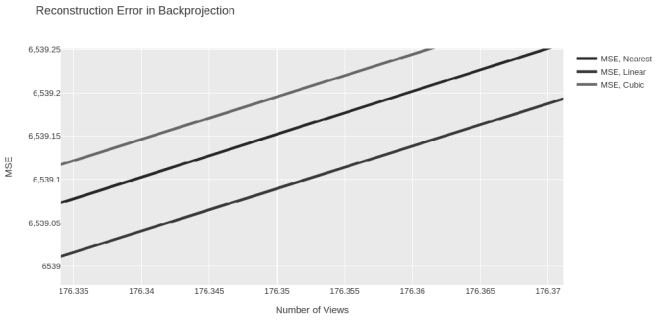
Nearest neighbor, linear, and cubic interpolations were used in the K-space for reconstructing the image from the sinogram. The Python implementation uses the Radon Transform function to obtain the sinogram of the image. The following samples from the results generated also include projections from different sets of angles.

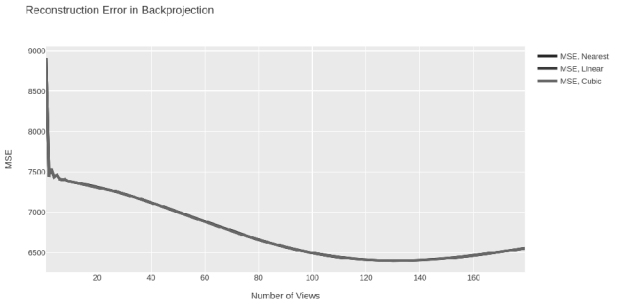
Observations: Backprojection produces an image which has a high density in the center. This is due to the fact that many images are being overlapped in this area. We observe that the resulting image is severely blurred.This effect comes from the overlapping of the Fourier-transformed images around the low frequency region. It is visually difficult to predict which interpolation gives the best result from the generated images. As the number of views increases, we observe better image reconstruction results. Lesser the number of detectors, lesser the

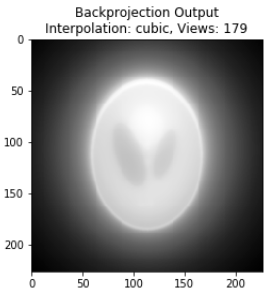
spatial resolution of the reconstructed image. For measuring the quality of the reconstruction, we use three measures in the Python implementation - MSE, PSNR, and SSIM.

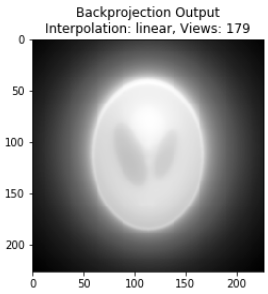
A. Quantitative Measurement in Backprojection: An Example

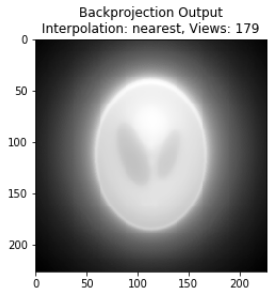
We have shown the comparison between different interpolation methods. In this example here, we have used the mean squared error metric. The interactive Plotly graph is available on the Jupyter notebook for MSE, PSNR, and SSIM measures. We observe (plot on the left) that the values are very close - cubic interpolation performs better in terms of MSE as the number of views get closer to 180^0. In general, we find that the reconstruction gets better as we increase the number of projections (plot on the right). The lowest value of MSE is obtained at near the point where the number of views equals the number of pixels in image dimensions.

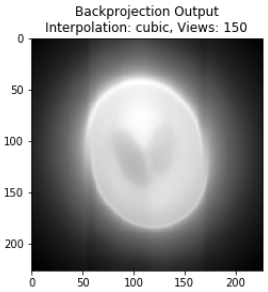


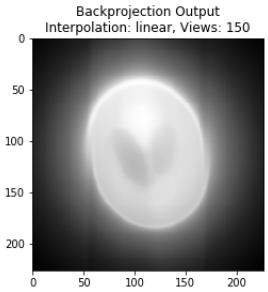


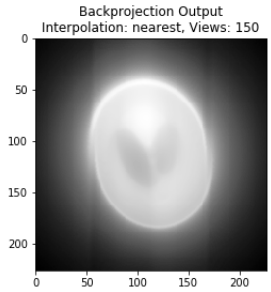






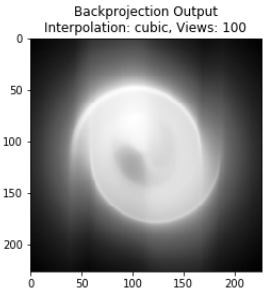


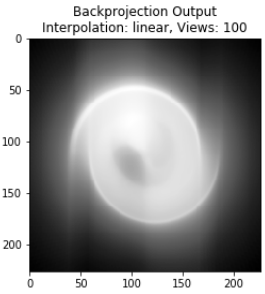


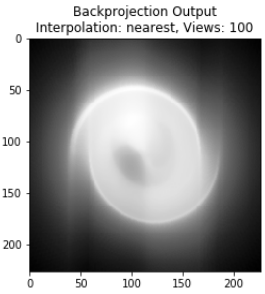


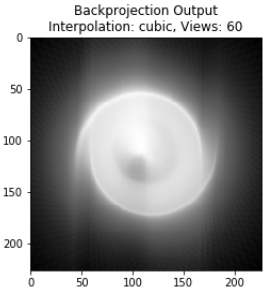
EC5.405 Medical Image Analysis

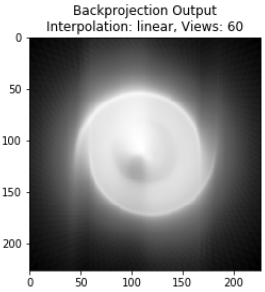
Assignment 01: Analytical Image Reconstruction

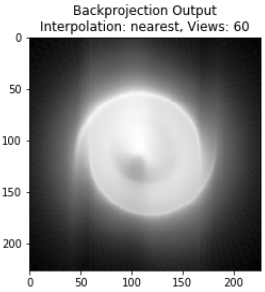


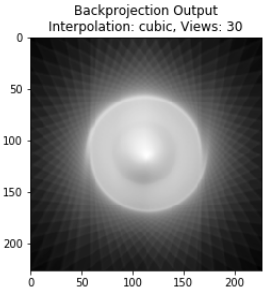


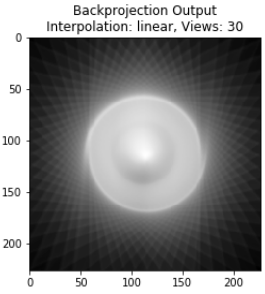


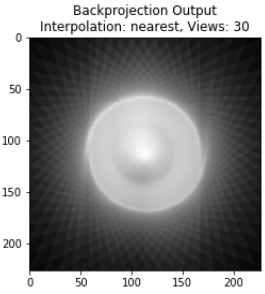


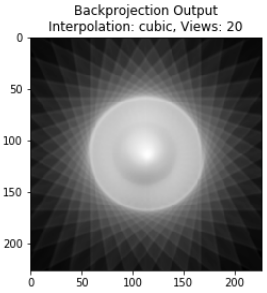


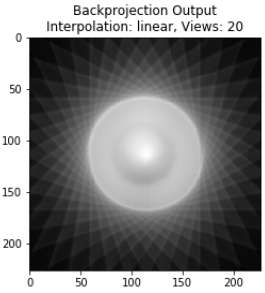


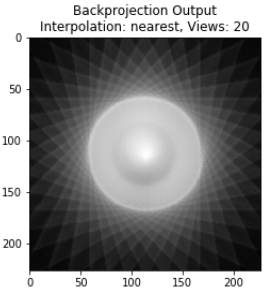


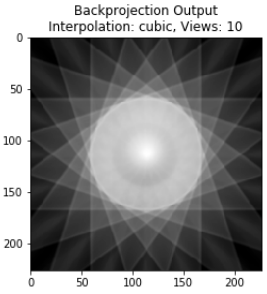


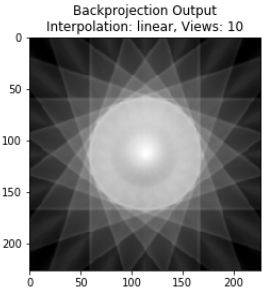


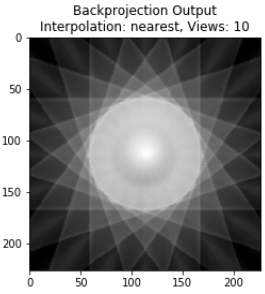


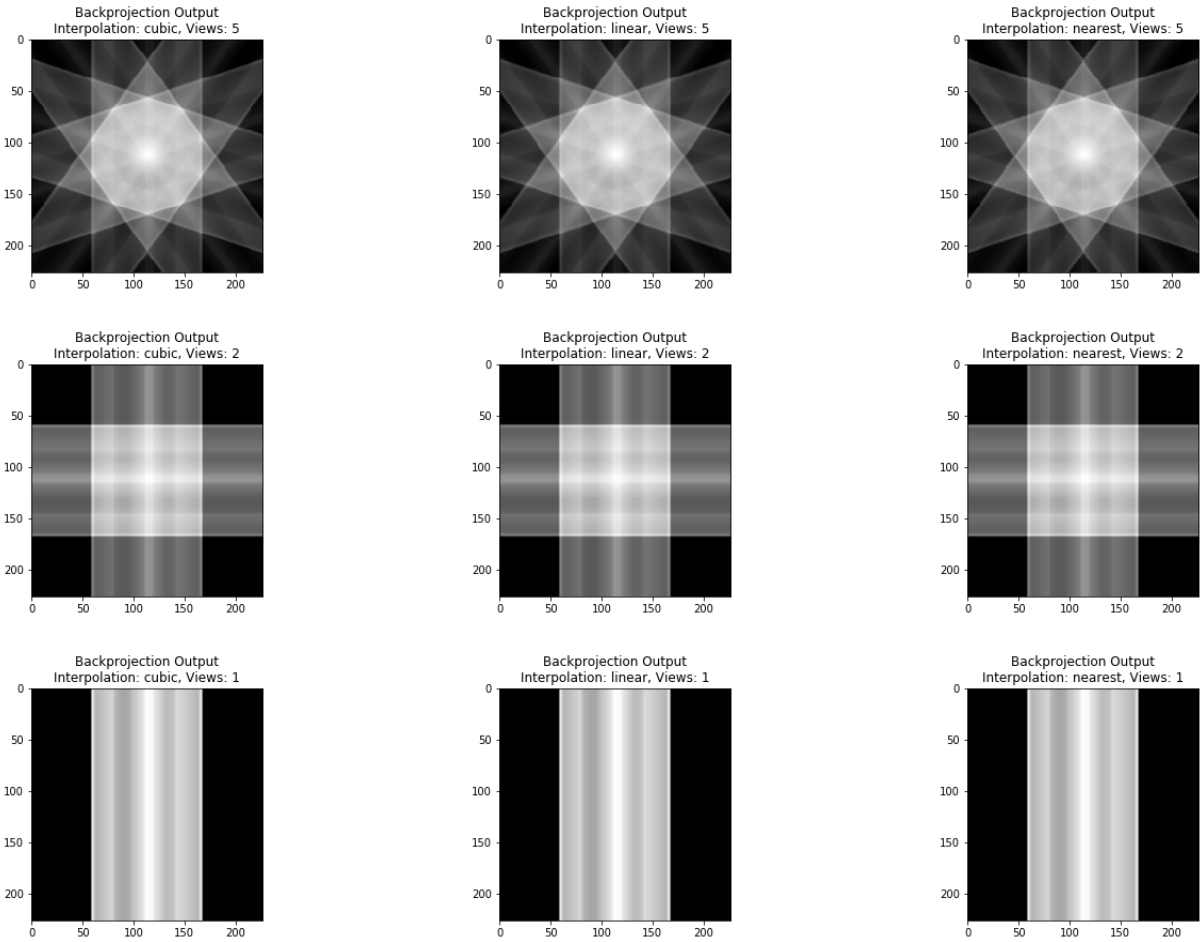












II. Filtered Backprojection

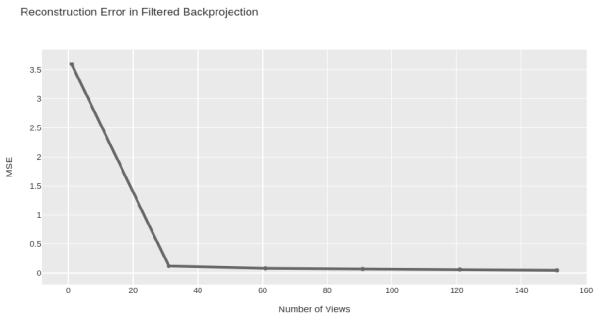
Nearest neighbor, linear, and cubic interpolations were used in the K-space for reconstructing the image from the sinogram. In addition, ramp, hamming, and hanning filters were the different filters used in the filtering process. The Python implementation uses the Radon Transform function to obtain the sinogram of the image. The following samples from the results generated also include projections from different sets of angles.

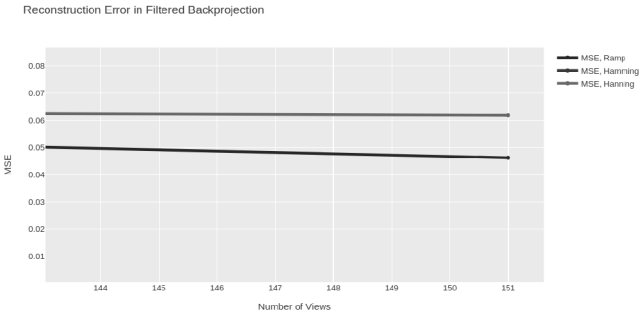
Filtered backprojection is implemented as an inverse Radon Transform with the sinogram filtered in the frequency domain using various filters.

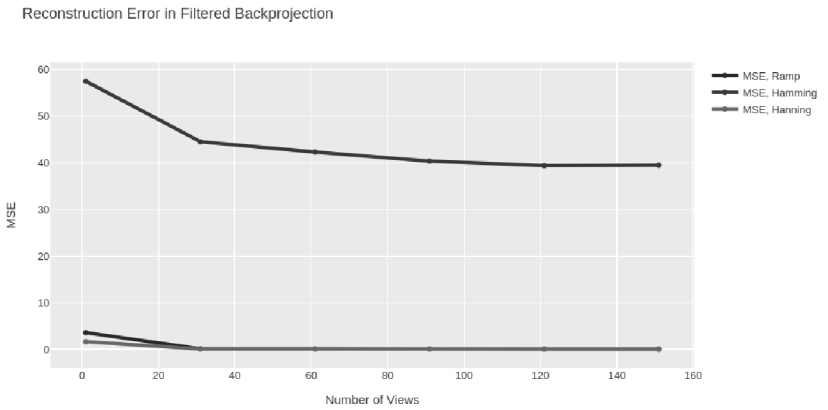
Observations: We use filtering to control the high density around the center and the blurring effects of the backprojection method. The ramp filter has the effect of filtering out low frequencies, and passing high frequencies. Hamming, and hanning filters are low-pass filters, and create an unwanted effect of smoothing out definitions. It is visually difficult to predict which interpolation method produces the best results. As the number of views increases, we observe better image reconstruction results. We find similar observations for all the three images used in the experiment. As the number of detectors are reduced, we find that the reconstructed

image quality is affected. As the number of detectors near the number of pixels in the object image, we find improvement in the reconstruction process. For measuring the quality of the reconstruction, we use three measures in the Python implementation - MSE, PSNR, and SSIM.

Quantitative Measurement in Filtered Backprojection: An Example







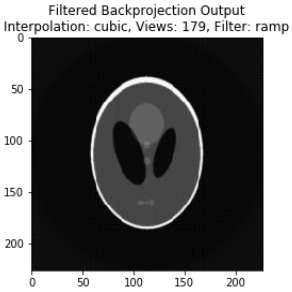
We have shown the comparison between different interpolation methods, and different filters. In this example here, we have used the mean squared error metric. The interactive Plotly graph is available on the Jupyter notebook for MSE, PSNR, and SSIM measures. We observe (first row left) that the values are very close - linear and cubic interpolation performs better in terms of MSE as the number of views get closer to 180^0. In general, we find that the reconstruction gets better as we increase the number of projections. The lowest value of MSE is obtained at near the point where the number of views equals the number of pixels in image dimensions. We see that the Hanning filter performs the worst, and the Ramp filter performs the best in reconstruction (third figure). Similar trends were observed for other sinograms as well. The plots can be found in the Jupyter notebooks.

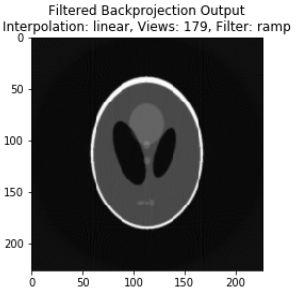
A. Ramp Filter (Shepp-Logan Phantom)

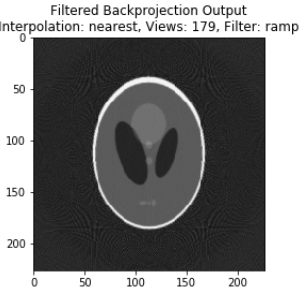
Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

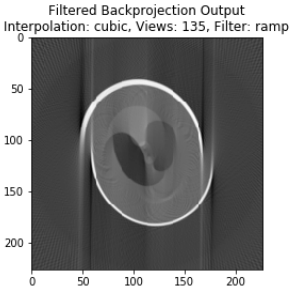
EC5.405 Medical Image Analysis

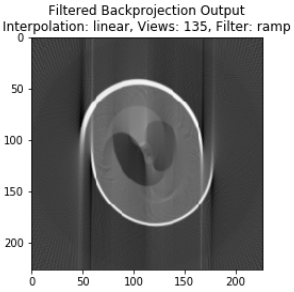
Assignment 01: Analytical Image Reconstruction

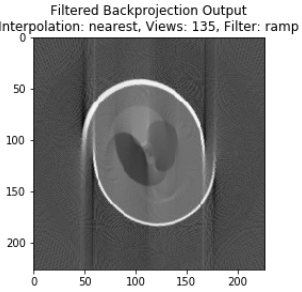


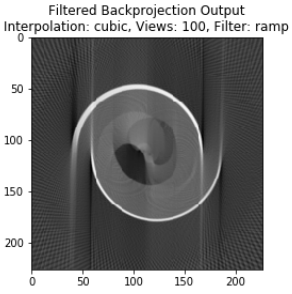


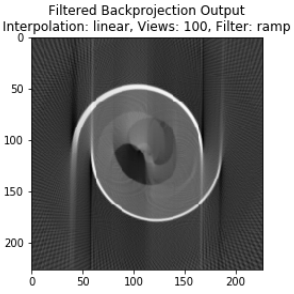


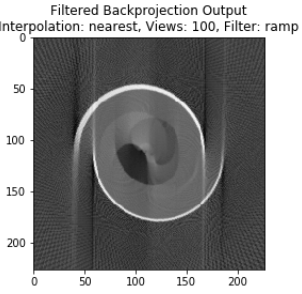


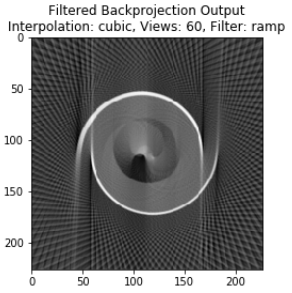


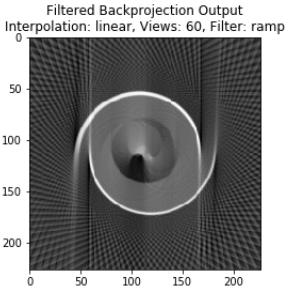


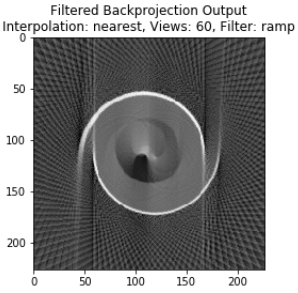


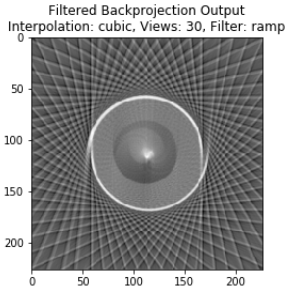


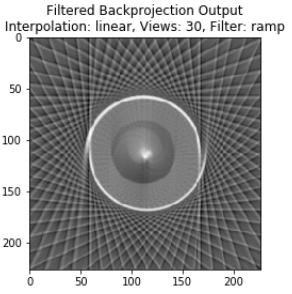


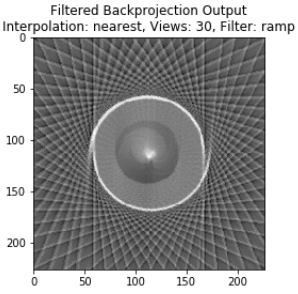


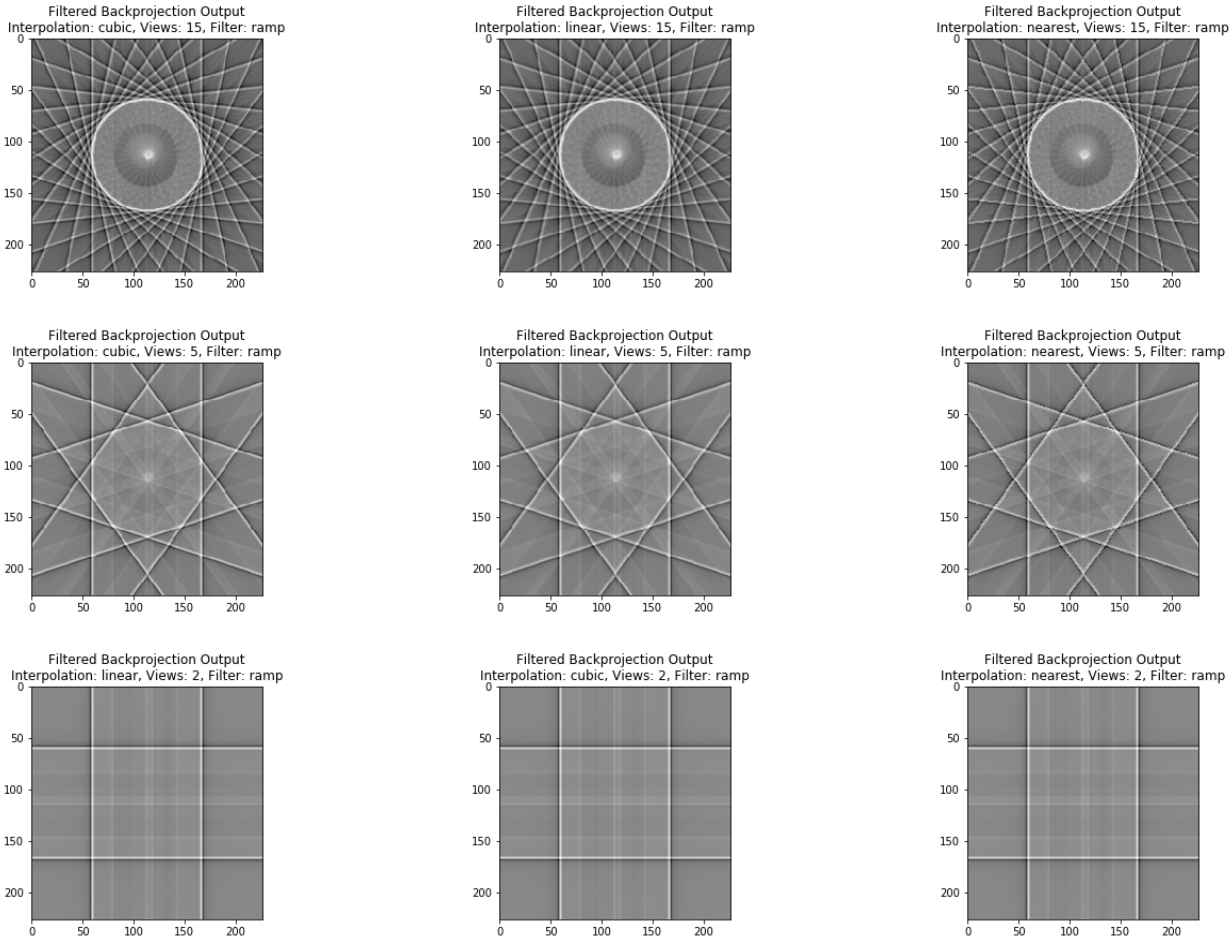










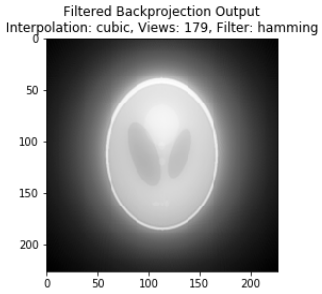


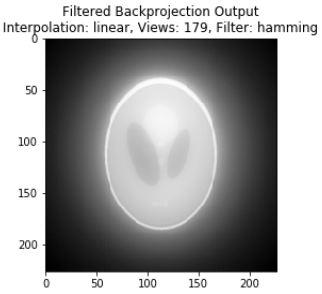
B. Hamming Filter (Shepp-Logan Phantom)

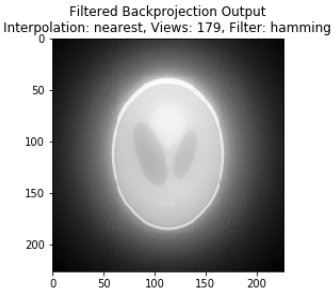
Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

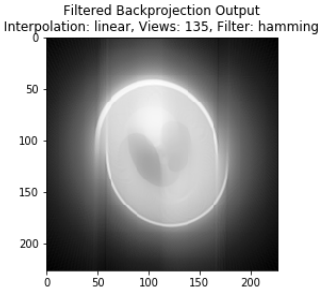
EC5.405 Medical Image Analysis

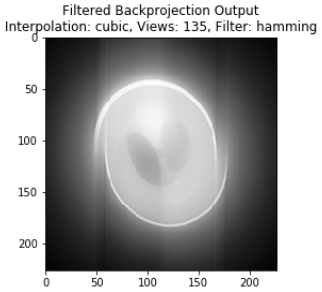
Assignment 01: Analytical Image Reconstruction

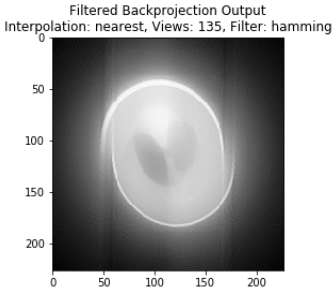


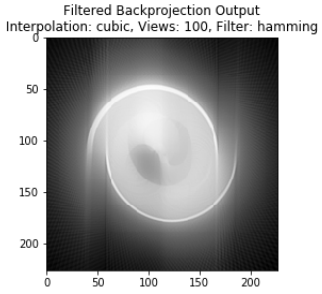


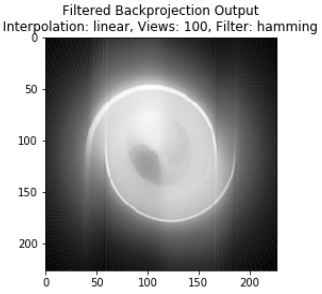


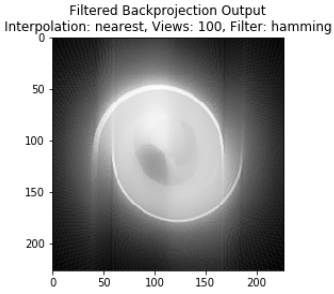


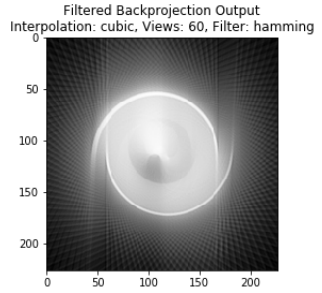


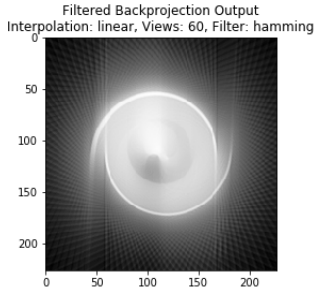


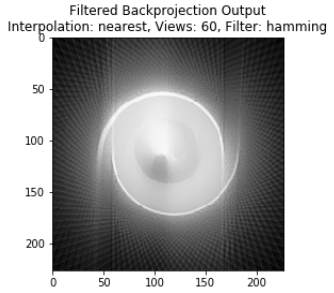


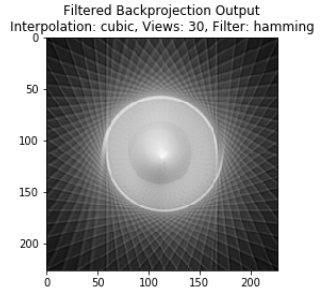


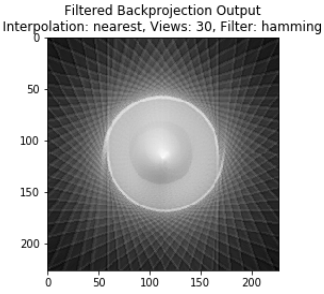


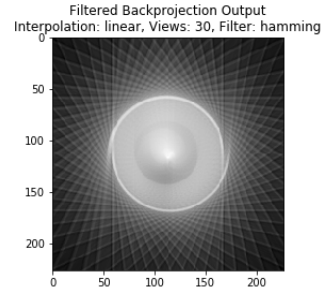






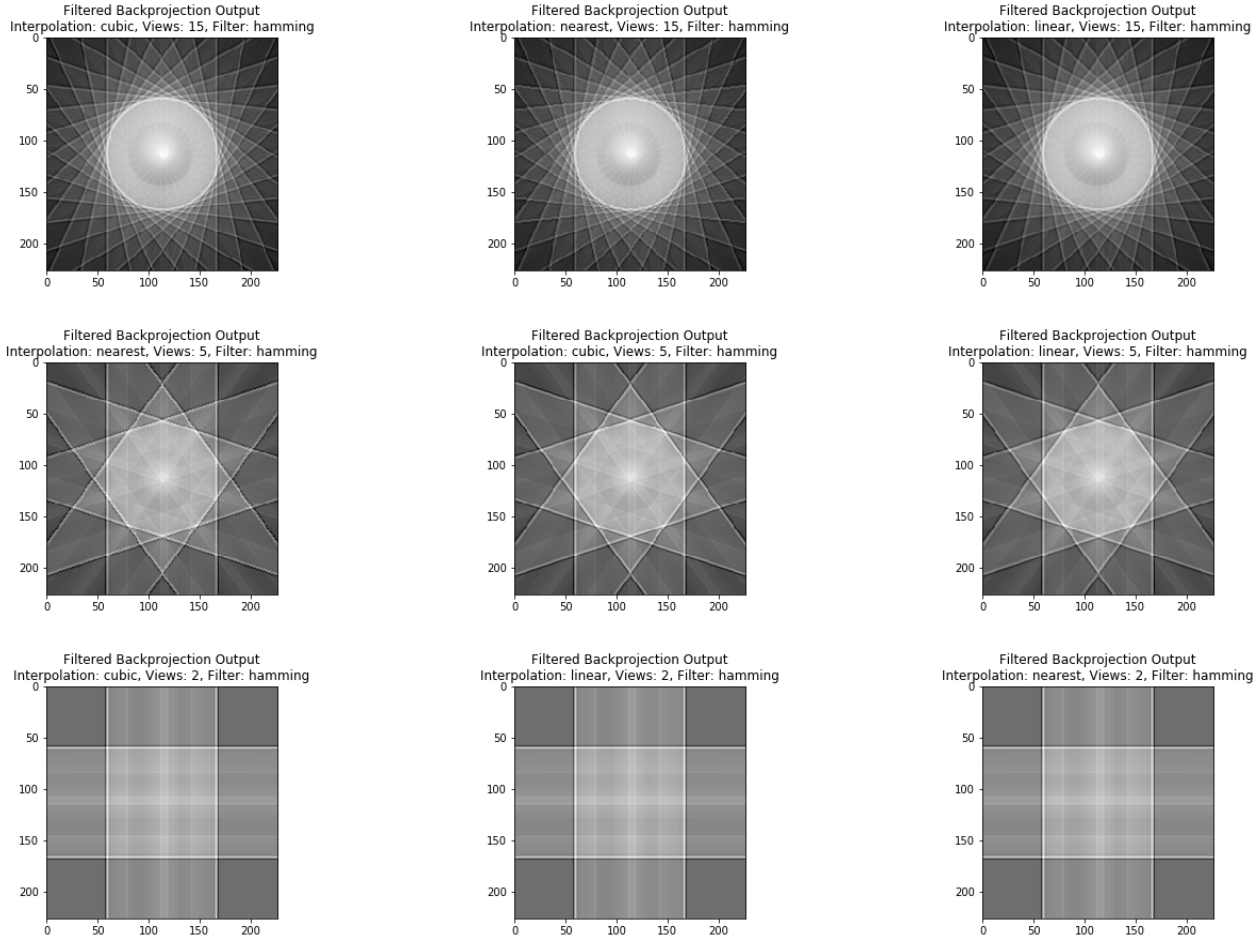






EC5.405 Medical Image Analysis

Assignment 01: Analytical Image Reconstruction

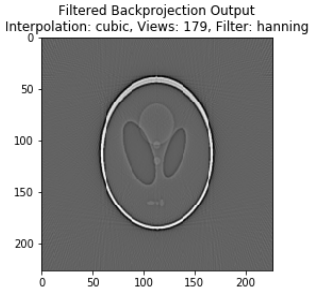


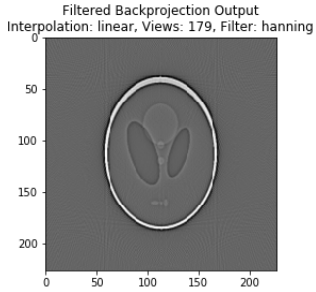
C. Hanning Filter (Shepp-Logan Phantom)

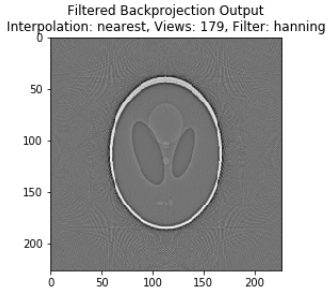
Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

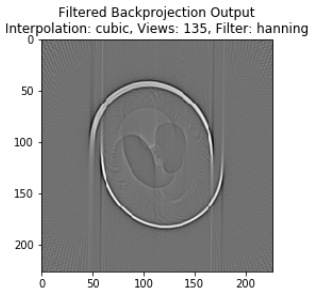
EC5.405 Medical Image Analysis

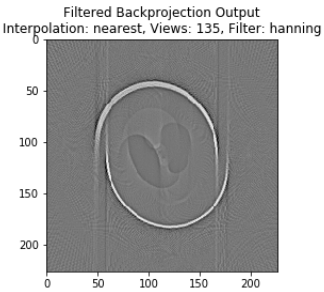
Assignment 01: Analytical Image Reconstruction

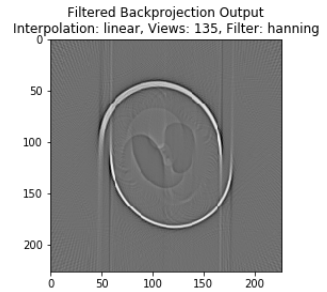


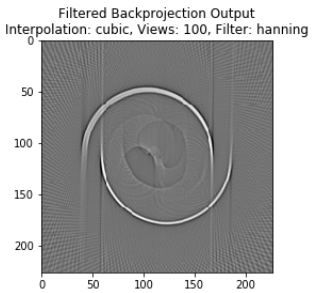


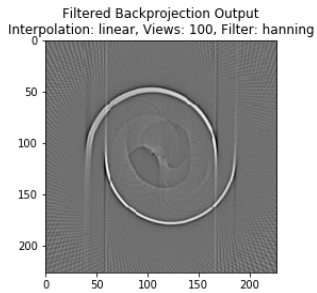


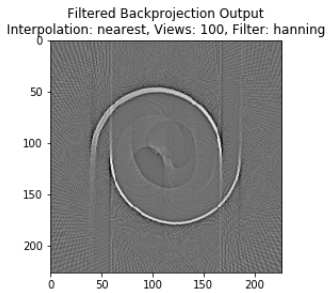


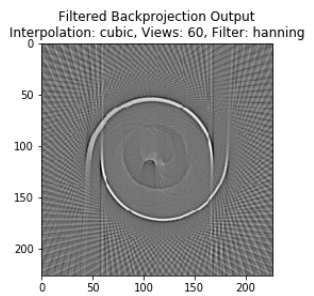


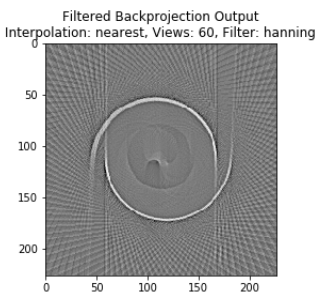


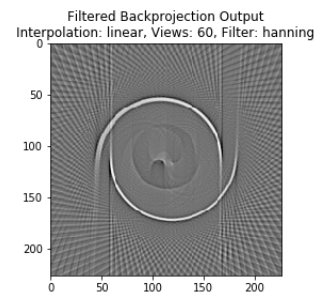


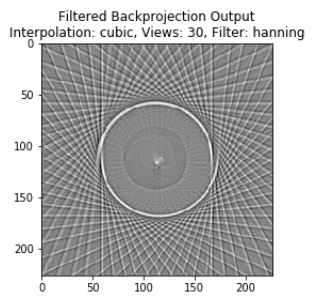


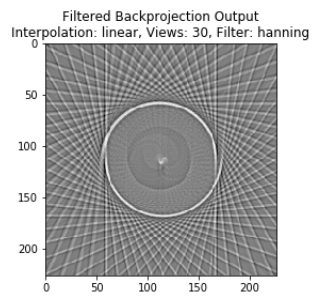


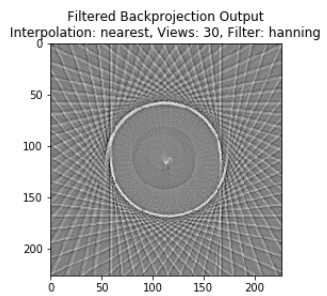


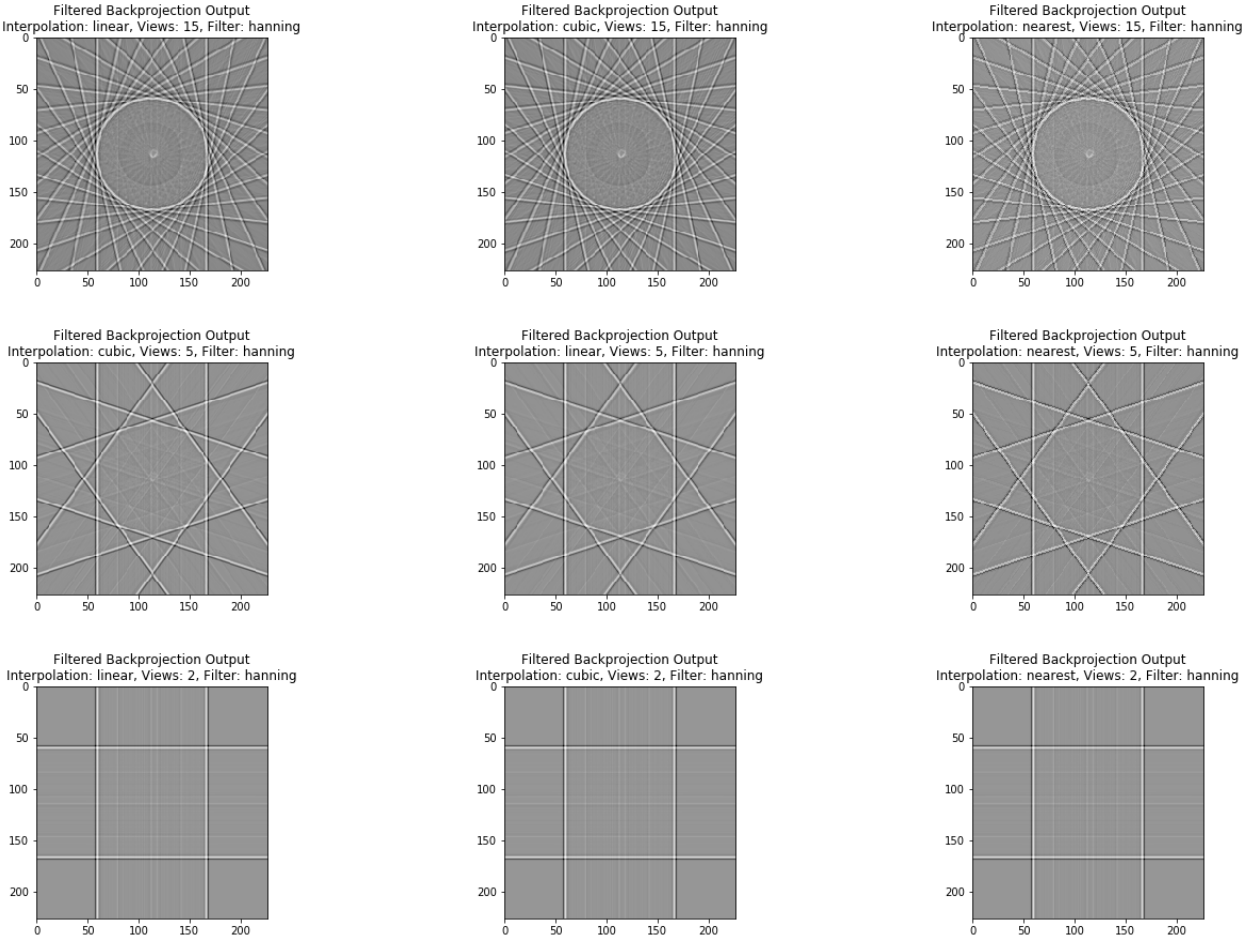












D. Direct Fourier Reconstruction

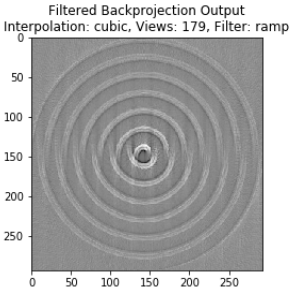
The basic version of direct fourier reconstruction requires interpolation to recover the object function. The interpolation step on the polar grid to obtain the data on the Cartesian grid introduces many errors, and artifacts which are not desirable. The interpolation, and filter observations are the same.

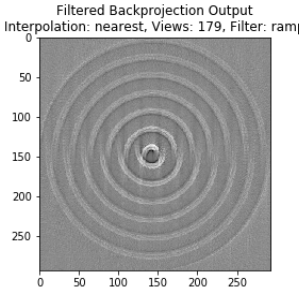
E. Ramp Filter (Nema Sinogram)

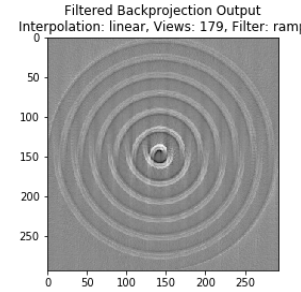
Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

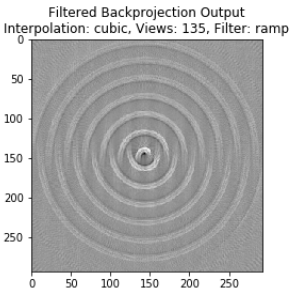
EC5.405 Medical Image Analysis

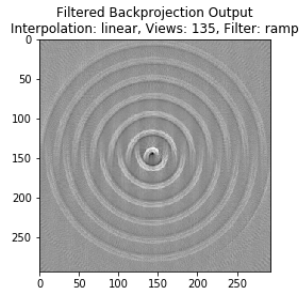
Assignment 01: Analytical Image Reconstruction

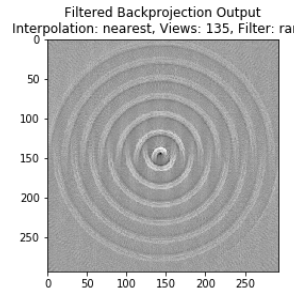


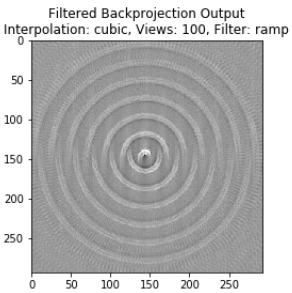


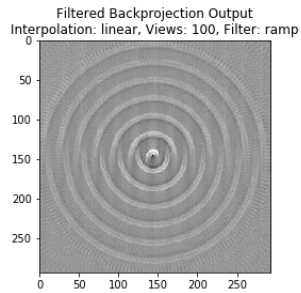


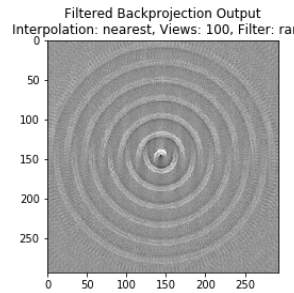


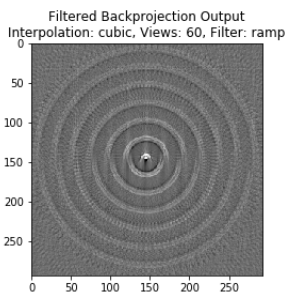


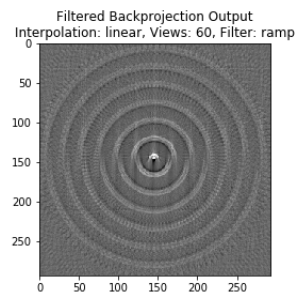


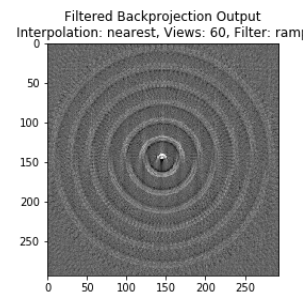


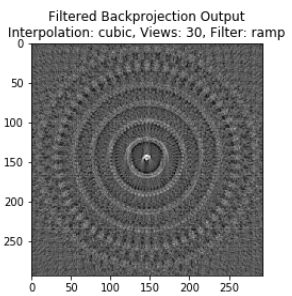


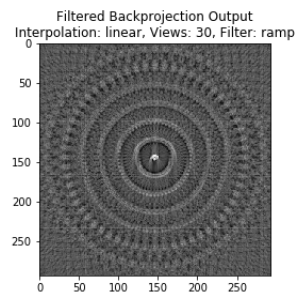


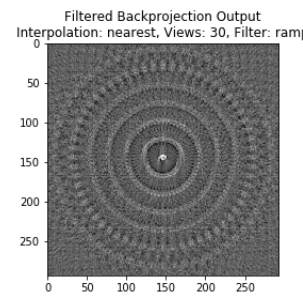


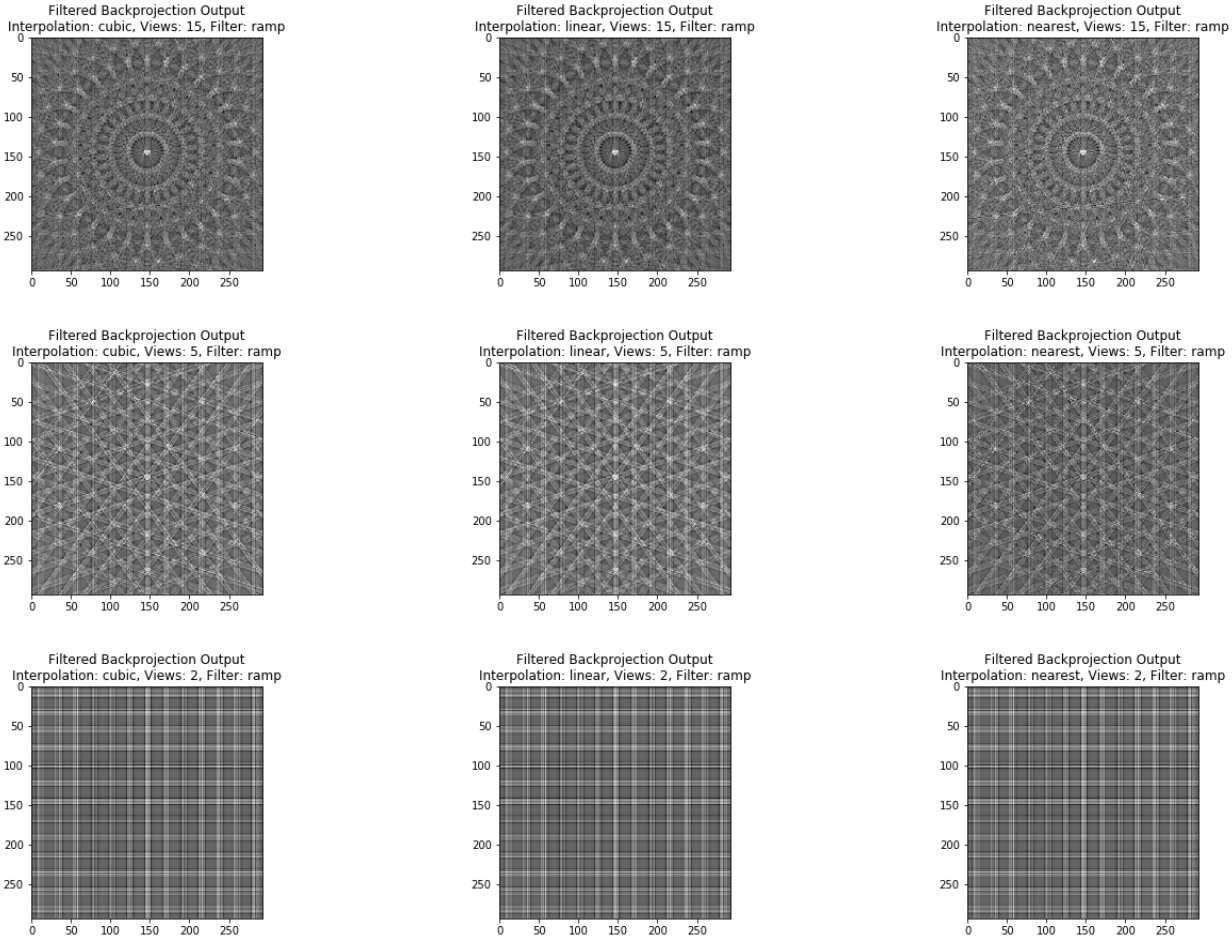






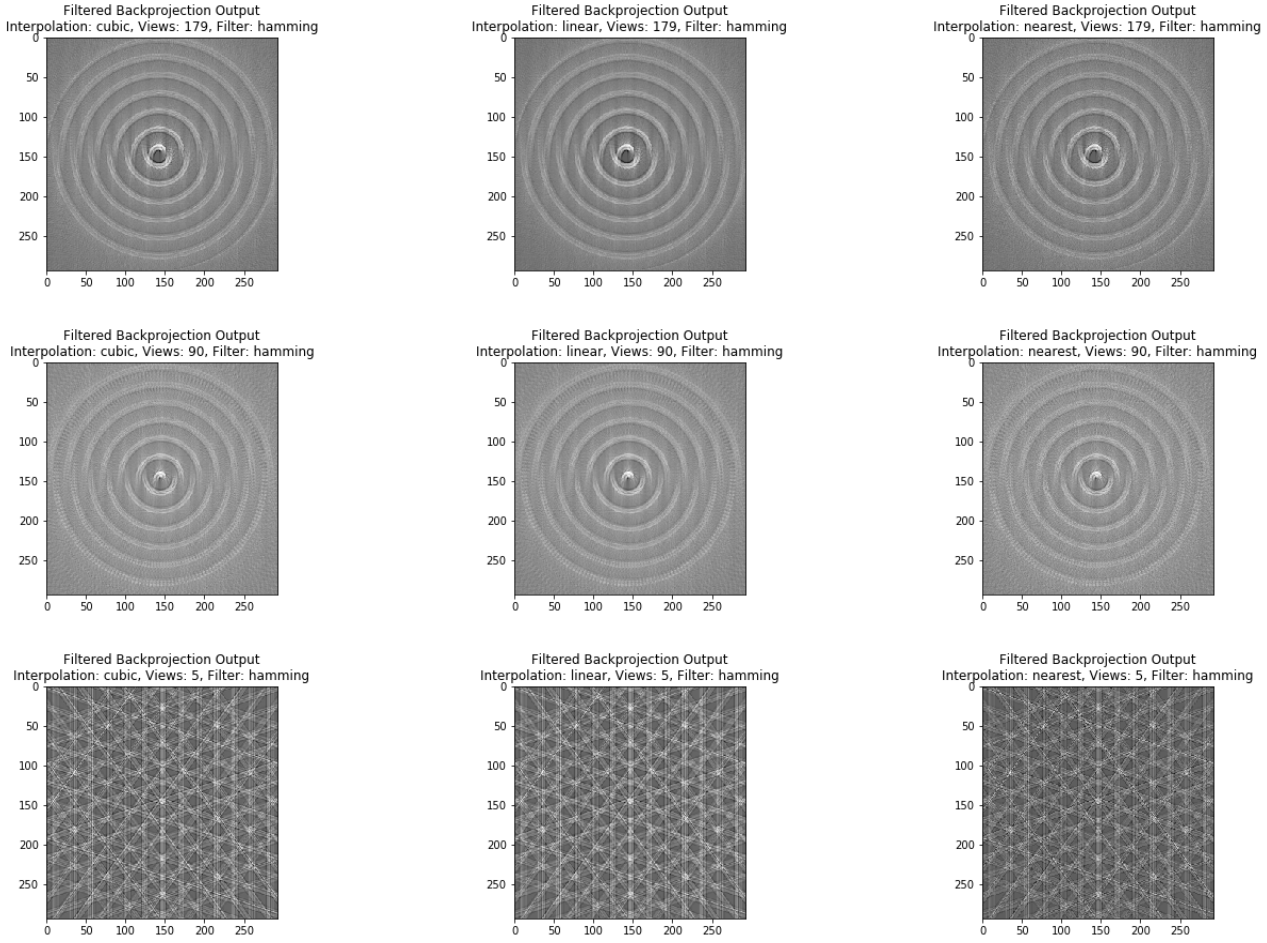






F. Hamming Filter (Nema Sinogram)

Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

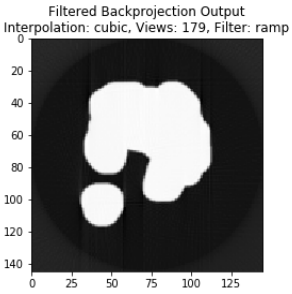


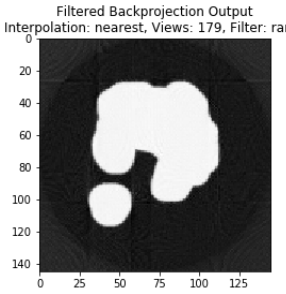
G. Ramp Filter (Custom Image)

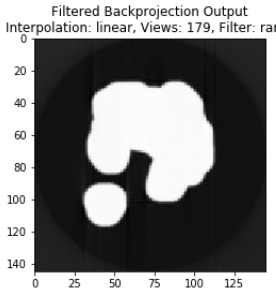
Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

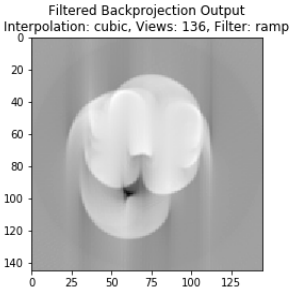
EC5.405 Medical Image Analysis

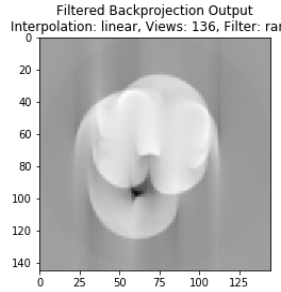
Assignment 01: Analytical Image Reconstruction

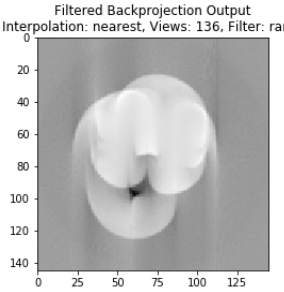


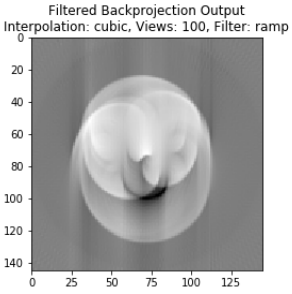


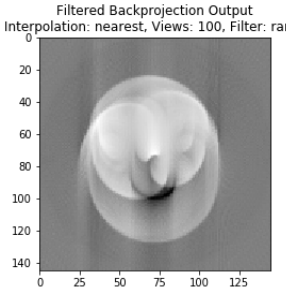


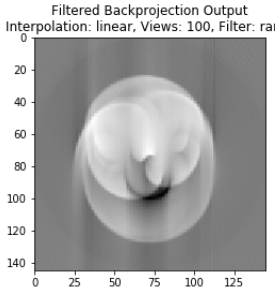


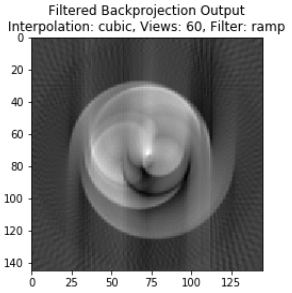


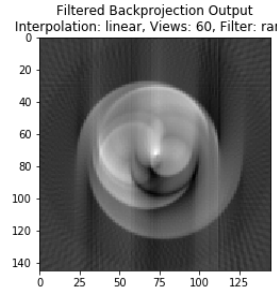


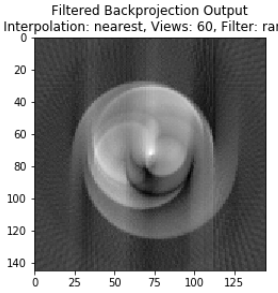


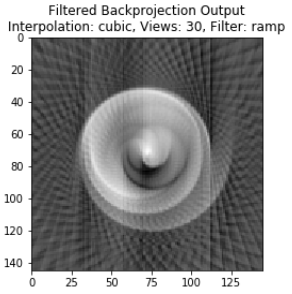


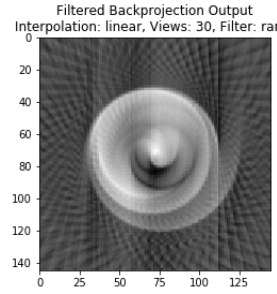


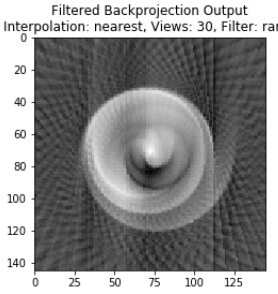


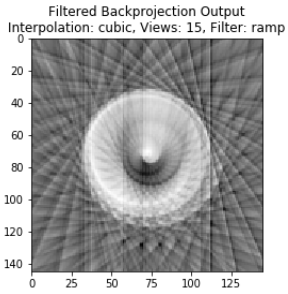


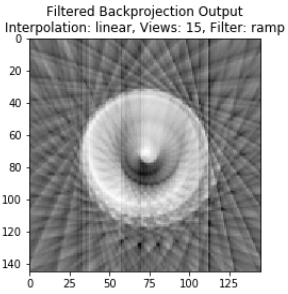


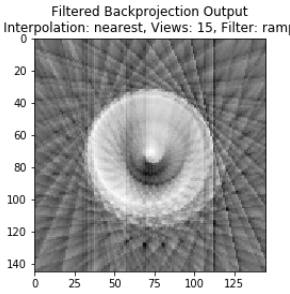


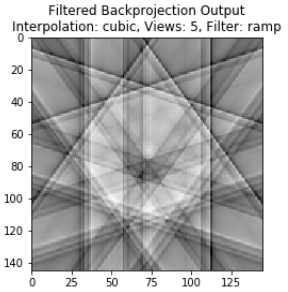


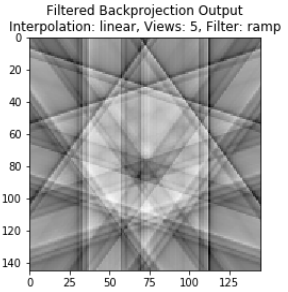


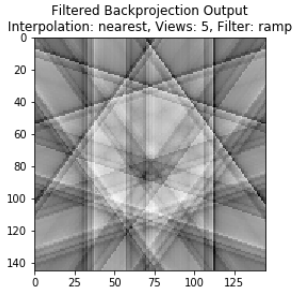


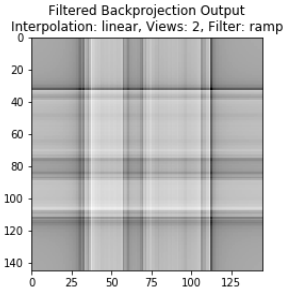


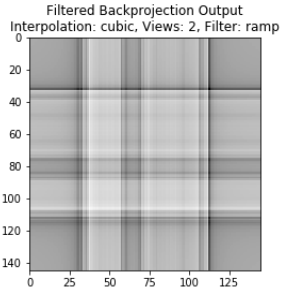


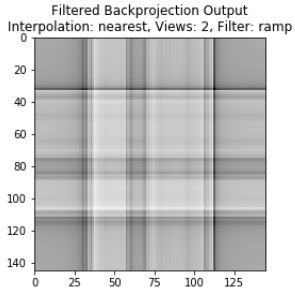






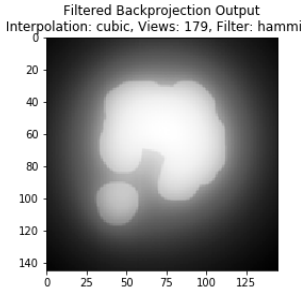


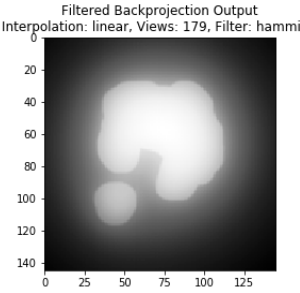


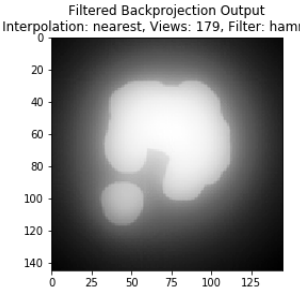


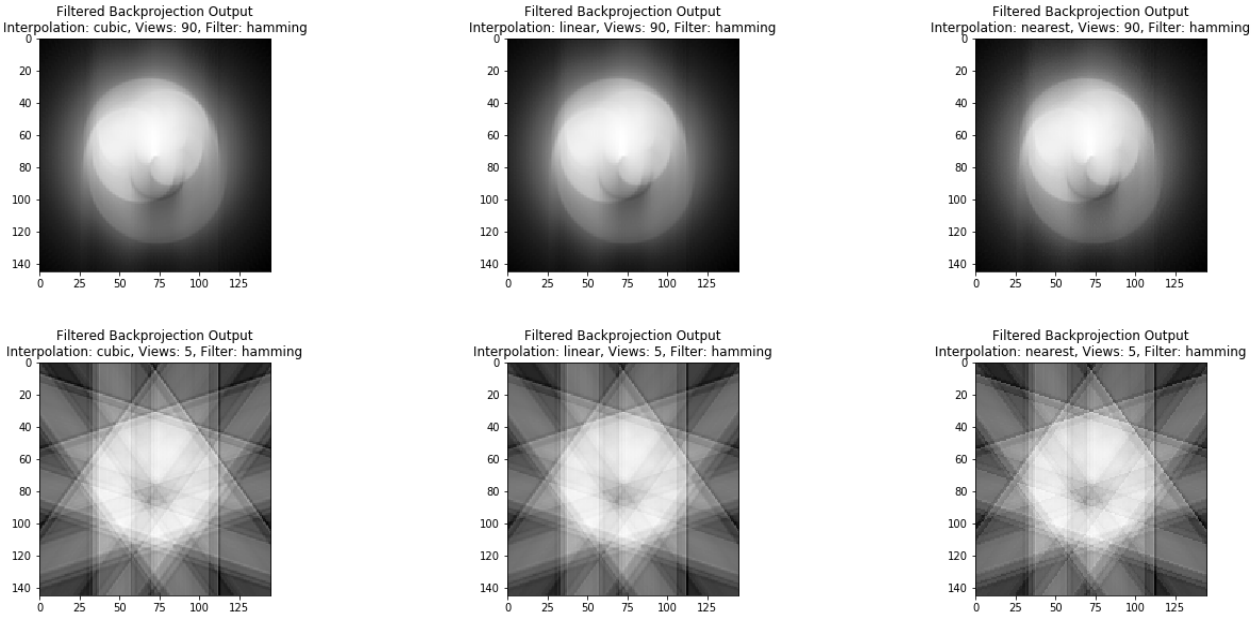
H. Hamming Filter (Custom Image)

Interpolations: Column A: Cubic Column B: Linear Column C: Nearest Neighbor

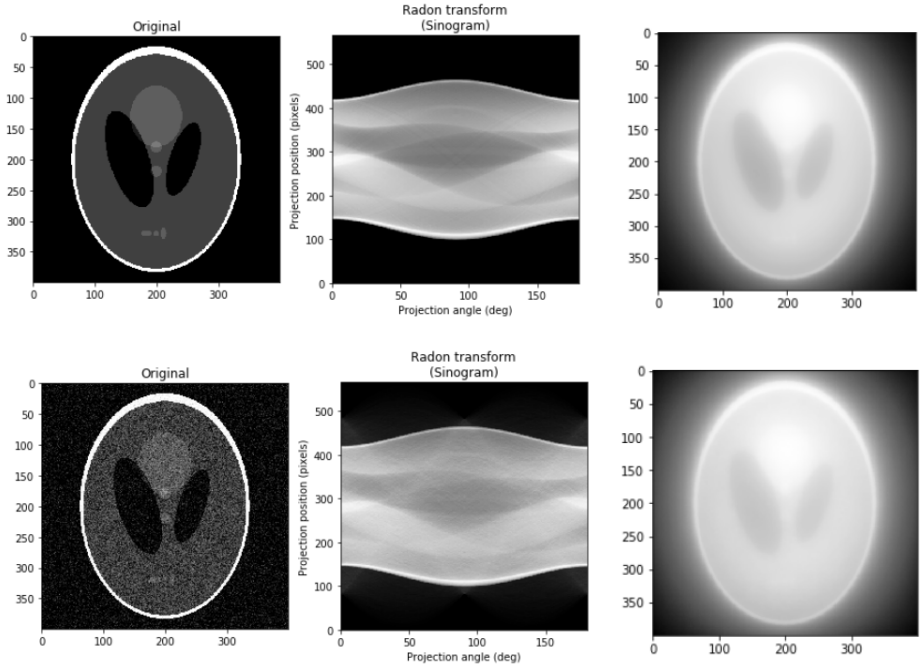




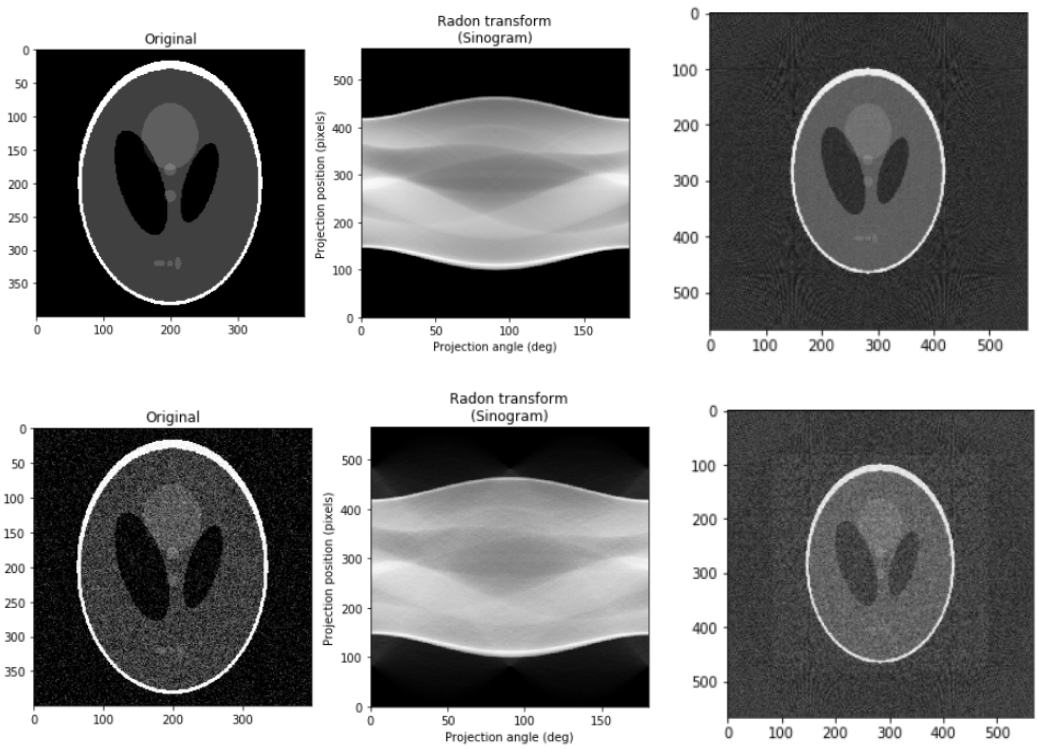




I. Impact of Noisy Shepp-Logan Phantom in Image Reconstruction



In the above example, in the first row, we have used Shepp-Logan phantom to produce the sinogram and then subsequently reconstruct the image using Backprojection. In the second row, we have used a noisy Shepp-Logan phantom, and we observe that the sinogram is noisy. The resulting reconstructed image is more blurred when we use a noisy phantom.



In the above example, in the first row, we have used Shepp-Logan phantom to produce the sinogram and then subsequently reconstruct the image using Filtered Backprojection. In the second row, we have used a noisy Shepp-Logan phantom, and we observe that the sinogram is noisy. The resulting reconstructed image is more blurred when we use a noisy phantom.