Divsion

Narendiran S

19-06-2021

1 Normal Divsion

Division is a process of repeated subtraction. If the Remainder is represented as R, the Divisor D and the Quotient Q. Let the i'th digit of the Quotient be q_i. So Q can be $\{q_(n-1), q_(n-1), \ldots, q_2, q_1, q_0\}$. If R(0) is the final Remainder, then ther general process of division can be expressed as,

$$R(i) = R(i+1) - q_i * D * 10^i$$

Let's look at an examaple of 3148/3. The steps of division can be seen below:

$$\begin{aligned}
3148 - 1 * 3 * 10^3 &= 148 & &\implies R(4) - q_3 * D * 10^3 = R(3) \\
148 - 0 * 3 * 10^2 &= 148 & &\implies R(3) - q_2 * D * 10^2 = R(2) \\
148 - 4 * 3 * 10^1 &= 28 & &\implies R(2) - q_1 * D * 10^1 = R(1) \\
28 - 9 * 3 * 10^0 &= 1 & &\implies R(1) - q_0 * D * 10^0 = R(0)
\end{aligned}$$

2 Restoring Division

Source: web.stanford.edu

Let's look into how to perform restroing division in decimal.

$$\begin{aligned}
q_3 = 1 \to R(3) &= & 3148 - 3 * 10^3 &= +148 \\
q_3 = 2 \to R(3) &= & 148 - 3 * 10^3 &= -2852 \text{ (not positive - restore)} \\
{\color{red} q_3 = 1} \to R(3) &= & -2852 + 3 * 10^3 &= +148 \text{ (move on)}
\end{aligned}$$

$$\begin{aligned}
q_2 = 1 \to R(2) &= & 148 - 3 * 10^2 &= -152 \text{ (not positive - restore)} \\
{\color{red} q_2 = 0} \to R(2) &= & -152 + 3 * 10^2 &= +148 \text{ (move on)}
\end{aligned}$$

$$
\begin{aligned}
q_1 = 1 \rightarrow R(1) &= \quad 148 - 3 * 10^1 = +118 \\
q_1 = 2 \rightarrow R(1) &= \quad 118 - 3 * 10^1 = +88 \\
q_1 = 3 \rightarrow R(1) &= \quad 88 - 3 * 10^1 = +58 \\
q_1 = 4 \rightarrow R(1) &= \quad 58 - 3 * 10^1 = +28 \\
q_1 = 5 \rightarrow R(1) &= \quad 28 - 3 * 10^1 = -2 \text{ (not positive - restore)} \\
{\color{red} q_1 = 4} \rightarrow R(1) &= \quad -2 + 3 * 10^1 = +28 \text{ (move on)}
\end{aligned}
$$

$$
\begin{aligned}
q_0 = 1 \rightarrow R(0) &= \quad 28 - 3 * 10^0 = +25 \\
q_0 = 2 \rightarrow R(0) &= \quad 25 - 3 * 10^0 = +22 \\
q_0 = 3 \rightarrow R(0) &= \quad 22 - 3 * 10^0 = +19 \\
q_0 = 4 \rightarrow R(0) &= \quad 19 - 3 * 10^0 = +16 \\
q_0 = 5 \rightarrow R(0) &= \quad 16 - 3 * 10^0 = +13 \\
q_0 = 6 \rightarrow R(0) &= \quad 13 - 3 * 10^0 = +10 \\
q_0 = 7 \rightarrow R(0) &= \quad 10 - 3 * 10^0 = +7 \\
q_0 = 8 \rightarrow R(0) &= \quad 7 - 3 * 10^0 = +4 \\
q_0 = 9 \rightarrow R(0) &= \quad 4 - 3 * 10^0 = +1 \\
q_0 = 10 \rightarrow R(0) &= \quad 1 - 3 * 10^0 = -2 \text{ (not positive - restore)} \\
{\color{red} q_0 = 9} \rightarrow R(0) &= \quad -1 + 3 * 10^0 = +1 \text{ (move on)}
\end{aligned}
$$

Applying this to binary by considering q_i's to be set $\{0, 1\}$. Now, the equation can be written as:

$$R(i) = R(i+1) - q_i * D * 2^i$$

If $R(i) \geq 0$, then assuming $q_i = 1$ was correct. If $R(i) < 0$, then assuming $q_i = 1$ was wrong, hence q_i should be 0 and restoration must be done.

Let's perform a divison of $67/5$. The steps of division can be seen below:

$$
\begin{aligned}
q_4 = 1 \rightarrow R(4) &= \quad 67 - 5 * 2^4 = -13 \text{ (not positive - restore)} \\
{\color{red} q_4 = 0} \rightarrow R(4) &= \quad -13 + 5 * 2^4 = +67 \text{ (move on)}
\end{aligned}
$$

$$
\begin{aligned}
{\color{red} q_3 = 1} \rightarrow R(3) &= \quad 67 - 5 * 2^3 = +27 \text{ (move on)} \\
{\color{red} q_2 = 1} \rightarrow R(2) &= \quad 27 - 5 * 2^2 = +7 \text{ (move on)} \\
q_1 = 1 \rightarrow R(1) &= \quad 7 - 5 * 2^1 = -3 \text{ (not positive - restore)} \\
{\color{red} q_1 = 0} \rightarrow R(1) &= \quad -3 + 5 * 2^1 = +7 \text{ (move on)}
\end{aligned}
$$

$$
{\color{red} q_0 = 1} \rightarrow R(0) = \quad 7 - 5 * 2^0 = +2 \text{ (move on)}
$$

2.1 Algorithm to Follow

Assuming N bit's of input.

1. Initlialize R with N bit zeros.
2. Initlialize Q with N bit Dividend.
3. Initlialize count with N.
4. Combine R and Q to form a 2N bit register nameley {R,Q}.
5. Perform the below operations till the count becomes zeros.
   - Shift {R,Q} left by 1.
   - Subtract M from R. (add 2's complement of M to R).
   - If `sign(R)` == 1
     - Restore R with it's previous value. (Add M to R).
     - Set Q[0] = 0
   - Else
     - Set Q[0] = 1
   - Decrement count by 1.
6. Finally, R gives the remainder and Q gives the Quotient.

2.2 Flow Chart