Divsion

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1 Normal Divsion

Division is a process of repeated subtraction. If the Remainder is represented as R, the Divisor D and the Quotient Q. Let the i'th digit of the Quotient be q_i. So Q can be $\{q_(n-1), q_(n-1), \ldots, q_2, q_1, q_0\}$. If R(0) is the final Remainder, then ther general process of division can be expressed as,

$$R(i) = R(i+1) - q_i * D * 10^i$$

Let's look at an examaple of 3148/3. The steps of division can be seen below:

$$\begin{aligned}
3148 - 1 * 3 * 10^3 &= 148 && \implies R(4) - q_3 * D * 10^3 = R(3)\\
148 - 0 * 3 * 10^2 &= 148 && \implies R(3) - q_2 * D * 10^2 = R(2)\\
148 - 4 * 3 * 10^1 &= 28 && \implies R(2) - q_1 * D * 10^1 = R(1)\\
28 - 9 * 3 * 10^0 &= 1 && \implies R(1) - q_0 * D * 10^0 = R(0)
\end{aligned}$$

2 Restoring Division

Source: web.stanford.edu

Let's look into how to perform restroing division in decimal.

$$\begin{aligned}
q_3 = 1 \rightarrow R(3) = &\quad 3148 - 3 * 10^3 = +148\\
q_3 = 2 \rightarrow R(3) = &\quad 148 - 3 * 10^3 = -2852 \text{ (not positive - restore)}\\
\color{red}{q_3 = 1} \rightarrow R(3) = &\quad -2852 + 3 * 10^3 = +148 \text{ (move on)}
\end{aligned}$$

$$\begin{aligned}
q_2 = 1 \rightarrow R(2) = &\quad 148 - 3 * 10^2 = -152 \text{ (not positive - restore)}\\
\color{red}{q_2 = 0} \rightarrow R(2) = &\quad -152 + 3 * 10^2 = +148 \text{ (move on)}
\end{aligned}$$

$$\begin{aligned}
q_1 = 1 \rightarrow R(1) = &\quad 148 - 3 * 10^1 = +118\\
q_1 = 2 \rightarrow R(1) = &\quad 118 - 3 * 10^1 = +88\\
q_1 = 3 \rightarrow R(1) = &\quad 88 - 3 * 10^1 = +58\\
q_1 = 4 \rightarrow R(1) = &\quad 58 - 3 * 10^1 = +28\\
q_1 = 5 \rightarrow R(1) = &\quad 28 - 3 * 10^1 = -2 \text{ (not positive - restore)}\\
\color{red}{q_1 = 4} \rightarrow R(1) = &\quad -2 + 3 * 10^1 = +28 \text{ (move on)}
\end{aligned}$$

$$
\begin{aligned}
q_0 = 1 \rightarrow R(0) &= & 28 - 3 * 10^0 &= +25 \\
q_0 = 2 \rightarrow R(0) &= & 25 - 3 * 10^0 &= +22 \\
q_0 = 3 \rightarrow R(0) &= & 22 - 3 * 10^0 &= +19 \\
q_0 = 4 \rightarrow R(0) &= & 19 - 3 * 10^0 &= +16 \\
q_0 = 5 \rightarrow R(0) &= & 16 - 3 * 10^0 &= +13 \\
q_0 = 6 \rightarrow R(0) &= & 13 - 3 * 10^0 &= +10 \\
q_0 = 7 \rightarrow R(0) &= & 10 - 3 * 10^0 &= +7 \\
q_0 = 8 \rightarrow R(0) &= & 7 - 3 * 10^0 &= +4 \\
q_0 = 9 \rightarrow R(0) &= & 4 - 3 * 10^0 &= +1 \\
q_0 = 10 \rightarrow R(0) &= & 1 - 3 * 10^0 &= -2 \text{ (not positive - restore)} \\
{\color{red} q_0 = 9} \rightarrow R(0) &= & -1 + 3 * 10^0 &= +1 \text{ (move on)}
\end{aligned}
$$

Applying this to binary by considering q_i's to be set $\{0, 1\}$. Now, the equation can be written as:

$$R(i) = R(i + 1) - q_i * D * 2^i$$

If $R(i) \geq 0$, then assuming $q_i = 1$ was correct. If $R(i) < 0$, then assuming $q_i = 1$ was wrong, hence q_i should be 0 and restoration must be done.

Let's perform a divison of 67/5. The steps of division can be seen below:

$$
\begin{aligned}
q_4 = 1 \rightarrow R(4) &= & 67 - 5 * 2^4 &= -13 \text{ (not positive - restore)} \\
{\color{red} q_4 = 0} \rightarrow R(4) &= & -13 + 5 * 2^4 &= +67 \text{ (move on)} \\
\\
{\color{red} q_3 = 1} \rightarrow R(3) &= & 67 - 5 * 2^3 &= +27 \text{ (move on)} \\
{\color{red} q_2 = 1} \rightarrow R(2) &= & 27 - 5 * 2^2 &= +7 \text{ (move on)} \\
q_1 = 1 \rightarrow R(1) &= & 7 - 5 * 2^1 &= -3 \text{ (not positive - restore)} \\
{\color{red} q_1 = 0} \rightarrow R(1) &= & -3 + 5 * 2^1 &= +7 \text{ (move on)} \\
\\
{\color{red} q_0 = 1} \rightarrow R(0) &= & 7 - 5 * 2^0 &= +2 \text{ (move on)}
\end{aligned}
$$

2.1 Algorithm to Follow

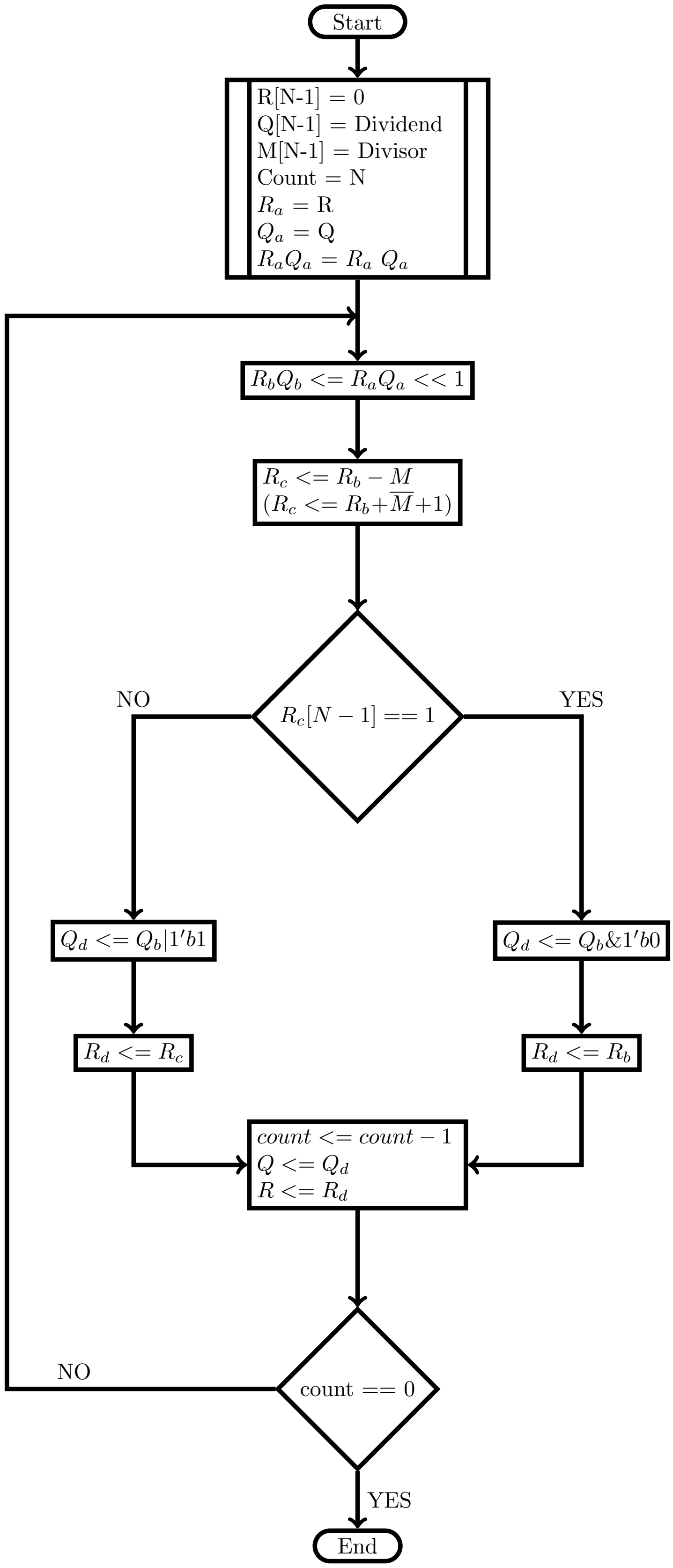
Assuming N bit's of input.

1. Initlialize R with N bit zeros.
2. Initlialize Q with N bit Dividend.
3. Initlialize count with N.
4. Combine R and Q to form a 2N bit register nameley {R,Q}.
5. Perform the below operations till the count becomes zeros.
   - Shift {R,Q} left by 1.
   - Subtract M from R. (add 2's complement of M to R).
   - If `sign(R)` == 1
     - Restore R with it's previous value. (Add M to R).
     - Set Q[0] = 0

- Else
  - Set Q[0] = 1
- Decrement count by 1.

6. Finally, R gives the remainder and Q gives the Quotient.

2.2 Flow Chart



2.3 Signed Division

Signed Division can be done by converting to unsigned and based on the sign value we add sign to Quotient.

- Find verb—sign = Dividend[N-1] xor Divisor[N-1]—.
- If Dividend[N-1] == 1, then Dividend is negative, convert to positive by performing two's complement.
- If Divisor[N-1] == 1, then Divisor is negative, convert to positive by performing two's complement.
- Finally, after division, if the sign is 1, then change the sign bit of Quotient to 1.

3 Non-Restoring Division

Here, there is no Restoring done. This is done by checking if the remainder is positive or negative and then Subtract. (which differs from restoring where subtraction is done and then checked)

Let's look into the binary divison of 67/3 using non-resoring division. The steps of division can be seen below:

$$
\begin{aligned}
q_4 = 1 \rightarrow R(4) = &\quad 67 - 5 * 2^4 = -13 \text{ (not positive)} \\
q_4 = 0 & \\
q_3 = 1 \rightarrow R(3) = &\quad -13 + 5 * 2^3 = 27 \\
q_2 = 1 \rightarrow R(2) = &\quad 27 - 5 * 2^2 = 7 \\
q_1 = 1 \rightarrow R(1) = &\quad 7 - 5 * 2^1 = -3 \text{ (not positive)} \\
q_1 = 0 & \\
q_0 = 1 \rightarrow R(0) = &\quad -3 + 5 * 2^0 = 2
\end{aligned}
$$

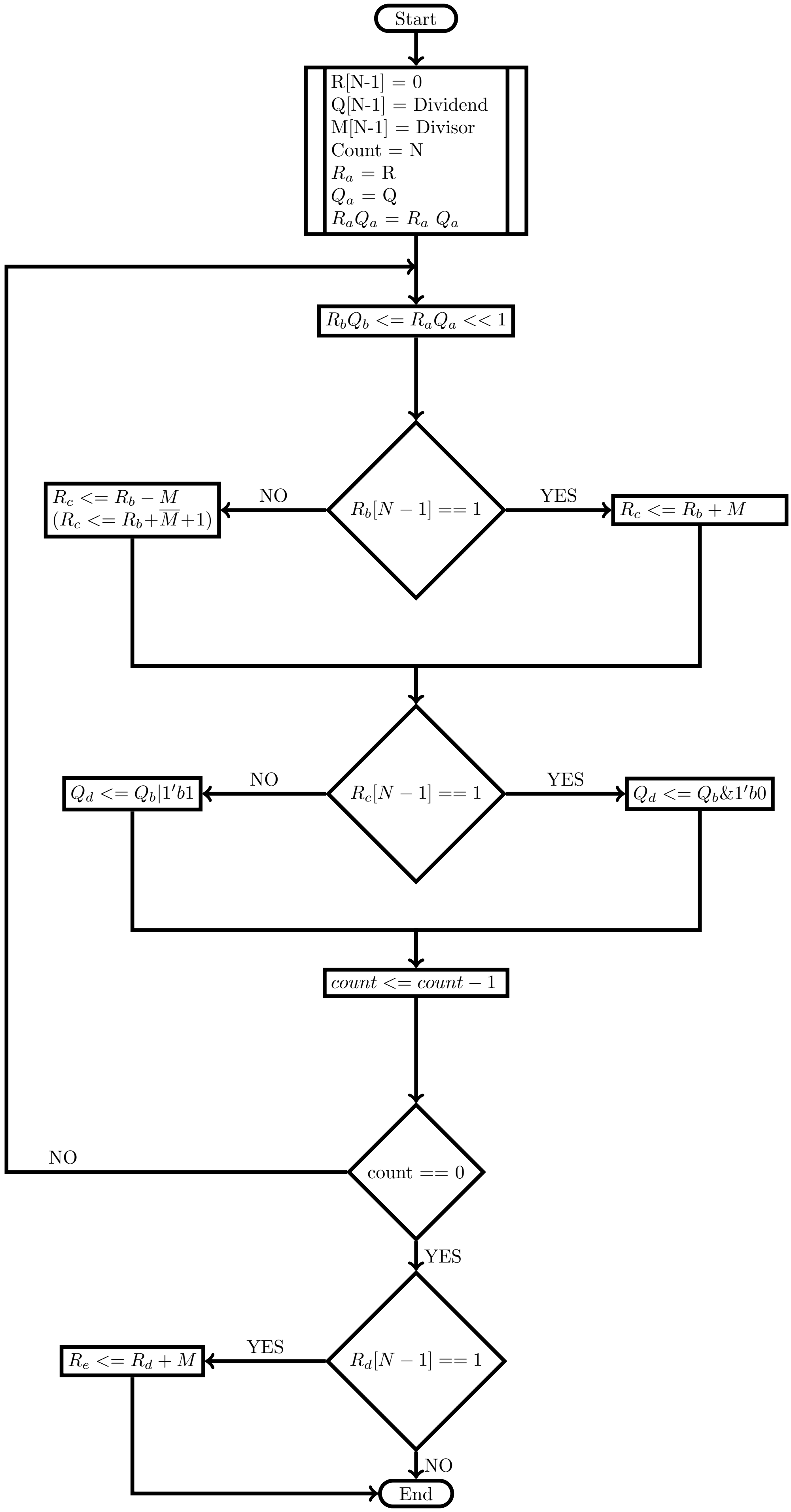
3.1 Algorithm to Follow

Assuming N bit's of input.

1. Initlialize R with N bit zeros.
2. Initlialize Q with N bit Dividend.
3. Initlialize count with N.
4. Combine R and Q to form a 2N bit register nameley {R,Q}.
5. Perform the below operations till the count becomes zeros.
   - If `sign(R)` == 1
     - Shift {R,Q} left by 1.
     - Add M to R.
   - Else
     - Shift {R,Q} left by 1.
     - Subtract M from R. (add 2's complement of M to R).
   - If `sign(R)` == 1
     - Set Q[0] = 0
   - Else
     - Set Q[0] = 1
   - Decrement count by 1.

6. If `sign(R)` $==1$
   - Add M to R.
7. Finally, R gives the remainder and Q gives the Quotient.

3.2 Flow Chart



- Find verb—sign = Dividend[N-1] xor Divisor[N-1]—.

- If Dividend[N-1] == 1, then Dividend is negative, convert to positive by performing two's complement.
- If Divisor[N-1] == 1, then Divisor is negative, convert to positive by performing two's complement.
- Finally, after division, if the sign is 1, then change the sign bit of Quotient to 1.