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Machine Learning

Diagrams

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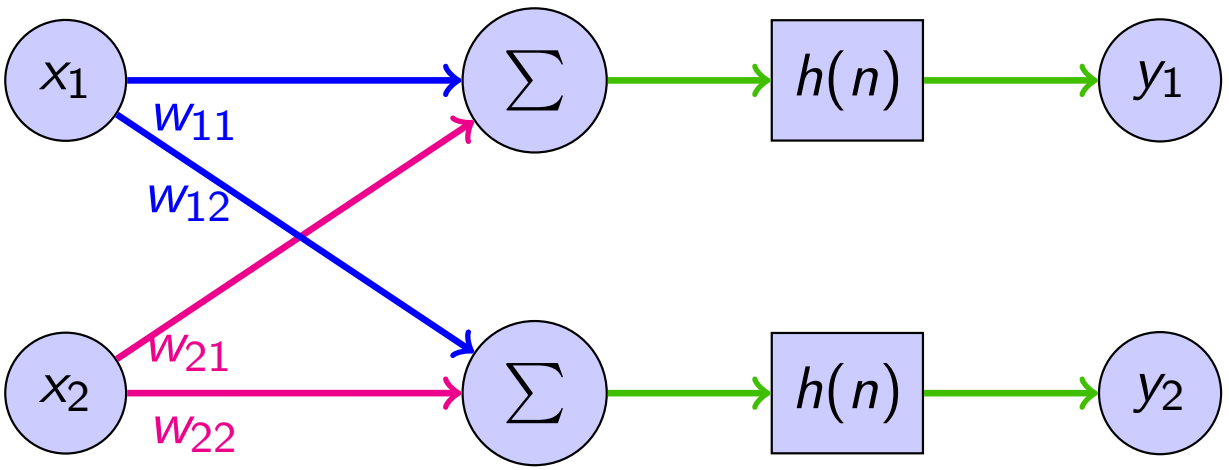
The instructor is gratefully acknowledging the authors who made their course materials freely available online.

Question 1

Consider the two perceptron model of the given neural network to classify the test sample while the training samples are given in the adjoining table.



x_1	x_2	class
1	1	bus
1	4	car
2	5	car
1	2	bus
4	2	plane
4	5	bicycle
5	4	bicycle
5	2	plane

1. Model the output in terms of y_1 and y_2.
2. Obtain the weights appropriately to classify the test patterns correctly.
3. Design the activation functions with threshold appropriately.
4. Classify the input $< 2.1, 4.5 >$. Show the relevant steps.

Answer 1

1. Model the output in terms of y_1 and y_2.

class	y_1	y_2
bus	0	0
car	0	1
plane	1	0
bicycle	1	1

Answer 1

Compute the weights and activation threshold for the perceptron.

Let $w_{11} = 1$

$w_{12} = 0$

$threshold = 3$

$$sum_1 = w_{11}x_1 + w_{21}x_2$$

$$y_1 = \begin{cases} 1 & \text{if } sum_1 \geq 3 \\ 0 & \text{if } sum_1 < 3 \end{cases}$$

x_1	x_2	sum_1	y_1
1	1	$1+0=1$	0
1	4	$1+0=1$	0
2	5	$2+0=2$	0
1	2	$1+0=1$	0
4	2	$4+0=4$	1
4	5	$4+0=4$	1
5	4	$5+0=5$	1
5	2	$5+0=5$	1

Answer 1

Compute the weights and activation threshold for the perceptron.

$$\text{Let} \quad w_{12} = 0$$
$$w_{22} = 1$$
$$threshold = 3$$
$$sum_2 = w_{12}x_1 + w_{22}x_2$$
$$y_2 = \begin{cases} 1 & \text{if } sum_2 \geq 3 \\ 0 & \text{if } sum_2 < 3 \end{cases}$$

x_1	x_2	sum_2	y_2
1	1	$0+1=1$	0
1	4	$0+4=4$	1
2	5	$0+5=5$	1
1	2	$0+2=2$	0
4	2	$0+2=2$	0
4	5	$0+5=5$	1
5	4	$0+4=4$	1
5	2	$0+2=2$	0

Answer 1

2. Obtain the weights appropriately to classify the test patterns correctly.

$$w_{11} = 1$$
$$w_{12} = 0$$
$$w_{21} = 0$$
$$w_{22} = 1$$

3. Design the activation functions with threshold appropriately.

$$threshold = 3$$

Answer 1

4. Classify the input $(2.1, 4.5)$.

$$
\begin{aligned}
sum_1 &= w_{11}x_1 + w_{21}x_2 \\
&= 1 * 2.1 + 0 * 4.5 = 2.1 \\
&< 3 \leftarrow threshold \\
y_1 &= 0 \\
sum_2 &= w_{12}x_1 + w_{22}x_2 \\
&= 0 * 2.1 + 1 * 4.5 = 4.5 \\
&> 3 \leftarrow threshold \\
y_2 &= 1 \\
y &= (0, 1) \\
&= Car
\end{aligned}
$$

Answer 1

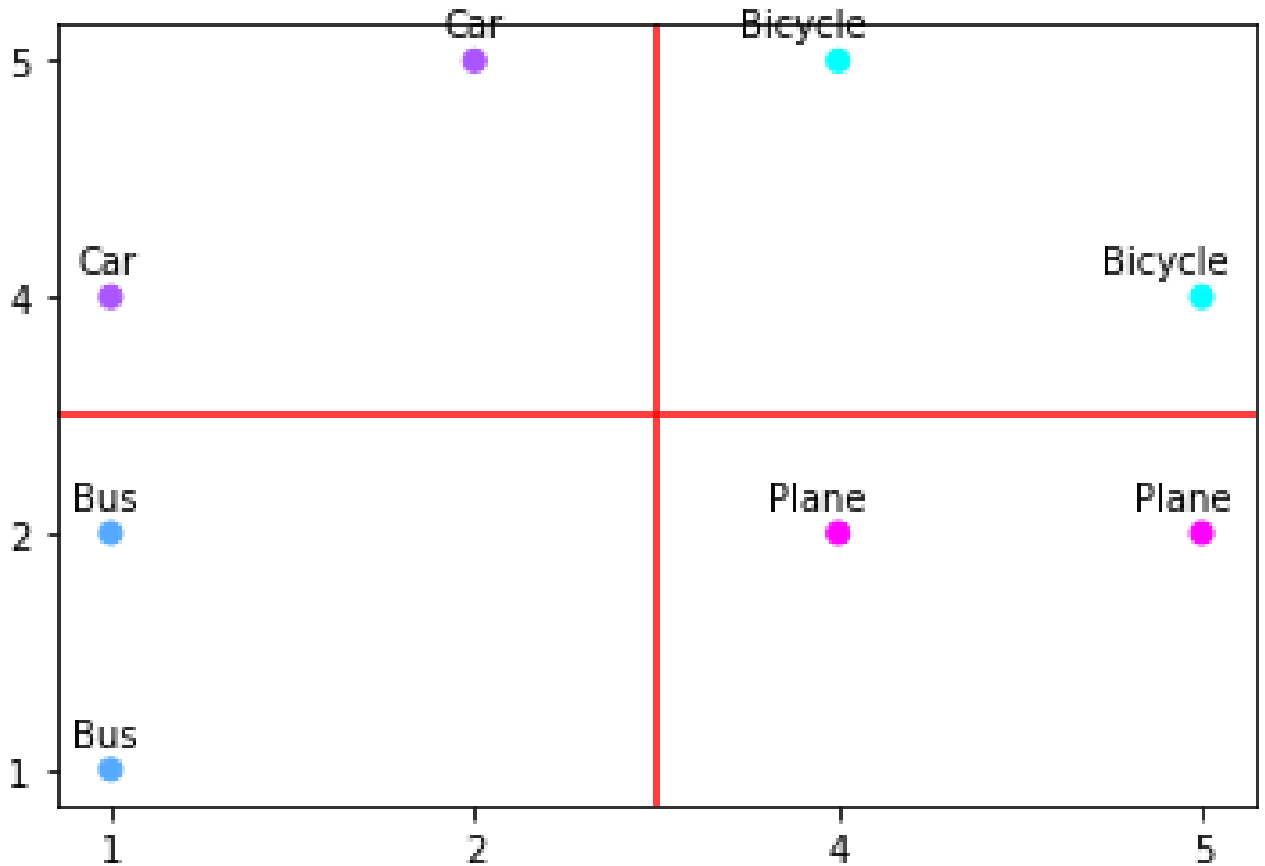


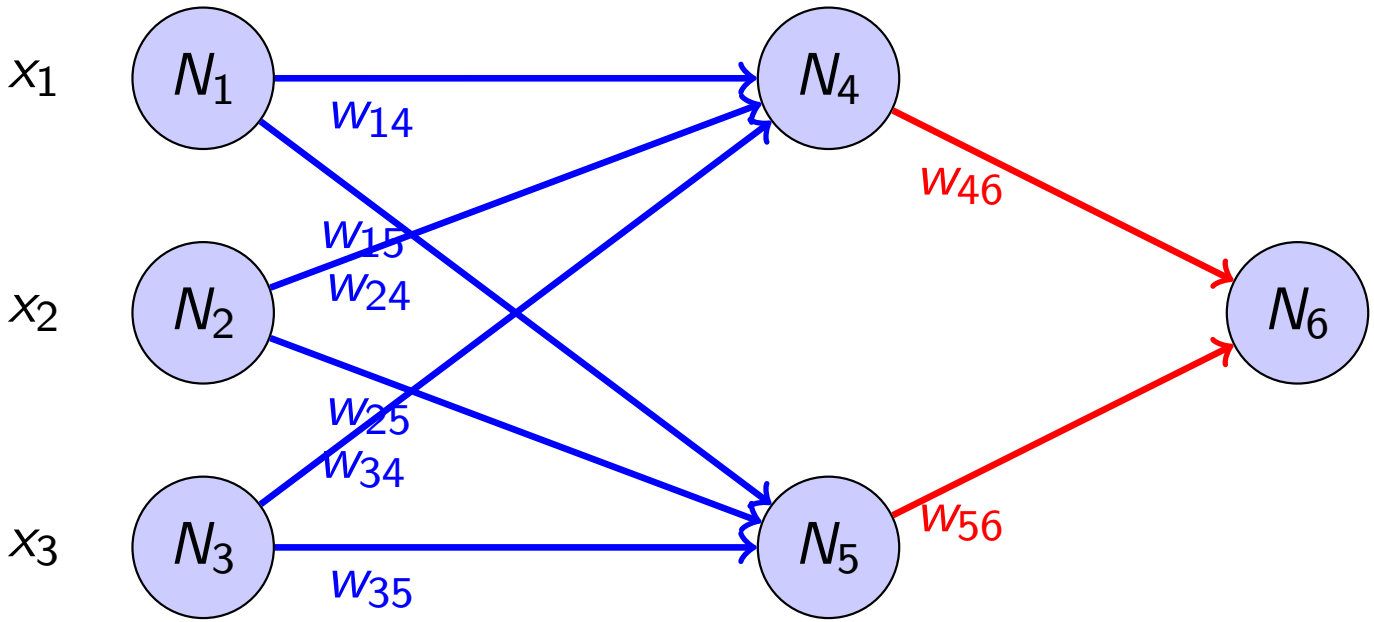
Figure: Graphical representation of the dataset

Question 2

Consider the given feed forward neural network. Let learning rate be 0.9. The inital weights and the biases are given in the table. Consider the following sample as first training example $X = (1, 0, 1)$ with class label as 1. Show

1. computation of net output at each node.
2. calculation of error at each node.
3. calculation of updation of weights and biases after one iteration.

Question 2



$b_4 = (-0.4)$ $b_5 = 0.1$ $b_6 = 0.1$

$w_{14} = 0.2$ $w_{15} = (-0.3)$ $w_{45} = (-0.3)$

$w_{24} = 0.4$ $w_{25} = 0.1$ $w_{56} = (-0.2)$

$w_{34} = (-0.5)$ $w_{35} = 0.2$

Answer 2

A] Computation of net output at each node.

$$z_j = \sum_{ij} w_{ij} x_i + b_i$$

$$a_j = \frac{1}{1 + e^{-z_j}}$$

N_4 $\quad z_4 = w_{14}x_1 + w_{24}x_2 + w_{34}x_3 + b_4 \qquad a_4 = \frac{1}{1+e^{-(-0.7)}} = 0.331$

$\quad z_4 = 0.2 * 1 + 0 - 0.5 * 1 - 0.4 = (-0.7)$

N_5 $\quad z_5 = w_{15}x_1 + w_{25}x_2 + w_{35}x_3 + b_5 \qquad a_5 = \frac{1}{1+e^{-(0.1)}} = 0.525$

$\quad z_5 = (-0.3) * 1 + 0 + 0.2 * 1 + 0.2 = 0.1$

N_6 $\quad z_6 = w_{46}a_4 + w_{56}a_5 + b_6 \qquad a_6 = \frac{1}{1+e^{-(-0.014)}} = 0.474$

$\quad z_6 = (-0.3) * 0.331 - 0.2 * 0.525 + 0.1$

$\quad z_6 = (-0.104)$

Answer 2

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B] Calculation of error at each node.

At output layer $E_j = a_j(1 - a_j)(y - a_j)$

At hidden layer $E_j = a_j(1 - a_j)\sum_k E_k w_{jk}$

N_6 $E_6 = a_6(1 - a_6)(y - a_6)$

$E_6 = 0.474(1 - 0.474)(1 - 0.474) = 0.131$

N_5 $E_5 = a_5(1 - a_5)(E_6 * w_{56})$

$E_5 = 0.525(1 - 0.525)(0.131 * (-0.2)) = (-0.0065)$

N_4 $E_4 = a_4(1 - a_4)(E_6 * w_{46})$

$E_4 = 0.331(1 - 0.331)(0.131 * (-0.3)) = (-0.0087)$

Answer 2

C] Calculation of updation of weights and biases after one iteration.

$$w_{ij} = w_{ij} + \alpha E_j a_i$$

$$b_j = b_j + \alpha E_j$$

$$\begin{aligned}
w_{46} &= (-0.3) + 0.9 * 0.131 * 0.331 && = 0.268 \\
w_{56} &= (-0.2) + 0.9 * 0.131 * 0.525 && = (-0.138) \\
w_{14} &= 0.2 + 0.9 * (-0.0087) * 1 && = 0.192 \\
w_{15} &= (-0.3) + 0.9 * (-0.0065) * 1 && = (-0.306) \\
w_{24} &= 0.4 + 0 && = 0.4 \\
w_{25} &= 0.1 + 0 && = 0.1 \\
w_{34} &= (-0.5) + 0.9 * (-0.0087) * 1 && = (-0.508) \\
w_{35} &= 0.2 + 0.9 * (-0.0065) * 1 && = 0.194 \\
b_4 &= (-0.4) + 0.9 * (-0.0087) && = (-0.408) \\
b_5 &= 0.2 + 0.9 * (-0.0065) && = 0.194 \\
b_6 &= 0.1 + 0.9 * 0.131 && = 0.218
\end{aligned}$$